Operant Subjectivity	•
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Some Consequences	•
of How We Measure ¹	•
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Abstract: Computer programs for Q Technique are a mixed blessing. As they reduce the work, on the one hand, we run a risk of becoming complacent if they are used in an unthinking manner. Hand computation forced intimate knowledge of how the technique worked, but this knowledge is no longer essential. The two best sources for students of Q are out-of-print, and contemporary books on factor analysis are written by people who do not understand Q. The community of scholars and researchers utilizing Q Method should collaborate to ensure that future students of the methodology rest upon a sound foundation.

Introduction

Many researchers today who use Q Methodology are vulnerable to overlooking the consequences of how we measure. This vulnerability can be linked directly to our reliance upon the computer programs we use to do the calculations. Dependence on the software, which surely must be worthy of our trust, will tend over time to invite a relaxed attitude, perhaps even an indifference, toward distinguishing between the Method itself and the Technique that is embedded in it. If these concerns are justified, understanding the mathematical operations in the several steps of Q Technique will tend, again over time, to disappear into obscurity. Additionally, because the computer programs have removed the labor that used to be necessary in the analysis steps, we are enabled to do more studies, and in taking on more studies we will tend to focus our attention on Methodological matter and be able to do so with little expenditure of energy on Technique.

Put simply, for the last decade or so we have not had to think through for ourselves the consequences of how we measure. Worse yet, those who follow us run a risk of not knowing or perhaps forgetting the effects the mathematical transformations have on

¹ Paper presented at "A Celebration of the Life and Work of William Stephenson," University of Durham, December 13, 1997. I am grateful to the Stricklin family for their permission to publish this conference paper. Apart from the formatting in the journal's house style and the correction of a few typos, the text remains unchanged. [*Ed.*]

² Michael Stricklin (1944-2022) was Emeritus Professor at the University of Lincoln-Nebraska. In 2022 he received the William Stephenson Award for his numerous contributions to the advancement of Q methodology, especially his development of PCQ software for the analysis of Q-sort data. [*Ed.*]

the data themselves. As author of PCQ, I must plead guilty to contributing to this situation.³

In this paper I want to share with you some of my recent thinking about consequences of how we measure. I want to invite members of the Q community to collaborate on a project making plain the operations involved in Q Technique but to carry through the project with an eye out for possibilities. I hope to attract your attention by giving here my reasons for concern, by pointing out briefly what I mean by underpinnings of Technique and by showing an example of what I mean by a possibility.

Why?

The ease of computation afforded by computer programs today is a mixed blessing. Manipulating many numbers, of course, is the forte of electronic computers, not of people. Thank goodness, because I would not go back to slide rules and hand calculators and graph paper for anything. Nor would I wish those devices on anyone else.

Before the availability of high-speed computers, though, our predecessors had no choice but to do it all by hand. What did that mean to them, in terms of the work? There are three main mathematical steps in Q Technique: calculating a correlation matrix, extracting factors from it and performing rotations of those factors. Each step used to be tedious and time-consuming. For example, a study with 50 items and 50 sorts yields a raw data matrix with $n \ge N=2,500$ entries. This means we must calculate (1/2)(n)(n-1) = 1,225 coefficients, and each coefficient results from a complicated computation. At the factor analysis step the amount of work becomes simply overwhelming for an individual to undertake by hand.

But the massive weight of doing the computations is not really my point.

I am afraid we run a risk, because practice is not necessary. We risk forgetting or at least becoming complacent. Today we can avoid knowing about what happens to the data between the time we collect the sorts and the final report generated by the software.

Students of factor analysis in earlier days had to know their mathematics. They could rightly take pride in becoming masters of doing the arithmetic and of finding and perfecting efficient algorithms for hand calculation. Today, on the other hand, we do not have to worry very much about anything that happens after the data have been entered correctly.

Yet there are better reasons for worry. These are more specific and closer at hand: The two major book-length treatments of Q Methodology are out of print. Students of the Method — and specifically its Technique — have few resources aside from *The Study of Behavior*, and *Political Subjectivity*.⁴ Many of the sources I have found to be most helpful are also out of print.

For example, information about judgmental and other forms of factor rotation is scattered. This information is to be found in the later chapters of books by Thurstone, Burt, Guilford, Thompson and others who have written more recently.

We are left with few assets of our own on rotation. The best resources are *The Study of Behavior* and *Political Subjectivity*. While I made a small contribution in a paper

⁴ A pdf of this book is now available at

³ See Stricklin, M. (1987). Some thoughts on completing computer programs for Q technique. *Operant Subjectivity*, *10*(4), 136-139. [*Ed.*]

https://qmethod.org/wp-content/uploads/2016/01/brown-1980-politicalsubjectivity.pdf [Ed.]

presented at a Q Conference a few years ago, it generated almost no attention. I should have done more, it would seem, but I did not have the same understanding then.

Apart from Stephenson and Brown, all the authorities on factor analysis are mathematically minded. None of them are Q Methodologists. These facts make the student's tasks double difficult. To adequately understand their explanations requires a heavy investment in mathematical training, and their only examples are of R studies. To deal with them seriously, one must be prepared to learn various styles of notation, think hard about the mathematics at issue, and then somehow translate all into the Q Methodology paradigm.

As of today we simply do not have adequate background resources on the Technique upon when we depend.

What Do I Mean by Underpinnings?

I should confess that I enjoy the computations and mathematical manipulation of Q data, but I believe there are few others today who do.

Why should we expect there to be very many? A small number should come as no surprise because Dr. Stephenson always played down the Technique. I believe he was saving his emphasis for the larger issues of the Methodology. Why invite us to work up to a high level of technical proficiency when most of us struggle to grasp the main problems?

Allow me to soften the implications of that question a little. In seminars I attended at Iowa, Dr. Stephenson, a master of hand computation, did not draw exceptional attention to Technique. He praised the computer programs of that time, the best of them being QUANAL (developed at Iowa by Norm Van Tubergen) and ROSETTA (developed at Missouri by Tom Danbury).

But I cannot forget his efficiency with a slide rule; many times I watched him in his office whipping through centroid solutions of my fellow students' data. I cannot help but think he enjoyed computing by hand. Maybe it was associated with a dream of his. I remember well that he spoke on several occasions about providing a Methodology (and a Technique) that a practicing clinical psychologist could operate in something resembling real-time in the office.

As we all know too well, Q Methodology is complicated. It is so complicated that the computer programs provide a little welcome relief — with them we are able to lower our level of attention to the technical aspects of the actual computations. This is a luxurious feeling one can enjoy when able to use high speed computers. With them we are freer to engage in convergent selective activities, to enjoy the communication pleasures inherent to Q.

Yet, as I said above, the computer programs are a mixed blessing. Except for *The Study of Behavior*, the books I referred to above give over much space to the procedures involved in the factor analysis and rotation steps. Thurstone, for example, gives large parts of 10 out of 21 chapters to them, Guilford gives almost 100 pages, Burt only 41. They had little choice, for most of them were writing before computers were available generally. Thurstone, Burt and Guilford wrote before computers were available at all.

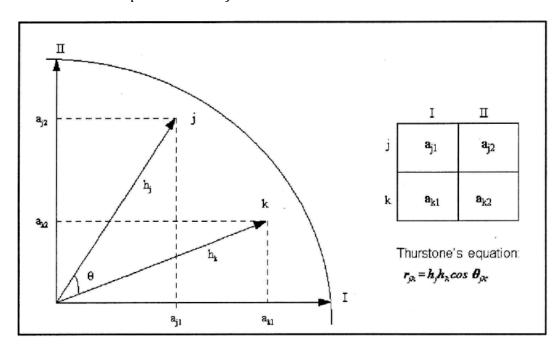
Their books were really intended as scholarly teaching devices for people who wanted to do factor analysis. And their books follow the same general sequence of presentation. Since solutions were necessarily performed by hand calculation, the authors first provided the equations, then gave formal derivations of them and finally wrote out recipe-like procedures, explaining through examples, in step-by-step fashion, what had to be done. Their books are bulky, formal and tedious. Naturally only a few of the readers of those books could accomplish all the tasks set before them. They did not have to be good at theory, but they had to be good at calculation because they had to be their own computers or had to oversee the students assisting them in the work.

Now I want to offer some examples of what I am referring to as the kind of underpinnings a student of Q should understand.

Consider a correlation coefficient. What is it, in non-technical terminology? It is a precise expression of a sort's relationship to another sort. But to arrive at this numerical expression, the scores from each sort must first be changed into a linear format. Once this has been accomplished all the mathematical rules that apply to lines can be brought to bear. From this point in the analysis until very near to the end-when factor loadings are used to assemble factor score arrays-numerical relationships are only expressed with linear equations. This is a powerful idea as Steve Brown gracefully lays out in *Political Subjectivity*.

Next, consider that every step of correlating, factor analysis and rotation can be expressed geometrically. This is so because linearity is assumed to apply to data in all the varieties of factor analysis. Additionally, most of these steps can be drawn graphically. Thurstone used geometric drawings to demonstrate his procedures. *Figure 1* is an example based upon one from *Multiple Factor Analysis*: Correlations as scalar products.

Figure 1



A Geometrical Representation of a Correlation

Thurstone explains that, due to the *a priori* assumption of linearity, each variable can be expressed as a vector (that is, a line segment) of a particular length and with a particular angular orientation from the origin of an arbitrarily defined two-dimensional figure. Thus, any two variables can be directly compared visually.

In *Figure 1*, two vectors, *j* and *k*, are drawn in reference to factors I and II. The factor loadings, a_{j1} and a_{j2} are shown as projections of variable *j* on the two axes, and similarly a_{k1} and a_{k2} of variable *k*. To find the correlation of *j* with *k*, multiply the length of vector

 h_j by the length of vector h_k ; then multiply the resulting value by the cosine of the angle between *j* and *k* at the origin. In Thurstone's notation the equation is

r_{jk}=h_jh_k cos _{€jk}.

But what is going on here? What does this transformation do to our original data? Thurstone, appropriately in his day, would never think to ask. But today we must. The answer to the second question is "It does nothing because of our assumption of linearity; there are many ways of expressing a correlation; this is so because of the linearity assumption." The first question, however, is more subtle: To answer we should know what we are doing when we accept the assumption that sorts should be compared in linear terms. I do not want to take this matter on in detail just yet. Suffice to say for now that a visual representation may be helpful whether or not one has an answer with which one is comfortable.

I turn next to a consideration of Centroid versus Principal Axis factor analysis. What makes them different enough that Stephenson preferred Centroid?

This turns out to be a very technical question. I have approached it many times over the last 20 years, and I want to share here an answer that at least satisfies me. I will give it in non-mathematical terms in hopes of not driving away too quickly those who do not care for understanding relationships with numbers. An analogy has helped me visualize the situation.⁵

Suppose there are a group of islands — we do not know how many — hidden somewhere off in the mist and further suppose that each island will have at least one inhabitant. Further suppose we know how many people are in the general area, but not every one of them is necessarily on an island. Finally, let us suppose we have two machines at our disposal to use for identification and measurement.

The simpler machine, the one easier to use, can find the islands and tell us the longest dimension of each one. It can tell us which persons are on which island and how far each of them is from the longest dimension (think of it as the center line or main road, if you like).

Now this machine has limitations. It cannot measure how much space an island occupies; that is, it cannot tell us how big an island is. Nor can the machine tell us very much about where the inhabitants are. It can tell us who is on an island but not how far apart or how close together they might be, only who lives on which side of the road and how far away from it each of them live.

This is what Centroid Analysis does. The island represents the data space, and the road down the center line of the island is analogous to a centroid factor. The inhabitants, of course, are analogous to the sorts, some of whom may not load on any factor. We say these sorts that have no loading on any factor are not on the concourse, or, to push my analogy a bit, they really are out to sea.

Let us suppose the other machine can provide all of the bits of information described above, and in addition this machine can tell us how wide an island is, how many square meters (or acres, or miles) make up an island. It can tell us how many inhabitants are on each, plus the size of each of their acreages. To top it all, this machine can tell us how all the islands are grouped together. Interestingly, this machine tells us that the *width* of the largest island turns out to be the *length* of the second largest, and the width of the

⁵ I am grateful to Alex Nesterenko, a fellow student of Stephenson at Iowa, for suggesting another version of this analogy to me many years ago.

second largest is the length of the third largest, and so on until all the islands are measured and the acreages of all the inhabitants are known. This is what Principal Axis does.

I am not saying here that an analogy should be accepted as convincing, but it does lead to another interesting question: In more formal terms, what do Centroid Analysis and Principal Axis accomplish? Let us consider Centroid Analysis first.

The name "centroid" is for me keenly descriptive because I can visualize it, and I have tried to share with you what I see through my analogy. A centroid can be thought of as a kind of grand average of the relationships between all the sorts, as they are represented by their correlation coefficients.

In other words, Centroid Analysis is a way of defining centers of gravity embedded in a correlation matrix. In physics, a center of gravity turns out to be where the weight tends to fall on average. For us this concept can be represented as a vector that spans the longest dimension of the data space. The factor loadings, then, are values expressing each sort's relationship with the centroid. Each loading represents a sort's contribution to the length of the centroid and thus can be expressed as the correlation of that sort with the centroid.

The more the sorts have in common with each other, the longer the centroid when expressed as a vector. If enough variance remains after subtracting out the influence of the first centroid, other centroids may be extracted. Each centroid, then, represents a different linear dimension hitherto unobserved in the correlations.

It is significant that a factor loading can be defined as the correlation of that sort with the centroid. This works out because the factor loadings are proportional to the factor and they can be measured in the same way shown above in *Figure 1*. It was Thurstone who recognized this relationship. It had been he who had first proposed Centroid Analysis in 1931. But his insight that loadings could be geometrically related to a centroid led him to go on, so he proposed the Principal Axis solution as a refinement of Centroid in a paper published a year later.

Taking advantage of the principle represented in *Figure 1*, he had found a way to more precision and a way of squeezing out more information from the correlation matrix. By transforming loadings into correlations with the factor, he found he could calculate the breadth of the factor space, too. Not only had he gained precision, he could also calculate how much of that factor space is associated with each sort. Using the rules of trigonometry he could also estimate how much area is accounted for by each sort.

He is mathematically correct in pursuing this course, and his work is the tap root of almost all subsequent developments in factor analysis.

So, which technique is to be preferred? My choice depends upon what I need to know and in what form I need to know it. Therefore, I do not need to turn to a cost/benefit ratio of the one compared to the other as a reason to make a choice. I never get there. It should be said, however, that before high-speed computers Principal Axis, which gives back more information but requires much more work, would have been prohibitively expensive.

In choosing Centroid Analysis, I want to suggest that Principal Axis provides too much information for my purposes and the additional information does not help me get to the next step in Q Methodology.

In a Q study, I only need to extract the factors and be able to identify which sorts go with each and in what proportion. I need this information to calculate the factor scores that I have been seeking all along. More than this is overkill. I do not need a Principal Axis solution because to attain it requires additional and complicated data transformations that give up unnecessary information.

While I am willing to allow multi-linear Q sorts to be transformed into simpler linear forms in the process of calculating correlation coefficients, I do so knowing these transformations will be reversed back to a multi-linear form when the factor scores are calculated. The additional information yielded by Principal Axis is not relevant to this process.

Yet there is another and more profound reason to select Centroid Analysis, and this is a matter of philosophy of science and not of mathematics.

Among the underpinnings of Principal Axis stands the fact that data are represented as points fixed in arbitrarily bounded mathematical space. This is why Principal Axis can produce a unique mathematical solution. It is a benefit for R researchers but is in my way of thinking a fatal flaw for the Q researcher.

I am uneasy with the determinancy of Principal Axis because it is out of anyone's control. The precision and elegance of Principal Axis are results consistent with the mathematics involved. I can only take this as a negative quality.

How can this be so? Centroid Analysis can never produce a unique solution, and this is what I want. Centroid factors are always indeterminate, meaning that an infinite number of equally valid solutions are possible. The indeterminance in Centroid is resolved by conditions imposed upon the study by the researcher for theoretical purposes. So, even with its limitations and that it appears to be closely similar to Principal Axis, I choose Centroid for reasons that have little to do with why others prefer Principal Axis.

After all, if proper care is not taken with theoretical matters in Q, it is the researcher's error. In Principal Axis, whom does one point to? There is nobody there.

Now I want to draw your attention to the most important reason of all. The mathematical precision to be gained through Principal Axis is a step back from what I believe to be the more important issues facing the Q Methodology community today. Principal Axis, as it is used in the social sciences, takes us deeper into the Newtonian paradigm, not closer to the quantum paradigm.

What Do I Mean by Possibilities?

Stephenson was trained as a physicist first and then as a psychologist. As far as I know he was the only one of this era so prepared. A little reflection on what this sequence led to may give us some useful insights on how it shaped his approach to experimentation and therefore on what being a Q Methodologist is all about.

His intellectual contemporaries, referring specifically to those with whom he tangled for much of his life, were mathematically-minded psychologists. I suspect they may have never seen Stephenson's point. Burt, Thurstone, Fisher and Thompson represent this group. He is referred to by them as a maverick, an iconoclast, an innovator. He was forever considered to be different and difficult. A key to understanding what made him different can be found by digressing a little on the topic of how physicists differ from mathematicians. I ask that you bear with me a little. I need to use a few quotations to make my argument clearer.

Burt, Thurstone and others in the 1930s were on the forefront of discovery in psychological methodology when Stephenson came onto the scene. He saw the situation differently than they did.

What was the nature of the difference? Stephenson experimented with theoretically imposed conditions and used factor analysis in the way a physicist uses apparatus. They did it the other way round.

Richard Feynman, the late American Nobel physicist, once wrote, "Physicists like to think that all you have to do is say, 'These are the conditions, now what happens next?'" This is exactly what Stephenson's training had prepared him to understand and had taught him to do. My reading suggests that his protagonists may not have understood this at all. They thought the argument was about *what to measure*. Stephenson thought it was about *what was to perform the measuring*. These are completely different problems that stemmed from completely different views of science.

Feynman continues,

But all our sister sciences have a completely different problem: in fact all the other things that are studied — history, geology, astronomical history — have a problem of this other kind. I find they are able to make predictions of a completely different type from those of a physicist. A physicist says, "In this condition I'll tell you what will happen next." But a geologist will say something like this — "I have dug in the ground and I have found certain kinds of bones. I predict that if you dig in the ground you will find a similar kind of bones." The historian, although he talks about the past, can do it by talking about the future. When he says that the French Revolution was in 1789, he means that if you look in another book about the French Revolution you will find the same date. What he does is to make a kind of prediction about something that he has never looked at before, documents that have still to be found. He predicts that the documents in which there is something written about Napoleon will coincide with what is written in the other documents.

Other psychological methodologists were digging in the ground while Stephenson was saying "Turn on the experimental apparatus and let us see what happens next."

It would appear the rub was there from the outset: he had brought with him a physicist's way of seeing reality that looked for possibilities which might lead to experimentation via factor analysis. He was bound to get into trouble, and were it not for his brilliance, a brilliance that even Burt had to acknowledge, it is doubtful he could have survived. Just at the moment when the best mathematical minds in psychology were defining research problems upon which they could justify testing out their new factor techniques, he came along and recognized possibilities they could not accept as problems.

A quotation from John D. Barrow's book *The World Within the World* points up the difference even more simply: "Right and wrong mean different things in science and mathematics. In the former, 'right' means correspondence with reality; in mathematics it means logical consistency."

I use the quotation here inferring that the mathematically-minded psychologists with whom Stephenson argued may have been more mathematicians than they were psychologists. Without doubt they were more interested in technique than they were methodology. It may have been this line of thinking that led him to insist that Q Technique is a subordinate part of Q Methodology.

I have indulged myself in speculations in the last seven paragraphs. But I do not regret it, for speculation and possibilities are the stuff of inquiry. Surely there is enough substance to suggest a need exists for additional scholarly inquiry into Stephenson's early work and into the conditions in which he worked. These matters are far beyond the scope and purpose of the present paper, but they interest me and have helped lead me to think seriously about how we measure.

So this brings me back to the main theme of this section: what do I mean by "possibilities" for Q Technique? I think we need to try to move ahead. His series of papers on quantum theory have directed our attention toward new areas. Q Technique may or may not be adequate for the tasks.

To move Q Technique forward, we must be open to rethinking what Q sorts represent and also, if we can find them, to open new ways to analysis. So, I want to end the paper with an example of another possibility, I am not sure it will lead anywhere, but I hope you find it interesting.

Consider a Q sort recorded on a sheet of paper. It is a form familiar to us, roughly like an upside-down step pyramid. *Figure 2* is an example. It is Sort I from the Lipset study.

Figure 2

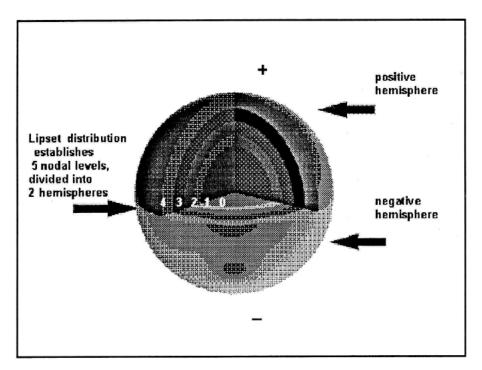
A Representation of a Lipset Sort in the Customary Format

-4	- 3	-2	-1	Ó	+1	+2	+ 3	+4	
16	17	3	1	2	6	13	14	27	
20	18	5	5	8	4	11	22	21	33
	30	31	10	7	12	28	23		
		32	15	9	24	29			
			19	26	25				
ipse	t Q StL	ıdy, So	ort 1						
l = 33 piles									

It has nine piles and 33 squares where the sorter writes in the item numbers. It is further labeled with a frequency of 2, 3, 4, 5, 5, 5, 4, 3, 2. It is a two-dimension object we customarily use to represent the sorter's multi-dimensional arrangement of the ideas at issue.

Now, for sake of exploring possibilities, try to visualize the recording instrument in three dimensions. *Figure 3* is such a representation. It is a hollow sphere with five nodal levels, numbered 0, 1, 2, 3, 4 and divided into two hemispheres, positive and negative.

Figure 3



A 3-Dimensional Representation of Lipset Data Space

Figure 4 is the state of the sphere before sorting. All 33 items can be visualized in the zero level.

Figure 4

A 3-Dimensional Representation of the Initial State of the Items

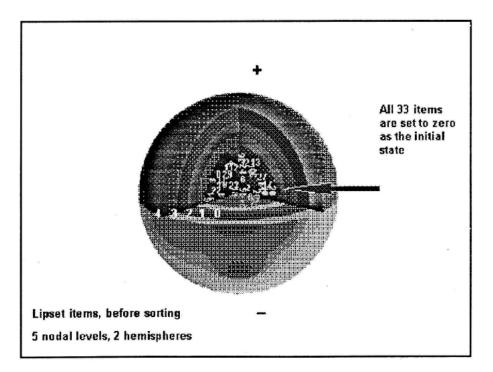
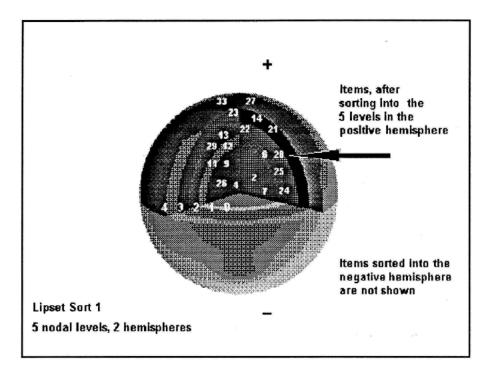


Figure 5



A Representation of Sort 1 After Sorting Has Been Completed

Figure 5 is the state of the sphere after sorting. Each item is shown now to be in a specific level corresponding to the distribution and frequency built in as a theoretical condition for the study.

I first visualized this image when I was in Brazil last June, sitting on the veranda of the bachelor officers' quarters in Teresina where I sketched out a Lipset Q sort inside a sphere. When I had finished and looked the sketches over, I thought to myself, "Gee, every time a person sorts it is like the Big Bang all over again." I could see the chaos in the pile of unorganized item numbers at the center. I marveled at the possibilities that the sorter's mind and hands would put into whatever order he or she would choose for whatever reason. It was then I appreciated more deeply than ever what Stephenson had been trying for years to teach us. Looking at the hollow spheres, I understood better what he meant when he said theory is basic to all: The conditions are those imposed by the theory and purposes of the researcher, but these are nothing without the minds and hands of the sorter.

Conclusions

There is a Portuguese word I learned this past summer: *vislumbrar*. It is a transitive verb referring to the act of glimpsing in fragments, in a flickering or wavering fashion. The Brazilian parallel to the Oxford English Dictionary offers "appercepting" as a synonym. How striking. How appropriate a word for Q Methodologists to know. I want to end with that word in mind.

We need to get on with preparing for the future students of Q Methodology. We need to do a better job of distinguishing between Method and Technique, and I believe we need to be creative in addressing both the underpinnings and possibilities of Q if we are serious about developing along the lines of quantum theory. Of course, we will come to a better understanding of our Methodology in the process.

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