Upconing Of A Salt-Water/Fresh Water Interface Below A Pumping Well

By

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PROJECT SUMMARY

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Introduction

This report presents an analytical solution for the upconing of a abrupt salt-water/fresh-water interface below a pumping well. Dispersion phenomenon arising from the displacement of a moving interface, or a finite transition zone between the invading and displaced fluids, can be superimposed on the analytical solution for the position of an abrupt interface. An interactive FORTRAN computer code has been developed which enables the user to modify parameters and to control the computational sequence. This interactive approach enables the user to gain insight into the effects of geometry and physical properties on the rate and extent of upconing and salinization of a well.

Mathematical Development

McWhorter (1972) presented the equations which describe the flow in saturated aquifers which are underlain by a zone of saline water and pointed out the difficulties in obtaining solutions to these problems. The complexity of the flow phenomenon has led many investigators to idealize the system as a fresh-water zone separated from an underlying salt-water zone by a sharp interface. In other words, the two fluids are assumed to be immiscible.

Schmorak and Mercado (1969) followed this approach and accounted for the mixing of the two fluids by superimposing the effects of dispersion on the transient solution for the position of an abrupt interface.

For the case of upconing beneath a pumping well partially penetrating a relatively thick confined aquifer as shown in Figure 1, Schmorak and Mercado (1969) presented Bear and Dagan's solution for the position of the interface as a function of time and radial distance from the pumping well as

$$X(r,t) = \frac{Q}{2\pi(\Delta\rho/\rho)K_x d} \quad \frac{1}{(1 + R^2)^{1/2}} - \frac{1}{((1 + \tau)^2 + R^2)^{1/2}} \qquad (1)$$

where R and τ are dimensionless distance and time parameters defined by

$$R = \frac{r}{d} \left(\frac{K_z}{K_x}\right)^{1/2} \qquad (2)$$

and

$$\tau = \frac{(\Delta\rho/\rho)K_z}{2\theta d} t \qquad (3)$$

Other notations are defined as follows (also refer to Figure 1):

- d = distance from the bottom of the well to the initial interface elevation (L)
- K_x, K_z = horizontal and vertical permeabilities, respectively (L/t)
- Q = well pumping rate (L^3/t)
- r = radial distance from well axis (L)
- t = time elapsed since start of pumping (t)
- X = rise of the interface above its initial position (L)

$\Delta\rho/\rho$ = dimensionless density difference between the two fluids, $(\rho_s - \rho_f)/\rho_f$

θ = porosity of the aquifer.

Application of the method of small pertubations restricts changes in the interface elevation to relatively small values. In terms of the physical problem, this restriction implies $d \ll l_f$ and $d \ll l_s$. Although the governing differential equations have been formulated for a confined aquifer, the results can be applied to unconfined systems if the drawdown is negligible compared to the saturated thickness of the fresh-water zone.

The upconing process as treated above assumes that the two fluids are immiscible and that the interface between them is abrupt. Actually, the interface is diffuse and a transition zone exists between the two fluids in which the concentration varies from the concentration in one fluid to the concentration in the other fluid over a finite distance. This transition zone is related to dispersion processes which alter the concentration profile across the moving interface.

Bear and Todd (1960) approximated the concentration profile as a function of position, X; the "interface" position, $\overline{X}$; the equivalent total distance the interface is displaced, $|\overline{\Delta x}|$, independent of direction; and the dispersivity, D_m. The concentration distribution is given by

$$\varepsilon(X) = \frac{1}{2} \text{ erfc } \frac{X-\overline{X}}{(2\sigma_0{}^2 + 4D_m|\overline{\Delta X}|^{1/2}} \tag{4}$$

where $2\sigma_0$ is the initial width of the transition zone.

Superposition of Dispersion on the Upconing of an Abrupt Interface

The position of the interface as a function of time and radial distance from the well is evaluated using Equation 1, which assumes an abrupt interface between the two fluids. This elevation is assumed to correspond to $X|_{\varepsilon=0.5}$, or the mean of the concentration distribution across the transition zone. The only difficulties in this approach occur for $\varepsilon(X) = 0.0$ and $\varepsilon(X) = 1.0$. Since

$$\varepsilon(X) = 0 \text{ for } X \to \infty \tag{5}$$

and

$$\varepsilon(X) = 1.0 \text{ for } X \to 0 \tag{6}$$

the transition zone would have an infinite width in theory. To overcome this physical impossibility, the width of the transition zone is arbitrarily set at five standard deviations. This range includes approximately 99 percent of the area under the concentration distribution curve.

Concentration in Pumped Water

The increase in concentration, or salinization, of pumped water is probably due to the intrusion of invading fluid above the critical depth. The linear approximation for the interface elevation is limited to elevations below the critical elevation and the dispersion concept should be limited to the zone below the critical depth. The complex mixing and flow phenomena above the critical depth, near the well screen, and within the well pipe are approximated expirically using the approach followed by Schmorak and Mercado (1969).

The average dimensionless concentration of the transition zone above the critical rise, $\overline{\varepsilon}(X > X_{cr})$, is approximated as one-half the concentration at the critical depth, or

$$\varepsilon(X > X_{cr}) = 0.5\ \varepsilon(X_{cr}) \tag{7}$$

The concentration in the pumped water, ε_W, is determined from dilution of the average transition-zone concentration above the critical depth with displaced fluid, or

$$\varepsilon_W = \phi\ \overline{\varepsilon}(X > X_{cr}) \tag{8}$$

where ϕ is an interception coefficient, or the fraction of transition zone fluid in the total volume pumped.

Computer Program

Two types of upconing problems are considered. The first involves the description of the expected interface elevation and the salinity of the pumped water as a function of time for a given pumping rate. The second problem addresses the maximum rate at which a well can be pumped without exceeding a specified salinity in the pumped water. Both types of problems are included in an interactive computer code. Data are required under two modes of operation - "Basic Input Data" and "Edit".

Basic input data are required to initiate a new problem using the UPCONE program. The data entries include the problem title, the physical properties of the aquifer and the two fluids, and the geometry of the system. The user is prompted for the required data through a series of input commands.

Once the basic input data have been entered, the problem as currently defined is listed and the program enters the "Edit" mode. The edit commands listed in Table 1 can be used to redefine the problem, execute elevation or pumping rate calculations, or terminate the program.

The program has been written in an unextended version of FORTRAN 77 and has been installed on microcomputers running under CP/M-80 and MS-DOS as well as a variety of minicomputers and mainframe machines. The major modifications in code to implement the program on a given system is the assignment of logical devices. Guidelines for these types of modifications are clearly identified in the source code.

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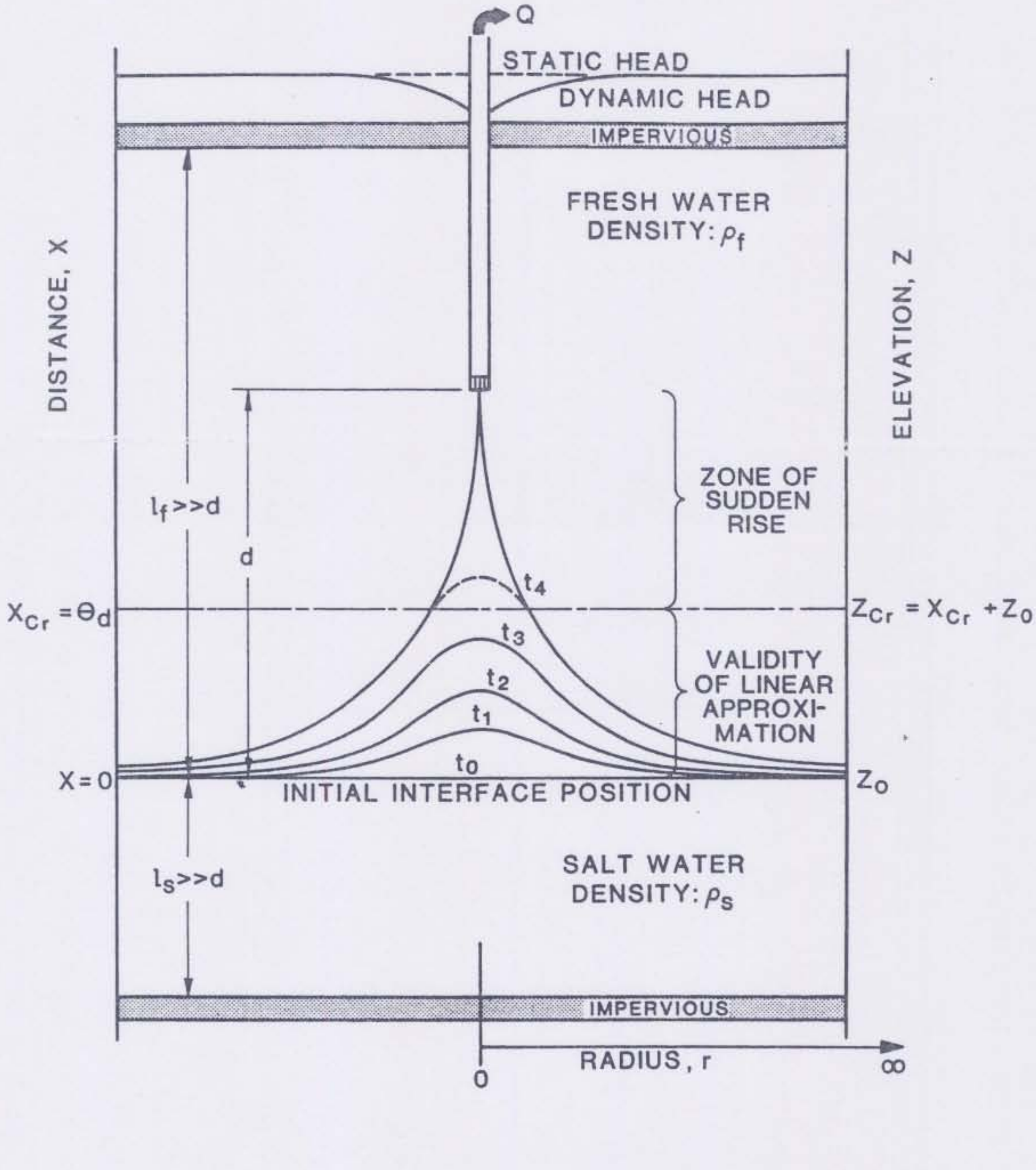

Figure 1. Upconing of an abrupt interface below a pumping well.

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ABSTRACT

Analytical solutions for the upconing of an abrupt salt-water/freshwater interface beneath a pumping well and for the concentration profile across a moving interface are developed for two types of upconing problems. The first considers the position of the interface and the salinity of the pumped water for a specified pumping rate. The second type of problem addresses the pumping schedules to prevent salinization of a well or to reach a predetermined salinity in the pumped water.

An interactive Fortran computer code has been developed to obtain solutions to both types of problems. The user is provided with options to modify the definition of a given problem, and, therefore, can gain some insight into the effects of geometry and physical properties on the rate and extent of upconing and the salinization of a well.

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INTRODUCTION

Relatively simple analytical models can often be used to solve ground-water contamination problems, depending upon the complexity of the system and the availability of field data. Analytical models can also be used to gain some insight to the expected behavior of a complex system before progressing to the application of more sophisticated numerical models. In general, relatively few input parameters are required to define a problem using an analytical model and numerical results can be calculated in a few seconds. Analytical models are well suited for interactive use, and in some instances can be programmed on hand-held calculators.

This report presents an analytical solution for the upconing of an abrupt salt-water/fresh-water interface below a pumping well. Dispersion phenomena arising from the displacement of a moving interface, or a finite transition zone between the invading and displaced fluids, can be superimposed on the analytical solution for the position of an abrupt interface. An interactive Fortran computer code has been developed which enables the user to modify input parameters and to control the computational sequence. This interactive approach enables the user to gain insight into the effects of geometry and physical properties on the rate and extent of upconing and salinization of a well.

SECTION I

Mathematical Development

McWhorter (1972) presented the equations which describe the flow in saturated aquifers which are underlain by a zone of saline water and pointed out the difficulties in obtaining solutions to these problems. The complexity of the flow phenomenon has led many investigators to idealize the system as a fresh-water zone separated from an underlying salt-water zone by a sharp interface. In other words, the two fluids are assumed to be immiscible. Schmorak and Mercado (1969) followed this approach and accounted for the mixing of the two fluids by superimposing the effects of dispersion on the transient solution for the position of an abrupt interface.

Upconing of an Abrupt Interface

The following discussion is based on the studies of Bear and Dagan as reported by Schmorak and Mercado (1969). The basic assumptions underlying the theoretical development are: (1) the porous medium is homogeneous and nondeformable, (2) the two fluids are incompressible, immiscible, and separated by an abrupt interface (a geometric surface), and (3) the flow obeys Darcy's law. The non-linear boundary condition along the interface between the two fluids constitutes the major difficulty with the immiscible formulation of the problem. Bear and Dagan used the method of small pertubations to obtain an approximate solution for the position of the interface which served as a tool for obtaining analytical solutions for cases involving small deviations from an initially steady interface.

For the case of upconing beneath a pumping well partially penetrating a relatively thick confined aquifer as shown in Figure 1, Schmorak and

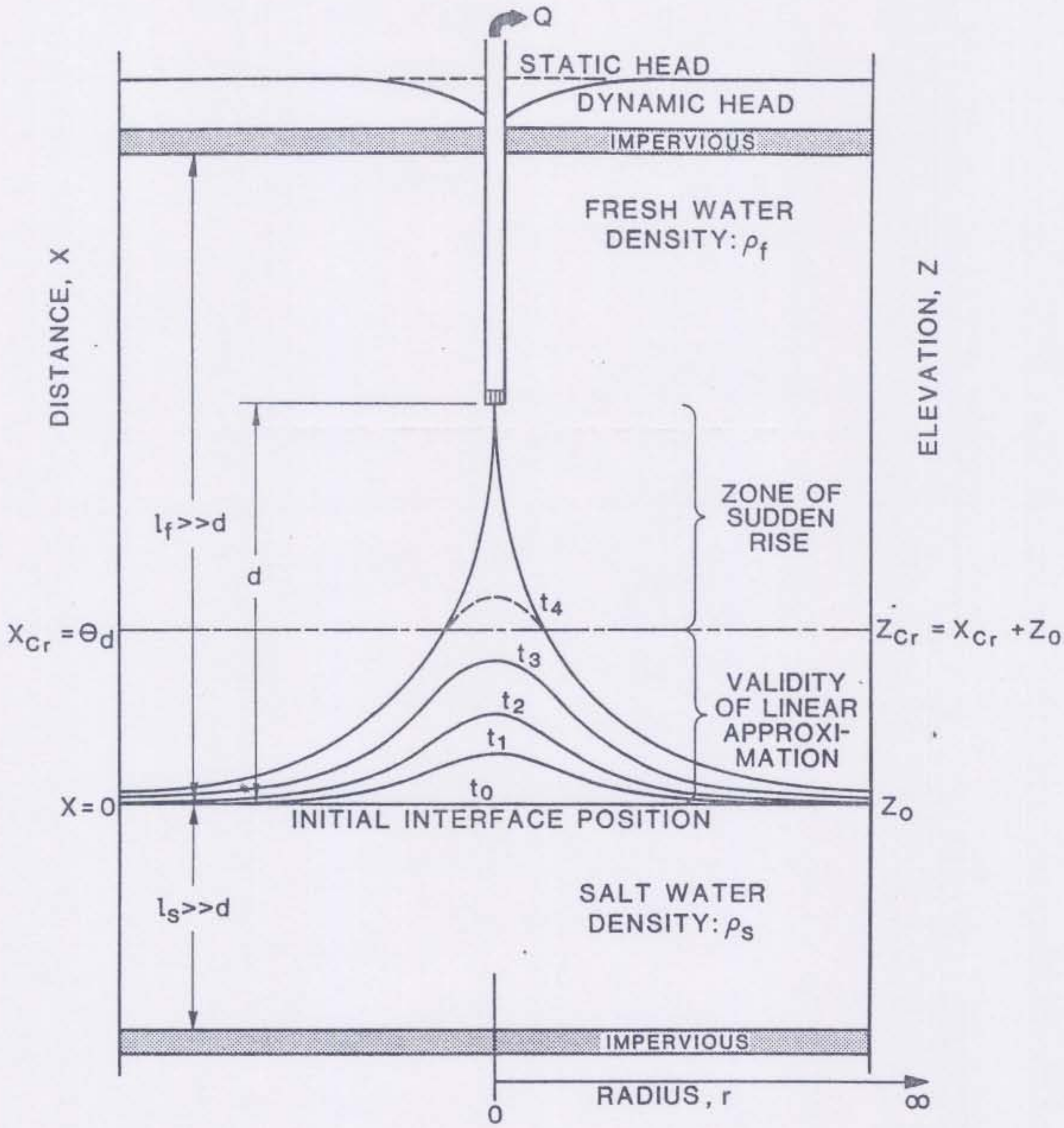

Figure 1. Upconing of an abrupt interface below a pumping well.

Mercado (1969) presented Bear and Dagan's solution for the position of the interface as a function of time and radial distance from the pumping well as

$$X(r,t) = \frac{Q}{2\pi(\Delta\rho/\rho)K_x d} \left\{ \frac{1}{(1 + R^2)^{1/2}} - \frac{1}{\left[(1 + \tau)^2 + R^2\right]^{1/2}} \right\} \quad (1)$$

where R and τ are dimensionless distance and time parameters defined by

$$R = \frac{r}{d}\left(\frac{K_z}{K_x}\right)^{1/2} \quad (2)$$

and

$$\tau = \frac{(\Delta\rho/\rho)K_z}{2\theta d}\, t \quad (3)$$

Other notations are defined as follows (also refer to Figure 1):

d	distance from the bottom of the well to the initial interface elevation (L)
K_x, K_z	horizontal and vertical permeabilities, respectively (L/t)
Q	well pumping rate (L^3/t)
r	radial distance from well axis (L)
t	time elapsed since start of pumping (t)
X	rise of the interface above its initial position (L)
$\Delta\rho/\rho$	dimensionless density difference between the two fluids, $(\rho_s - \rho_f)/\rho_f$
θ	porosity of the aquifer.

Application of the method of small pertubations restricts changes in the interface elevation to relatively small values. In terms of the physical problem, this restriction implies $d \ll l_f$ and $d \ll l_s$. Although the governing differential equations have been formulated for a confined aquifer, the results can be applied to unconfined systems if the drawdown is negligible compared to the saturated thickness of the fresh-water zone.

The linear relationship, Equation 1, between the rise of the interface and the pumping rate is limited to a certain "critical rise," X_{cr}. This limitation arises from linear approximation of the boundary conditions. As the interface approaches this critical rise, the rate of rise increases. Above the critical rise the interface reaches the pumping well with a sudden jump. Muskat (1946) defines the zone of accelerated rise for $X/d > 0.48$ and the critical rise within the limits of $X/d \sim 0.60$ to 0.75. Schmorak and Mercado (1966) recommend application of the linear approximation for $X/d < \sim 0.5$. Sahni (1972) investigated the zone of instability of the interface using both numerical and physical models and recommended design criteria for skimming wells.

An abrupt interface such that (1) salinization of the pumping well occurs only for $X > X_{cr} = fd$ where f is the fractional critical rise, and (2) Equation 1 is valid for $0 \leq X \leq X_{cr}$ will be assumed in this report. Thus, the maximum permissible pumping rate which will ensure salt-free water can be obtained from Equation 1.

For $r = 0$ and $t \to \infty$

$$X(0,\infty) = \frac{Q}{2\pi d(\Delta\rho/\rho)K_x} \qquad (4)$$

and

$$Q_{max} = 2\theta d(\Delta\rho/\rho)K_x X_{cr} \qquad (5)$$

For a decaying mound, an imaginary recharge well is superimposed at time $t = t^*$ corresponding to the end of the pumping period, and for $t > t^*$

$$X(r,t) = \frac{Q}{2\pi(\Delta\rho/\rho)K_x d}\left(\frac{1}{\left[(1+\tau_1)^2 + R^2\right]^{1/2}} - \frac{1}{\left[(1+\tau)^2 + R^2\right]^{1/2}}\right) \qquad (6)$$

where

$$\tau_1 = \frac{(\Delta\rho/\rho)K_z}{2\theta d}(t-t^*) \qquad (7)$$

Values of R and τ are evaluated using Equations 2 and 3 respectively.

Dispersion

The upconing process as treated above assumes that the two fluids are immiscible and that the interface between them is abrupt. Actually, the interface is diffuse and a transition zone exists between the two fluids in which the concentration varies from the concentration in one fluid to the concentration in the other fluid over a finite distance. This transition zone is related to dispersion processes which alter the concentration profile across the moving interface.

Bear and Todd (1960) approximated the concentration profile as a function of position, X; the "interface" position, $\overline{X}$; the equivalent total distance the interface is displaced, $|\Delta\overline{X}|$, independent of direction; and the dispersivity, D_m. The correlation is given by

$$\varepsilon(X) = \frac{1}{2}\,\text{ERFC}\left(\frac{X-\overline{X}}{2(D_m|\Delta\overline{X}|)^{\frac{1}{2}}}\right) \qquad (8)$$

where $\varepsilon(X)$ is a dimensionless, or relative, concentration defined as

$$\varepsilon(X) = \frac{C - C_b}{C_x - C_b} \qquad (9)$$

C = measured concentration at X

C_s = concentration of the invading fluid

C_b = background concentration of the displaced fluid.

Now, Equation 9 is a normal distribution function with a mean $\overline{X}$ and a standard deviation

$$\sigma = (2D_m|\Delta\overline{X}|)^{\frac{1}{2}} \qquad (10)$$

or

$$P_r \{X \geq x\} = \frac{1}{\sigma(2\pi)^{\frac{1}{2}}} \int_x^{\infty} \text{EXP} \left(- \frac{(X-\overline{X})^2}{2\sigma^2} \right) dx \qquad (11)$$

From the definition of the error function,

$$P_r \{X \geq x\} = \frac{1}{2} \text{ ERFC} \left(\frac{X-\overline{X}}{\sigma \sqrt{2}} \right) \qquad (12)$$

The two-parameter distribution is completely defined once the mean, $\overline{X}$, and standard deviation, σ, are known.

The mean of the distribution is assumed to be the rise of the interface as determined from Equation 1, or

$$\overline{X} = X\big|_{\varepsilon=0.5} = X(r,t) \qquad (13)$$

The standard deviation is defined as

$$2\sigma = \left(X\big|_{\varepsilon=0.841} - X\big|_{\varepsilon=0.159} \right) \qquad (14)$$

Now, the width of the transition zone is a function of the total distance traveled, $|\overline{\Delta X}|$, (independent of direction) and the dispersivity as given by Equation 10, or

$$2\sigma = 2(2D_m|\overline{\Delta X}|)^{\frac{1}{2}} \qquad (15)$$

For an interface with an initial transition width, $2\sigma_o$, when raised by a distance, $\overline{\Delta X}$,

$$\sigma_1 = \left(\sigma_o{}^2 + (2D_m|\overline{\Delta X}|) \right)^{\frac{1}{2}} \qquad (16)$$

The concentration distribution function then becomes

$$\varepsilon(X) = \frac{1}{2} \text{ ERFC} \left(\frac{X-\overline{X}}{\sigma_1 \sqrt{2}} \right) \qquad (17)$$

or

$$\varepsilon(X) = \frac{1}{2} \text{ ERFC} \left(\frac{X-\overline{X}}{\left(2\sigma_o{}^2 + 4D_m|\overline{\Delta X}| \right)^{\frac{1}{2}}} \right) \qquad (18)$$

Two important points should be noted concerning the preceding discussion of dispersion. First, the "initial width" of the transition zone has been defined as two standard deviations of the dimensionless concentration distribution. This definition has been adopted for convenience and serves to define the standard deviation of the concentration distribution across the initial transition zone. Secondly, the dispersion concept should be limited to the zone below the critical depth. This point will be considered in more detail in the following paragraphs.

Superposition of Dispersion on the Upconing of an Abrupt Interface

The position of the interface as a function of time and radial distance from the well is evaluated using Equation 1, which assumes an abrupt interface between the two fluids. This elevation is assumed to correspond to $X|_{\varepsilon=0.5}$, or the mean of the concentration distribution across the transition zone. In other words,

$$\overline{X} = X(r,t) = \frac{Q}{2\pi(\Delta\rho/\rho)K_x d}\left(\frac{1}{\left[(1+\tau_1)^2 + R^2\right]^{\frac{1}{2}}} - \frac{1}{\left[(1+\tau)^2 + R^2\right]^{\frac{1}{2}}}\right) \tag{6}$$

assuming an abrupt interface. The effect of dispersion arising from the displacement of the interface by a distance

$$|\overline{\Delta X}| = |X(r,t \le t^*)| + |X(r,t > t^*)| \tag{20}$$

is superimposed to estimate the concentration distribution across the interface using Equation 18, or

$$\varepsilon(X) = \frac{1}{2}\,\mathrm{ERFC}\left(\frac{X - \overline{X}}{\left(2\sigma_o^{\,2} + 4D_m|\overline{\Delta X}|\right)^{\frac{1}{2}}}\right) \tag{18}$$

The only difficulties in the approach occur for $\varepsilon(X) = 0.0$ and $\varepsilon(X) = 1.0$. Since

$$\varepsilon(X) = 0 \text{ for } X \to \infty$$

and

$$\varepsilon(X) = 1.0 \text{ for } X \to 0 \tag{22}$$

the transition zone would have an infinite width in theory. To overcome this physical impossibility, the width of the transition zone is arbitarily set at five standard deviations. This range includes approximately 99 percent of the area under the concentration distribution curve. Thus

$$\varepsilon(X) \simeq 0 \text{ for } X = \overline{X} + 2.5\ \sigma_1 \tag{23}$$

and

$$\varepsilon(X) \simeq 1 \text{ for } X = \overline{X} - 2.5\ \sigma_1 \tag{24}$$

Note that these limits differ from those used to define the "initial width" of the transition zone defined by Equation 14, or

$$2\sigma_o = \left(X|_{\varepsilon=0.841} - X|_{\varepsilon=0.159} \right) \tag{25}$$

Concentration in Pumped Water

The increase in concentration, or salinization, of pumped water is probably due to the intrusion of invading fluid above the critical depth. Data for two pumping tests on a coastal aquifer in the Ashqelon area of Israel indicated that the increase in salinity of the pumped water was approximately proportional to the average salinity above the critical depth (Schmorak and Mercado, 1969).

Previous discussion has emphasized that the linear approximation for the interface elevation is limited to elevations below the critical elevation and that the dispersion concept should be limited to the zone below the critical depth. The complex mixing and flow phenomena above the critical depth, near the well screen, and within the well pipe are approximated expirically using the approach followed by Schmorak and Mercado (1969).

The average dimensionless concentration of the transition zone above the critical rise, $\overline{\varepsilon}(X > X_{cr})$, is approximated as one-half the concentration at the critical depth, or

$$\varepsilon(X > X_{cr}) = 0.5\ \varepsilon(X_{cr}) \tag{26}$$

The concentration in the pumped water, ε_w, is determined from dilution of the average transition-zone concentration above the critical depth with displaced fluid, or

$$\varepsilon_w = \phi\ \overline{\varepsilon}(X > X_{cr}) \tag{27}$$

where ϕ is an interception coefficient, or the fraction of transition zone fluid in the total volume pumped.

SECTION II

Applications

Two types of upconing problems are considered. The first involves the description of the expected interface elevation and the salinity of the pumped water as a function of time for a given pumping rate. The second problem addresses the maximum rate at which a well can be pumped without exceeding a specified salinity in the pumped water. Both types of problems are discussed in the following paragraphs.

Case I - Estimation of Interface Elevations for a Given Pumping Rate

Case I problems are solved in a fairly straight-forward manner. Once the physical properties of the aquifer and the initial conditions have been specified, Equation 1 or Equation 6 is solved for the position of the abrupt interface, or the mean of the transition zone, i.e.,

$$X(r,t) = X\big|_{\varepsilon=0.5} = \overline{X} \qquad (13)$$

In terms of elevations,

$$Z(r,t) = X(r,t) + Z_o \qquad (28)$$

where Z_o is the initial interface elevation.

The concentration profile across the transition zone is evaluated using Equation 18. The salinity of the pumped water is determined from the dilution of the average transition zone salinity above the critical rise. From Equations 26 and 27

$$\varepsilon_w = 0.5\ \phi\ \varepsilon(X_{cr}) \qquad (29)$$

and from the definition of dimensionless concentration (Equation 9)

$$C_w = \varepsilon_w\ (C_s - C_b) + C_b \qquad (30)$$

where C_w is the concentration in the pumped water.

<u>Case II - Estimation the Maximum Permissible Pumping Rate to Prevent Salinization of a Well</u>

Case II problems present some difficulty as the pumping rate, Q, is unknown. Thus, the elevation of the interface, $\bar{X}$, and the total displacement of the interface, $|\overline{\Delta X}|$, are also unknown. Equation 18 must be solved for the maximum permissible rise in interface elevation such that

$$\varepsilon(X_{cr}=fd) \leq \varepsilon_{max} \tag{31}$$

where

$$\varepsilon_{max} = \frac{\varepsilon_w}{0.5\phi} \tag{32}$$

Assuming a constant, steady pumping rate the total displacement of the interface will be equal to the rise of the interface or

$$|\overline{\Delta X}| = \bar{X}$$

and Equation 18 can be written as

$$\varepsilon_{max} = \frac{1}{2}\,\mathrm{ERFC}\left(\frac{X_{cr} - \bar{X}}{\left(2\sigma_o^2 + 4D_m\bar{X}\right)^{\frac{1}{2}}}\right) \tag{33}$$

Equation 33 must be solved for $\bar{X}$ using trial-and-error procedures. The maximum permissible rise in the interface elevation,

$$X_{max} \leq X_{cr}=fd \tag{34}$$

is then corrected for the concentration profile as

$$X^*_{max} = X_{max} - (fd-\bar{X}) \tag{35}$$

and

$$Z^*_{max} = X^*_{max} + Z_o \tag{36}$$

The maximum permissible steady-state pumping rate is then obtained using Equation 5, or

$$Q^*_{max} = 2\pi d(\Delta\rho/\rho)K_x X^*_{max} \tag{37}$$

where X^*_{max} depends only upon the critical rise and the dispersion pattern.

The time required to reach a predetermined salinity in the well can be estimated by rewriting Equation 1 as

$$t(C_w) = \frac{2\theta d}{(\Delta\rho/\rho)K_z}\left\{\frac{1}{1 - \left[2\pi(\Delta\rho/\rho)K_x dX^*_{max}\right]/Q} - 1\right\} \tag{38}$$

Substituting Equation 37 into Equation 38 yields

$$t(C_w) = \frac{2\theta d}{(\Delta\rho/\rho)K_z}\left\{\frac{1}{1-Q^*_{max}/Q} - 1\right\} \tag{39}$$

which can be used to estimate the time required to reach a predetermined salinity in the pumped water for pumping rates, Q, greater than the maximum steady-state pumping rate, Q^*_{max}.

An interactive computational code has been developed to calculate interface elevations, and concentrations for both Case I and Case II problems using the approach described above. The computer program is discussed in Section IV of this report.

Example Problem - Upconing Below a Coastal Collector Well

The application of the analytical model will be demonstrated using the field data for Test B on the coastal collector well Semadar 1 in the Ashqelon area of Israel (Schmorak and Mercado, 1969). Test B consisted of pumping Semadar 1 at a rate of 348 m^3/day for a period of 84 days. The upconing and decay of the salt-water/fresh-water interface were monitored by measuring the

TABLE 1

Parameters for Semadar 1 - Test B

Fresh-water density, ρ_f	1.00 g/cm^3
Salt-water density, ρ_s	1.03 g/cm^3
Porosity, θ	0.33
Horizontal permeability, K_x	14.7 m/day
Vertical permeability, K_z	14.7 m/day
Initial interface elevation, Z_o	-30.75 m MSL
Distance from bottom of well to initial interface, d	15.5 m
Fractional critical rise, f	0.4
Chloride concentration in salt water, C_s	22,000. ppm Cl
Chloride concentration in fresh water, C_b	145. ppm Cl
Dispersivity, D_m	0.5 m
Initial width of transition zone, $2\sigma_o$	3.5 m
Interception coefficient, ϕ	0.08
Pumping rate, Q	348.m^3/dy
Pumping period, t^*	84. dy

Observation wells:

Identification	Radius, m
T-2	4.5
T-3	12.4
T-4	16.7
T-5	33.9

salinity profiles in four observation wells. Samples of the pumped water were collected periodically and analyzed for chlorides. The properties of the fluids and the aquifer as estimated by Schmorak and Mercado are summarized in Table 1.

Program UPCONE was used to calculate the transient interface elevations and chloride concentrations in the pumped water. The input data dialog and printed results for this problem are presented in Appendix D.

The predicted and observed interface elevations after 16, 57, and 84 days of pumping are shown in Figure 2. The observed interface elevations correspond to elevations for a 50 percent relative concentration of sea water, or

$$\varepsilon = \frac{C - C_b}{C_s - C_b} = 0.5$$

The predicted and observed interface elevations match fairly well with the exception of the values for observation well T-2. Well T-2 is located 4.5 meters from the pumping well and penetrates to an elevation of 31.10 meters below MSL. However, this well is screened to an elevation of only 29.02 meters below MSL, and Schmorak and Mercado (1969) indicate that saline water was entrapped at the bottom of the well from a previous pumping test. The predicted rise of the interface at well T-2 also approaches the critical elevation for a fractional critical rise of 0.4. Thus, the observed interface elevation could be in a zone of accelerated rise. Finally, the reported concentration gradients were very steep in well T-2, and small errors in concentration measurements could lead to large errors in estimating the position of the interface.

The predicted upconing and recovery curves for each of the four observation wells are shown in Figure 3. With the exception of well T-2 the

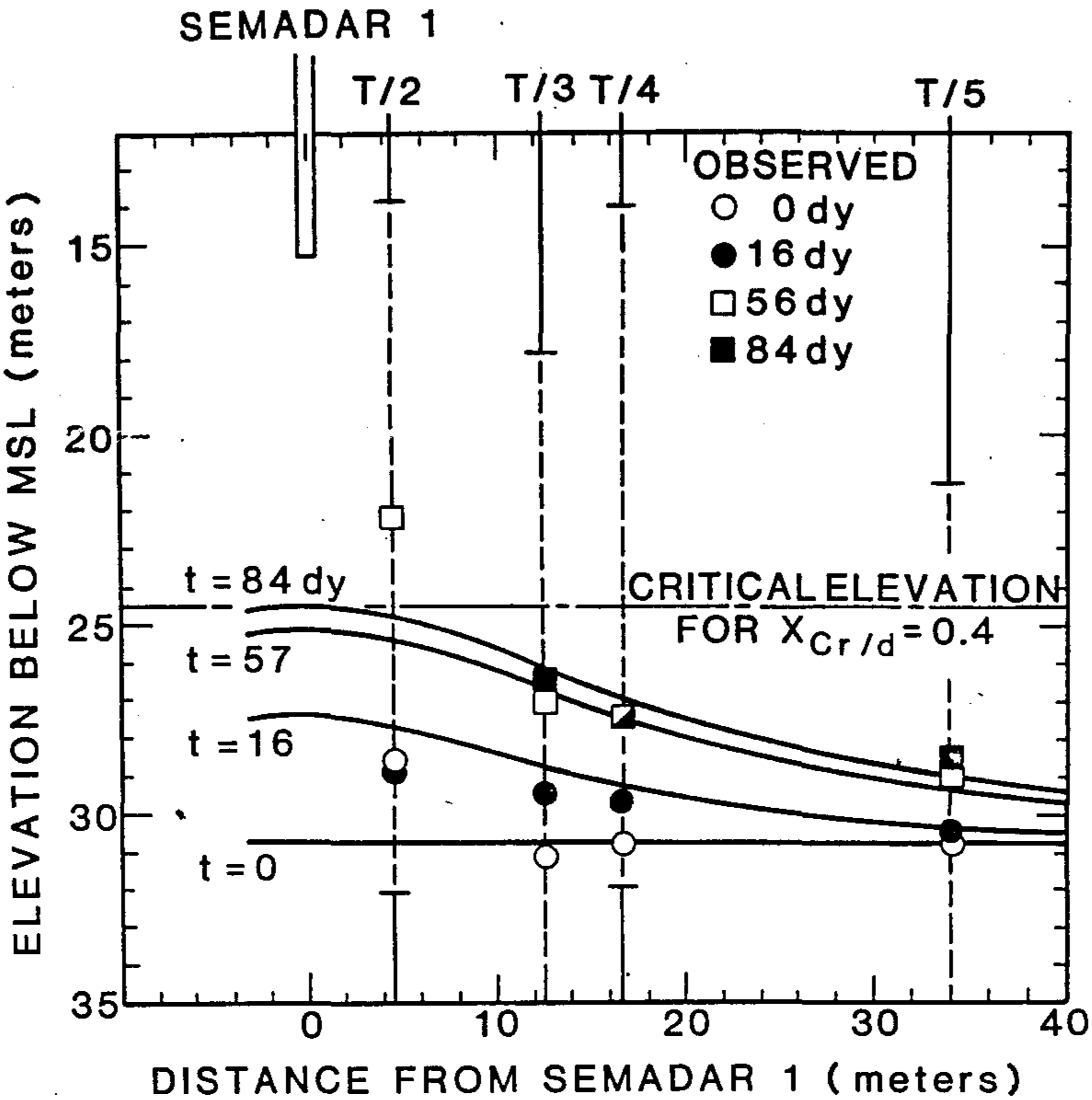

Figure 2. Predicted and observed interface elevations for Test B of Semadar 1.

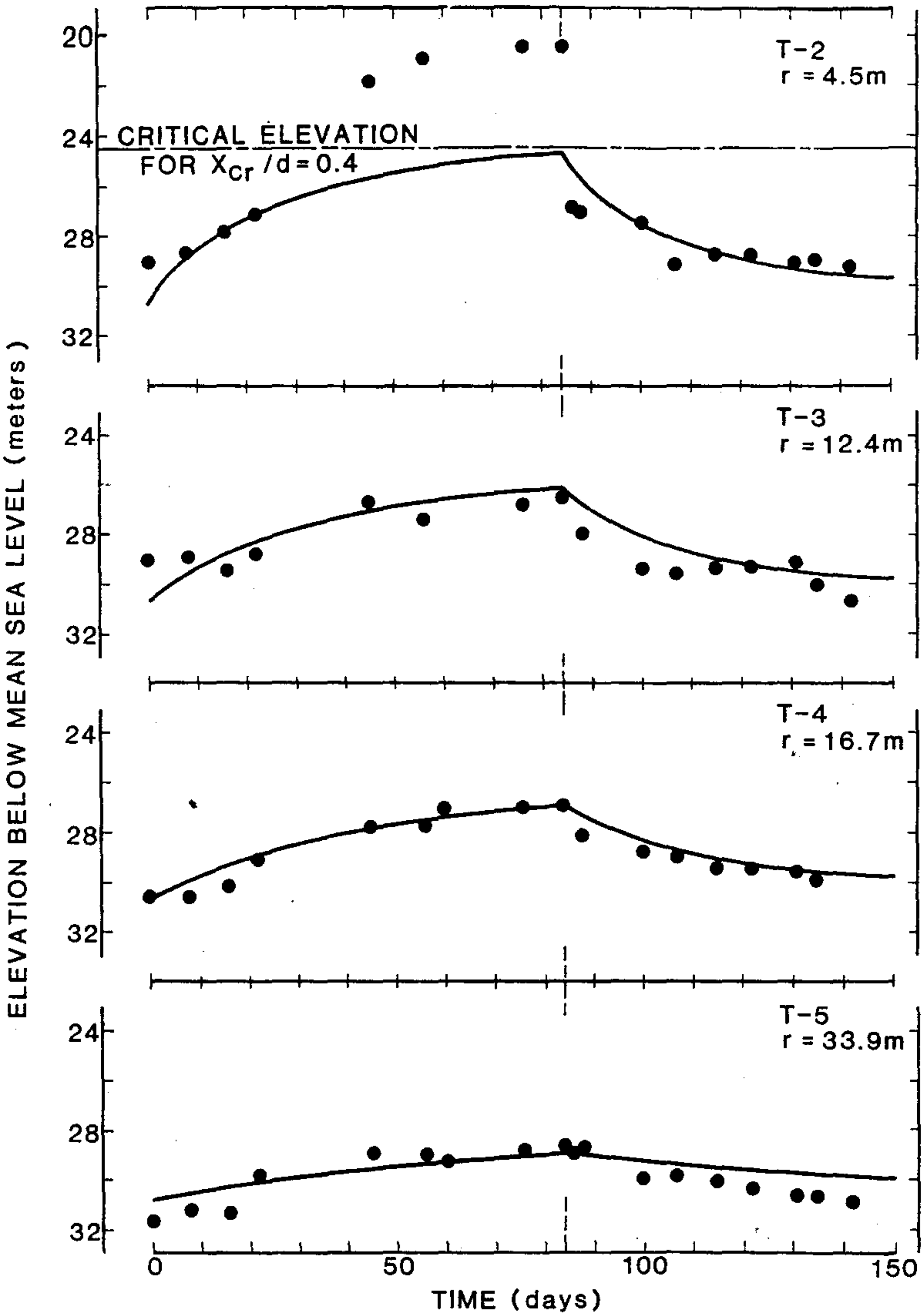

Figure 3. Predicted and observed upconing and recovery curves for Test B of Semadar 1.

predicted upconing curves follow the observed interface elevations fairly well. The recession curves for all four observation wells are also in fair agreement with the field data.

Initial observed relative concentrations for wells T-3, T-4, and T-5 are plotted on Figures 4a and 4b. The predicted concentration profile across the transition zone using an initial transition zone width, $2\sigma_o$, of 3.5 meters is also shown. This value represents an average of the widths of the transition zones at the three observation wells.

The parameters listed in Table 1 were used to predict the concentration of chlorides in the pumped water as a function of time. The results of the simulation are summarized in Figure 5 and agree very well with the observed values.

No effort has been made in this report to "optimize" model input parameters or to quantify the "goodness of fit" between observed and predicted values of elevations, concentration profiles, or salinity of the pumped water. However, a qualitive comparison of the predicted and observed values as shown in Figures 2, 3, and 5 indicate that the assumptions incorporated in the analytical model approach the field conditions.

Program UPCONE was also used to develop salinity/maximum pumping rate relationships and salinity/time relationships for Semadar 1. These relationships correspond to Case II types of problems. The corrected critical interface elevations, Z^*_{max}, and maximum steady-state pumping rates, Q^*_{max}, for several values of predetermined salinity in the pumped water are presented in Table 2. The time required to reach the specified salinity in the pumped water were also calculated for two optional pumping rates. These pumping rates correspond to Test A and Test B of Semadar 1.

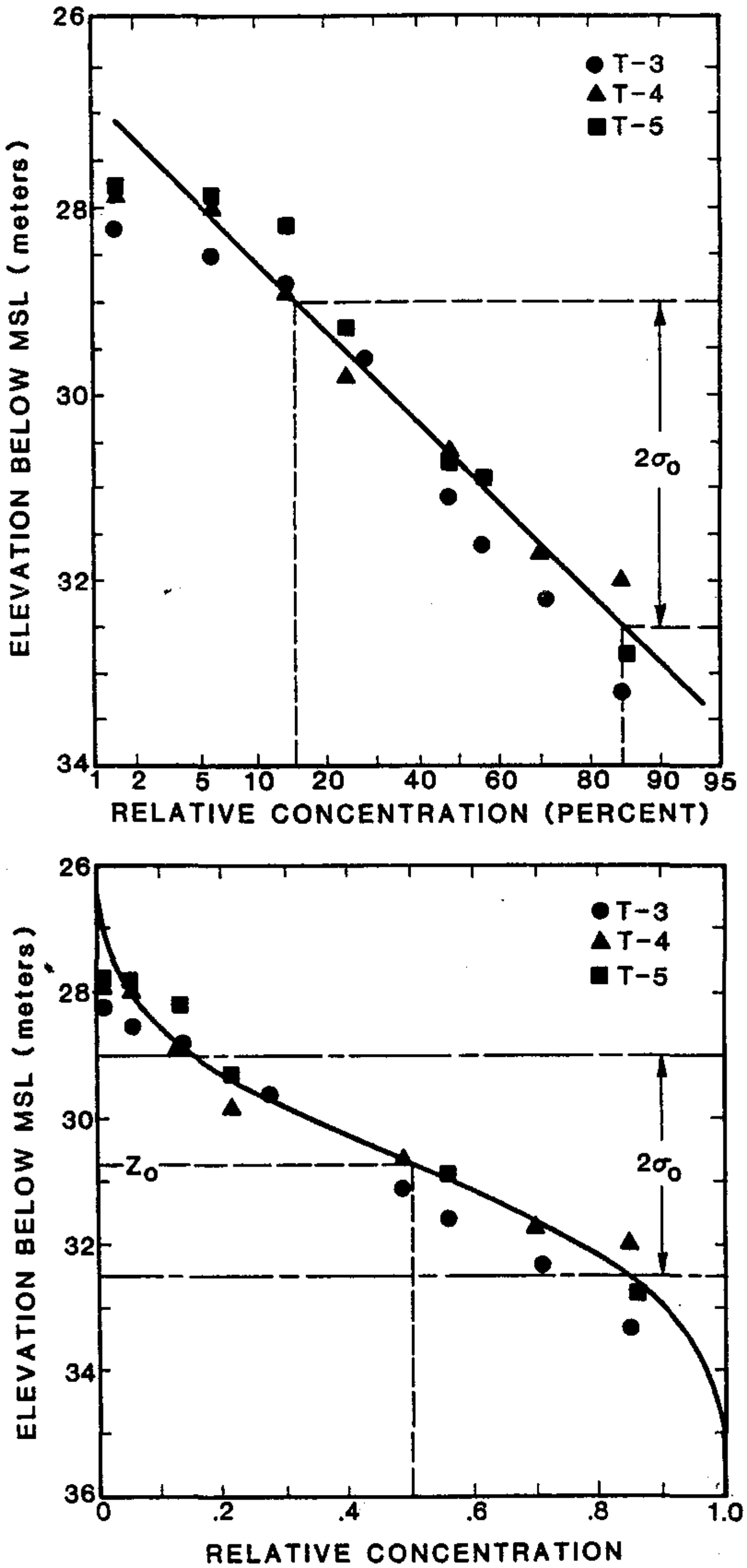

Figure 4. Concentration profile across initial transition zone.
(a) Relative concentration on probability scale.
(b) Relative concentration on arithmetic scale.

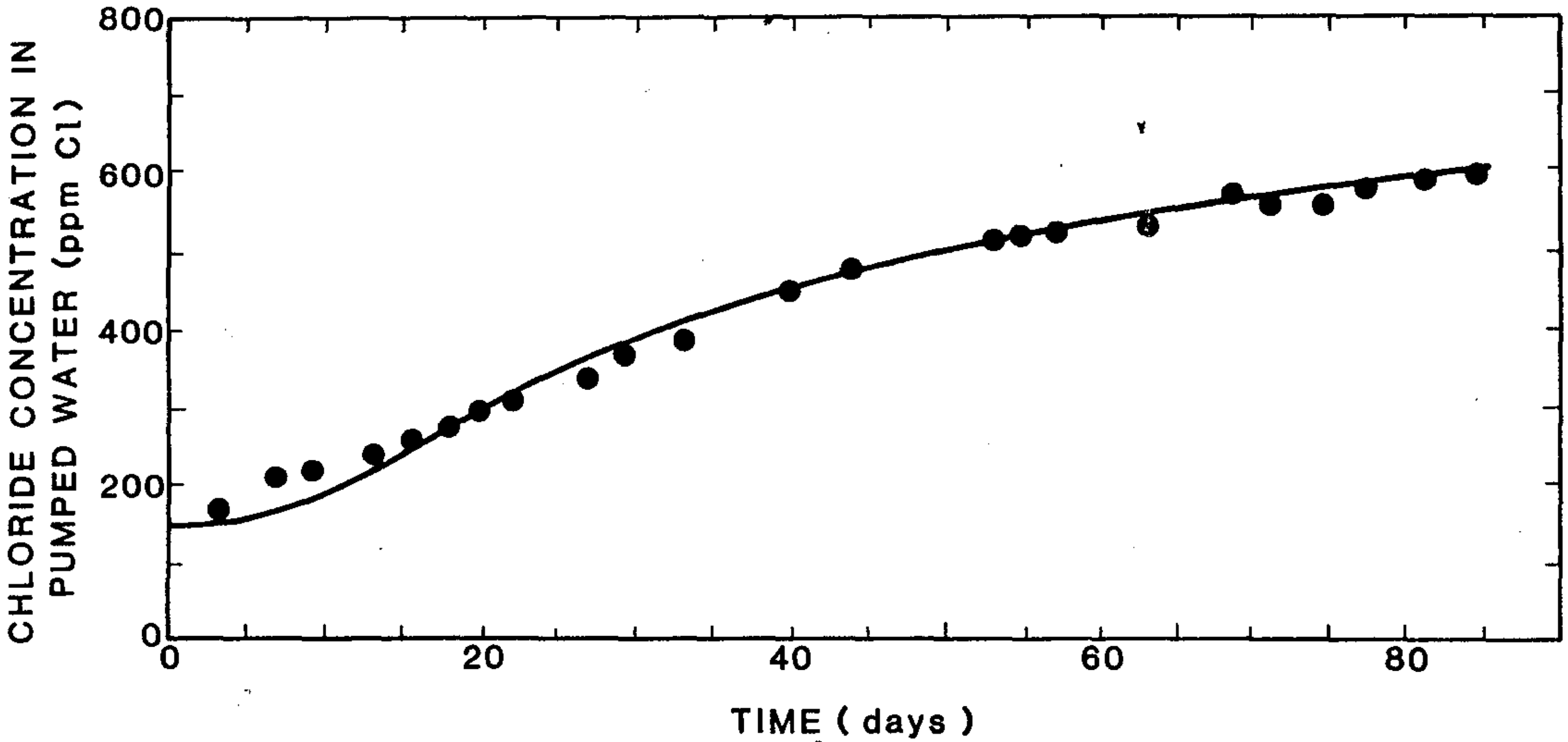

Figure 5. Predicted and observed chloride concentrations in pumped water for Test B on Semadar 1.

TABLE 2

Salinity/Maximum Pumping Rate and Salinity/Time

Relationships For Semadar 1 (X_{cr} = 0.4d)

Relative Concentration of Salt Water (Dimensionless)	Chloride Concentration In Pumped Water (ppm CL)	X^*_{max} (meters)	Q^*_{max} (m^3/day)	Time: Test A, Q = 575 m^3/day (days)	Time: Test B, Q = 348 m^3/day (days)
0.001	166.85	2.85	79.57	3.73	6.88
.003	210.56	3.73	117.40	5.95	11.81
.005	254.27	4.30	141.66	7.58	15.92
.010	363.55	5.36	187.34	11.21	27.05
.020	582.10	7.20	266.28	20.01	75.59
.030	800.65	9.49†	364.76†	40.25	-

(†) $X^*_{max} > X_{cr}$ = 6.20 meters

The example problems using the data for Test B of Semadar 1 described above are intended to support, in general, the validity of the theoretical approach and to demonstrate the application of the analytical model to a typical upconing problem. The reader interested in methods which might be used to develop input parameters for the model are referred to the Schmorak and Mercado (1969) discussion of the field investigation and interpretation of the field data.

SECTION III

Program UPCONE

Program UPCONE evaluates the position of an abrupt salt-water/fresh-water interface beneath a pumping well as a function of time and radial distance from the well. The program has also been written to (1) superimpose the effects of dispersion on the abrupt interface to estimate the concentration profile across the interface and the salinity of the pumped water, or (2) estimate the maximum pumping rate or time required to reach a specified salinity in the well. The program has been designed for interactive use and requires data input under two modes of operation--"Basic Input Data" and "Edit."

Basic Input Data

Basic input data are required to initiate a new problem using the UP-CONE program. The data entries include the problem title, the physical properties of the aquifer and the two fluids, and the geometry of the system. The user is prompted for the required data through a series of input commands described below. Numeric data should be entered through the keyboard with decimal points, and multiple data entries should be separated by a comma(s). The first basic input command is

```
ENTER TITLE
?
```

Any valid keyboard characters can be used. The first 60 characters will be retained for further problem identification.

The next two input commands are used to define the units of all variables used in the calculations and printouts of the results. Any consistent set of units may be used. The two commands are

ENTER UNITS FOR LENGTH (2 CHARACTERS)

?

ENTER UNITS FOR TIME (2 CHARACTERS)

?

Any valid keyboard characters can be used. The first two characters will be retained for identification of length and time units.

The next series of input commands are used to specify the physical properties of the fluids and the aquifer. Input data errors which may interrupt the computational sequence are detected by the program and a command is issued to reenter the data for the appropriate variable. The series of commands are as follows:

ENTER FRESH-WATER AND SALT-WATER DENSITIES

?,?

The densities, or specific gravities, of the two fluids may be entered in any units so long as the units are identical for both entries.

ENTER AQUIFER POROSITY

?

Enter the volume void fraction as a decimal value greater than zero and less than one.

ENTER HORIZONTAL PERMEABILITY (L/t)

?

ENTER VERTICAL PERMEABILITY (L/t)

?

Horizontal and vertical permeabilities must be entered with dimensions of L/t in the units requested. Numerical values for both entries must be greater than zero.

ENTER INITIAL INTERFACE ELEVATION (L)

?

The initial interface elevation may be either positive, zero, or negative, depending upon the location of the reference elevation with respect to the initial abrupt interface (or the elevation of the mean concentration of the transition zone). The elevation must be entered in the units requested.

ENTER DISTANCE FROM BOTTOM OF WELL TO INITIAL INTERFACE (L)

?

The entry must be positive and in the units requested.

ENTER FRACTIONAL CRITICAL RISE

?

The fractional critical rise must be a decimal value greater than zero and less than one.

The next basic input command is used to select the option of performing concentration calculations. The command is

CONCENTRATION CALCULATIONS? (Y/N)

If the user does not respond with Y, the problem title and parameters which have been specified are listed. The critical rise and the maximum steady-state pumping rate which will maintain the interface at the critical elevation are evaluated and the results are printed. The program then enters the "Edit" mode.

If the response to the last basic input command is Y, the user will be requested for additional basic input data required to carry out the concentration calculations. The first command is

ENTER UNITS FOR CONCENTRATION (6 CHARACTERS)

?

Any valid keyboard characters can be used. The first 6 characters will be used to specify concentrations for following data entries and printed results. The following commands are:

ENTER SALT-WATER AND BACKGROUND CONCENTRATIONS (M/L^3)

?,?

The concentration of any desired component in the invading fluid and the displaced fluid are entered. If the user wishes to work in terms of dimensionless concentrations, enter a value of 1.0 for the salt-water concentration and a value of 0.0 for the background concentration.

ENTER DISPERSIVITY (L)

?

The dispersivity has dimensions of L and must be entered in the units requested. Numerical values must be greater than zero.

ENTER INITIAL WIDTH OF TRANSITION ZONE (M)

?

The width of the transition zone is defined as two standard deviations of the concentration distribution function across the transition zone, as discussed in Section II of this report. For an initially abrupt interface enter a value of zero.

ENTER INTERCEPTION COEFFICIENT

?

The interception coefficient is the fraction of transition zone water in the total volume pumped. Enter a decimal fraction greater than zero and less than one.

At this point the program will list the problem title and parameters as they are currently specified along with the critical rise and the maximum permissible pumping rate for an abrupt interface. The program then enters the "Edit" mode.

Edit Commands

Once the basic input data have been entered, the problem as currently defined is listed and the program enters the "Edit" mode. The edit commands are listed in Table 3. The request for information is

ENTER NEXT COMMAND

?

One of the responses from Table 1 should be given. If the response is incorrect or improperly formated the statement

ERROR IN LAST COMMAND--REENTER

?

is issued. Error messages for invalid numeric data will be issued as described under Basic Input Data.

The request for information will be repeated until one of the responses EL, PR, LI, NP or DN is entered.

EL will initiate the calculation of interface elevations for a specified pumping rate. The user is given the option of calculating the concentration profile across the transition zone by responding to the following command

CONCENTRATION CALCULATIONS ? (Y/N)

If the response is Y and the data required for concentration calculations have not been entered previously, the user is prompted for the required information using "Basic Input" commands.

For the initial use of the EL edit command, the following requests for data are issued:

ENTER PUMPING RATE (CU L/t) AND PERIOD (t)
?,?

Enter the well pumping rate and the pumping period in the units requested. Both entries must be positive and separated by a comma.

Two additional requests for data are used to define the coordinates of the observation points. The first is

ENTER TFIRST, TLAST, DELTAT (t)
?,?,?

Input units for the time variables must be in the units requested. TFIRST must not be negative value. A zero entry for DELTAT will result in interface elevations at a single time. The second request is

```
ENTER RFIRST, RLAST, DELTAR (L)
?,?,?
```

The numerical values used to define the radial coordinates of the observation points may be positive or negative. The results of the calculations will be printed from RFIRST to RLAST.

The pumping rate and observation coordinate parameters are listed and a request

```
CONTINUE ? (Y/N)
```

is issued. If the response is not Y, the program returns to the edit mode. If the response is Y, the program proceeds to the computation of interface elevations at the specified times and radial distances from the well and prints the results. If concentration calculations were requested, the concentration in the pumped water and the concentration distribution across the transition zone are also evaluated and printed for the specified times.

On subsequent use of the EL edit command, the pumping rate and observation coordinate data as currently defined will be listed and a request to continue will be issued.

PR initiates the calculation of maximum permissible interface elevations and pumping rates for a specified concentration in the pumped water. If the data required for concentration have not been entered previously, the user is prompted for the required data using "Basic Input Data" dialog. The following request for information is then issued:

ENTER MAXIMUM PERMISSIBLE CONCENTRATION IN PUMPED WATER (M/L^3)

?

Enter the concentration in the units requested. The numerical value must not be negative.

The program then lists the problem as currently defined and evaluates the maximum permissible interface elevation and pumping rate. The following request for information is then issued:

ENTER OPTIONAL PUMPING RATE (L^3/t)

?

If a value greater than the maximum pumping rate is entered, the time to reach the specified concentration in the pumped water will be calculated, and the command reissued. If a value less than the maximum pumping rate is entered the program returns to the edit mode.

LI will list the problem as currently defined.

NP will request a complete new problem definition using the "Basic Input Data" dialog.

DN will terminate the program.

Although many tests for valid input data and properly formulated edit commands have been embedded in the computer code, the user is encouraged to correct "keyboard errors" before the data are transmitted. This practice will serve to minimize the frustration of program termination as a result of fatal errors during execution of the numerical computations.

TABLE 3

Edit Commands for UPCONE

Command	Variable Changed/Execution
CO	Salt-water and background concentrations
CR	Fractional critical rise
DI	Dispersivity
DT	Depth from bottom of well to initial interface
FD	Fluid densities
IC	Interception coefficient
KX	Horizontal permeability
KZ	Vertical permeability
OB	Time and radius coordinates
PO	Porosity
QP	Pumping rate and time
RC	Radius coordinates
TC	Time coordinates
TW	Initial width of transition zone
ZO	Initial interface elevation
EL	Interface elevation calculations
DN	Terminate program
LI	List problem definition
NP	New problem
PR	Pumping rate calculations

REFERENCES

Abramowitz, M. and I. A. Stegun. 1966. Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables. National Bureau of Standards Applied Mathematics Series 55, U. S. Department of Commerce, 1046 pp.

Bear, J. and D. K. Todd. 1960. "The Transition Zone between Fresh and Salt Waters in Coastal Aquifers." Contribution No. 29, Water Resources Center, University of California, Berkeley, CA.

Carnahan, B., H. A. Luther and J. O. Wilkes. 1969. Applied Numerical Methods. John Wiley and Sons, New York, NY.

McWhorter, D. B. 1972. "Steady and Unsteady Flow of Fresh Water in Saline Aquifers." Water Management Technical Report No. 20, Engineering Research Center, Colorado State University, Fort Collins, CO.

Schmorak, S. and A. Mercado. 1969. "Upconing of Fresh Water-Sea Water Interface Below Pumping Wells, Field Study." Water Resources Research, Vol. 5, No. 6, pp 1290-1311.

APPENDIX A

Example Problems

The following pages contain the documentation of the upconing simulation used to generate the predicted interface/time and concentration/time relationships for the example problem discussed in Section II of this report.

```
  ENTER TITLE
  ?
SEMADAR 1 -- TEST B
  ENTER UNITS FOR LENGTH (2 CHARACTERS)
  ?
M
  ENTER UNITS FOR TIME (2 CHARACTERS)
  ?
DY
  ENTER FRESH-WATER AND SALT-WATER DENSITIES
  ?,?
1.00,1.03
  ENTER AQUIFER POROSITY
  ?
0.33
  ENTER HORIZONTAL PERMEABILITY (M /DY)
  ?
14.7
  ENTER VERTICAL PERMEABILITY (M /DY)
  ?
14.7
  ENTER INITIAL INTERFACE ELEVATION (M )
  ?
-30.75
  ENTER DISTANCE FROM BOTTOM OF WELL TO INITIAL INTERFACE (M )
  ?
15.5
  ENTER FRACTIONAL CRITICAL RISE
  ?
0.4

  CONCENTRATION CALCULATIONS ? (Y/N)
N
```

```
 SEMADAR 1 -- TEST B

   DENSITY OF FRESH WATER                                  1.0000
   DENSITY OF SALT WATER                                   1.0300

   AQUIFER POROSITY                                         .3300
   HORIZONTAL PERMEABILITY (M /DY)                        14.7000
   VERTICAL PERMEABILITY (M /DY)                          14.7000

   INITIAL INTERFACE ELEVATION (M )                      -30.7500
   DISTANCE FROM BOTTOM OF WELL TO INTERFACE (M )         15.5000
   FRACTIONAL CRITICAL RISE                                 .4000
   CRITICAL RISE (M )                                      6.2000
   CRITICAL ELEVATION (M )                               -24.5500

   MAXIMUM STEADY-STATE PUMPING RATE (CU M /DY)          266.2818




  ENTER NEXT COMMAND
  ?
EL

  CONCENTRATION CALCULATIONS ? (Y/N)
N

  ENTER PUMPING RATE (CU M /DY) AND PERIOD (DY)
  ?,?
348.,84.
  ENTER TFIRST, TLAST, DELTAT (DY)
  ?,?,?
0.,57.,16.
  ENTER RFIRST, RLAST, DELTAR (M )
  ?,?,?
0.,40.,5.

   PUMPING RATE (CU M /DY)                               348.0000
   PUMPING PERIOD (DY)                                    84.0000

   TFIRST =     .0000   TLAST =   57.0000   DELTAT =    16.0000
   RFIRST =     .0000   RLAST =   40.0000   DELTAR =     5.0000

  NOTE:  INTERFACE WILL RISE TO CRITICAL ELEVATION IN      75.59 DY

  CONTINUE ? (Y/N)
Y
```

PUMPING RATE: 348.00 CU M /DY FOR 84.00 DY

INTERFACE ELEVATIONS (M)

* R (M) * T (DY) *	.00	5.00	10.00	15.00	20.00	25.00	30.00
.00	-30.75	-30.75	-30.75	-30.75	-30.75	-30.75	-30.75
16.00	-27.44	-27.75	-28.42	-29.09	-29.60	-29.95	-30.18
32.00	-26.05	-26.41	-27.23	-28.08	-28.78	-29.30	-29.67
48.00	-25.29	-25.66	-26.52	-27.45	-28.22	-28.82	-29.26
57.00	-24.99	-25.37	-26.25	-27.18	-27.98	-28.60	-29.08

INTERFACE ELEVATIONS (M) (CONTINUED)

```
   *
    * R (M )
      *        35.00     40.00
 T (DY) *
         *
       .00   -30.75    -30.75
     16.00   -30.34    -30.45
     32.00   -29.94    -30.13
     48.00   -29.60    -29.84
     57.00   -29.43    -29.70


 ENTER NEXT COMMAND
 ?
OB
 ENTER TFIRST, TLAST, DELTAT (DY)
 ?,?,?
0.,160.,5.
 ENTER RFIRST, RLAST, DELTAR (M )
 ?,?,?
4.5,0.,0.


 ENTER NEXT COMMAND
 ?
EL


 CONCENTRATION CALCULATIONS ? (Y/N)
N


    PUMPING RATE (CU M /DY)                                  348.0000
    PUMPING PERIOD (DY)                                       84.0000

    TFIRST =      .0000   TLAST =  160.0000   DELTAT =     5.0000
    RFIRST =     4.5000   RLAST =     .0000   DELTAR =      .0000

 NOTE:  INTERFACE WILL RISE TO CRITICAL ELEVATION IN      75.59 DY

 CONTINUE ? (Y/N)
Y
```

```
                 PUMPING RATE:      348.00 CU M /DY FOR     84.00 DY

                 INTERFACE ELEVATIONS (M )

   *
     * R (M )
       *          4.50
 T (DY) *
         *
       .00   -30.75
      5.00   -29.45
     10.00   -28.52
     15.00   -27.81
     20.00   -27.27
     25.00   -26.83
     30.00   -26.47
     35.00   -26.18
     40.00   -25.93
     45.00   -25.71
     50.00   -25.53
     55.00   -25.36
     60.00   -25.22
     65.00   -25.09
     70.00   -24.98
     75.00   -24.88
     80.00   -24.79
     85.00   -25.00
     90.00   -26.13
     95.00   -26.94
    100.00   -27.55
    105.00   -28.01
    110.00   -28.37
    115.00   -28.67
    120.00   -28.91
    125.00   -29.11
    130.00   -29.27
    135.00   -29.41
    140.00   -29.54
    145.00   -29.64
    150.00   -29.73
    155.00   -29.81
    160.00   -29.88

 * NOTE: CRITICAL ELEVATION OF  -24.55 M  EXCEEDED AT R=0 AND T=   75.59 DY


 ENTER NEXT COMMAND
 ?
RC
 ENTER RFIRST, RLAST, DELTAR (M )
 ?,?,?
12.4,0.,0.


 ENTER NEXT COMMAND
 ?
EL


 CONCENTRATION CALCULATIONS ? (Y/N)
N


    PUMPING RATE (CU M /DY)                              348.0000
    PUMPING PERIOD (DY)                                   84.0000

    TFIRST =     .0000   TLAST =  160.0000   DELTAT =    5.0000
    RFIRST =   12.4000   RLAST =     .0000   DELTAR =     .0000

 NOTE:  INTERFACE WILL RISE TO CRITICAL ELEVATION IN      75.59 DY

 CONTINUE ? (Y/N)
Y
```

```
                    PUMPING RATE:       348.00 CU M /DY FOR      84.00 DY

                    INTERFACE ELEVATIONS (M )

     *
       * R (M )
         *         12.40
   T (DY) *
           *
          .00    -30.75
         5.00    -29.99
        10.00    -29.36
        15.00    -28.85
        20.00    -28.42
        25.00    -28.06
        30.00    -27.76
        35.00    -27.50
        40.00    -27.28
        45.00    -27.08
        50.00    -26.91
        55.00    -26.76
        60.00    -26.63
        65.00    -26.51
        70.00    -26.40
        75.00    -26.30
        80.00    -26.22
        85.00    -26.30
        90.00    -26.96
        95.00    -27.49
       100.00    -27.92
       105.00    -28.28
       110.00    -28.57
       115.00    -28.82
       120.00    -29.02
       125.00    -29.20
       130.00    -29.34
       135.00    -29.47
       140.00    -29.58
       145.00    -29.68
       150.00    -29.77
       155.00    -29.84
       160.00    -29.91

   * NOTE: CRITICAL ELEVATION OF  -24.55 M  EXCEEDED AT R=0 AND T=   75.59 DY


   ENTER NEXT COMMAND
   ?
 RC
   ENTER RFIRST, RLAST, DELTAR (M )
   ?,?,?
 16.7,0.,0.


   ENTER NEXT COMMAND
   ?
 EL


   CONCENTRATION CALCULATIONS ? (Y/N)
 N


      PUMPING RATE (CU M /DY)                              348.0000
      PUMPING PERIOD (DY)                                   84.0000

      TFIRST =      .0000   TLAST =  160.0000   DELTAT =     5.0000
      RFIRST =    16.7000   RLAST =      .0000   DELTAR =      .0000

   NOTE:  INTERFACE WILL RISE TO CRITICAL ELEVATION IN     75.59 DY

   CONTINUE ? (Y/N)
 Y
```

```
                    PUMPING RATE:       348.00 CU M /DY FOR      84.00 DY

                    INTERFACE ELEVATIONS (M )

   *
     * R (M )
       *        16.70
  T (DY) *
          *
         .00   -30.75
        5.00   -30.23
       10.00   -29.76
       15.00   -29.36
       20.00   -29.00
       25.00   -28.70
       30.00   -28.44
       35.00   -28.21
       40.00   -28.00
       45.00   -27.83
       50.00   -27.67
       55.00   -27.53
       60.00   -27.40
       65.00   -27.29
       70.00   -27.19
       75.00   -27.09
       80.00   -27.01
       85.00   -27.04
       90.00   -27.48
       95.00   -27.87
      100.00   -28.20
      105.00   -28.49
      110.00   -28.73
      115.00   -28.94
      120.00   -29.12
      125.00   -29.27
      130.00   -29.41
      135.00   -29.52
      140.00   -29.63
      145.00   -29.72
      150.00   -29.80
      155.00   -29.87
      160.00   -29.93

  * NOTE: CRITICAL ELEVATION OF  -24.55 M  EXCEEDED AT R=0 AND T=   75.59 DY


  ENTER NEXT COMMAND
  ?
RC
  ENTER RFIRST, RLAST, DELTAR (M )
  ?,?,?
33.9


  ENTER NEXT COMMAND
  ?
EL


  CONCENTRATION CALCULATIONS ? (Y/N)
N


     PUMPING RATE (CU M /DY)                             348.0000
     PUMPING PERIOD (DY)                                  84.0000

     TFIRST =      .0000   TLAST =  160.0000   DELTAT =     5.0000
     RFIRST =    33.9000   RLAST =      .0000   DELTAR =      .0000

  NOTE:  INTERFACE WILL RISE TO CRITICAL ELEVATION IN      75.59 DY

  CONTINUE ? (Y/N)
 Y
```

[illegible]

```
                     INTERFACE ELEVATIONS (M )

     *
       * R (M )
          *          33.90
   T (DY) *
             *
            .00    -30.75
           5.00    -30.62
          10.00    -30.48
          15.00    -30.34
          20.00    -30.20
          25.00    -30.07
          30.00    -29.94
          35.00    -29.82
          40.00    -29.70
          45.00    -29.59
          50.00    -29.49
          55.00    -29.40
          60.00    -29.31
          65.00    -29.23
          70.00    -29.15
          75.00    -29.08
          80.00    -29.02
          85.00    -28.98
          90.00    -29.05
          95.00    -29.14
         100.00    -29.23
         105.00    -29.32
         110.00    -29.41
         115.00    -29.49
         120.00    -29.58
         125.00    -29.65
         130.00    -29.72
         135.00    -29.79
         140.00    -29.85
         145.00    -29.91
         150.00    -29.96
         155.00    -30.01
         160.00    -30.05

   * NOTE: CRITICAL ELEVATION OF  -24.55 M  EXCEEDED AT R=0 AND T=   75.59 DY


   ENTER NEXT COMMAND
   ?
 OB
   ENTER TFIRST, TLAST, DELTAT (DY)
   ?,?,?
 0.,84.,5.
   ENTER RFIRST, RLAST, DELTAR (M )
   ?,?,?
 0.


   ENTER NEXT COMMAND
   ?
 EL


   CONCENTRATION CALCULATIONS ? (Y/N)
 Y
   ENTER UNITS FOR CONCENTRATION (6 CHARACTERS)
   ?
 PPM CL
   ENTER SALT-WATER AND BACKGROUND CONCENTRATIONS (PPM CL)
   ?,?
 22000.,145.
   ENTER DISPERSIVITY (M )
   ?
 0.5
   ENTER INITIAL WIDTH OF TRANSITION ZONE (M )
   ?
 3.5
   ENTER INTERCEPTION COEFFICIENT
   ?
 0.08
```

```
SEMADAR 1 -- TEST B

  DENSITY OF FRESH WATER                                      1.0000
  DENSITY OF SALT WATER                                       1.0300

  AQUIFER POROSITY                                             .3300
  HORIZONTAL PERMEABILITY (M /DY)                            14.7000
  VERTICAL PERMEABILITY (M /DY)                              14.7000

  INITIAL INTERFACE ELEVATION (M )                          -30.7500
  DISTANCE FROM BOTTOM OF WELL TO INTERFACE (M )             15.5000
  FRACTIONAL CRITICAL RISE                                     .4000
  CRITICAL RISE (M )                                          6.2000
  CRITICAL ELEVATION (M )                                   -24.5500

  MAXIMUM STEADY-STATE PUMPING RATE (CU M /DY)              266.2818



  CONCENTRATION IN SALT WATER (PPM CL)                    22000.0000
  BACKGROUND CONCENTRATION (PPM CL)                         145.0000
  INITIAL WIDTH OF TRANSITION ZONE (M )                       3.5000

  DISPERSIVITY (M )                                            .5000
  INTERCEPTION COEFFICIENT                                     .0800



  PUMPING RATE (CU M /DY)                                   348.0000
  PUMPING PERIOD (DY)                                        84.0000

  TFIRST =      .0000   TLAST =    84.0000   DELTAT =     5.0000
  RFIRST =      .0000   RLAST =      .0000   DELTAR =      .0000

NOTE:  INTERFACE WILL RISE TO CRITICAL ELEVATION IN      75.59 DY

CONTINUE ? (Y/N)
Y
```

PUMPING RATE: 348.00 CU M /DY FOR 84.00 DY

INTERFACE ELEVATIONS (M)

T (DY) \ R (M)	.00
.00	-30.75
5.00	-29.31
10.00	-28.31
15.00	-27.57
20.00	-27.00
25.00	-26.55
30.00	-26.18
35.00	-25.88
40.00	-25.62
45.00	-25.40
50.00	-25.22
55.00	-25.05
60.00	-24.91
65.00	-24.78
70.00	-24.66
75.00	-24.56
80.00	-24.47*
84.00	-24.40*

* NOTE: CRITICAL ELEVATION OF -24.55 M EXCEEDED AT R=0 AND T= 75.59 DY

CONCENTRATION IN WELL AND PROFILES BENEATH WELL (PPM CL)

T (DY)	C(WELL) E(WELL)	ELEVATION FOR C/(E) (M)					
		145.0 (.0)	2330.5 (.1)	4516.0 (.2)	6701.5 (.3)	8887.0 (.4)	11072.5 (.5)
.00	145.17 (.0000)	-26.4	-28.5	-29.3	-29.8	-30.3	-30.8
5.00	155.81 (.0005)	-24.0*	-26.6	-27.5	-28.2	-28.8	-29.3
10.00	192.67 (.0022)	-22.4*	-25.3	-26.3	-27.1	-27.7	-28.3
15.00	244.28 (.0045)	-21.3*	-24.4*	-25.5	-26.3	-26.9	-27.6
20.00	297.22 (.0070)	-20.5*	-23.7*	-24.8	-25.6	-26.3	-27.0
25.00	345.51 (.0092)	-19.8*	-23.1*	-24.3*	-25.1	-25.9	-26.5
30.00	387.60 (.0111)	-19.3*	-22.6*	-23.9*	-24.7	-25.5	-26.2
35.00	423.69 (.0128)	-18.8*	-22.3*	-23.5*	-24.4*	-25.2	-25.9
40.00	454.52 (.0142)	-18.5*	-22.0*	-23.2*	-24.1*	-24.9	-25.6
45.00	480.92 (.0154)	-18.2*	-21.7*	-23.0*	-23.9*	-24.7	-25.4
50.00	503.65 (.0164)	-17.9*	-21.5*	-22.7*	-23.7*	-24.5*	-25.2
55.00	523.35 (.0173)	-17.7*	-21.3*	-22.6*	-23.5*	-24.3*	-25.1
60.00	540.53 (.0181)	-17.4*	-21.1*	-22.4*	-23.3*	-24.2*	-24.9
65.00	555.62 (.0188)	-17.3*	-20.9*	-22.2*	-23.2*	-24.0*	-24.8
70.00	568.94 (.0194)	-17.1*	-20.8*	-22.1*	-23.1*	-23.9*	-24.7
75.00	580.79 (.0199)	-17.0*	-20.7*	-22.0*	-23.0*	-23.8*	-24.6
80.00	591.38 (.0204)	-16.8*	-20.6*	-21.9*	-22.9*	-23.7*	-24.5*
84.00	599.06 (.0208)	-16.7*	-20.5*	-21.8*	-22.8*	-23.6*	-24.4*

* NOTE: THE DISPERSION CONCEPT SHOULD BE LIMITED TO THE ZONE BELOW THE CRITICAL ELEVATION OF -24.5500 M

CONCENTRATION IN WELL AND PROFILES BENEATH WELL (PPM CL)

T (DY)	C(WELL) E(WELL)	ELEVATION FOR C/(E) (M) 11072.5 (.5)	13258.0 (.6)	15443.5 (.7)	17629.0 (.8)	19814.5 (.9)	22000.0 (1.0)
.00	145.17 (.0000)	-30.8	-31.2	-31.7	-32.2	-33.0	-35.1
5.00	155.81 (.0005)	-29.3	-29.9	-30.4	-31.1	-32.0	-34.6
10.00	192.67 (.0022)	-28.3	-28.9	-29.5	-30.3	-31.3	-34.2
15.00	244.28 (.0045)	-27.6	-28.2	-28.9	-29.7	-30.8	-33.8
20.00	297.22 (.0070)	-27.0	-27.7	-28.4	-29.2	-30.3	-33.5
25.00	345.51 (.0092)	-26.5	-27.2	-28.0	-28.8	-30.0	-33.3
30.00	387.60 (.0111)	-26.2	-26.9	-27.6	-28.5	-29.7	-33.1
35.00	423.69 (.0128)	-25.9	-26.6	-27.4	-28.2	-29.5	-32.9
40.00	454.52 (.0142)	-25.6	-26.3	-27.1	-28.0	-29.3	-32.8
45.00	480.92 (.0154)	-25.4	-26.1	-26.9	-27.8	-29.1	-32.7
50.00	503.65 (.0164)	-25.2	-26.0	-26.8	-27.7	-29.0	-32.5
55.00	523.35 (.0173)	-25.1	-25.8	-26.6	-27.5	-28.8	-32.5
60.00	540.53 (.0181)	-24.9	-25.7	-26.5	-27.4	-28.7	-32.4
65.00	555.62 (.0188)	-24.8	-25.5	-26.4	-27.3	-28.6	-32.3
70.00	568.94 (.0194)	-24.7	-25.4	-26.3	-27.2	-28.5	-32.2
75.00	580.79 (.0199)	-24.6	-25.3	-26.2	-27.1	-28.5	-32.2
80.00	591.38 (.0204)	-24.5*	-25.2	-26.1	-27.0	-28.4	-32.1
84.00	599.06 (.0208)	-24.4*	-25.2	-26.0	-27.0	-28.3	-32.1

* NOTE: THE DISPERSION CONCEPT SHOULD BE LIMITED TO THE ZONE BELOW THE CRITICAL ELEVATION OF -24.5500 M

ENTER NEXT COMMAND
?
NP

```
 ENTER TITLE
  ?
SEMADAR 1  SALINITY/TIME RELATIONSHIPS
  ENTER UNITS FOR LENGTH (2 CHARACTERS)
  ?
M
  ENTER UNITS FOR TIME (2 CHARACTERS)
  ?
DY
  ENTER FRESH-WATER AND SALT-WATER DENSITIES
  ?,?
1.00,1.03
  ENTER AQUIFER POROSITY
  ?
0.33
  ENTER HORIZONTAL PERMEABILITY (M /DY)
  ?
14.7
  ENTER VERTICAL PERMEABILITY (M /DY)
  ?
14.7
  ENTER INITIAL INTERFACE ELEVATION (M )
  ?
-30.75
  ENTER DISTANCE FROM BOTTOM OF WELL TO INITIAL INTERFACE (M )
  ?
15.5
  ENTER FRACTIONAL CRITICAL RISE
  ?
0.4


  CONCENTRATION CALCULATIONS ? (Y/N)
Y
  ENTER UNITS FOR CONCENTRATION (6 CHARACTERS)
  ?
PPM CL
  ENTER SALT-WATER AND BACKGROUND CONCENTRATIONS (PPM CL)
  ?,?
22000.,145.
  ENTER DISPERSIVITY (M )
  ?
0.5
  ENTER INITIAL WIDTH OF TRANSITION ZONE (M )
  ?
3.5
  ENTER INTERCEPTION COEFFICIENT
  ?
0.08
```

SEMADAR 1 SALINITY/TIME RELATIONSHIPS

DENSITY OF FRESH WATER	1.0000
DENSITY OF SALT WATER	1.0300
AQUIFER POROSITY	.3300
HORIZONTAL PERMEABILITY (M /DY)	14.7000
VERTICAL PERMEABILITY (M /DY)	14.7000
INITIAL INTERFACE ELEVATION (M)	-30.7500
DISTANCE FROM BOTTOM OF WELL TO INTERFACE (M)	15.5000
FRACTIONAL CRITICAL RISE	.4000
CRITICAL RISE (M)	6.2000
CRITICAL ELEVATION (M)	-24.5500
MAXIMUM STEADY-STATE PUMPING RATE (CU M /DY)	266.2818

CONCENTRATION IN SALT WATER (PPM CL)	22000.0000
BACKGROUND CONCENTRATION (PPM CL)	145.0000
INITIAL WIDTH OF TRANSITION ZONE (M)	3.5000
DISPERSIVITY (M)	.5000
INTERCEPTION COEFFICIENT	.0800

```
  ENTER NEXT COMMAND
  ?
PR

  ENTER MAXIMUM PERMISSIBLE CONCENTRATION IN PUMPED WATER (PPM CL)
  ?
166.85
```

```
SEMADAR 1  SALINITY/TIME RELATIONSHIPS

  DENSITY OF FRESH WATER                                      1.0000
  DENSITY OF SALT WATER                                       1.0300

  AQUIFER POROSITY                                             .3300
  HORIZONTAL PERMEABILITY (M /DY)                            14.7000
  VERTICAL PERMEABILITY (M /DY)                              14.7000

  INITIAL INTERFACE ELEVATION (M )                          -30.7500
  DISTANCE FROM BOTTOM OF WELL TO INTERFACE (M )             15.5000
  FRACTIONAL CRITICAL RISE                                     .4000
  CRITICAL RISE (M )                                          6.2000
  CRITICAL ELEVATION (M )                                   -24.5500

  CONCENTRATION IN SALT WATER (PPM CL)                    22000.0000
  BACKGROUND CONCENTRATION (PPM CL)                         145.0000
  INITIAL WIDTH OF TRANSITION.ZONE (M )                       3.5000

  DISPERSIVITY (M )                                            .5000
  INTERCEPTION COEFFICIENT                                     .0800



  MAXIMUM CONCENTRATION IN PUMPED WATER (PPM CL)            166.8500
  MAXIMUM RELATIVE CONCENTRATION                               .0010
  MAXIMUM INTERFACE ELEVATION (M )                          -28.8973
  MAXIMUM PERMISSIBLE PUMPING RATE (CU M /DY)                79.5689

 ENTER OPTIONAL PUMPING RATE (CU M /DY)
 ?
348.

 PUMPING RATE (CU M /DY)         348.0000
 TIME TO REACH CMAX (DY)           6.8762

 ENTER OPTIONAL PUMPING RATE (CU M /DY)
 ?
575.

 PUMPING RATE (CU M /DY)         575.0000
 TIME TO REACH CMAX (DY)           3.7256

 ENTER OPTIONAL PUMPING RATE (CU M /DY)
 ?
0.

 ENTER NEXT COMMAND
 ?
PR

 ENTER MAXIMUM PERMISSIBLE CONCENTRATION IN PUMPED WATER (PPM CL)
 ?
210.56
```

```
SEMADAR I  SALINITY/TIME RELATIONSHIPS

  DENSITY OF FRESH WATER                                     1.0000
  DENSITY OF SALT WATER                                      1.0300

  AQUIFER POROSITY                                            .3300
  HORIZONTAL PERMEABILITY (M /DY)                           14.7000
  VERTICAL PERMEABILITY (M /DY)                             14.7000

  INITIAL INTERFACE ELEVATION (M )                         -30.7500
  DISTANCE FROM BOTTOM OF WELL TO INTERFACE (M )            15.5000
  FRACTIONAL CRITICAL RISE                                    .4000
  CRITICAL RISE (M )                                         6.2000
  CRITICAL ELEVATION (M )                                  -24.5500

  CONCENTRATION IN SALT WATER (PPM CL)                   22000.0000
  BACKGROUND CONCENTRATION (PPM CL)                        145.0000
  INITIAL WIDTH OF TRANSITION ZONE (M )                      3.5000

  DISPERSIVITY (M )                                           .5000
  INTERCEPTION COEFFICIENT                                    .0800



  MAXIMUM CONCENTRATION IN PUMPED WATER (PPM CL)           210.5600
  MAXIMUM RELATIVE CONCENTRATION                              .0030
  MAXIMUM INTERFACE ELEVATION (M )                         -28.0165
  MAXIMUM PERMISSIBLE PUMPING RATE (CU M /DY)              117.4009

 ENTER OPTIONAL PUMPING RATE (CU M /DY)
 ?
348.


 PUMPING RATE (CU M /DY)         348.0000
 TIME TO REACH CMAX (DY)          11.8100

 ENTER OPTIONAL PUMPING RATE (CU M /DY)
 ?
575.


 PUMPING RATE (CU M /DY)         575.0000
 TIME TO REACH CMAX (DY)           5.9515

 ENTER OPTIONAL PUMPING RATE (CU M /DY)
 ?
0.


 ENTER NEXT COMMAND
 ?
PR


 ENTER MAXIMUM PERMISSIBLE CONCENTRATION IN PUMPED WATER (PPM CL)
 ?
245.27
```

```
SEMADAR 1  SALINITY/TIME RELATIONSHIPS

   DENSITY OF FRESH WATER                                      1.0000
   DENSITY OF SALT WATER                                       1.0300

   AQUIFER POROSITY                                             .3300
   HORIZONTAL PERMEABILITY (M /DY)                            14.7000
   VERTICAL PERMEABILITY (M /DY)                              14.7000

   INITIAL INTERFACE ELEVATION (M )                          -30.7500
   DISTANCE FROM BOTTOM OF WELL TO INTERFACE (M )             15.5000
   FRACTIONAL CRITICAL RISE                                     .4000
   CRITICAL RISE (M )                                          6.2000
   CRITICAL ELEVATION (M )                                   -24.5500

   CONCENTRATION IN SALT WATER (PPM CL)                    22000.0000
   BACKGROUND CONCENTRATION (PPM CL)                         145.0000
   INITIAL WIDTH OF TRANSITION ZONE (M )                       3.5000

   DISPERSIVITY (M )                                            .5000
   INTERCEPTION COEFFICIENT                                     .0800



   MAXIMUM CONCENTRATION IN PUMPED WATER (PPM CL)            245.2700
   MAXIMUM RELATIVE CONCENTRATION                               .0046
   MAXIMUM INTERFACE ELEVATION (M )                          -27.5565
   MAXIMUM PERMISSIBLE PUMPING RATE (CU M /DY)               137.1553

 ENTER OPTIONAL PUMPING RATE (CU M /DY)
 ?
348.

 PUMPING RATE (CU M /DY)          348.0000
 TIME TO REACH CMAX (DY)           15.0899

 ENTER OPTIONAL PUMPING RATE (CU M /DY)
 ?
575.

 PUMPING RATE (CU M /DY)          575.0000
 TIME TO REACH CMAX (DY)            7.2666

 ENTER OPTIONAL PUMPING RATE (CU M /DY)
 ?
0.

 ENTER NEXT COMMAND
 ?
PR

 ENTER MAXIMUM PERMISSIBLE CONCENTRATION IN PUMPED WATER (PPM CL)
 ?
363.55
```

```
SEMADAR 1  SALINITY/TIME RELATIONSHIPS

  DENSITY OF FRESH WATER                                     1.0000
  DENSITY OF SALT WATER                                      1.0300

  AQUIFER POROSITY                                            .3300
  HORIZONTAL PERMEABILITY (M /DY)                           14.7000
  VERTICAL PERMEABILITY (M /DY)                             14.7000

  INITIAL INTERFACE ELEVATION (M )                         -30.7500
  DISTANCE FROM BOTTOM OF WELL TO INTERFACE (M )            15.5000
  FRACTIONAL CRITICAL RISE                                    .4000
  CRITICAL RISE (M )                                         6.2000
  CRITICAL ELEVATION (M )                                  -24.5500

  CONCENTRATION IN SALT WATER (PPM CL)                   22000.0000
  BACKGROUND CONCENTRATION (PPM CL)                        145.0000
  INITIAL WIDTH OF TRANSITION ZONE (M )                      3.5000

  DISPERSIVITY (M )                                           .5000
  INTERCEPTION COEFFICIENT                                    .0800



  MAXIMUM CONCENTRATION IN PUMPED WATER (PPM CL)           363.5500
  MAXIMUM RELATIVE CONCENTRATION                              .0100
  MAXIMUM INTERFACE ELEVATION (M )                         -26.3880
  MAXIMUM PERMISSIBLE PUMPING RATE (CU M /DY)              187.3442

 ENTER OPTIONAL PUMPING RATE (CU M /DY)
 ?
348.

 PUMPING RATE (CU M /DY)          348.0000
 TIME TO REACH CMAX (DY)           27.0509

 ENTER OPTIONAL PUMPING RATE (CU M /DY)
 ?
575.

 PUMPING RATE (CU M /DY)          575.0000
 TIME TO REACH CMAX (DY)           11.2107

 ENTER OPTIONAL PUMPING RATE (CU M /DY)
 ?
0.

 ENTER NEXT COMMAND
 ?
PR

 ENTER MAXIMUM PERMISSIBLE CONCENTRATION IN PUMPED WATER (PPM CL)
 ?
582.1
```

```
SEMADAR 1  SALINITY/TIME RELATIONSHIPS

   DENSITY OF FRESH WATER                                    1.0000
   DENSITY OF SALT WATER                                     1.0300

   AQUIFER POROSITY                                           .3300
   HORIZONTAL PERMEABILITY (M /DY)                          14.7000
   VERTICAL PERMEABILITY (M /DY)                            14.7000

   INITIAL INTERFACE ELEVATION (M )                        -30.7500
   DISTANCE FROM BOTTOM OF WELL TO INTERFACE (M )           15.5000
   FRACTIONAL CRITICAL RISE                                   .4000
   CRITICAL RISE (M )                                        6.2000
   CRITICAL ELEVATION (M )                                 -24.5500

   CONCENTRATION IN SALT WATER (PPM CL)                  22000.0000
   BACKGROUND CONCENTRATION (PPM CL)                       145.0000
   INITIAL WIDTH OF TRANSITION ZONE (M )                     3.5000

   DISPERSIVITY (M )                                          .5000
   INTERCEPTION COEFFICIENT                                   .0800



   MAXIMUM CONCENTRATION IN PUMPED WATER (PPM CL)          582.1000
   MAXIMUM RELATIVE CONCENTRATION                             .0200
   MAXIMUM INTERFACE ELEVATION (M )                        -24.5511
   MAXIMUM PERMISSIBLE PUMPING RATE (CU M /DY)             266.2366

 ENTER OPTIONAL PUMPING RATE (CU M /DY)
 ?
348.

 PUMPING RATE (CU M /DY)          348.0000
 TIME TO REACH CMAX (DY)           75.5346

 ENTER OPTIONAL PUMPING RATE (CU M /DY)
 ?
575.

 PUMPING RATE (CU M /DY)          575.0000
 TIME TO REACH CMAX (DY)           20.0023

 ENTER OPTIONAL PUMPING RATE (CU M /DY)
 ?
0.

 ENTER NEXT COMMAND
 ?
PR

 ENTER MAXIMUM PERMISSIBLE CONCENTRATION IN PUMPED WATER (PPM CL)
 ?
800.65
```

```
SEMADAR 1  SALINITY/TIME RELATIONSHIPS

   DENSITY OF FRESH WATER                                      1.0000
   DENSITY OF SALT WATER                                       1.0300

   AQUIFER POROSITY                                             .3300
   HORIZONTAL PERMEABILITY (M /DY)                            14.7000
   VERTICAL PERMEABILITY (M /DY)                              14.7000

   INITIAL INTERFACE ELEVATION (M )                          -30.7500
   DISTANCE FROM BOTTOM OF WELL TO INTERFACE (M )             15.5000
   FRACTIONAL CRITICAL RISE                                     .4000
   CRITICAL RISE (M )                                          6.2000
   CRITICAL ELEVATION (M )                                   -24.5500

   CONCENTRATION IN SALT WATER (PPM CL)                    22000.0000
   BACKGROUND CONCENTRATION (PPM CL)                         145.0000
   INITIAL WIDTH OF TRANSITION ZONE (M )                       3.5000

   DISPERSIVITY (M )                                            .5000
   INTERCEPTION COEFFICIENT                                     .0800



   MAXIMUM CONCENTRATION IN PUMPED WATER (PPM CL)            800.6500
   MAXIMUM RELATIVE CONCENTRATION                               .0300
   MAXIMUM INTERFACE ELEVATION (M )                          -22.2571*
   MAXIMUM PERMISSIBLE PUMPING RATE (CU M /DY)               364.7604

 * NOTE: THE DISPERSION CONCEPT SHOULD BE LIMITED TO THE ZONE BELOW
         THE CRITICAL ELEVATION OF     -24.5500 M
 (MAXIMUM CONCENTRATIONS IN PUMPED WATER LESS THAN     582.10 PPM CL)


 ENTER OPTIONAL PUMPING RATE (CU M /DY)
  ?
575.


 PUMPING RATE (CU M /DY)          575.0000
 TIME TO REACH CMAX (DY)           40.2467

 ENTER OPTIONAL PUMPING RATE (CU M /DY)
  ?
0.


 ENTER NEXT COMMAND
  ?
PR


 ENTER MAXIMUM PERMISSIBLE CONCENTRATION IN PUMPED WATER (PPM CL)
  ?
1019.2
```

```
 SEMADAR 1  SALINITY/TIME RELATIONSHIPS

   DENSITY OF FRESH WATER                                       1.0000
   DENSITY OF SALT WATER                                        1.0300

   AQUIFER POROSITY                                              .3300
   HORIZONTAL PERMEABILITY (M /DY)                             14.7000
   VERTICAL PERMEABILITY (M /DY)                               14.7000

   INITIAL INTERFACE ELEVATION (M )                           -30.7500
   DISTANCE FROM BOTTOM OF WELL TO INTERFACE (M )              15.5000
   FRACTIONAL CRITICAL RISE                                      .4000
   CRITICAL RISE (M )                                           6.2000
   CRITICAL ELEVATION (M )                                    -24.5500

   CONCENTRATION IN SALT WATER (PPM CL)                     22000.0000
   BACKGROUND CONCENTRATION (PPM CL)                          145.0000
   INITIAL WIDTH OF TRANSITION ZONE (M )                        3.5000

   DISPERSIVITY (M )                                             .5000
   INTERCEPTION COEFFICIENT                                      .0800



   MAXIMUM CONCENTRATION IN PUMPED WATER (PPM CL)  1019.2000
   MAXIMUM RELATIVE CONCENTRATION                             .0400
   MAXIMUM INTERFACE ELEVATION (M )                         -13.1997*
   MAXIMUM PERMISSIBLE PUMPING RATE (CU M /DY)             753.7639

 * NOTE: THE DISPERSION CONCEPT SHOULD BE LIMITED TO THE ZONE BELOW
          THE CRITICAL ELEVATION OF     -24.5500 M
 (MAXIMUM CONCENTRATIONS IN PUMPED WATER LESS THAN     582.10 PPM CL)


 ENTER OPTIONAL PUMPING RATE (CU M /DY)
  ?
0.

 ENTER NEXT COMMAND
  ?
DN
Stop - Program terminated.

C>
```

APPENDIX B

Description of Program UPCONE

Program UPCONE has been written in an unextended Fortran computer code in an effort to make the program transportable between computer systems. The code consists of a main program and two function subroutines. The program has been documented internally through the liberal use of comment statements.

The main program is divided into three sections. Section I provides for the "Basic Input Data" as described in Section III of this report. The numerical evaluation of interface elevations, concentrations, and maximum pumping rates is accomplished in Section II of the main program which contains the computational algorithms for Case I and Case II types of problems. Section III provides for problem redefinition and control of execution under the "Edit" mode.

Two function subroutines are required to calculate the concentration distribution across the transition zone or to evaluate the maximum pumping rate for a specified salinity of the pumped water. Function ERFC (Z) is a rational approximation of the complimentary error function of the argument Z.

Function IERFC (X, Z, ZMIN, ZMAX) uses a *regula falsi* root finding technique to find the argument, Z, for a specified value of the complimentary error function, X. The last two arguments, ZMIN and ZMAX, define the lower and upper limits of the initial search interval.

APPENDIX C

Listing of Program UPCONE

```
C     PROGRAM UPCONE
C     VERSION 1.0
C     JAN WAGNER
C     SCHOOL OF CHEMICAL ENGINEERING
C     OKLAHOMA STATE UNIVERSITY
C     STILLWATER, OK  74078
C     TELEPHONE (405) 624-5280
C
C     JANUARY, 1982
C
      DIMENSION A(30),C(11),CW(51),E(11),EW(51),IC(20),KFLG(11),
     1R(26),T(51),Z(26,26)
      REAL*4 KX,KZ
      DATA KHAR1,KHAR2,NY/' ','*','Y'/
      DATA IC/'FD','PO','KX','KZ','ZO','DT','CR','CO','DI','WT','IC',
     1'OB','TC','RC','QP','LI','EL','PR','NP','DN'/
C
C     READ DEVICE: NI=1       WRITE DEVICE: NO=1
      NI = 1
      NO = 1
C
C     MAXIMUM NUMBERS OF OBSERVATION POINTS FOR TIME AND RADIUS
C     HAVE BEEN SET AT 50 AND 25, RESPECTIVELY
C      DIMENSION CW(M),EW(M),R(N),T(N),Z(M,N)
C          -- WHERE M=MAXTIM+1 AND N=MAXPTS+1
      MAXTIM = 50
      MAXPTS = 25
C
C     INITIALIZE PROGRAM FLOW PARAMETERS
    1 IEDIT = 1
      KCON = 1
      KELE = 1
C
C ***** SECTION I -- BASIC INPUT DATA
C
C     READ TITLE
      WRITE(NO,5)
    5 FORMAT('1',3X,'ENTER TITLE',/,3X,'?')
      READ(NI,10) (A(I),I=1,30)
   10 FORMAT(30A2)
C
C     DEFINE UNITS
      WRITE(NO,15)
   15 FORMAT(3X,'ENTER UNITS FOR LENGTH (2 CHARACTERS)',/,3X,'?')
      READ(NI,20) IL
   20 FORMAT(A2)
      WRITE(NO,25)
   25 FORMAT(3X,'ENTER UNITS FOR TIME (2 CHARACTERS)',/,3X,'?')
      READ(NI,20) ITU
C
C     FLUID DENSITIES
   29 WRITE(NO,30)
   30 FORMAT(3X,'ENTER FRESH-WATER AND SALT-WATER DENSITIES',/,3X,'?,?')
      READ(NI,35) RHOF,RHOS
   35 FORMAT(2F10.0)
   40 IF(RHOS.GT.0.0) GO TO 55
      WRITE(NO,45)
   45 FORMAT(3X,'SALT-WATER DENSITY MUST BE GREATER THAN ZERO',
     1' -- REENTER',/,3X,'?')
      READ(NI,50) RHOS
   50 FORMAT(F10.0)
      GO TO 40
   55 IF(RHOF.GT.0.0.AND.RHOF.LT.RHOS) GO TO 69
      WRITE(NO,60)
   60 FORMAT(3X,'FRESH-WATER DENSITY MUST BE GREATER THAN ZERO',
     1/,6X,'AND LESS THAN SALT-WATER DENSITY -- REENTER',/,3X,'?')
      READ(NI,50) RHOF
      GO TO 55
   69 GO TO (70,700), IEDIT
C
```

```
C      POROSITY
   70 WRITE(NO,75)
   75 FORMAT(3X,'ENTER AQUIFER POROSITY',/,3X,'?')
   80 READ(NI,50) PO
      IF(PO.GT.0.0.AND.PO.LT.1.0) GO TO 89
      WRITE(NO,85)
   85 FORMAT(3X,'POROSITY MUST BE GREATER THAN ZERO AND LESS THAN',
     1' ONE -- REENTER',/,3X,'?')
      GO TO 80
   89 GO TO (90,700), IEDIT
C
C      HORIZONTAL PERMEABILITY
   90 WRITE(NO,95) IL,ITU
   95 FORMAT(3X,'ENTER HORIZONTAL PERMEABILITY (',A2,'/',A2,') ',
     1/,3X,'?')
  100 READ(NI,50) KX
      IF(KX.GT.0.0) GO TO 110
      WRITE(NO,105)
  105 FORMAT(3X,'HORIZONTAL PERMEABILITY MUST BE GREATER THAN',
     1' ZERO -- REENTER',/,3X,'?')
      GO TO 100
  110 GO TO (111,700), IEDIT
C
C      VERTICAL PERMEABILITY
  111 WRITE(NO,112) IL,ITU
  112 FORMAT(3X,'ENTER VERTICAL PERMEABILITY (',1A2,'/',A2,') ',
     1/,3X,'?')
  114 READ(NI,50) KZ
      IF(KZ.GT.0.0) GO TO 119
      WRITE(NO,115)
  115 FORMAT(3X,'VERTICAL PERMEABILITY MUST BE GREATER THAN ZERO',
     1' -- REENTER',/,3X,'?')
      GO TO 114
  119 GO TO (120,700), IEDIT
C
C      INITIAL INTERFACE ELEVATION
  120 WRITE(NO,125) IL
  125 FORMAT(3X,'ENTER INITIAL INTERFACE ELEVATION (',A2,')',/,3X,'?')
      READ(NI,50) ZO
      GO TO (129,700), IEDIT
C
C      DISTANCE FROM BOTTOM OF WELL TO INITIAL INTERFACE
  129 WRITE(NO,130) IL
  130 FORMAT(3X,'ENTER DISTANCE FROM BOTTOM OF WELL TO INITIAL ',
     1'INTERFACE (',A2,') ',/,3X,'?')
  135 READ(NI,50) D
      IF(D.GT.0.0) GO TO 144
      WRITE(NO,140)
  140 FORMAT(3X,'DISTANCE MUST BE GREATER THAN ZERO -- REENTER',/,
     13X,'?')
      GO TO 135
  144 GO TO (145,700), IEDIT
C
C      FRACTIONAL CRITICAL RISE
  145 WRITE(NO,150)
  150 FORMAT(3X,'ENTER FRACTIONAL CRITICAL RISE',/,3X,'?')
  155 READ(NI,50) THETA
      IF(THETA.GT.0.0.AND.THETA.LT.1.0) GO TO 164
      WRITE(NO,160)
  160 FORMAT(3X,'FRACTION MUST BE GREATER THAN ZERO AND LESS THAN',
     1' ONE -- REENTER',/,3X,'?')
      GO TO 155
  164 XCR = THETA*D
      ZCR = XCR + ZO
C      MAXIMUM STEADY-STATE PUMPING RATE
      QMAXSS = 6.283185*((RHOS-RHOF)/RHOF)*KX*D*XCR
  165 GO TO (169,700), IEDIT
C
C
C      DATA FOR CONCENTRATION CALCULATIONS
```

```
  169 WRITE(NO,170)
  170 FORMAT('0',2X,'CONCENTRATION CALCULATIONS ? (Y/N)')
      READ(NI,175) ICON
  175 FORMAT(A1)
C
      IF(ICON.NE.NY) GO TO 275
C
  176 KCON = 2
      WRITE(NO,180)
  180 FORMAT(3X,'ENTER UNITS FOR CONCENTRATION (6 CHARACTERS)',
     1/,3X,'?')
      READ(NI,185) IM1,IM2,IM3
  185 FORMAT(3A2)
C
C     SALT-WATER AND BACKGROUND CONCENTRATIONS
  188 IF(KCON.EQ.2) GO TO 189
      WRITE(NO,180)
      READ(NI,185) IM1,IM2,IM3
  189 WRITE(NO,190) IM1,IM2,IM3
  190 FORMAT(3X,'ENTER SALT-WATER AND BACKGROUND CONCENTRATIONS (',
     13A2,') ',/,3X,'?,?')
      READ(NI,35) CO,CB
  195 IF(CO.GE.1.0) GO TO 205
      WRITE(NO,200)
  200 FORMAT(3X,'SALT-WATER CONCENTRATION MUST BE GREATER THAN',
     1' OR EQUAL TO ONE -- REENTER',/,3X,'?')
      READ(NI,50) CO
      GO TO 195
  205 IF(CB.GE.0.0.AND.CB.LT.CO) GO TO 214
      WRITE(NO,210)
  210 FORMAT(3X,'BACKGROUND CONCENTRATION MUST BE GREATER THAN',
     1' OR EQUAL TO ZERO',/,6X,'AND LESS THAN SALT-WATER',
     2' CONCENTRATION -- REENTER',/,3X,'?')
      READ(NI,50) CB
      GO TO 205
  214 GO TO (215,700,215,215), IEDIT
C
C     DISPERSIVITY
  215 WRITE(NO,220) IL
  220 FORMAT(3X,'ENTER DISPERSIVITY (',A2,') ',/,3X,'?')
  225 READ(NI,50) DM
      IF(DM.GT.0.0) GO TO 234
      WRITE(NO,230)
  230 FORMAT(3X,'DISPERSIVITY MUST BE GREATER THAN ZERO -- REENTER',
     1/,3X,'?')
      GO TO 225
  234 GO TO (235,700,235,235), IEDIT
C
C     INITIAL WIDTH OF TRANSITION ZONE
  235 WRITE(NO,240) IL
  240 FORMAT(3X,'ENTER INITIAL WIDTH OF TRANSITION ZONE (',A2,') ',
     1/,3X,'?')
  245 READ(NI,50) SO2
      IF(SO2.GE.0.0) GO TO 254
      WRITE(NO,250)
  250 FORMAT(3X,'INITIAL WIDTH MUST BE GREATER THAN ZERO -- REENTER',
     1/,3X,'?')
      GO TO 245
  254 GO TO (255,700,255,255), IEDIT
C
C     INTERCEPTION COEFFICIENT
  255 WRITE(NO,260)
  260 FORMAT(3X,'ENTER INTERCEPTION COEFFICIENT',/,3X,'?')
  265 READ(NI,50) PHI
      IF(PHI.GT.0.0.AND.PHI.LT.1.0) GO TO 274
      WRITE(NO,270)
  270 FORMAT(3X,'COEFFICIENT MUST BE GREATER THAN ZERO AND LESS THAN',
     1' ONE -- REENTER',/,3X,'?')
      GO TO 265
  274 GO TO (275,700,275,510), IEDIT
```

```
C
C     LIST PROBLEM DEFINITION
C
  275 CONTINUE
      WRITE(NO,280) (A(I),I=1,30), RHOF,RHOS,PO,IL,ITU,KX,IL,ITU,KZ
  280 FORMAT('1',3X,30A2,//,
     16X,'DENSITY OF FRESH WATER',25X,F10.4,/,
     26X,'DENSITY OF SALT WATER',26X,F10.4,//,
     36X,'AQUIFER POROSITY',31X,F10.4,/,
     46X,'HORIZONTAL PERMEABILITY (',A2,'/',A2,') ',15X,F10.4,/,
     56X,'VERTICAL PERMEABILITY (',A2,'/',A2,') ',17X,F10.4)
      WRITE(NO,281)IL, ZO,IL,D,THETA,IL,XCR,IL,ZCR
  281 FORMAT('0',
     15X,'INITIAL INTERFACE ELEVATION (',A2,') ',14X,F10.4,/,
     26X,'DISTANCE FROM BOTTOM OF WELL TO INTERFACE (',A2,') ',F10.4,/,
     36X,'FRACTIONAL CRITICAL RISE ',22X,F10.4,/,
     46X,'CRITICAL RISE (',A2,') ',28X,F10.4,/,
     56X,'CRITICAL ELEVATION (',A2,') ',23X,F10.4)
C
      WRITE(NO,285) IL,ITU,QMAXSS
  285 FORMAT('0',5X,'MAXIMUM STEADY-STATE PUMPING RATE (CU ',A2,'/',
     1A2,')    ',F10.4,/,'0',/,'0')
C
      IF(ICON.NE.NY.OR.KCON.EQ.1) GO TO 700
C
      WRITE(NO,290) IM1,IM2,IM3,CO,IM1,IM2,IM3,CB,IL,SO2,IL,DM,PHI
  290 FORMAT('0',5X,'CONCENTRATION IN SALT WATER (',3A2,') ',10X,F10.4,/
     16X,'BACKGROUND CONCENTRATION (',3A2,') ',13X,F10.4,/,
     26X,'INITIAL WIDTH OF TRANSITION ZONE ('A2,') ',9X,F10.4,//,
     36X,'DISPERSIVITY (',A2,') ',29X,F10.4,/,
     46X,'INTERCEPTION COEFFICIENT',23X,F10.4,/,'0',/,'0')
C
      GO TO (700,700,301), IEDIT
C
C
C ***** SECTION II -- NUMERICAL EVALUATION OF INTERFACE ELEVATIONS
C
C
C     CASE I PROBLEMS -- EVALUATE INTERFACE ELEVATIONS AND CONCENTRATION
C
C
  300 CONTINUE
      IEDIT = 3

C     PARAMETERS FOR INTERFACE ELEVATION CALCULATIONS
  301 IF(KELE.EQ.2) GO TO 329
      KELE = 2
C
C     PUMPING RATE AND PERIOD
  302 WRITE(NO,305) IL,ITU,ITU
  305 FORMAT('0',2X,'ENTER PUMPING RATE (CU ',A2,'/',A2,') AND',
     1' PERIOD (',A2,')',/,3X,'?,?')
      READ(NI,35) Q,TPUMP
  306 IF(Q.GT.0.0) GO TO 308
      WRITE(NO,307)
  307 FORMAT(3X,'PUMPING RATE MUST BE GREATER THAN ZERO -- REENTER',
     1/,3X,'?')
      READ(NI,50) Q
      GO TO 306
  308 IF(TPUMP.GT.0.0) GO TO 310
      WRITE(NO,309)
  309 FORMAT(3X,'PUMPING PERIOD MUST BE GREATER THAN ZERO -- REENTER',
     1/,3X,'?')
      READ(NI,50) TPUMP
      GO TO 308
  310 GO TO (700,700,312), IEDIT
C
C     COORDINATES OF OBSERVATION POINTS -- TIME AND RADIUS
  311 IEDIT = 1
  312 WRITE(NO,313) ITU
```

```
 313 FORMAT(3X,'ENTER TFIRST, TLAST, DELTAT (',A2,') ',/,3X,'?,?,?')
     READ(NI,315) TF,TL,DELT
 315 FORMAT(3F10.0)
     DELT = ABS(DELT)
 316 IF(TF.GE.0.0.AND.DELT.LE.1.0E-06) GO TO 320
     IF(TF.GE.0.0) GO TO 318
     WRITE(NO,317)
 317 FORMAT(3X,'TFIRST MUST NOT BE LESS THAN ZERO -- REENTER',
    1/,3X,'?')
     READ(NI,50) TF
     GO TO 316
 318 IF(TL.GE.0.0) GO TO 321
     WRITE(NO,319)
 319 FORMAT(3X,'TLAST MUST NOT BE LESS THAN ZERO -- REENTER',
    1/,3X,'?')
     READ(NI,50) TL
     GO TO 318
 320 TL = TF
 321 GO TO (322,700,322), IEDIT
C
 322 WRITE(NO,323) IL
 323 FORMAT(3X,'ENTER RFIRST, RLAST, DELTAR (',A2,') ',/,3X,'?,?,?')
     READ(NI,325) RF,RL,DELR
 325 FORMAT(3F10.0)
     DELR = ABS(DELR)
     GO TO (700,700,329), IEDIT
C
 329 WRITE(NO,330) IL,ITU,Q,ITU,TPUMP,TF,TL,DELT,RF,RL,DELR
 330 FORMAT('0',5X,'PUMPING RATE (CU ',A2,'/',A2,') ',23X,F10.4,/,
    16X,'PUMPING PERIOD (',A2,') ',27X,F10.4,//,
    26X,'TFIRST =',F10.4,3X,'TLAST =',F10.4,3X,'DELTAT =',F10.4,/,
    36X,'RFIRST =',F10.4,3X,'RLAST =',F10.4,3X,'DELTAR =',F10.4)
     IF(Q.LE.QMAXSS) GO TO 332
     TCR = ((2.*PO*D)/(((RHOS-RHOF)/RHOF)*KZ))*((1./(1.-QMAXSS/Q))-1.)
     WRITE(NO,331) TCR,ITU
 331 FORMAT('0',2X,'NOTE:  INTERFACE WILL RISE TO CRITICAL ELEVATION',
    1' IN',F10.2,A3)
 332 WRITE(NO,333)
 333 FORMAT('0',2X,'CONTINUE ? (Y/N)')
     READ(NI,175) JFLOW
     IF(JFLOW.NE.NY) GO TO 700
C
C     RADIUS COORDINATES
 335 CONTINUE
     IR = 1
     R(IR) = RF
     IF(DELR.LE.1.0E-06) GO TO 345
     DIF = RL - RF
     IF(ABS(DIF).LE.1.0E-06) GO TO 345
     IF(DIF.LE.0.0) DELR = -DELR
     NPTS = DIF/DELR
     REM = DIF - DELR*FLOAT(NPTS)
     TOL = 1.0E-06*ABS(DIF)
     NPTS = NPTS + 1
     IF(NPTS.LE.MAXPTS) GO TO 337
     WRITE(NO,336) NPTS,MAXPTS
 336 FORMAT(3X,I3,' RADIUS OBSERVATION POINTS EXCEED MAXIMUM OF',I4)
     GO TO 700
 337 CONTINUE
     DO 340  IR=2,NPTS
      R(IR) = R(IR-1) + DELR
 340 CONTINUE
     IR = NPTS
     IF(ABS(REM).LT.TOL) GO TO 345
     IR = IR + 1
     R(IR) = RL
 345 CONTINUE
C
C     TIME COORDINATES
     IT = 1
```

```
      T(IT) = TF
      IF(DELT.LE.1.0E-06) GO TO 355
      DIF = TL - TF
      IF(ABS(DIF).LE.1.0E-06) GO TO 355
      IF(DIF.LE.0.0) DELT = -DELT
      NPTS = DIF/DELT
      REM = DIF - DELT*FLOAT(NPTS)
      TOL = 1.0E-06*ABS(DIF)
      NPTS = NPTS + 1
      IF(NPTS.LE.MAXTIM) GO TO 347
      WRITE(NO,346) NPTS,MAXTIM
  346 FORMAT(3X,I3,' TIME OBSERVATION POINTS EXCEED MAXIMUM OF',I4)
      GO TO 700
  347 CONTINUE
      DO 350  IT=2,NPTS
       T(IT) = T(IT-1) + DELT
  350 CONTINUE
      IT = NPTS
      IF(ABS(REM).LT.TOL) GO TO 355
      IT = IT + 1
      T(IT) = TL
  355 CONTINUE
      TMAX = TL
      IF(TF.GT.TL) TMAX=TF
C
C
      COEF = Q/(6.2832*((RHOS-RHOF)/RHOF)*KX*D)
      CONR = SQRT(KZ/KX)/D
      CONT = ((RHOS-RHOF)/RHOF)*KZ/(2.0*PO*D)
      DO 365  I=1,IT
       TAU = CONT*T(I)
       TAU1 = CONT*(T(I)-TPUMP)
       IF(T(I).LE.TPUMP) TAU1=0.0
       XRO = COEF*(1.0/(1.0+TAU1) - 1.0/(1.0+TAU))
       ZRO = XRO + ZO
       DO 360  J=1,IR
          RDIM = CONR*R(J)
          Z(I,J) = COEF*((1.0/SQRT((1.0+TAU1)**2 + RDIM**2))
     1
  360  CONTINUE
      IF(ZRO.GT.ZCR) IPR=2
  365 CONTINUE
  370 IT = I
C
C     PRINT INTERFACE ELEVATIONS
      WRITE(NO,375) Q,IL,ITU,TPUMP,ITU,IL,IL
  375 FORMAT('1',18X,'PUMPING RATE:',F12.2,' CU ',A2,'/',A2,' FOR',
     1F10.2,A3,/,'0',18X,'INTERFACE ELEVATIONS (',A2,') ',//,
     23X,' *',/,3X,'    * R (',A2,')')
      LIM1 = 1
      LIM2 = 7
  380 IF(LIM2.GT.IR) LIM2=IR
      WRITE(NO,385) (R(L),L=LIM1,LIM2)
  385 FORMAT(3X,'       *     ',7F9.2)
      WRITE(NO,390) ITU
  390 FORMAT(3X,'T (',A2,') *',/,12X,'*')
      DO 400 I=1,IT
       DO 393  L=LIM1,LIM2
       KFLG(L) = KHAR1
       IF(Z(I,L).GT.ZCR) KFLG(L)=KHAR2
  393  CONTINUE
       WRITE(NO,395) T(I),(Z(I,L),KFLG(L),L=LIM1,LIM2)
  395  FORMAT(5X,F8.2,1X,7(F8.2,A1))
  40AX.GE.TCR)
     1
  405 FORMAT('0',2X,'* NOTE: CRITICAL ELEVATION OF',F8.2,A3,
     1' EXCEEDED AT R=0 AND T=',F8.2,A3)
      IF(LIM2.EQ.IR) GO TO 415
      LIM1 = LIM1 + 7
      LIM2 = LIM2 + 7
```

```
      WRITE(NO,410) IL,IL
  410 FORMAT('1',18X,'INTERFACE ELEVATIONS (',A2,') (CONTINUED)',//,
     13X,' *',/,3X,'     * R (',A2,')')
      GO TO 380
  415 CONTINUE
      IF(ICON.NE.NY) GO TO 700
C
C     CONCENTRATION PROFILES
      SO = SO2/2.0
      E(1) = 0.0
      C(1) = CB
      DO 420  K=2,11
       E(K) = E(K-1) + 0.1
       C(K) = E(K)*(CO-CB) + CB
  420 CONTINUE
C
UP = CONT*TPUMP
       XBAR1 = COEF*(1.0-1.0/(1.0+TAUP))
       TAU1 = CONT*(T(I)-TPUMP)
       XTOT = COEF*((1.0/(1.0+TAU1)) - (1.0/(1.0+TAU)))
       XBAR2 = XBAR1 - XTOT
       XBAR = XBAR1 + XBAR2
       GO TO 440
  435  XTOT = COEF*(1.0 - 1.0/(1.0+TAU))
       XBAR = XTOT
  440  CONTINUE
       S1 = SQRT(SO**2 + 2.0*DM*XBAR)
       ARG = 10.0
       IF(S1.GT.0.0) ARG = (XCR-XTOT)/(1.414214*S1)
       EZCR = 0.5*ERFC(ARG)
       EW(I) = 0.5*EZCR*PHI
       CW(I) = EW(I)*(CO-CB) + CB
       Z(I,1) = XTOT + 2.5*S1 + ZO
       Z(I,11) = XTOT - 2.5*S1 + ZO
       XLIM1 = 2.0
       XLIM2 = 0.0
       DO 445  K=2,5
          CERF = 2.0*E(K)
          CALL IERFC(CERF,ARG,XLIM1,XLIM2)
          DIST = 1.41421*S1*ARG
          Z(I,K) = XTOT + DIST + ZO
          L = 12-K
          Z(I,L) = XTOT - DIST + ZO
  445  CONTINUE
       Z(I,6) = XTOT + ZO
  446 CONTINUE
      LIM1 = 1
      LIM2 = 6
 IN WELL AND PROFILES BENEATH WELL',
     1' (',3A2,')',//,11X,'T      C(WELL)',14X,'ELEVATION FOR C/(E)  (',
     2A2,') ',/,9X,'(',A2,')    E(WELL)')
      WRITE(NO,430) (C(K),K=LIM1,LIM2),(E(K),K=LIM1,LIM2)
  430 FORMAT(24X,6(F8.1,1X),/,24X,6('    (',F3.1,')'))
  431 DO 455  I=1,IT
       DO 449  K=LIM1,LIM2
          KFLG(K) =  KHAR1
          IF(Z(I,K).GT.ZCR) KFLG(K) = KHAR2
  449  CONTINUE
       WRITE(NO,450) T(I),CW(I),(Z(I,K),KFLG(K),K=LIM1,LIM2),EW(I)
  450  FORMAT(/,6X,F8.2,F9.2,1X,6(F8.1,A1),/,16X,('(',F6.4,')'))
  455 CONTINUE
      WRITE(NO,457) ZCR,IL
  457 FORMAT('0',2X,'* NOTE: THE DISPERSION CONCEPT SHOULD BE LIMITED',
     1' TO THE ZONE BELOW',/,11X,' THE CRITICAL ELEVATION OF',F12.4,A3)
      IF(LIM2.EQ.11) GO TO 700
      LIM1 = 6
      LIM2 = 11
      GO TO 447
C
C
```

```
NCENTRATION IN PUMPED WATER
C
  500 CONTINUE
      IEDIT = 4
      IF(KCON.NE.2) GO TO 176
  510 WRITE(NO,515) IM1,IM2,IM3
  515 FORMAT('0',2X,'ENTER MAXIMUM PERMISSIBLE CONCENTRATION IN PUMPED',
     1' WATER (',3A2,')',/,3X,'?')
  516 READ(NI,50) CMAX
      IF(CMAX.GE.CB.AND.CMAX.LT.CO) GO TO 518
      WRITE(NO,517)
  517 FORMAT(3X,'CONCENTRATION MUST BE GREATER THAN OR EQUAL TO CB',/,
     13X,'AND LESS THAN CO -- REENTER',
     2/,3X,'?')
      GO TO 516
  518 CONTINUE
C
C
      SO = SO2/2.0
      EMAX = (CMAX-CB)/(CO-CB)
      EXCR = EMAX/(0.5*PHI)
      ELIM = 0.0
      IF(SO.GT.0.0) ELIM = 0.5*ERFC(XCR/(1.1414214*SO))
      CINIT = 0.5*PHI*ELIM*(CO-CB) + CB
      IF(EXCR.LT.ELIM) GO TO 550
      IF(EXCR.LE.0.0.OR.EXCR.GE.1.0) GO TO 520
      CERF = 2.0*EXCR
      XLIM1 = 3.0
      XLIM2 = 0.0
      CALL IERFC(CERF,ARG,XLIM1,XLIM2)
      B = -4.0*ARG*ARG*DM - 2.0*XCR
      CON = -2.0*ARG*ARG*SO*SO + XCR*XCR
      GO TO 521
  520 B = -12.5*DM - 2.0*XCR
      CON = -6.25*SO*SO + XCR*XCR
  521 CONTINUE
      ROOT = B*B - 4.0*CON
      IF(ROOT.LT.0.0) GO TO 650
      XBAR1 = (-B-(ROOT**0.5))/2.0
      XBAR2 = (-B+(ROOT**0.5))/2.0
      XBAR = XBAR1
      IF(EXCR.GT.0.5) XBAR=XBAR2
      ZMAX = XBAR + ZO
      JFLG = KHAR1
      IF(ZMAX.GT.ZCR) JFLG=KHAR2
      QMAX = 6.283185*((RHOS-RHOF)/RHOF)*KX*D*XBAR
C
      WRITE(NO,280) (A(I),I=1,30),RHOF,RHOS,PO,IL,ITU,KX,IL,ITU,KZ
      WRITE(NO,281) IL,ZO,IL,D,THETA,IL,XCR,IL,ZCR
      WRITE(NO,290) IM1,IM2,IM3,CO,IM1,IM2,IM3,CB,IL,SO2,IL,DM,PHI
      WRITE(NO,525) IM1,IM2,IM3,CMAX,EMAX,IL,ZMAX,JFLG,IL,ITU,QMAX
  525 FORMAT('0',5X,'MAXIMUM CONCENTRATION IN PUMPED WATER (',3A2,')',
     11X,F10.4,/,
     26X,'MAXIMUM RELATIVE CONCENTRATION',17X,F10.4,/,
     36X,'MAXIMUM INTERFACE ELEVATION (',A2,')',15X,F10.4,A1,/,
     46X,'MAXIMUM PERMISSIBLE PUMPING RATE (CU ',A2,'/',A2,')',4X,
     5F10.4)
      IF(ZMAX.LE.ZCR) GO TO 530
      WRITE(NO,457) ZCR,IL
      CLIM = 0.5*(0.5*PHI)*(CO-CB) + CB
      WRITE(NO,527) CLIM,IM1,IM2,IM3
  527 FORMAT(3X,'(MAXIMUM CONCENTRATIONS IN PUMPED WATER LESS THAN',
     1F10.2,A3,2A2,') ',/,'0')
C
  530 WRITE(NO,535) IL,ITU
  535 FORMAT('0',2X,'ENTER OPTIONAL PUMPING RATE (CU ',A2,'/',A2,')',
     1/,3X,'?')
      READ(NI,50) QP
      IF(QP.LE.QMAX) GO TO 600
      TIME = ((2.*PO*D)/(((RHOS-RHOF)/RHOF)*KZ))*((1./(1.-QMAX/QP))-1.)
```

```
      WRITE(NO,545) IL,ITU,QP,ITU,TIME
  545 FORMAT('O',2X,'PUMPING RATE (CU ',A2,'/',A2,')',6X,F10.4,/
     1
      GO TO 530
C
  550 WRITE(NO,555) SO2,IL,CINIT,IM1,IM2,IM3
  555 FORMAT('O',2X,'CMAX EXCEEDED FOR AN INITIAL TRANSITION ',
     1'ZONE WIDTH OF',F10.4,A3,/,
     23X,'INITIAL CONCENTRATION IN PUMPED WATER IS',F10.4,A3,2A2)
  600 CONTINUE
      GO TO 700
C
  650 WRITE(NO,655)
  655 FORMAT(3X,'IMAGINARY ROOT OBTAINED FOR XMAX')
      GO TO 700
C
C
C ***** SECTION III -- PROBLEM REDEFINITION AND CONTROL OF EXECUTION
C
C
  700 CONTINUE
      IEDIT = 2
      XCR = THETA*D
      ZCR = XCR + ZO
      QMAXSS = 6.283185*((RHOS-RHOF)/RHOF)*KX*D*XCR
      WRITE(NO,705)
  705 FORMAT(//,3X,'ENTER NEXT COMMAND',/,3X,'?')
  710 READ(NI,715) NEXT
  715 FORMAT(A2)
C
      DO 720  I=1,20
       IF(NEXT.EQ.IC(I)) GO TO 730
  720 CONTINUE
      WRITE(NO,725)
  725 FORMAT(3X,'ERROR IN LAST COMMAND -- REENTER',/,3X,'?')
      GO TO 710
  730 GO TO (29,70,90,111,120,129,145,188,215,235,255,
     1311,312,322,302,275,300,500,1,1000),I
C
 1000 STOP
      END
```

APPENDIX D

Listing of Function Subroutines

```
      FUNCTION ERFC(Z)
C     RATIONAL APPROXIMATION OF THE COMPLIMENTARY ERROR FUNCTION
C     SEE SECTION 7.1 OF ABRAMOWITZ AND STEGUN (1966)
      IF(ABS(Z).GT.7.5) GO TO 30
C     THE FOLLOWING IDENTITIES ARE USED TO HANDLE NEGATIVE ARGUMENTS
C        ERFC( Z) = 1 - ERF(Z)
C        ERFC(-Z) = -ERFC(Z) = 1 + ERF(Z)
C
      X = Z
C     NEGATIVE ARGUMENTS
      IF (Z.LT.0.0) X = -Z
      ERFC =       1.0/((1.0 + 0.070523*X + 0.042282*(X**2)
     1                 + 0.009270*(X**3) + 0.000152*(X**4)
     2                 + 0.000276*(X**5) + 0.000043*(X**6))**16)
C     NOTE: 2-ERF(X) = ERFC(-X) = ERFC(Z) FOR Z<0
      IF (Z.LT.0.0) ERFC = 2.0 - ERFC
      RETURN
C
C     FOR Z>7.5, ERFC(Z)<2.32E-22 AND IS SET TO 0
   30 ERFC = 0.0
C     FOR Z < -7.5  ERFC(Z) IS SET TO 2.0
      ERFC = 2.0
      RETURN
      END
```

```
C     INVERSE COMPLIMENTARY ERROR FUNCTION
      SUBROUTINE IERFC(CERF,X,XLH,XRH)
C     INVERSE COMPLIMENTARY ERROR FUNCTION
C     A REGULA FALSI ROOT-FINDING TECHNIQUE IS USED TO
C     LOCATE X FOR A SPECIFIED VALUE OF CERF.  XLH AND
C     XRH DEFINE THE INITIAL SEARCH INTERVAL.
C     SEE CARNAHAN, LUTHER AND WILKES (1969) FOR A
C     DISCUSSION OF THE METHOD.
      FLH = CERF - ERFC(XLH)
      FRH = CERF - ERFC(XRH)
   10 XEST = (XLH*FRH - XRH*FLH)/(FRH - FLH)
      FEST = CERF - ERFC(XEST)
      IF(ABS(FEST).GT.1.0E-04) GO TO 20
      X = XEST
      RETURN
   20 IF(FEST.GT.0.0.AND.FRH.GT.0.0) GO TO 30
      XLH = XEST
      FLH = CERF - ERFC(XLH)
      GO TO 10
   30 XRH = XEST
      FRH = CERF - ERFC(XRH)
      GO TO 10
      END
```