A Note on Measuring Changes in Q Factor Loadings

Expositor

ABSTRACT: The published literature contains no known formulas for assessing changes in factor loadings in Q-methodology studies, but a formula recommended by Stephenson and located in an unpublished dissertation by Rawlins provides a solution. The formula is applied to a single case reported by Stephenson and another reported by Dryzek, and the value of the formula for assessing change is discussed.

It is sometimes necessary to readminister a Q sort which was given to a person previously, and on these occasions the question arises as to whether the differences in the two performances reflect random fluctuations or systematic change. This is explicit in experimentation. For example, a client seeking counseling might be asked to describe his or her self and ideal self as part of the diagnostic process, and then be instructed to repeat the process after six months of psychotherapy. Has the self-ideal correlation changed?

Different ways of obtaining and analyzing data pose different problems in analysis. For instance, the above example could be expanded to include 10 clients who are in psychotherapy and a control group of 10 who are not: the mean difference in self-ideal

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correlations for the two groups could then be assessed using a one-tailed t test (df=18) for independent samples. Or the mean self-ideal correlation for the 10 clients at time-1 could be compared to their own mean at time-2 (one-tailed t, df=9, dependent sample). Normative strategies such as this are obviously R-methodological, and their statistics of appraisal are well known (cf. Johnson & Jackson, 1959, pp. 348-358, for a discussion of standard errors for correlation coefficients obtained under various conditions).

Matters become more complicated, however, when Q sorts are factor analyzed in the usual Q-methodological way, since a factor loading under these conditions only represents that portion of a person's performance associated with the factor: hence assessment is of change or not in a *segment* of the person's response. [1] Statistics are of course unnecessary when a person moves from one factor at time-1 to another at time-2: in these instances, that change has occurred is obvious; however, changes of this clarity and magnitude are apt to be exceptions rather than the rule.

Far more common are situations in which the person remains on the same factor, but with an enhanced or diminished factor loading. If the factor loading has moved from 0.70 to 0.80, does the change (d=0.10) represent a significant shift *on the same factor*? Has the person's factor-A attitude changed in the sense of having undergone enhancement? Or, in the event the loading has diminished, has the attitude undergone deterioration?

The published literature gives little guidance in instances such as these. Brown's (1980) *Political Subjectivity* draws near with the following comment:

> Similar procedures would be followed ... in determining whether the same person had increased his association on a factor following a second administration of the same Q sort.... The two loadings on factor a would then be f_{a1} and f_{a2}, each of which would have a standard error. Since the two Q sorts were given by the same person, their likely correlation would have to

[1] A person's Q-sort responses at times-1 and -2 will be saturated to a greater or lesser extent on all of the factors; the loading on any one factor therefore expresses that proportion of the person's response which that factor can explain, the remainder of the variability being explainable by other factors (plus specificity and error).

> be taken into account. Equation (10.3) would therefore be the most appropriate standard error estimate. (p. 301)

The formula to which Brown refers (on p. 298) is as follows:

$$\sigma_d = \sqrt{[\sigma_a^2 + \sigma_b^2 - 2(r_{ab})(\sigma_a)(\sigma_b)]} \quad [1]$$

where σ_d is the standard error of the difference in factor loadings, σ_a is the standard error of a zero-order loading for the first factor, σ_b is the standard error for the second, and r_{ab} is the correlation. The problem of course is in finding a value for r_{ab}, which is not simply the correlation between the two Q sorts, but only that portion of their overall correlation which is subsumed by the factor (see footnote 1).

A proposed solution to this problem is to be found in an unpublished doctoral dissertation by Mary Jane Rawlins (1964, p. 177), one of William Stephenson's students. According to Rawlins, Stephenson suggested formula [1], with *r* being obtained by taking the cross-product of the two factor loadings. Hence, with two Q sorts loaded on the same factor 0.70 and 0.50, respectively, that part of their intercorrelation expressed by the factor would be (0.70)(0.50)=0.35.

Before turning to an illustration, it is important to draw a circle around what is actually being proposed, which, it bears stressing, is of a statistical nature only. Hence concern is not with the important matter of theoretical rotation: the magnitude of the factor loadings is obviously dependent on the rotations chosen, but we can assume that all of those decisions have already been made and that the task before us is to assess the factor loadings which have resulted. Nor does concern touch on the epistemology of experimentation per se: a detectable change in factor loadings may or may not bespeak a potent intervening cause or even an actual change in the person's subjective state, which is a conclusion that must rely on more than just statistics -- e.g., on proposition sets and on whether or not the change is schematic (Expositor, 1985, 1987), no less than on the facts and problems at issue (Stephenson, 1984). Nor, finally, is concern with the two Q sorts (at time-1 and time-2) in their entirety, i.e., apart from their location in factor space: it is of course important to examine these two responses in detail, but statistics are already known for comparing one whole Q sort with another (e.g, Brown, 1980, p. 299). The sole purpose of equation [1] is to provide a measure (under the rules of probability) of the range of chance-like fluctuations so as to be able to assess actual differences between loadings on the same factor, much as Stephenson (1978) has already shown for factor scores.

For purposes of illustration, consider the case of "Rogerg," in *The Study of Behavior* (Stephenson, 1953, p. 262). Rogerg provided several Q sorts between July and September, among them "my self" at the outset and again at the conclusion of all the sortings: the first obtained a loading of 0.74 and the latter 0.64 on the same factor. Does this difference of d=0.10 represent a significant decline in the self's saturation with the factor? That is, even though the self has remained firmly attached to the factor, has there been significant movement nonetheless?

The standard error for a zero-order loading is $\sigma_r=1/\sqrt{N}$ (where N is the Q sample size), but for loadings other than zero an approximate value is given by

$$\sigma_r=(1 - r^2)/\sqrt{N} \qquad [2]$$

With N=60 statements, the standard errors for the above two loadings (0.74 and 0.64) are therefore 0.06 and 0.08, respectively, and the standard error of the difference, using expression [1], is

$$\sigma_d=\sqrt{[.06^2+.08^2 - 2(.74\times.64)(.06)(.08)]}=0.07$$

In normalized terms, $z=d/\sigma_d=(.74-.64)/.07=1.41$, which falls short of significance. Hence there is little evidence that Rogerg's self underwent substantial change between the July and September Q sortings.

Or consider the case of Ms. X reported by Dryzek (1990, p. 183). X ranked a set of policy statements under a variety of conditions of instruction, resulting in three factors. Her own personal preferences were obtained at the outset and again at the end, and these two conditions loaded on the same factor by the amounts 0.83 and 0.93. Dryzek does not report the size of the Q sample, but if we arbitrarily suppose it to be of the order N=40, then expression [2] produces standard errors for the two loadings of 0.05 and 0.03, respectively, and expression [1] produces a standard error of the difference of σ_d=0.04. The difference between the loadings is again d=0.10, and the normalized value is $z=d/\sigma_d$=0.10/0.04=2.82 (p<.01), which is significant. Hence, evidence exists that Ms. X's personal preferences shifted in a significant way even though both performances were firmly associated with the same factor. What we might seek to make of this difference is of course subject to the caveats introduced previously.

But it is also to be noted that Ms. X's personal preferences were likewise loaded on the third factor to the extent of -0.24 and 0.21. Neither of these loadings is itself significant (i.e., significantly different from zero), and so we might be prone to overlook them; the distance between them is significant, however (d=0.45, z=2.07, p<.05), and indicates that this segment of Ms. X's preferences underwent alteration as well.

This may seem like much ado about nothing: neither of the loadings in and of itself is significantly different from zero after all; however, a different rotation focused on these two Q sorts could have resulted in significant loadings. Even so, statistical significance can only hoist red flags, but cannot otherwise substitute for a more careful examination of the Q sorts themselves to determine which items shifted and which did not, and whether there is some interpretation or conclusions that will encompass these changes.

The advantages of the above formula are twofold. First, in the usual case involving Q sorts from several persons, it permits detection of those individuals in an experiment who have in fact changed. In analysis of variance studies, a significant F-ratio indicates that subjects in the treatment group have obtained significantly different scores from the controls *on the average*, but under some conditions only a few high scoring individuals are required in order to produce that effect; moreover, an insignificant outcome does not enable the scientist to determine if the treatment had a sizeable impact on specific individuals even if not on the group as a whole. The above procedures enable us to identify individual changers even when the experimental group as a whole did not change significantly.

Second, in single-case studies, expression [1] permits us to detect which aspects of a person's performance have undergone change. In the case of Ms. X above, it would have been quite possible for her personal Q sort to have been on the first factor at both time-1 and time-2, and for the two loadings to have been quite high and insignificantly different -- and for there still to have been a significant shift in her stance vis-a-vis one of the other factors in her make-up. As above, X's loadings on the third factor were -0.24 and 0.21, neither of which was significant, but the difference between them was. If the investigator's attention is riveted on the dominating first factor, the significant change on the third might go unnoticed. Expression [1] gives us an opportunity to assess such changes, as small and unimportant as they may seem at first blush.

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In future issues...

William Stephenson, *Introduction to Q-Methodology*.

Paul Grosswiler, *Some Methodological Considerations on the Use of Multimedia Q-sample Items*.