RESEARCH ARTICLE

Variable and equality sign misconceptions in K-12 algebra

PATRICK FRIDAY OBOT and MANDANA DELAVARI

ABSTRACT

Algebra forms the foundation for higher mathematics and is a key component of K–12 education. Most students struggle to conceptualize and apply algebraic principles, which leads to misconceptions that hinder their mathematical progress. This conceptual study reviews algebraic misconceptions about variables and equality signs and presents potential interventions aimed at supporting students in developing accurate conceptual understanding. This conceptual study underscores the need for continued innovative strategies in mathematics education to support learners in achieving algebraic proficiency.

KEYWORDS

Misconceptions, algebraic misconceptions, variables, equality sign

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Algebraic misconceptions, particularly those related to variables and the equal sign, have profound educational implications that extend across K–12 mathematics education. Persistent errors rooted in these misconceptions frequently manifest across grade levels, evolving into significant barriers to algebraic proficiency. These misconceptions shape how students interact with abstract concepts and approach algebraic problem-solving, which can hinder students' achievement in algebra. Persistent errors and failures stemming from conceptual misunderstandings can lead to frustration and disengagement. Many students grapple with understanding critical concepts in algebra, resulting in pervasive misconceptions that hinder students' confidence in learning algebra. This is particularly concerning during the crucial shift from arithmetic to algebraic content.

Teachers often face challenges in identifying the root causes of students' errors, as misconceptions can manifest in ways that appear procedural rather than conceptual. For instance, a student who incorrectly simplifies $2x + 3x = 5$ might not necessarily lack procedural knowledge but might also misunderstand the concept of combining like terms. Recognizing and addressing algebraic misconceptions is crucial for fostering mathematics learning that supports conceptual understanding and prepares all learners for success.

Aim and Scope

This conceptual study aims to explore algebraic misconceptions surrounding variables and the equal sign within K–12 mathematics education. It presents interventions from literature that remediate algebraic misconceptions and foster deeper conceptual understanding. Furthermore, it aims to inform curriculum design, enhance instructional strategies, and contribute to systemic improvements in mathematics education.

Methodology

This conceptual study employed a systematic and literature review methodology to synthesize findings on algebraic misconceptions about interpreting variables and the equal sign in K–12 education. The researchers identified relevant studies through major academic databases using specific keywords such as "algebra," "algebraic variables," "equality sign," "algebra misconceptions," and "errors in algebra." Only empirical studies focusing on K–12 algebra misconceptions or intervention and strategies were included. The identified articles covered aspects of algebraic misconceptions about interpreting variables and the equal sign. This methodology ensures a holistic understanding of the problem, offering practical insights for educators, curriculum designers, and policymakers committed to improving algebraic learning outcomes.

Common Algebraic Misconceptions

Algebra is a cornerstone of mathematical reasoning and problem-solving, yet it remains challenging for many students. Misconceptions in algebra are not just simple mistakes but reflect underlying misunderstandings of mathematical concepts. The instructional approaches commonly used in traditional classrooms often intensify the challenges associated with learning algebra, which learners encounter from early grades, and it usually evolves throughout their educational journey if not properly addressed. Literature has revealed a wide range of misconceptions in algebra and noted the need to understand and address them (Booth et al., 2017; Brodie, 2014; Bush, 2013; Cholily et al., 2020; Egodawatte, 2011; Enu & Ngcobo-Ndlovu, 2020; Luneta & Makonye, 2010; McNeil, 2014; Muchoko, 2019; Pournara et al., 2016; Stemele, 2024; Welder, 2012).

Some studies focus on misconceptions about specific grade levels, and limited research has attempted to argue if the misconceptions persist across K–12 education or are only prominent in a particular grade level. Stephens (2021) highlights students' challenges in using variable notation to represent arithmetic properties, functional relationships, and related unknown quantities. Fitria et al. (2023) noted that the most prominent misconception among year 8 students in algebra is their understanding of variables. Other commonly studied algebraic misconceptions include equality, variables, negativity, order of operations, fractions, and functions (Booth, 2017).

Misconceptions often present themselves in diverse forms; for instance, several prevalent error categories among grade 9 students are noted to include errors in problem-solving, conjoining, and cancellation, which reflect gaps in both procedural proficiency and conceptual understanding (Mathaba, 2024). Several instructional interventions have been documented in the literature to aid the understanding of algebra (Chan et al., 2022; Bajwa, 2019; Rakes, 2010; Hawthorne, 2023; Star et al., 2015; Stephens et al., 2022). Despite these interventions and methods of instruction in algebra, misconceptions persist. This conceptual study reviews two algebra misconceptions about variables and the equal sign and provides research-based recommendations for teachers to adapt their instruction and proactively address these misconceptions.

Misconceptions About Variables

Children as young as 6 can use variable notation meaningfully to express relationships between co-varying quantities (Brizuela et al., 2013). Learners typically struggle with the abstract nature of variables when first encountering them in mathematics. Rather than recognizing variables as flexible symbols that can represent any number, they tend to assign fixed, concrete meanings to them. This concrete interpretation makes it harder for students to work effectively with variables in equations and understand their broader mathematical applications (Obot, 2023). As students progress academically, misconceptions dealing with the use of variables can be carried over to the next academic year, thereby becoming a barrier to their success in algebra. Rather than random mistakes, these errors often reflect specific stages in mathematical understanding and highlight particular conceptual hurdles that learners face during their development.

Using letters as variables in algebra introduces challenges that can lead to misconceptions. Although, in early grades, students' grasp of variables can be quite rudimentary, variables have long been noted to be misconstrued as specific objects or labels (Asquith et al., 2007; Clement, 1982; MacGregor & Stacey, 2007; Stacey & MacGregor, 1997; Usiskin, 1988). This struggle with the abstract nature of variables could be due to their prior mathematical experiences and thinking errors. This misconception can also stem from insufficient scaffolding of the cognitive processes involved in learning algebra. When students miss the necessary experiences and structured opportunities to bridge this gap, it usually hinders their ability to understand variables as general symbols representing unknown or varying quantities (HR, 2023; Kieran, 2006; Knuth & Alibali, 2005; McNeil et al., 2010).

Sahin (2011) reveals different misconceptions and mistakes of elementary school students about the concept of 'variable' including overlooking the variables; not being able to find the connection between the verbal expressions and the variables; reducing the variables to constants; attributing digits to the variable in multiplication; confusing the "x" unknown with the multiplication sign; and not using parenthesis. When working with variables in algebra, students might completely ignore the variable, assign it a random value, or think of it as representing a physical object rather than a number. Some students also associate letters with their place in the alphabet or treat them as specific fixed numbers. When students write something like $12p + 20h = 32$, they sometimes ignore what p and h represent and just add the numbers they can see. This shows they don't understand that variables represent unknown values (Moss et al., 2018).

Another issue comes from how we often use first letters to represent mathematical concepts (like “h” for height). This can make students think letters always stand for words rather than numbers. Some students might simplify 2x + 5 to just 2 + 5, leaving out the variable entirely (Brizuela et al., 2015). Others might see “2b” and think it means “2 boys” instead of “2 times b.” Sometimes, when students see expressions like 2x + 3y, they incorrectly combine them into 5xy, incorrectly applying addition rules they learned earlier. Changing how we teach basic arithmetic could help students better understand mathematical equivalence and move away from these incorrect patterns of thinking (McNeil, 2015).

Misconceptions About Equality

One major obstacle to learning early algebra in K–12 education is students' misunderstanding of what the equal sign means, which can seriously hinder their mathematical development. Before formal instruction, Blanton (2018) revealed that young children usually hold an operational view of the equal sign that can persist throughout instruction. Students typically view it as a directive to “compute” rather than a symbol of equivalence. Vermeulen (2017) revealed that students lack a well-developed relational conception of the equal sign, which limits their ability to describe the meaning of the equal sign correctly. Sumpter (2022) examined grade 7 students' understanding of the equal sign, revealing that many perceive it operationally as a prompt to perform a calculation rather than relationally, indicating equivalence between two expressions. Moreover, younger students often assume that the number directly following the equal sign represents the answer (Alibali, 1999; Falkner et al., 1999; Li et al., 2008).

This revelation of how students initially express an operational view of the equal sign, according to Sumpter (2022), supports other findings that it can be difficult for students to provide a relational definition spontaneously. While this type of arithmetic thinking may be sufficient during the early years, it causes significant problems once students are asked to think algebraically (Booth & Koedinger, 2008; Knuth et al., 2006). This deficiency in equal sign understanding and interpretation can affect early algebra learning and performance (Byrd et al., 2015; Jacobs et al., 2007; Kieran, 1992; Knuth et al., 2006; Lee et al., 2020). The correct understanding of the meaning of the equal sign is imperative to manipulate and solve algebraic equations (Carpenter et al., 2003; Kieran, 1981). Xu et al. (2023) noted that when students grasp that the equal sign signifies an equivalence relationship; they can leverage this understanding to create adaptable strategies for solving nonstandard equivalence problems, which supports the view that the development of equivalence knowledge involves reasoning skills (Miller Singley & Bunge, 2014; Morsanyi et al., 2018).

Addressing Student Algebraic Misconceptions

Literature reveals integrated and multi-faceted approaches to address algebraic misconceptions. Interventions targeting variables and the equal sign share common themes, including the importance of early exposure, the use of visual and interactive tools, and the integration of real-world contexts. Various interventions exist that focus on enhancing students' conceptual understanding of algebra (Hiebert et al., 1996; Ma, 2010; Rittle-Johnson & Star, 2007; Xin et al., 2008). When integrating variables into problem-solving contexts, students develop a foundational understanding of their abstract nature. Xu et al. (2023) expanded on this by highlighting the importance of visual tools and manipulatives, such as dynamic software, which allow learners to interact with variables in real time. These tools provide immediate feedback, enabling students to visualize and correct errors in their reasoning.

One particularly successful method for teaching algebra combines improving students' understanding of concepts with developing their practical problem-solving abilities built on three key learning principles (Blanton et al., 2018). First, it uses self-explanation, where students actively explain concepts to themselves during their learning process (Chi, 2013). Second, it employs worked examples—showing students step-by-step solutions to problems before they attempt similar ones independently. Third, it creates cognitive dissonance by presenting incorrect solutions for students to analyze. When students examine these errors, they learn to identify what makes a solution wrong and become less likely to make similar mistakes themselves (Ohlsson, 1996; Siegler, 2002). This process helps students recognize and fix their own misunderstandings.

Fitria and Susanto (2023) demonstrated that tasks designed to progressively build on students' prior knowledge of arithmetic and algebra significantly enhance their ability to generalize patterns using variables. This approach aligns with HR and Parta's (2023) findings, which underscore the necessity of cognitive conflict strategies. Educators can facilitate deeper conceptual change by presenting students with tasks that target existing misconceptions.

Real-world applications have emerged as a powerful strategy for enhancing engagement and comprehension. For instance, algebraic expressions can be linked to everyday scenarios, such as calculating costs or analyzing trends, which helps students see variables as dynamic and versatile (Chan et al., 2022). By contextualizing algebraic concepts within practical scenarios, educators can make abstract ideas more accessible and relatable. For example, using a *pan balance scale* as a concrete representation to support understanding of equivalence has been reported to be effective (Hiebert & Carpenter, 1992; Magruder & Mohr-Schroeder, 2013). Similarly, Matthews and Fuchs (2020) argued that real-world contexts make variables more relatable, fostering both engagement and a deeper conceptual grasp. This approach also aligns with the findings of Fitria and Susanto (2023), who emphasized the importance of meaningful connections between mathematical concepts and students' lived experiences. Additionally, using interdisciplinary projects that combine algebra with science or economics can broaden students' appreciation of variables and equivalence (HR & Parta, 2023).

Teachers' understanding of algebraic concepts significantly impacts their ability to address misconceptions. Effective professional development programs should: focus on teaching strategies that prioritize conceptual understanding over procedural fluency (Jacobs et al., 2007); train teachers to identify and address common algebraic misconceptions using diagnostic assessments (Fitria & Susanto, 2023); encourage teachers to reflect on their instructional approaches and adapt them based on student feedback (Stephens et al., 2021); and facilitate peer collaboration where teachers share best practices and collectively solve instructional challenges (Star et al., 2015).

Studies underscore the significance of early and continuous interventions. Introducing abstract concepts like variables and equivalence through patterns and functional relationships at an early stage lays a strong foundation for advanced algebraic reasoning (Blanton et al., 2015; Brizuela et al., 2013). This approach prevents misconceptions from becoming entrenched and ensures a smoother transition to higher-level mathematics. Additionally, early interventions should involve explicit discussions on the symbolic nature of algebra to prevent procedural habits from overshadowing conceptual understanding.

Blanton et al. (2018) explored the impact of introducing non-standard equations early in mathematical instruction. By presenting equations in formats that deviate from conventional layouts such as 7 = 3 + 4, students are prompted to reconceptualize the equal sign as a symbol of equivalence rather than computation which is in line with several studies (Falkner et al., 1999; Kieran, 1981; Knuth et al., 2005; Rittle-Johnson & Alibali, 1999). This finding is supported by Sumpter and Löwenhielm (2022), who demonstrated that engaging students in discussions about balance and using physical manipulatives like balance scales significantly improves their relational reasoning. Algebra tiles manipulative in solving linear equations in one variable have the potential to improve performance in mathematics (Chaurasia, 2019; Salifu, 2022).

Technology has also emerged as a critical tool in addressing algebraic misconceptions. Xu et al. (2023) highlighted the role of interactive simulations in helping students visualize equivalence. These digital tools enhance comprehension and provide individualized learning opportunities, catering to diverse student needs. Furthermore, Matthews and Fuchs (2020) revealed the importance of integrating games and digital platforms that require students to solve equivalence problems, promoting both relational reasoning and enjoyment. The role of technology has been particularly prominent in recent interventions. Tools such as dynamic software and simulations facilitate visualization and also enable personalized learning experiences (Chan et al., 2022). These technologies empower students to experiment with mathematical concepts, fostering a deeper and more intuitive understanding. Furthermore, digital platforms often include adaptive features, allowing for tailored instruction based on individual student needs (Matthews & Fuchs, 2020). Leveraging dynamic software tools like GeoGebra to visualize relationships involving variables could enable students to manipulate variables and observe their effects on equations in real time.

Professional development for teachers has been identified as a cornerstone of effective intervention. Literature emphasized the importance of equipping educators with diagnostic tools to identify and address misconceptions about the equal sign (Stephens et al., 2022; HR & Parta, 2023). Training programs focusing on conceptual teaching and reflective practices enable teachers to adapt their instruction based on student feedback, fostering a deeper understanding of equivalence. Similarly, Jacobs et al. (2007) emphasized collaborative professional development sessions where teachers analyze student work to pinpoint and address common errors. By empowering educators with the tools and strategies needed to address misconceptions, these programs ensure that interventions are both effective and sustainable.

Educational Implications

This conceptual study underscores the importance of early and sustained interventions to address algebraic misconceptions. Curricula should introduce algebraic reasoning in elementary grades using age-appropriate tasks, design tasks that gradually build students' understanding of variables and equivalence, and embed assessments that track students' conceptual growth and provide actionable insights for instruction. Such design can help identify and address misconceptions before they become deeply ingrained. However, no instructional design can fully eliminate the emergence of misconceptions. Teachers and curriculum are inseparable—teachers are the bridge through which planned curriculum becomes enacted in the classroom. Therefore, to respond effectively, we must support our teachers—whose instructional choices and pedagogical approaches directly shape students' conceptual development and reasoning. Professional development programs must include intentional opportunities to inform teachers about common algebraic misconceptions—such as those explored in this study, including misunderstandings of variables and the equal sign—and equip them with research-based strategies to adapt their instruction accordingly. Through this dual commitment to thoughtful curriculum and instructional design, we can support students' algebraic thinking more effectively and meaningfully.

Conclusion

This study identifies several misconceptions from the literature to include challenges in using variable notation to represent arithmetic properties, functional relationships, and related unknown quantities. This supports Fitria et al. (2023) that the most prominent misconception among year 8 students in algebra is their understanding of variables. This study also revealed that other commonly studied algebraic misconceptions include equality, variables, negativity, order of operations, fractions, and functions (Booth, 2017; Stephens, 2021). Understanding the progression of algebraic misconceptions across K–12 is essential for improving mathematics education. Educators can implement targeted strategies to support students' development by identifying common errors and their cognitive origins.

Addressing algebraic misconceptions requires a holistic approach that combines early intervention, targeted instruction, diagnostic assessments, innovative teaching tools, and professional development. Research highlights the importance of fostering a relational understanding of variables and the equal sign through strategies that are interactive, contextual, and diagnostic. By fostering a deeper understanding of these concepts, educators can help students overcome barriers to algebraic reasoning. Implementing evidence-based interventions, leveraging technological innovations, and prioritizing professional development, educators can significantly enhance students' algebraic proficiency. These efforts ensure that students develop a robust foundation in algebra, empowering them to excel in mathematics and beyond.

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