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Mathematics Teachers' Perceptions of Practice: A Q-Methodology Study

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Abstract: The instructional practices enacted by mathematics teachers have the most powerful impact on students' learning. In our study, we analyzed mathematics teachers' perceptions of their instructional practices, specifically related to their use of actions that support high-leverage practices. Q Methodology was used to investigate the divergent perceptions of mathematics teachers' teaching practices. Employing principal component analysis with varimax rotation, five factors were extracted that represented the perceptions held by 38 elementary, middle-level, and high school teachers from Pennsylvania, North Carolina, and Mississippi. We identified the five factors as: Promoting Students' Productivity; Using High-Level Tasks; Promoting Sense-Making and Reasoning; Encouraging Mathematical Representation; and Acknowledging Students in Time. The mathematics teachers' perceptions of their teaching actions that support high-leverage practices will benefit mathematics coaches, mathematics educators, professional development providers, and the teachers themselves.

Keywords: high-leverage practices, perceptions of instruction, Q methodology, Q sort

Introduction

Research indicates that teachers' classroom instructional practices have a powerful impact on student achievement (e.g., Clotfelter, Ladd, & Vigdor, 2007; Rivkin, Hanushek, & Kain, 2005). In fact, what teachers do in the classroom has a greater effect on students' learning than do the teachers' own personal beliefs about learning (Creemers & Kyriakides, 2006). Efforts to improve the quality of teaching in the U.S. have emphasized the need for ambitious teaching practices that aim "to teach all kinds of students to not only to know academic subjects, but also to be able to use what they know in working on authentic problems in academic domains" (Lampert, Boerst, & Graziani, 2011, p. 1362). Ambitious teaching requires teachers to implement teaching practices that elicit thinking, facilitate productive discourse, and promote deep understanding to enhance students' learning. These practices center on the teacher

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attending to and elevating students thinking or "leveraging" student thinking (Singer-Gabella, Shahan, & Kim, 2016, p.412). Mathematics educators have identified specific teaching practices, referred to as high-leverage practices, that address the need for ambitious teaching by improving classroom instruction and focusing on students' deep understanding of mathematics (Ball & Forzani, 2009; Grossman, 2013; Lampert et al., 2013). Examples of such practices include posing cognitively demanding mathematical tasks (Henningsen & Stein, 1997; Stein, Remillard, & Smith, 2007); promoting classroom discourse to enhance students' mathematical learning (Franke, Kazemi, & Battey, 2007; Smith & Stein, 2011); eliciting and responding to students' ideas (Jacobs, Lamb, & Philipp, 2010; Sleep & Boerst, 2012); orienting students to instructional goals (Hiebert & Morris, 2009; Sleep, 2012); supporting students' conversations about mathematics concepts (Boaler, 2006; Hufferd-Ackles, Fuson, & Sherin, 2004; Kazemi & Stipek, 2001); using and making connections among representations (Ball & Bass, 2003; Fuson, Kalchman, & Bransford, 2005; Stylianou & Silver, 2004); engaging students in productive struggle (Henningsen & Stein, 1997; Hiebert & Grouws, 2007); and questioning strategies that promote students' thinking (Herbel-Eisenmann & Cirillo, 2009).

Implementing these types of high-leverage teaching practices plays an important role in promoting student achievement. However, there is little research on teachers' perceptions of their implementation of high-leverage practices in the mathematics classroom. In the present study, we used Q methodology as the research methodology (McKeown & Thomas, 1988; 2013) to investigate the divergent perceptions of mathematics teachers' teaching practices.

Studying Mathematics Teaching Practices

Researchers have conducted numerous studies to examine mathematics teachers' instructional practices in the classroom (Boston, 2012; McGee, Wang, & Polly, 2003; Wilhelm & Kim, 2015). However, developing and implementing such systems continues to be problematic for two reasons. First, the use of observational systems (Hill, Charalambos, & Kraft, 2012) can offer insight into what occurs in a teacher's classroom. However, these systems have been questioned because the studies that support their use exhibit methodological weaknesses that include the quality of the implementation, validity of the results, reliability of the measures, and costs of implementation (Kane & Staiger, 2012). Second, researchers often base teacher surveys on self-reporting data with items measured on a Likert scale that limit the respondent's thoughtful expression to circling the researcher-supplied choice with which that respondent most agrees or disagrees. More importantly, researchers have questioned the validity of the data obtained with such Likert scales. Specifically, these items may yield socially-desirable responses rather than a response that reflects a respondent's true belief (Jamieson, 2004; Kazdin, 1998). For example, a teacher could circle "Strongly Agree" when asked to respond to the statement "I identify how the goals fit within a mathematics learning progression to a high degree." This response would be biased if the teacher circled the response choice because they felt that their principal believed that all outstanding teachers would circle this choice. Researchers will continue to collect self-report data because these data are less expensive and less intrusive to collect than are many alternatives. Teachers' self-reports of their classroom teaching actions can be informative if researchers understand that each teacher's perspective is subjective and socially constructed by the individual (Lather, 2006).

In an effort to address these limitations and contribute to the knowledge base regarding the extent to which teachers implement high-leverage practices, the present study used Q methodology. Originally formulated and developed by William Stephenson (1935, 1953), in recent years its affinities with social constructivism have been elaborated by Paul Stenner and Simon Watts (Stenner, 2008; Watts & Stenner, 2012). The purpose of the study was to explore the divergent perspectives of mathematics teachers' teaching practices. Specifically, participants construct their meaning of a set of statements and interpret them based on their own reference before sorting them on a grid (Ramlo & Newman, 2011). The researchers then conduct a qualitative analysis of the factors that emerge from a factor analysis and construct their interpretations.

Researchers have used Q Methodology to study Norwegian primary and secondary school teachers' perceptions of the actions they take to support students with learning disabilities who attend inclusive classrooms (Subba, et al., 2017). Earlier, researchers used Q Methodology to design and evaluate an activity that focuses on middle school teachers' mathematics knowledge with respect to teaching and building the mathematics self-efficacy of a culturally and socially diverse, middle-school student population (Stevens, Harris, Aguirre-Munoz, & Cobbs, 2014). Focusing on a larger objective, researchers have used Q Methodology as the focus of a workshop devoted to eliciting and deliberating on participants' perspectives (Yoshizawaa, Iwase, Okumoto, Tahara, & Takahashie, 2016). For the present study, we used Q methodology to explore mathematics teachers' perceptions of the extent they implement specific teaching actions. Insight into mathematics teachers' perceptions of their high-leverage teaching practices can help identify teachers' instructional strengths and areas for growth (Ball & Forzani, 2010).

Purpose of the Study

Effective teaching of mathematics across all grade levels is essential to ensure all students learn mathematics and can think mathematically (National Council of Teachers of Mathematics, 2014, p. vii). The implementation of high-leverage teaching practices in today's classrooms ensures the provision of those practices that promote the deep learning of mathematics and which are most likely to affect student learning (Ball & Forzani, 2010, p. 45). To that end, the purpose of this Q-methodology study was to examine teachers' perspectives of the extent to which they implement high-leverage practices. The process involved having teachers construct their meaning of a set of statements that represented teaching actions (the Q sample) and sort them in terms of a quasi-normal forced choice grid, based on their perceptions of the extent to which they implement the teaching actions (the Q sort). We explored the following research question: What common perceptions do the participating mathematics teachers have regarding their implementation of high-leverage teaching practices? To answer this question, we used Q Methodology because it offered a valid and sound approach to identify factors that represent the commonly shared perceptions of mathematics teachers' classroom teaching practices (Brown & Good, 2010; Carlson & Hyde, 2003).

Method

Participants

We recruited a convenience sample of mathematics teachers (n = 38) who taught 4th through 10th grades from five local public-school districts in three U.S. states. We selected the districts based on their proximity to the researchers and the willingness of the district to agree to have teachers engaged in the study. Table 1 presents selected characteristics of our participating teachers. The participants' years of classroom teaching experience ranged from 1 to 30 years (mean = 9.3; median = 7.5). Table 2 provides information on the participants' school districts and the student population of the districts. All participants signed documented informed consent forms and received a $25 Amazon gift card to compensate them for their time.

Table 1: Selected Characteristics of the Study Participants

State	Gender		Total
	Female	Male	
Mississippi	11	4	15
North Carolina	8	2	10
Pennsylvania	9	4	13
Total	28	10	38

Table 2: School District Demographics and Student Populations

State	No.Teachers	District Demographics	Student Population
Mississippi	7	Rural, high poverty	66% Black, 30% White, 3% Asia 1% Latin
	8	Rural, high poverty	79% Black, 30% White, 1% Hispanic
North Carolina	10	Suburban, high poverty	33% Black, 33% Latinx, 34% White
Pennsylvania	7	Suburban, low poverty	75% White, 14% Asian, 7% Latinx, 4% Black
	6	Urban, high poverty	57% Black, 33% Latinx, 3% White, 7% Other

Proponents of Q Methodology have suggested that study participants be individuals who are familiar with and have a distinct view on the topic. A large number of participants are not required. Indeed, Brown has argued that the emphasis when recruiting study participants is on "the nature of the segments of subjectivity that exist and the extent to which they are similar and dissimilar" (as cited in DuPlessis, 2005, p. 151). Although Q Methodologists recommend that the sample of participants (P set) reflects the variability exhibited by the population of potential participants, the proportion of participants who share a particular characteristic is not important (Brown, 1980).

Q Sample

The 37 statements in the present study's Q sample served as a representation of the concourse. The concourse was a set of teacher actions related to implementing high-leverage practices identified in the premier landmark publication *Principles to Actions: Ensuring Mathematical Success for All* (PtA) (National Council of Teachers of Mathematics [NCTM], 2014) document. Specifically, we based these 37 teacher actions on the summary of teaching actions that the PtA writing team identified, based on what research has shown are actions to support the implementation of high-leverage teaching practices. Collectively, the teacher actions help improve instruction by providing a common language and deeper understanding of how high-leverage teaching practices can be implemented.

Each teacher action was typed on cardstock that was sized to fit in the cells of a 9" X 20.5" symmetrical Q grid. The grid was designed so that participants were required to place each of the 37 cards in a cell under one of the 11 columns labeled from -5 (Least Characteristic of My Teaching) to +5 (Most Characteristic of My Teaching). We chose the number of cells displayed in each column of the grid in order to ensure that each participant identified the same number of similarly ranked statements. We used a random-number generator to assign a number between 1 and 37 to each statement. We then printed this number on the card to help identify the statement when we analyzed the data. We informed the participants that the number on the card did not represent any value or ranking and that it was simply present so that they could later record on a paper version of the grid, the cell where they had placed each card. The *Appendix* provides the Q sample, the associated random number, and the mathematical practice associated with each teacher action.

Given that Q sorts are entirely subjective, and participants arrange the printed statements according to the way they make sense of the statements, there is no need for researchers to use an external criterion to validate the responses (Brown & Good, 2010).

Q-Sorting Process

Three researchers from the research team met via Skype before participant recruitment in order to review the study materials and protocols and make sure that the researchers followed the same procedure at each site. When the study was fielded, one Q sort session was held in Mississippi (15 teachers), two in North Carolina (5 teachers in each group), and two in Pennsylvania (6 teachers in one group and 7 teachers in the other). One research team member was present at each session in order to answer participants' questions or provide clarification. In one session, the participants needed clarification of the teacher action "Providing students with opportunities for distributed practice of procedures" (NCTM, 2014, p. 48). The researcher described this as "spacing" the practice over time.

Each session began with participants working independently, reading all 37 cards on which a teacher action was printed and then sorting the cards into three piles: (a) actions most characteristic of their teaching, (b) actions least characteristic of their teaching, and (c) actions they were not sure about. The participants then used the Q grid to place all the cards, assigning a particular value to each.

To add depth to the analysis, we asked the participants to complete a short questionnaire and report their gender, number of years of teaching, certification area, school, grade level teaching, and type of school. We also asked the participants to use this short form to respond to four open-ended questions: 1. Which statements were the easiest to place and why? 2. Which statements were the hardest to place and why? 3. What experiences have shaped how you implement the teacher actions in your classroom? and 4. What reactions or thoughts did you have as you were deciding where to place the statements? The researchers conducted interviews with a random sample of four or five participants at each site and asked participants to elaborate on the reasons why they placed the statements as they had, specifically those on the extreme ends of the 11-point scale. The interviews also offered the participants an opportunity to verbalize their perceived motivations for implementing or not implementing certain teacher actions in their classrooms. The researchers recorded the interviews and two graduate students and one of the researchers transcribed them.

Data Analysis

To answer the research question, we conducted a principal components factor analysis of participants' Q sorts (McKeown & Thomas, 1988) to extract factors that reflected the participants' perceptions of classroom teaching practices. Specifically, we first calculated Pearson correlation coefficients among the 38 Q sorts. We then used Zabala's (2014, 2015) R software program, qmethod, to perform the principal components factor analysis with varimax rotation of the 38 X 38 correlation matrix. Zabala's program reports how many participants had statistically significant loadings on a factor, the variance explained by each extracted factor, and the factor's composite reliability. Q sorts that load significantly on a given factor do so because they exhibit a common pattern (Stenner, Cooper, & Skevington, 2003, p. 2164). It is important to note that with a Q-Methodology factor analysis, researchers use the loadings on the factors that emerge to identify the participants who share a common perspective, as opposed to the researcher using an a priori set of criteria to group participants (Brown, 1980, p. 208).

We ran the R software program eight times; and with each run, we extracted an increasing number of factors. Specifically, we extracted four to eleven factors, ported the extensive output to Excel, and then engaged in a detailed comparative study of the output. This analysis entailed identification of the consensus and distinguishing statements that resulted from each factor extraction and selecting the number of factors that best reflected the participating teachers' perspectives as represented by the Q sorts that loaded on the same factor (McKeown & Thomas, 2013, p. 60). We also considered (a) the set of factors that reflected the maximum number of participants and (b) the set that accounted for the largest amount of explained variance. We used these two criteria to decide which extraction yielded the best solution.

The decision to extract a given number of factors rests largely on the persuasiveness of the interpretation that can be given to (a) the factors that are extracted and (b) the story that can be told upon examination of the distinguishing and consensus statements. Finding the distinguishing and consensus statements is one of the features unique to a Q-Method factor analysis. Distinguishing statements are those for which all of the absolute differences between the factor z-scores calculated for each pair of factors are statistically significant (Zabala, 2015, p. 168). The reasoning is as follows: If the difference between the factor z-scores of a statement is statistically significant (when compared to the standard error of the difference), then what both factors indicate about that statement is distinct; when none of the differences between the factor z-scores of a statement for any pair of factors is significant, then the statement is a consensus statement for which the corresponding respondents view the statement similarly.

In Q Methodology, the percentage of variance explained is not as relevant a measure as it is in the non-dominant R-mode factor analysis because it does not represent a population-based percentage given the manner in which researchers study participants. Instead, the percentage of variance reflects the extent to which various factors exist and the extent to which there are similarities and differences among the factors (Akhtar-

Danesh, 2018; Cuppen, Breukers, Hisschemöller, & Bergsma, 2010). To use language specific to the present study, the percent of explained variance reflects the degree of homogeneity among the diverse group of participants we sampled, and the statistically significant loadings associated with a participant's Q sort indicate which participants hold common beliefs or perceptions of their practices.

Moreover, in Q-mode factor analysis, the analyst uses factor z-scores to assess the relations between the statements and factors. Specifically, the z-scores indicate the extent to which each factor is identified with a statement. The z-score is a weighted average of the raw scores given by the Q sorts that are "flagged" for that statement. When Zabala's qmethod program flags a Q sort for a given factor, it means that the analyst can use the values associated with the flagged Q sort for subsequent calculations (e.g., for obtaining statement scores that reflect the scale used with the Q grid). In R-mode factor analysis, researchers can calculate a participant's factor scores (i.e., the participant's score on each latent factor). However, in Q-mode factor analysis, researchers can calculate a statement's factor scores (i.e., the statement's score on each latent factor). In a Q-mode factor analysis, each extracted factor is a perspective.

For the present study, the researchers interpreted and labeled each factor based on the distinguishing statements specific to that factor, using a set of guidelines by Pett, Lackey, and Sullivan (2003). The statements with the highest and lowest scores on the sorting continuum are of particular interest because they represent the most characteristic and least characteristic high-leverage practices of the participants who loaded on the factor (Valenta & Wigger, 1997).

The researchers also considered the supporting qualitative statements that participants provided after they had completed their sorting task and had given their materials to the researchers. The researchers coded participants' responses to the open-ended questions and interviews according to various motivations they perceived influenced their implementation of the teacher actions (Saldaña, 2013). This process entailed identifying key words, phrases, or contexts that supported the participants' motivations for the degree of implementation of various practices. When there was a difference in the coding, the researchers discussed the data and mutually agreed upon a motivation. Next, the researchers used an Excel spreadsheet to list the teacher characteristics, qualitative statements, and the motivations identified (Meyer & Avery, 2008). The Excel file enabled the researchers to arrange the open-ended and interview comments according to the Q sorts that loaded on specific factors and identify comments the teachers shared about their motivations for implementing various actions. The researchers chose the names for each factor based on how they represented the perceived motivations reflected in the qualitative data by the participants whose Q sorts loaded on each factor.

Results

Using the two selection criteria discussed in the preceding section we chose the five-factor solution. The factors explained approximately 47% of the total variance with 29 of the 38 Q sorts loading significantly onto the five factors. More importantly, we believe that the five-factor solution reflected the best representation of participants' Q sorts in terms of their shared perceptions of motivations for implementing the high-leverage practices (cf. McKeown & Thomas, 1988). Table 3 provides a summary of the number of participant Q-sorts that loaded onto each factor as well as the eigenvalues, explained variance, and reliability associated with the five factors.

Table 3: Summary of the Five-Factor Extraction

Factor	Number loading	Eigenvalues	% Explained Variance	Reliability
1	12	4.92	12.95	0.98
2	5	4.06	10.69	0.95
3	5	3.53	9.29	0.95
4	4	2.72	7.15	0.94
5	3	2.48	6.51	0.92

Table 4 provides the factor matrix associated with the participant demographics used to define the sorts. Visual inspection of the program output indicated that factor loadings of +/-0.40 were significant at the $p < 0.05$ level.

Table 4: Factor Matrix with Participant Demographics

Number/Gender		Years of Teaching	School Setting	Grade Level	Factor Loadings				
					1	2	3	4	5
2	F	1	S	MS	0.62*	−0.08	0.36	−0.21	−0.05
3	M	20	S	EL	0.71*	0.09	0.18	0.17	0.05
4	F	6	S	MS	0.44*	0.40	−0.03	0.00	−0.18
6	M	18	S	HS	0.47*	−0.10	−0.12	−0.14	0.07
8	F	31	U	HS	0.52*	−0.40	−0.03	0.03	−0.18
12	F	1	U	EL	0.68*	0.23	−0.13	0.00	−0.21
13	F	8	U	EL	0.42*	0.27	0.11	−0.09	−0.01
23	F	4	S	EL	−0.53*	0.07	−0.07	0.40	0.08
24	F	9	R	EL	0.60*	0.40	0.04	0.25	0.02
33	F	2	R	HS	0.53*	0.40	0.08	−0.13	0.07
36	M	1	R	HS	0.45*	0.01	−0.05	−0.18	−0.02
37	M	4	R	HS	0.47*	0.14	−0.40	0.03	0.26
26	F	8	R	EL	0.16	0.65*	0.17	−0.10	−0.20
27	F	10	R	EL	−0.04	0.60*	−0.08	−0.22	0.21
29	F	6	R	EL	0.14	0.60*	−0.03	0.11	−0.06
31	F	1	R	EL	0.24	0.53*	0.17	0.11	0.41
34	F	2	R	HS	0.02	0.65*	0.00	0.04	−0.24

Number/Gender		Years of Teaching	School Setting	Grade Level	Factor Loadings				
					1	2	3	4	5
15	F	14	S	EL	0.34	−0.11	0.66*	0.17	0.13
16	M	15	S	EL	−0.04	0.20	0.75*	−0.14	−0.10
17	F	9	S	EL	−0.10	0.07	0.61*	0.06	0.29
20	F	6	S	EL	0.37	0.32	0.57*	−0.23	−0.12
22	F	1	S	EL	−0.05	−0.08	0.78*	0.10	−0.01
5	F	18	S	HS	0.04	0.00	−0.01	0.50*	−0.03
7	M	12	S	HS	0.18	0.04	−0.21	0.44*	0.22
14	F	15	S	EL	0.10	−0.09	−0.15	0.74*	0.15
25	F	7	R	HS	0.27	−0.12	0.03	0.43*	0.03
10	M	20	U	EL	0.40	0.01	0.03	−0.03	0.49*
19	F	15	S	EL	−0.06	−0.09	0.01	0.28	0.66*
38	F	12	R	HS	0.11	0.20	0.07	0.05	0.70*
1	F	6	S	MS	0.39	0.40	0.39	0.14	0.16
9	F	20	U	EL	0.40	0.21	−0.23	0.26	−0.18
11	F	2	U	EL	−0.26	−0.04	0.15	−0.18	0.32
18	F	15	S	EL	0.30	−0.30	0.33	0.40	0.11
21	M	15	S	EL	−0.05	0.40	0.32	0.39	0.05
28	F	15	R	EL	0.15	0.35	−0.40	0.14	−0.37
30	F	12	R	EL	−0.05	0.45	0.08	−0.40	0.40
32	M	15	R	HS	0.01	−0.11	−0.18	−0.23	−0.24
35	M	7	R	HS	0.35	0.17	0.03	−0.29	0.26

*$p < 0.05$

The five-factor solution did not reflect every participant's Q sort. Indeed, the five-factor solution only included 29 of the 38 Q sorts. Although the nine-factor solution explained 66% of the total variance, it only reflected 23 of the 38 Q sorts. Moreover, only two Q sorts loaded on each of four extracted factors, and one Q sort loaded on one of the extracted factors. We observed similar undesirable characteristics until we examined the five-factor solution. For the sake of completeness, we note that the four-factor solution explained 40.5% of the variance and reflected 28 of the 38 Q sorts; and that the three-factor solution explained 33.5% of the variance and reflected 27 of the 38 Q sorts.

Because nine Q sorts did not load on any extracted factor associated with the five-factor solution they were excluded from further analysis and interpretation (Armatas, Venn, & Watson, 2014; Watts & Stenner, 2012). The occurrence of Q sorts that do not load on any factor is not a remarkable feature of any Q Methodology study for two reasons. First, it should not be surprising to find that a small convenience sample of individuals provides evidence of one or more unique perspectives on a topic. Second, Q Methodology, by design, challenges its practitioners to concentrate on factors defined by two or more statistically significant Q sorts. Using this criterion, we decided that we would not accept the nine-factor solution that explained more of the variance but had a factor defined by just one Q sort.

The factors reveal five shared common perspectives these teachers hold regarding their implementation of high-leverage teaching practices: Factor 1 – Promoting Students' Productivity; Factor 2 – Using High-Level Tasks; Factor 3 – Promoting Sense-Making and Reasoning; Factor 4 – Encouraging Mathematical Representations; and Factor 5 – Acknowledging Students in Time. The researchers analyzed the interview transcripts along with the factor arrays to provide insight into the teachers' motivations for their perceptions of the teacher actions (Van Exel & de Graaf, 2005).

Factor 1: Promoting Students' Productivity

We interpreted the first factor, which accounted for 12.95% of the total variance, as "Promoting Students' Productivity." This orientation is comprised of 12 participants (five secondary, two middle-level, and five elementary teachers) with an average of 8.75 years of teaching. The teachers in this group reported that four statements were most characteristic of their teacher actions: *Anticipate what students might struggle with during a lesson and be prepared to support them productively through the struggle* (statement 21) [21]; *Give students time to struggle with tasks, and ask questions that scaffold students' thinking without stepping in to do the work for them* [15]; *Ask students to discuss and explain why the procedures that they are using work to solve particular problems* [2]; and *Ask intentional questions that make the mathematics more visible and accessible for student examination and discussion* [17].

Both of the participating female middle school teachers who loaded on this factor shared how they encourage students who may struggle. One teacher wrote, "By identifying what students will struggle with prior to the learning experience, I am able to alter the lesson in ways they will connect the material to them personally. By doing so, they have the opportunity to be successful" [21]. She added in the interview, "I can't always predict where my students may struggle but I do my best to not give them hints or tell them what to do. I try to pose questions to help scaffold their thinking and connect to what they already know. Probing and giving more opportunities for reasoning and posing higher-cognitive demanding activities requires appropriate time. But I try to plan for this." The other teacher who taught in a suburban district explained, "If students do not understand why they are taking particular steps to solve a problem or are not able to explain their thought process, they do not demonstrate they fully understand the material" [2]. "This is where I try to pose a question for them to think about or have them turn and talk to their tablemates." A female secondary teacher shared, "It is hard to plan for what students will understand but I try to think about how they will know about what I am teaching." She also wrote, "I plan the questions I might ask in my lesson plan and try to anticipate how I might push my students' thinking."

Several teachers in this group identified *Facilitate discourse among students by positioning them as authors of ideas, who explain, and defend their approaches* [34] as least characteristic of their teaching. One male high school teacher who taught in a suburban district explained, "I tend to rely on my Algebra pacing guide to keep up with where I need to be. I don't always take the time to ask questions or let students explain because I have so much content to cover." A female elementary teacher stated, "Some actions I know I should do but I know my students aren't ready to do these things." She later added, "I know I should give them time to talk to each other and get them to talk about how they solved a problem."

Factor 2: Using High-Level Tasks

The Q sorts of five teachers (four elementary and one secondary), all female who taught in high poverty, rural school districts with an average of 5.4 years of teaching, comprised this common orientation, which accounted for 10.7% of the total variance. We interpreted this factor as "Using High-Level Tasks." The teachers in this group found three statements were either most or least characteristic of their teaching: *Pose tasks on a regular basis that require a high level of cognitive demand* [3]*; Select tasks that allow students to decide which representations to use in making sense of the problems* [24]; and *Select tasks that provide multiple entry points through the use of varied tools and representations* [32].

The secondary teacher shared that she found tasks were a way to get students to "talk about the procedures and compare their thinking to others." In her interview she added, "Tasks hold everything together. Without tasks that are good it is almost impossible to do the rest of these practices." The teacher also commented, "Students love to explore and try to figure out problems. If I choose a task that doesn't motivate them or that is too easy or too hard those are the days that they get off task very easily." One elementary teacher wrote, "The selection of tasks is something that I have focused on a lot. Students need to have tasks that allow them to have a lot of freedom and choice in strategies."

On the other hand, two elementary teachers shared comments regarding how the use of tasks was not characteristic of their teaching. One elementary teacher explained, "I quickly placed the planning, discussion, and modeling teaching actions as most like me, but I struggle with using tasks in my classroom and implementing essential features of mathematics." She added, "It is one of the hardest actions because I can't find the time to do them [tasks] but know I should. Also, classroom management issues, as well as students' levels of mathematical content knowledge, shape how I plan my lessons." The other elementary teacher shared that she avoided "high-level tasks" because her students could not do them. She also responded to the open-ended question, "What experiences shape how you think about using the teaching practices in your classroom?" by stating that her students' low performance levels shape her decisions regarding what she does in her classroom. She wrote, "The students I had this year were very low and needed to go back to basic skills."

Factor 3: Encouraging Sense Making and Reasoning

We interpreted the third factor, which accounted for 9.3% of the total variance, as "Encouraging Sense Making and Reasoning." The Q sorts of five elementary teachers with an average of nine years of teaching who taught in high poverty, suburban districts comprised this orientation. The teachers in this group found most characteristic of their teaching the following three teacher actions: *Praise students for their efforts in making* *sense of mathematical ideas and perseverance in reasoning through problems* [1]; *Engage students in purposeful sharing of mathematical ideas, reasoning, and approaches using varied representations* [5]; and *Make certain to ask questions that go beyond gathering information to probe thinking and require explanation and justification* [31].

A male elementary teacher with fifteen years of experience shared how important it is to have his students engaged, "My whole classroom is built on students sharing strategies and ideas." He expressed the belief that giving students opportunities to work on problems and tasks in small groups helps them talk about the math and think about what they needed to do. One female elementary school teacher stated, "One of my favorite actions is engaging students in purposeful sharing of math ideas and praising students who are willing to put effort and persevere in making sense of mathematics. I like this activity because it clarifies for me what I like the most about teaching and what makes me want to go to work every morning and this is actually working with students." A female elementary teacher wrote, "Praising students is the only way I have found them to work for me on hard problems. It doesn't seem like much, but it has made my students harder workers." Another female teacher commented, "Actions I placed under most like me are things like praise students, wait time, discussion, and motivating students. I seem to focus more on these actions during the lesson than prior knowledge and final assessments."

The Q sorts of the five elementary teachers identified the teacher action, *Motivate students' learning of mathematics through opportunities for exploring and solving problems that build on and extend their current mathematical understanding* [28] as least characteristic of their teaching. One female, fourth-grade teacher who loaded on this factor expressed, "My students tend to easily get distracted when they work on problems in small groups. They are more engaged when I have the whole class do things like choral respond or use their whiteboards." Another female, elementary teacher commented, "While I know some problems are good, my students struggle with them too much. I know I should be doing them more."

Factor 4: Encouraging Mathematical Representation

We interpreted the fourth factor, which accounted for 7.2% of the total variance, as "Encouraging Mathematical Representation." The Q sorts of four participants (two female and one male high school teachers and one elementary female teacher) with an average of 13 years of teaching comprised this orientation. The teachers in this group found most characteristic of their teaching the following three teacher actions: *Ask students to make math drawings or use other visual supports to explain and justify their reasoning* [9]; *Encourage students to use varied approaches and strategies to make sense of and solve tasks* [26]; and *Use visual models to support students' understanding of general methods* [35].

One female, high school teacher who taught in a suburban district stated that the easiest actions to place on the Q sort were those that engage students, ask students, encourage students, and address the use of visual aids. "I always use visual representations and encourage students to use it too (drawings, manipulatives, etc.)." Another female, high school teacher who taught in a rural district wrote, "I found it easy to place the statements that deal with multiple representations of math models. I usually always try to have students use various strategies to solve tasks." The female, elementary teacher explained, "I use visual models every day because many of my students are visual learners. They need a visual to understand a concept." She added, "I have a large group of IEP and ESL students. I provide many visual models so that my concepts are not so abstract."

Several teachers in this group found the following two statements least characteristic of their teaching: *Use the mathematics goals to guide lesson planning and reflection and to make in-the-moment decisions during instruction* [23]; and *Help students realize that confusion and errors are a natural part of learning, by facilitating discussions on mistakes, misconceptions, and struggles* [25].

One female, high school teacher wrote, "I feel like I do include goals in my lessons, but I am not sure that I do it every time." She also shared, "I try to plan for making connections across representations, but I tend to just go with one approach. I figure if they can do it one way they will do well on the [state] test." The other female, high school teacher shared that she does not take the time to discuss students' misconceptions. She stated during the interview, "I'm afraid to show students' misconceptions for fear of confusing students."

Factor 5: Acknowledging Students in Time

We interpreted the fifth factor, which accounted for 6.5% of the total variance, as "Acknowledging Students in Time." The Q sorts of three teachers (one secondary female, one elementary male and one elementary female) with an average of 15.7 years of teaching comprised this factor. The teachers in this group found two statements most characteristic of their teacher actions: *Make in-the-moment decisions on how to respond to students with questions and prompts that prove, scaffold, and extend* [10]; and *Advance student understanding by asking questions that build on, but do not take over or funnel, student thinking* [19].

The female, secondary teacher shared, "Letting students solve problems on their own is something that I have tried doing this year. It has led to more independent thinkers." She wrote in her open-ended responses, "I do a great deal of cooperative learning to ensure students are active in making sense of concepts." One female, elementary teacher stated, "I try to help students when they struggle by asking them questions to scaffold their learning. I know this is a strategy that can help them."

Teachers in this group identified several statements as least characteristic of their teaching: *Allow sufficient wait time so that more students can formulate and offer responses* [8]; *Give students time to struggle with tasks, and ask questions that scaffold students' thinking without stepping in to do the work for them* [15]. The female, elementary school teacher noted, "Giving students time to come to understanding, I realize my teaching practices are lacking in this element." She added during her interview, "I know discussing math and talking about math is most important, but I struggle to find the time to do these things. I would like to give students time to struggle but there just isn't enough time." The male, elementary teacher wrote, "I placed the actions related to wait time and giving students time under least like me. I struggle with finding the balance between the essential material (content) and finding time to allow students to really think about the mathematics concepts. I am strong on getting through the lesson but weak on how to really help students understand."

Discussion

The purpose of the study was not to imply that one classroom teaching practice is more important than another. Rather, the purpose was to have teachers identify the extent to which they believe they enact some practices more than others and to identify the shared factors that emphasize the participants' perceptions of their implementation of high-leverage teaching practices (Ball & Forzani, 2009, p. 497). Indeed, having participants perform the cognitive task of discriminating between teaching practices they believe to be of equal value is what gives the Q sort its power to identify factors that reveal shared perspectives. We interpreted five factors extracted through the Q methodology principal components factor analysis as: Promoting Students' Productivity; Using High-Level Tasks; Promoting Sense-Making and Reasoning; Encouraging Mathematical Representation; and Acknowledging Students in Time.

Teachers associated with the factor *Promoting Students' Productivity* indicated that they engage their students in productive struggle to advance their thinking and help them learn how to persevere (Hiebert & Grouws, 2007). These educators recognize the value in preparing lessons that include planning specific questions to probe the thinking of the students who may struggle or have misconceptions. This type of planning enables the teachers to pose intentional questions aimed at making the students' thinking visible so that the teachers can identify where confusion or gaps in knowledge exist (Herbel-Eisenmann & Cirillo, 2009). Teachers who loaded onto this factor shared the perspective that they find time constraints and the need to follow a pacing guide prevent them from taking the time to have students explain their mathematical reasoning or knowledge. At the elementary level, the teachers stated that their students are either not ready for discussions with other students or that behavior issues prevent them from encouraging students to explain their thinking to one another.

The teachers who loaded on the factor, *Using High-Level Tasks*, recognize the need to consider what their students are doing as they engage in solving mathematical tasks before responding or making in-the-moment decisions (Lampert & Graziani, 2009). These educators plan for and implement tasks to motivate and actively engage their students in reasoning and problem solving but may struggle with finding tasks that meet their students' ability levels. Comments from the participating teachers from high-poverty school districts, who loaded onto this factor, resonate with studies conducted by Boaler (2002) and Silver et al., (2005) that found teachers often struggle to cover the curriculum and that they avoid the implementation of high-level tasks with low-ability level students.

The teachers who loaded on the factor, *Promoting Sense-Making and Reasoning*, engage their students in purposeful discussions of mathematical strategies and ideas. The student discussions often address mistakes and misconceptions to help their students learn and help them plan for possible interventions or next steps. These educators use praise to motivate their students to remain engaged and to persevere when working on mathematical problems. Moreover, these educators feel a sense of accomplishment when students are productively struggling to solve problems. However, some teachers who loaded onto this factor avoid engaging their students in mathematical discussions because of students' behavioral issues or because students see their struggle with math as a weakness due to a lack of mathematical knowledge.

The teachers who loaded on the factor *Encouraging Mathematical Representation* use visual representations to help students understand mathematical concepts, connect the concrete manipulatives to the abstract, and solve problems (Stylianou & Silver, 2004). These teachers shared a belief that visual representations are essential to helping their students learn and understand mathematical concepts. They encourage their students to use visual representations to solve problems and try to connect the visual representations to other representational models. These educators believe that the use of visual models helps their English language learners and struggling learners to visualize and understand the mathematics. Some of the teachers who loaded onto this factor also struggled to promote the use of representations due to time constraints, poor planning, and state-testing pressures.

The teachers who loaded on the factor *Acknowledging Students in Time* tend to struggle with taking time to encourage students to think and make sense of the mathematics through problem solving and investigations (Boaler, 2006). As these teachers plan lessons, they try to plan questions and prompts that will support their students productively and allow their students to engage deeply with the mathematics. These teachers' value encouraging students to solve problems on their own and think for themselves. They try to use questions to scaffold their students' learning so students do not struggle too much. However, some of these teachers find the use of wait time to allow students time to think and make sense of a problem or question is difficult due to pressures to get through the lessons.

The teachers' Q grids were constructed (Papert & Harel, 1991) using a combination of their knowledge and beliefs about students' and their ability to promote students' productive struggle (Borko & Putnam, 1996); knowledge of mathematical content and their use of mathematical tasks and problems (Ball, Thames, & Phelps, 2008); their ability to use students' prior knowledge and understandings to promote discussions and engagements (Hill, Ball, & Schilling, 2008); and their ability to encourage the use of representations and strategies (Ball et al., 2008; Hill, et al., 2008; Shulman, 1987). The qualitative statements reported by the participating teachers provided insight into the different motivations behind the placement of the cards. For example, the participants expressed differences in how they value the engagement of students in meaningful mathematical activities, select and use multiple representations to demonstrate a concept, and the extent to which they explicitly connect the instructional activity to the learning goals for students. Differences such as these align with findings from previous research studies that examined various teachers' mathematics instruction (Ball & Bass, 2003; Hiebert & Morris, 2009; Hill et al., 2008; Jensen, Bartell, & Berk, 2009). The difference in the participants' perceptions may be explained by reasons such as the teachers' mathematical knowledge (Ball & Bass, 2003; Doerr & English, 2006), the teachers' confidence about mathematics (Senger, 1999); the curriculum expectations and implementation (Remillard, 2005); and the composition of the students' ability-levels (Boaler, 2002). Further research on the motivations for these differing perspectives may be needed to help ensure that the teaching practices that most affect student learning occur in every mathematics classroom (Ball & Forzani, 2010).

Our analysis revealed that the participating teachers' motivations also included struggles with the pacing of curriculum due to accountability measures, students' motivation, and classroom management. When teachers do not enact high-leverage teaching practices in their classrooms, mathematics education researchers often find such practices are sacrificed for traditional teaching methods that emphasize the need to "cover" the material (Cobb & Jackson, 2011; Hammerness, 2004; McKinney, Chappell, Berry, & Hickman, 2009; McLaughlin & Talbert, 1993). Studies have affirmed that teachers find the need to follow curriculum pacing guides as well as the limited amount of class time dedicated to mathematics instruction prevents them from implementing high-leverage practices (Hiebert et al., 2003: Remillard, 2005; Resnick & Zurawsky, 2005; Son, 2008; Stein, Grover & Henningsen, 1996). Boaler (2002, 2015) also expressed concern that teachers may not enact high-leverage teaching practices because they believe that the students do not have the ability, they need to complete successfully demanding mathematical tasks. Qualitative statements made by some of the teachers support the reliance on explicit teaching instruction to prepare their students to do well on standardized tests because administrators use test results to assess the performance of students and teachers (Cobb & Jackson, 2011).

Teachers shared the view that the challenges they face with classroom management acknowledges the realities that exist in many classrooms in promoting cognitively demanding tasks with all students (Henningsen & Stein, 1997). Several teachers indicated that their students' inability to work productively in groups or their lack of prior mathematical knowledge often caused confusion or frustration with high-level tasks. This pressured the teachers to reduce the cognitive demand of the task in order to maintain classroom management. This raises the concerns addressed by some researchers (Burris et al., 2008; Pogrow, 1988) that the teachers might be preventing these students from achieving self-confidence in their mathematical ability. Engaging students in cognitively challenging tasks can help them see the benefits of persisting and promote their mathematics self-efficacy.

The Q Sort task engaged teachers in a purposeful reflection on their teaching and provided an opportunity for them to gain self-awareness of their teaching actions (Bengtsson, 1995; Franz, Wilburne, Polly, & Wagstaff, 2017; Osterman & Kottkamp, 1993; Wilburne, Polly, Franz, & Wagstaff, 2018). As they constructed their Q grids. they became aware of the teacher actions they implement more often than others and were enabled to question their reasons for placing some actions as most characteristic of their teaching and some as least characteristic.. Participants' responses to the open-ended questions included: "It was a good reflection of my teaching practices." "This was tough. I learned what I value in my teaching and where I need to grow." "This activity confirms that I have so much room for improvement." By being aware of their teaching actions and reflective of which teacher actions they implement more than others, these participants can improve their effectiveness in the classroom (Cohen & Ball, 1999; Ferraro, 2000).

Implications of the Study

Q Methodology offers researchers a means to study teachers' beliefs, perspectives, feelings, or opinions (Brown, 1980) about their teaching. More importantly, completing a Q sort requires teachers to reflect on their teaching. Identifying mathematics teachers' perspectives on their practices could provide insight into their beliefs about how students learn, based on actions such as how they situate and adapt their work for specific groups of students (Ball & Forzani, 2009), or how they modify the use of mathematical tasks (Hill, Ball, & Schilling, 2008; Stein, Remillard, & Smith, 2007). Q Methodology can provide a powerful approach to promote teachers' reflections on their teaching and to examine shared insights more effectively than can responses to items on a Likert scale. Reflection activities such as the Q sort are essential to the work of teaching and provide insight into the successes and challenges teachers face to implement high-leverage practices (Schön, 1983).

The present study may help researchers, teacher preparation programs, mathematics coaches, and professional development providers to see the value of Q Methodology to identify teachers' perspectives on their teaching and to design professional development materials, services, and recommendations to fit the specific situation (McNeil, Newman, & Steinhauser, 2005). By tailoring services, professional development may be more effective and meaningful to teachers.

Limitations

Although the present findings have the potential to impact the mathematics education of all students, we must acknowledge the study's limitations. There are a limited number of studies in mathematics education that have used Q Methodology. Examples of such studies include examining pre-university students' difficulties with problem solving (Ayop & Tarmizi, 2015), middle school mathematics teachers' mathematical knowledge and self-efficacy following a professional development activity (Stevens, Harris, Aguirre-Munoz, & Cobbs, 2009), and the views of community college students on learning mathematics in terms of their epistemological beliefs (Wheeler & Montgomery, 2009). Thus, it is a relatively new research approach in the field of mathematics education and, as such, researchers may be tempted to view Q Methodology through the lens used in hypothetico-deductive (R-methodological) research (McKeown & Thomas, 2013, p. 70). In Q studies, the purpose is not to estimate a population statistic but to identify and discern individuals' perceptions on a specific topic from the vantage point of self-reference (McKeown & Thomas, 2013, p. 1). "The objective of Q-Methodology is to be able to describe typical representations of different perspectives rather than find the proportion of individuals with specific perspectives" (Akhtar-Danesh, Baumann, & Cordingley, 2008, p. 763). The results of a Q study are not as dependent on the size of the sample as in R-methodology studies. As a result, the findings are specific to the respective study and not generalizable in the usual way

In the present study, the Q sort involves a self-report of the participants' perceptions of their teaching. We did not verify these reports with any records of practice, classroom observations, lesson plans, or student work samples. Future investigations of this line of research could include these data sources and examine the relationship between teachers' report as assessed by the Q sort and other data sources.

In addition, while this study reveals the shared perspectives of the study participants, it may not capture the views of all mathematics educators. Additional perspectives may emerge by conducting Q sorts with teachers from across various locations. However, replication of the study would not necessarily produce similar results.

Conclusion

Q methodology was used in this study to explore the perspectives of mathematics teachers' teaching practices because of its focus on the analysis of the variety of perspectives and not the individual teachers who hold them. As teachers do not share the same view of their teaching, they differ in their perception of the extent to which they implement different classroom practices. The Q-methodology process loads the disparity of views onto factors that represent the commonly shared perspectives. The constructivist approach used in Q methodology provides the basis for re-constructing the teachers' perspectives of their teaching to identify patterns and provide interpretation of the factors. In our study, we identified five factors that a group of mathematics teachers from school districts in three U.S. states commonly held about their perceptions of their teaching. In addition, we used a qualitative analysis of the participants' interviews and open-ended responses to identify their motivations for the extent to which they implement the high-leverage teaching practices that were presented. As research posits, enactment of high-leverage teaching practices in today's mathematics classrooms is essential to support the ambitious teaching needed to help all students achieve success in mathematics. Knowledge of the factors that describe the common perceptions of the participating teachers' high-leverage practices and their motivations for implementing such actions can inform the mathematics education field and define professional-development needs for in-service mathematics teachers.

The present study also shows how the Q sort can be used as a tool for teachers to reflect on their implementation of high-leverage teaching practices. The teachers found the activity allowed them to think about the practices they need to expand upon and those they may need to change. The activity also helped them recognize issues that prevent them from implementing high-leverage practices in their classroom. The intentional focus on one's teaching using a similar Q sort may lead teachers to explore strategies that can promote more equitable and effective teaching practices that are aimed at helping each of their students achieve success with mathematics.

References

Akhtar-Danesh, N. (2018). qfactor: A command for Q-Methodology. *The Stata Journal, 18*(2), 432-446.

Akhtar-Danesh, N., Baumann, A., & Cordingley, L. (2008). Q-Methodology in nursing research: A promising method for the study of subjectivity. *Western Journal of Nursing Research, 30*(6), 759-773.

Armatas, C. A., Venn, T. J., & Watson, A. E. (2014). Applying Q-methodology to select and define attributes for non-marketing variation: A case study from Northwestern Wyoming, United State. *Ecological Economics, 107*(1), 447-456.

Ayop, R. A., & Tarmizi, R. A. (2015). Pre-university students' difficulties in problem solving by using Q-methodology. *Journal of Science and Mathematics Education in Malaysia, 5*(2), 1-11.

Ball, D. L., & Bass, H. (2003). Making mathematics reasonable in school. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A research companion to Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Ball, D. L., & Forzani, F. M. (2009). The work of teachers and the challenges of teacher education. *Journal of Teacher Education, 60*(5), 497-511.

Ball, D. L., & Forzani, F. M. (2010). Teaching skillful teaching. *Educational Leadership, 68*(4), 40-45.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389–407

Bengtsson, J. (1995). What is reflection? On reflection in the teaching profession and teacher education. *Teachers and Teaching: Theory and Practice, 1*(1), 23-32.

Boaler, J. (2002). *Experiencing school mathematics: Revised and expanded edition.* Mahweh, NJ: Lawrence Erlbaum Assoc. Inc.

Boaler, J. (2006). Opening our ideas: How a detracked mathematics approach promoted respect, responsibility and high achievement. *Theory into Practice, 45*(1), 1–11.

Boaler, J. (2015). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages, and innovative teaching.* New York: Jossey-Bass.

Borko, H., & Putnam, R. (1996). Learning to teach. In D. Berliner & R. Calfee (Eds.), *Handbook of educational psychology* (pp. 673-708). New York: Macmillan.

Boston, M. (2012). Assessing instructional quality in mathematics. *The Elementary School Journal*, 113(1), 76–104.

Brown, S. R. (1980). *Political subjectivity: Applications of Q-Methodology in political science*. New Haven, CT: Yale University Press.

Brown, S. R., & Good, J. M. M. (2010). Q-Methodology. In N. J. Salkind (Ed.), *Encyclopedia of research design*. Thousand Oaks, CA: Sage.

Burris, C. C., Wiley, E., Welner, K. G., & Murphy, J. (2008). Accountability, rigor, and detracking: Achievement effects of embracing a challenging curriculum as a universal good for all students. *Teachers College Record, 110*(3), 571–607.

Carlson, J. M., & Hyde, M. S. (2003). *Doing empirical political research*. Boston, MA: Houghton Mifflin.

Clotfelter, C. T., Ladd, H. F., & Vigdor, J. L. (2007). Teacher credentials and student achievement in high school: A cross-subject analysis with student fixed effects. *Economics of Education Review, 26*(6), 673-782.

Cobb, P., & Jackson, K. (2011). Towards an empirically grounded theory of action for improving the quality of mathematics teaching at scale. *Mathematics Teacher Education and Development, 13*(1), 6-33.

Cohen, D. K., & Ball, D. L. (1999). *Instruction, capacity, and improvement*. (CPRE Research Report No. RR—043). Philadelphia: University of Pennsylvania, Consortium for Policy Research in Education.

Creemers, B. P. M., & Kyriakides, L. (2006). Critical analysis of the current approaches to modeling educational effectiveness: The importance of establishing a dynamic model. *School Effectiveness and School Improvement, 17*(3), 347-366.

Cuppen, E., Breukers, S., Hisschemöller, M., & Bergsma, E. (2010). Articulating divergent perspectives: Q Methodology in a participatory integrated assessment on energy options from biomass in the Netherlands. *Institute for Environmental Sciences Working Paper,* Amsterdam, The Netherlands.

Doerr, H. M., & English, L. D. (2006). Middle grade teachers' learning through students' engagement with modeling tasks. *Journal of Mathematics Teacher Education*, *9*(1), 5-32.

DuPlessis, C. (2005). *A theoretical framework of corporate online communication: A marketing public relations (MPR) perspective.* (Doctoral dissertation). Retrieved from UNISA. International Repository. http://uir.unisa.ac.za/handle/10500/2271

Ferraro, J. M. (2000). Reflecting practice and professional development. *ERIC Digest, No*: ED451733.

Franke, M. L., Kazemi, E., & Battey, D. (2007). Mathematics teaching and classroom practice. In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 225-256). Charlotte, NC: Information Age Publishing.

Franz, D. P., Wilburne, J. M., Polly, D., & Wagstaff, D. A. (2017). The teacher action Q-sort: A card-sorting tool for professional learning. *Journal of Mathematics Leadership 18*(2), 3-14.

Fuson, K. C., Kalchman, M., & Bransford, J. D. (2005). Mathematical understanding: An introduction. In M. S. Donovan and J. D. Bransford (Eds.), *How students learn: History, math, and science in the classroom* (pp. 217-256). Washington, DC: National Academy Press.

Grossman, P. (2013). *Leveraging an observation protocol for instructional improvement: Objective and perspectives.* Paper presented at the annual meeting of the American Educational Research Association, San Francisco, CA.

Hammerness, K. (2004). Teaching with vision: How one teacher negotiates the tension between high ideals and standardized testing. *Teacher Education Quarterly, 31*(4), 33-43.

Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education*, *28*(5), 524-549.

Herbel-Eisenmann, B., & Cirillo, M. (Eds.). (2009). *Promoting purposeful discourse*: *Teacher research in mathematics classrooms*. Reston, VA: NCTM.

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371-404). Charlotte, NC: Information Age Publishing.

Hiebert, J., & Morris, A. K. (2009). Building a knowledge base for teacher education: An experience in K-8 mathematics teacher preparation. *Elementary School Journal, 109*(5), 475-490.

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education, 39*(4), 372–400.

Hill, H. C., Charlamabos, C. Y., & Kraft, M. A. (2012). When rater reliability is not enough: Teacher observation systems and a case for the generalizability study. *Educational Researcher, 41*(2), 56-64.

Hufferd-Ackles, K., Fuson, K., & Sherin, M. G. (2004). Describing levels and components of a math-talk learning community. *Journal for Research in Mathematics Education*, *35*(2), 81–116.

Jacobs, V. R., Lamb, L. L., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education, 41*(2), 169-202.

Jamieson, S. (2004). Likert scales: How to (ab)use them. *Med Education, 38*(12): 1217-1218.

Jensen, A., Bartell, T., & Berk, D. (2009). The role of learning goals in building a knowledge base for elementary mathematics teacher education. *Elementary School Journal, 109*(5), 525-536.

Kane, T. J., & Staiger, D. O. (2012). Gathering feedback for teachers: Combining high-quality observations with student surveys and achievement gains. Retrieved from http://www.metporoject.org/downloads/MET_Gathering_Feedback_Practioner_Brief.pdf

Kazdin, A. (1998). *Research design in clinical psychology* (3rd ed.). Boston, MA: Allyn & Bacon.

Kazemi, E., & Stipek, D. (2001). Promoting conceptual thinking in four upper-elementary mathematics classrooms. *The Elementary School Journal, 102*(1), 59-80.

Lambert, M., Boerst, T. A., & Graziani, F. (2011). Organizational resources in the service of school-wide ambitious teaching practice. *Teachers College Record, 113*(7), 1361-1400.

Lampert, M., Franke, M., Kazemi, E., Ghousseini, H., Turrou, A., Beasley, H., & Crowe, K. (2013). Keeping it complex: Using rehearsals to support novice teacher learning of ambitious teaching. *Journal of Teacher Education, 64*(3), 226-243.

Lampert, M., & Graziani, F. (2009). Instructional activities as a tool for teachers' and teacher educators' learning. *Elementary School Journal, 109*(5), 491-509.

Lather, P. (2006). Foucauldian scientificity: Rethinking the nexus of qualitative research and educational policy analysis. *Qualitative Studies in Education*, *19*(6), 783-791.

McGee, J. R., Wang, C., & Polly, D. (2013). Guiding teachers in the use of a standards-based mathematics curriculum: Perceptions and subsequent instructional practices after an intensive professional development program. *School Science and Mathematics*, *113*(1), 16-28.

McKeown, B., & Thomas, D. (1988). *Q methodology*. Newbury Park, CA: Sage.

McKeown, B., & Thomas, D. (2013). *Q methodology,* Second edition. Newbury Park, CA: Sage.

McKinney, S. E., Chappell, S., Berry, R. Q., & Hickman, B. T. (2009). An examination of the instructional practices of mathematics teachers in urban schools. *Preventing School Failure, 53*(4), 278-284.

McLaughlin, M. W., & Talbert, J. E. (1993). *Contexts that matter for teaching and learning.* Stanford, CA: Stanford University Center for Research on the Context of Secondary School Teaching.

McNeil, K., Newman, I., & Steinhauser, J. (2005). *How to be involved in program evaluation: What every administrator needs to know.* Lanham, MD: Scarecrow Education.

Meyer, D. Z., & Avery, L. M. (2008). *Excel as a qualitative analysis tool.* doi:10.1177/1525822X08323985

National Council of Teachers of Mathematics (2014). *Principles to actions: Ensuring mathematical success for all students.* Reston, VA: Author.

Osterman, K. F., & Kottkamp, R. B. (1993). *Reflective practice for teachers.* Newbury Park, CA: Corwin Press.

Papert, S., & Harel, I. (1991). *Constructionism.* Norward, NJ: Ablex Publishing.

Pett, M. A., Lackey, N. R., & Sullivan, J. J. (2003). *Making sense of factor analysis: The use of factor analysis for instrument development in health care research.* Thousand Oaks, CA: Sage Publications.

Pogrow, S. A. (1988). *Evaluation of the Chapter 1 higher order thinking skills project.* Report prepared for the Program Effectiveness Panel of the National Diffusion Network. Tucson, University of Arizona, College of Education.

Ramlo, S. E., & Newman, I. (2011). Q-methodology and its position in the mixed-methods continuum. *Operant Subjectivity: The International Journal of Q-Methodology, 34*(3), 172-191.

Remillard, J. T. (2005). Examining key concepts in research on teachers' use of mathematics curricula. *Review of Educational Research, 75*(2), 211-246.

Resnick, L., & Zurawsky, C. (2005). Getting back on course: Fixing standards-based reform and accountability. *American Educator, 29*(1), 8-46.

Rivkin, S. G., Hanushek, E. A., & Kain, J. F. (2005). Teachers, schools, and academic achievement. *Econometrica, 73*(2), 417-458.

Saldaña, J. (2013). *The coding manual for qualitative research*. Second edition. Washington, DC: Sage Publications.

Schön, D. A. (1983). *The reflective practitioner: How professionals think in action*, New York: Basic Books.

Senger, E. S. (1999). Reflective reform in mathematics: The recursive nature of teacher change. *Educational Studies in Mathematics, 37*(3), 199-221.

Shulman, L. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57(*1), 1-21.

Silver, E. A., Mills, V., Castro, A., Ghousseini, H., & Stylianides, G. J. (2005). *Complementary approaches to mathematics teacher professional development: Integrating case analysis and lesson study in the BI-FOCAL project*. Paper presented at the 15th ICMI study: The professional education and development of teachers of mathematics.

Sleep, L. (2012). The work of steering instruction toward the mathematical point: A decomposition of teaching practice. *American Educational Research Journal, 49*(5), 935–970. https://doi.org/10.3102/0002831212448095

Sleep, L., & Boerst, T. A. (2012). Preparing beginning teachers to elicit and interpret students' mathematical thinking. *Teaching and Teacher Education, 28*(7), 1038-1048.

Smith, M. S., & Stein, M. K. (2011). *5 Practices for orchestrating productive mathematics discussions.* Reston, VA: National Council of Teachers of Mathematics.

Son, J.-W. (2008). *Elementary teachers' mathematics textbook use in terms of cognitive demands and influential factors: A mixed method study* (Doctoral dissertation). Retrieved from ProQuest Dissertations and Theses database. (UMI No. 3332018)

Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal, 33*(2), 455–488. doi:10.3102/00028312033002455

Stein, M. K., Remillard, J. T., & Smith, M. S. (2007). How curriculum influences student learning. In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 319–369). Charlotte, NC: Information Age Publishing.

Stenner, P. (2008). A. N. Whitehead and subjectivity. *Subjectivity, 22*(1), 90-109.

Stenner, P., Cooper, D., & Skevington, S. (2003). Putting the Q into quality of life: The identification of subjective constructions of health-related quality of life using Q methodology. *Social Science and Medicine, 57*(11), 2161-2172.

Stephenson, W. (1935). Technique of factor analysis. *Nature*, *136*, 297.

Stephenson, W. (1953). *The study of behavior:* Q-technique and its methodology. Chicago: University of Chicago Press.

Stevens, T., Harris, G., Aguirre-Munoz, Z., & Cobbs, L. (2009). A case study approach to increasing teachers' mathematics knowledge for teaching and strategies for building students' math self-efficacy. *International Journal of Math Education in Science and Technology, 40(*7), 903-914.

Stylianou, D. A., & Silver, E. A. (2004). The role of visual representations in advanced mathematical problem solving: An examination of expert-novice similarities and differences. *Mathematical Thinking and Learning, 6*(4), 353-387.

Subba, L., Bru E., & Thorsen, A.A. (2017). Primary and lower secondary school teachers' perceptions of how they manage to support students with learning difficulties in inclusive classrooms. *Operant Subjectivity: The International Journal of Q Methodology, 39*(3-4), 25-45.

Van Eeten, M. (1998). *Dialogues of the deaf: Defining new agendas for environmental deadlocks*. Delft: Eburon.

Valenta, A. L., & Wigger, U. (1997). Q-methodology: Definition and application in health care informatics. *Journal of the American Medical Informatics Association, 4*(6), 501-510.

Van Exel, J., & de Graaf, G. (2005). Q methodology: A sneak preview. Retrieved from: https://qmethodblog.files.wordpress.com/2016/01/qmethodologyasneakpreviewreferenceupdate.pdf

Watts, S., & Stenner, P. (2012). *Doing Q methodological research: Theory, method, and interpretation.* London: Sage Publications.

Wheeler, D. L., & Montgomery, D. (2009). Community college students' views on learning mathematics in terms of their epistemological beliefs: A Q-method study. *Educational Studies in Mathematics, 72*(3), 289-306. doi: 10:1007/s10649-009-9192-2

Wilburne, J. M., Polly, D., Franz, D. P., & Wagstaff, D. A. (2018). Mathematics teachers' perceptions of their teaching actions: A Q-methodology study. *School Science and Mathematics, 118*(6), 232-243.

Wilhelm, A. G., & Kim, S. (2015). Generalizing from observations of mathematics teachers' instructional practice using the instructional quality assessment. *Journal for Research in Mathematics Education, 46*(3), 270-279.

Yoshizawaa, G., Iwase, M., Okumoto, M., Tahara, K., & Takahashie, S. (2016). Q Workshop: An application of Q Methodology for visualizing, deliberating and learning contrasting perspectives. *International Journal of Environmental & Science Education*, *11*(13), 6277-6302.

Zabala, A. (2014). qmethod: A package to explore human perspectives using Q Methodology. *The R Journal, 6*(2), 163-173.

Zabala, A. (2015). *qmethod: An R package to analyse Q method data. R package version 1.3.1.* University of Cambridge, Cambridge, UK. Retrieved from http://CRAN.R-project.org/package=qmethod

Appendix A
Mathematical Teacher Actions by Statement Number Identification and Teaching Practice

Statement Number	*Teacher Action*	*Principles to Actions Teaching Practice**
1	Praise students for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems.	VII
2	Ask students to discuss and explain why the procedures that they are using work to solve particular problems.	VI
3	Pose tasks on a regular basis that require a high level of cognitive demand.	II
4	Introduce forms of representations that can be useful to students.	III
5	Engage students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations.	IV
6	Establish clear goals that articulate the mathematics students are learning as a result of instruction in a lesson, over a series of lessons, or throughout a unit.	I
7	Discuss and refer to the mathematical purpose and goal of a lesson during instruction to ensure that students understand how the current work contributes to their learning.	I
8	Allow sufficient wait time so that more students can formulate and offer responses.	V

Statement Number	*Teacher Action*	*Principles to Actions Teaching Practice**
9	Ask students to make math drawings or use other visual supports to explain and justify their reasoning.	III
10	Make in-the-moment decisions on how to respond to students with questions and prompts that prove, scaffold, and extend.	VIII
11	Support students in exploring tasks without taking over student thinking.	II
12	Focus students' attention on the structure or essential features of mathematical ideas that appear, regardless of the representation.	III
13	Design ways to elicit and assess students' abilities to use representations meaningfully to solve problems.	III
14	Provide students with opportunities for distributed practice of procedures.	VI
15	Give students time to struggle with tasks and ask questions that scaffold students' thinking without stepping in to do the work for them.	VII
16	Interpret student thinking to assess mathematical understanding, reasoning, and methods.	VIII
17	Ask intentional questions that make the mathematics more visible and accessible for student examination and discussion.	V
18	Provide students with opportunities to use their own reasoning strategies and methods for solving problems.	VI
19	Advance student understanding by asking questions that build on, but do not take over or funnel, student thinking.	V
20	Identify what counts as evidence of student progress toward mathematics learning goals.	VIII
21	Anticipate what students might struggle with during a lesson and be prepared to support them productively through the struggle.	VII
22	Elicit and gather evidence of student understanding at strategic points during instruction.	VIII

Statement Number	*Teacher Action*	*Principles to Actions Teaching Practice**
23	Use the mathematics goals to guide lesson planning and reflection and to make in-the-moment decisions during instruction.	I
24	Select tasks that allow students to decide which representations to use in making sense of the problems.	III
25	Help students realize that confusion and errors are a natural part of learning, by facilitating discussions on mistakes, misconceptions, and struggles.	VII
26	Encourage students to use varied approaches and strategies to make sense of and solve tasks.	II
27	Identify how the goals fit within a mathematics learning progression.	I
28	Motivate students' learning of mathematics through opportunities for exploring and solving problems that build on and extend their current mathematical understanding.	II
29	Ensure progress toward mathematical goals by making explicit connections to student approaches and reasoning.	IV
30	Connect student-generated strategies and methods to more efficient procedures as appropriate.	VI
31	Make certain to ask questions that go beyond gathering information to probe thinking and require explanation and justification.	V
32	Select tasks that provide multiple entry points through the use of varied tools and representations.	II
33	Allocate substantial instructional time for students to use, discuss, and make connections among representations.	III
34	Facilitate discourse among students by positioning them as authors of ideas, who explain, and defend their approaches.	IV
35	Use visual models to support students' understanding of general methods.	VI
36	Reflect on evidence of student learning to inform the planning of next instructional steps.	VIII

Statement Number	*Teacher Action*	*Principles to Actions Teaching Practice**
37	Select and sequence student approaches and solution strategies for whole-class analysis and discussion.	IV

*(NCTM, 2014)

Appendix B
Identification of the "Principles to Actions" (NCTM, 2014) Teaching Practices and Q-Statements with Factor Alignment

Practice Number	Teaching Practice	Q-Statements & Factor Alignment
I	Establish mathematical goals to focus learning	● Establishing clear goals that articulate the mathematics that students are learning as a result of instruction in a lesson, over a series of lessons, or throughout a unit. ● Identifying how the goals fit within a mathematical learning progression. ● Discussing and referring to the mathematical purpose and goal of a lesson during instruction to ensure that students understand how the current work contributes to their learning. ● Using mathematical goals to guide lesson planning and reflection and to make in-the-moment decisions during instruction. [Factor 4].
II	Implement tasks that promote reasoning and problem solving	● Motivating students' learning of mathematics through opportunities for exploring and solving problems that build on and extend their current mathematical understanding. [Factor 3]. ● Selecting tasks that provide multiple entry points through the use of varied tools and representations. [Factor 2]. ● Posing tasks on a regular basis that require a high level of cognitive demand. [Factor 2]. ● Supporting students in exploring tasks without taking over student thinking. ● Encouraging students to use varied approaches and strategies to make sense and solve tasks. [Factor 4].

III	Use and connect mathematical representations	● Selecting tasks that allow students to decide which representation to use in making sense of the problems. [Factor 2]. ● Allocating substantial instructional time for students to use, discuss, and make connections among representations. ● Introducing forms of representations that can be useful to students. ● Asking students to make math drawings or use other visual supports to explain and justify their reasoning. [Factor 4]. ● Focusing students' attention on the structure or essential features of mathematical ideas that appear, regardless of the representation. ● Designing ways to elicit and assess students' abilities to use representations meaningfully to solve problems.
IV	Facilitate meaningful mathematical discussions	● Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations. [Factor 3]. ● Selecting and sequencing student approaches and solution strategies for whole-class analysis and discussion. ● Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches. [Factor 1]. ● Ensuring progress toward mathematical goals by making explicit connections to students approaches and reasoning.
V	Pose purposeful questions	● Advancing student understanding by asking questions that build on, but do not take over or funnel, student thinking. [Factor 5]. ● Making certain to ask questions that go beyond gathering information to probing thinking and requiring explanation and justification. [Factor 3]. ● Asking intentional questions that make the mathematics more visible and accessible for student examination and discussion. [Factor 1]. ● Allowing sufficient wait time so that more students can formulate and offer responses. [Factor 5].

VI	Build procedural fluency from conceptual understanding	● Providing students with opportunities to use their own reasoning strategies and methods for solving problems. ● Asking students to discuss and explain why the procedures that they are using work to solve particular problems. [Factor 1]. ● Connecting student-generated strategies and methods to more efficient procedures as appropriate. ● Using visual models to support students' understanding of general methods. [Factor 4]. ● Providing students with opportunities for distributed practice of procedures.
VII	Support productive struggle in learning mathematics	● Anticipating what students might struggle with during a lesson and being prepared to support them productively through the struggle. [Factor 1]. ● Giving students time to struggle with tasks, and asking questions that scaffold students' thinking without stepping in to do the work for them. [Factor 5]. ● Helping students realize that confusion and errors are a natural part of learning, by facilitating discussions on mistakes, misconceptions, and struggles. [Factor 4]. ● Praising students for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems. [Factor 3].
VIII	Elicit and use evidence of student thinking	● Identifying what counts as evidence of student progress toward mathematics learning goals. ● Eliciting and gathering evidence of student understanding at strategic points during instruction. ● Interpreting student thinking to assess mathematical understanding, reasoning, and methods. ● Making in-the-moment decisions on how to respond to students with questions and prompts that probe, scaffold, and extend. [Factor 5]. ● Reflecting on evidence of student learning to inform the planning of next instructional steps.