Technical Project Completion Report
Project No. B-010-Okla.

ECONOMIC EFFICIENCY IN THE ALLOCATION OF IRRIGATION WATER OVER TIME

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by
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ABSTRACT

The acreage of irrigated crop production in the region overlying the Central Ogallala Formation has increased from approximately 70,000 in 1950 to 1.5 million in 1965. Annual water withdrawals have exceeded estimated annual recharge since 1954. Resource availability and the demand for the crops produced in the area suggest irrigation will continue to expand.

Projections made assuming current institutional restrictions indicate the number of acres irrigated annually will increase to a peak of 3.4 million in 1990 and then decline to 2.8 million by the year 2000. The projected rate of development utilizes 43.5 percent of the stock water supply by the year 2000, making irrigation uneconomic on approximately 44 percent of the area.

Estimated primary economic benefits are approximately $50 million per year for the 1970-1990 period and then decline rapidly. Application of a multi-stage sequential decision model indicated the projected rates of withdrawal during the remainder of this century do not exceed the rates required to maximize the present value of net primary benefits for the study area.

Analysis of institutional constraints indicates imposing a quantity restriction of 1.5 acre feet annually per acre irrigated would reduce primary economic benefits both per acre irrigated and per unit of water used. However, imposing a tax of $6.00 per acre foot of water used above 1.5 acre feet per acre irrigated would increase benefits both per acre irrigated and per unit of water used. Additional analysis of alternative taxing arrangements is needed before a specific plan is recommended for use in the Central Ogallala.

KEYWORDS

Water Allocation, Ground Water, Irrigation Water, Economic Efficiency, Economics, Direct Benefits, Farm Management, Water Demand, Irrigation Practices, Forecasting, Oklahoma, Texas, Kansas, Ground Water Mining, Dynamic Programming, Linear Programming, Computer Models, Simulation Analysis, Soil-Water-Plant Relationships, Great Plains, Water Policy, Water Resources Development, Progressive Taxes, Water Management (Applied), Yield Equations

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INTRODUCTION

The Ogallala Formation is an unconsolidated aquifer named after the town Ogallala, Nebraska. This formation underlies most of the Great Plains area, extending from the southern half of South Dakota to a few miles north of the Pecos River in southern Texas. The sediments that compose the formation are believed to have been eroded from the Rocky Mountains and carried by streams to be deposited in the eroded and dissected surfaces of the pre-Ogallala rocks ranging in age from Permian to Cretaceous.[1] The easterly gradient of the base of the formation is attributed to this phenomenon. After the cuts and dips in these rocks were filled, water continued to shift and deposit the sediment over the entire area described above. Conditions changed over time and streams started to cut into the unconsolidated deposits [7, pp. 7-8]. The North Platt River, the Arkansas River and the Canadian River have cut completely through the formation into the older rocks. Consequently, unconnected distinct subdivisions of the Ogallala Formation can be identified.

This study is concerned with water use in the central part of the Ogallala Formation bounded by the Arkansas River on the north and the Canadian River on the south (Figure 1). Some portions of the area are underlain by other formations (e.g., the Dakota and Cheyenne sandstones on the west) which also provide water for users living in the area. Some portions of the Central Ogallala are very thin (such as the extreme eastern

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[1] Permian era, 235,000,000 years ago, reptiles developed, conifers were abundant; Cretaceous era, 130,000,000 years ago, extinction of dinosaurs.

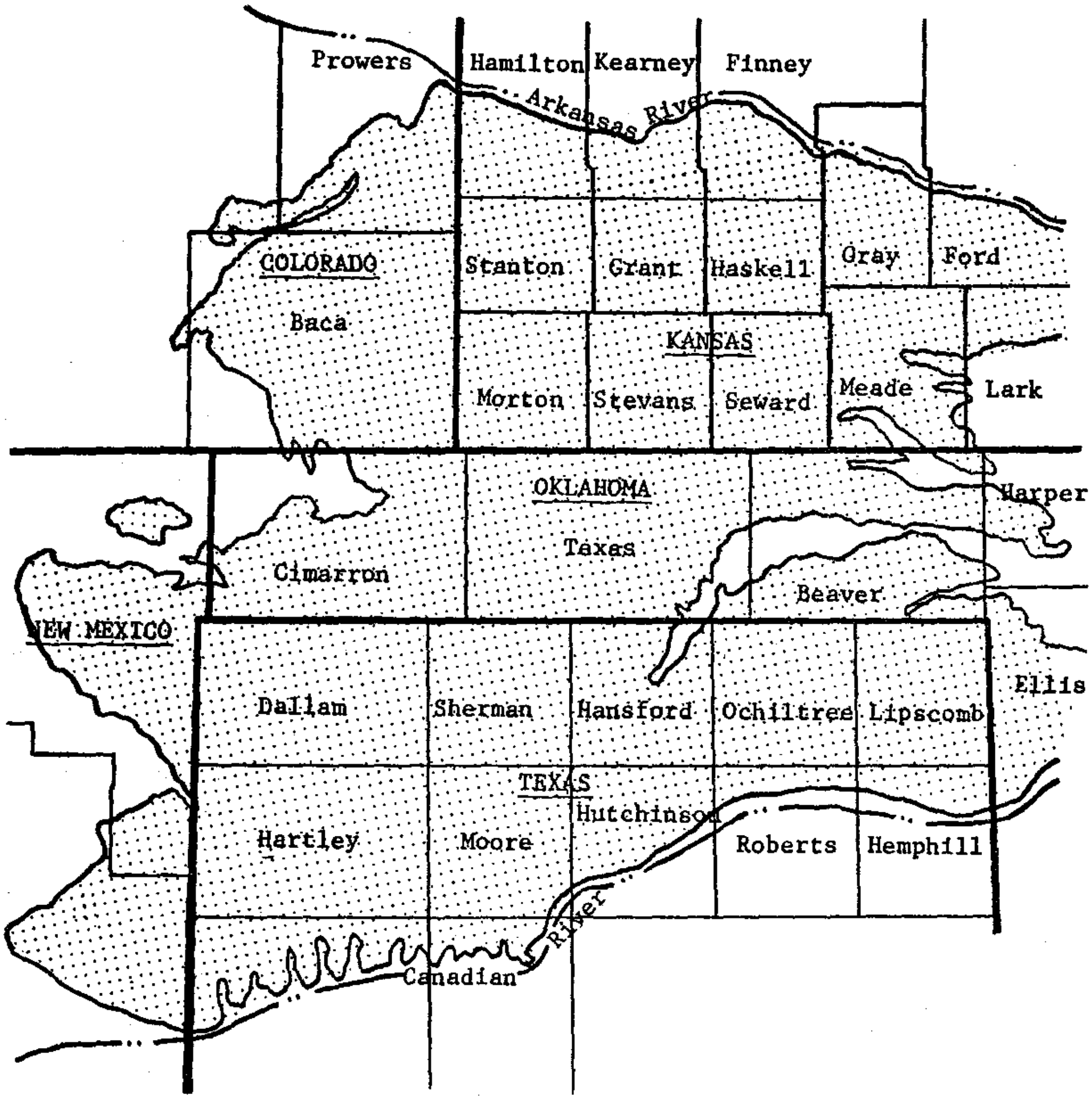

Figure 1. The Study Area

parts) and are not capable of supplying large quantities of water. Thus the boundaries of the study area were limited to those portions of the Central Ogallala that (1) represent either large actual or potential use and (2) that obtain their water supply primarily from the Ogallala. The boundaries of the study area are shown in Figure 1. The portion of the Central Ogallala considered in this project includes a small portion of two counties in southeastern Colorado, eight counties in southwestern Kansas, the three Panhandle counties of Oklahoma and seven counties in the northern part of the Texas High Plains. The land area overlying this hydrologic subdivision is approximately 17,500 miles.

Agriculture is the major primary industry upon which the economy of the study area is dependent. The production of wheat, grain sorghum and beef cattle dominates the type of agriculture practiced. Due to the relative shortness of the growing season, high-valued cash crops like cotton and peanuts cannot be grown successfully in much of the area. The area does not appear to have a competitive advantage in the production of high value fruit or vegetable crops. Thus it does not appear the region will shift from production of feed grains, food grains and beef. Wheat and sorghum are the principal crops. In the census year 1964, these two crops accounted for 92.1 percent of the total irrigated acreage of the eight main irrigated crops in the area and 98.6 percent of the total dryland acreage of the eight crops [4, p. 185].

In recent years cattle feeding in the study area has been characterized by rapidly increasing numbers of large-scale commercial feedlots. The number of cattle fed in such enterprises in the study area have increased from one and one-fourth million in 1967 to two and one-fourth million in

1969 [76]. The area overlying the Central Ogallala, once a large feed grain surplus area, currently consumes as much feed grain as produced in the average year. This area is expected to have continued growth in the cattle feeding industry. This growth is expected to provide a market for the feed grains and forages produced in the area from an expanded irrigated acreage.

Irrigation wells to tap the Central Ogallala Formation were drilled as early as 1932, but the greatest development has occurred since 1950. The advent of large economical and efficient pumping systems coupled with the severe drought of 1952-56 accelerated the growth of irrigation. The protion of the study area in Texas experienced the most rapid growth in irrigation both in absolute and relative terms followed by Oklahoma, Kansas and Colorado in that order. The breakdown of irrigation development for the period 1950-1965 by state is given in Table I. Figure 2 shows the growth of irrigated acres for each state. During the 1950-65 period the number of irrigated acres increased from 17,000 to 1,003,000 in Texas, from 10,000 to 117,000 in Oklahoma, from 34,000 to 279,000 in Kansas and from 9,000 to 29,000 in Colorado. By 1965, 13.7 percent of the total study area was irrigated.

The net volume of water withdrawn from the Central Ogallala Formation has continued to increase with the expansion of irrigation. While average annual recharge is estimated to be about .27 million acre feet [4,p.193], the annual withdrawal of water from the aquifer in recent years has exceeded two million acre feet. As shown in the last column of Table I, the first overdraft of the aquifer occurred around 1954 when a net of 113,650 acre feet of water was pumped. By 1965 the overdraft had increased to over 2.7 million acre feet per year. The amount of water withdrawn for

TABLE I

ESTIMATED NUMBER OF IRRIGATED ACRES AND ACRE FEET OF GROUND WATER APPLIED IN THE STUDY AREA 1950-1965[1]

	Colorado		Kansas		Oklahoma		Texas		Total		Year to Year Change in Total		
Year	Irrigated Acres	Acre Feet Withdrawn	Irrigated Acres	Acre Feet Withdrawn	Irrigated Acres	Acre Feet Withdrawn	Irrigated Acres	Acre Feet Withdrawn	Irrigated Acres	Acre Feet Withdrawn	Irrigated Acres	Acre Feet Withdrawn	Net Withdrawal[2]
1950	8,584	14,593	34,012	57,820	10,024	17,041	16,944	28,805	69,564	118,259	11,203	23,084	-151,819
1951	9,027	15,797	42,085	73,649	10,458	18,302	19,197	33,595	80,767	141,343	16,193	46,758	-128.735
1952	9,470	18,372	53,376	103,549	10,892	21,130	23,222	45,050	96,960	188,101	32,214	70,247	- 81,977
1953	9,913	19,826	67,120	134,240	16,985	33,970	35,156	70,312	129,174	258,348	64,626	125,376	- 11,730
1954	10,356	20,505	86,904	172,070	23,078	45,694	73,462	145,455	193,800	383,724	110,109	254,486	113,646
1955	12,097	25,404	135,745	285,065	35,478	74,504	120,589	253,237	303,909	638,210	196,716	618,358	368,132
1956	13,838	34,733	150,942	378,864	64,456	161,785	271,389	681,186	500,625	1,256,568	119,379	14,440	986,490
1957	15,580	31,939	235,693	483,171	69,124	141,704	299,607	614,194	620,004	1,271,008	13,941	-167,943	1,000,093
1958	16,213	28,211	249,573	434,257	61,567	107,127	306,592	533,470	633,945	1,103,065	28,337	29,437	832,987
1959	16,846	28,807	256,409	438,459	63,280	108,209	325,747	557,027	662,282	1,132,502	39,439	60,424	862,424
1960	18,940	32,198	270,670	460,139	63,390	107,763	348,721	592,826	701,721	1,192,926	29,356	93,770	922,848
1961	21,034	37,020	279,516	491,948	63,500	111,760	367,027	645,968	731,077	1,286,696	64,462	153,230	1,016,618
1962	23,128	41,862	299,865	542,756	63,609	115,132	408,937	740,176	795,539	1,439,926	188,937	588,937	1,169,848
1963	25,222	51,957	322,176	663,683	73,962	152,362	563,116	1,160,018	984,476	2,028,020	297,428	958,817	1,749,942
1964	27,314	63,642	347,999	810,838	95,443	222,382	811,148	1,889,975	1,281,904	2,986,837	246,885	-663,078	2,708,759
1965	29,406	44,697	379,248	576,457	116,925	177,726	1,003,210	1,524,879	1,528,789	2,323,759			

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[1] Estimated rate of water application taken from "Ground Water in the Cimarron River Basin", 1966, prepared by the U. S. Geologic Survey Water Resource Division for the U. S. Corps of Engineers, Tulsa District, p. 33.

[2] Acre feet applied minus recharge. Negative figures indicate a net addition to storage.

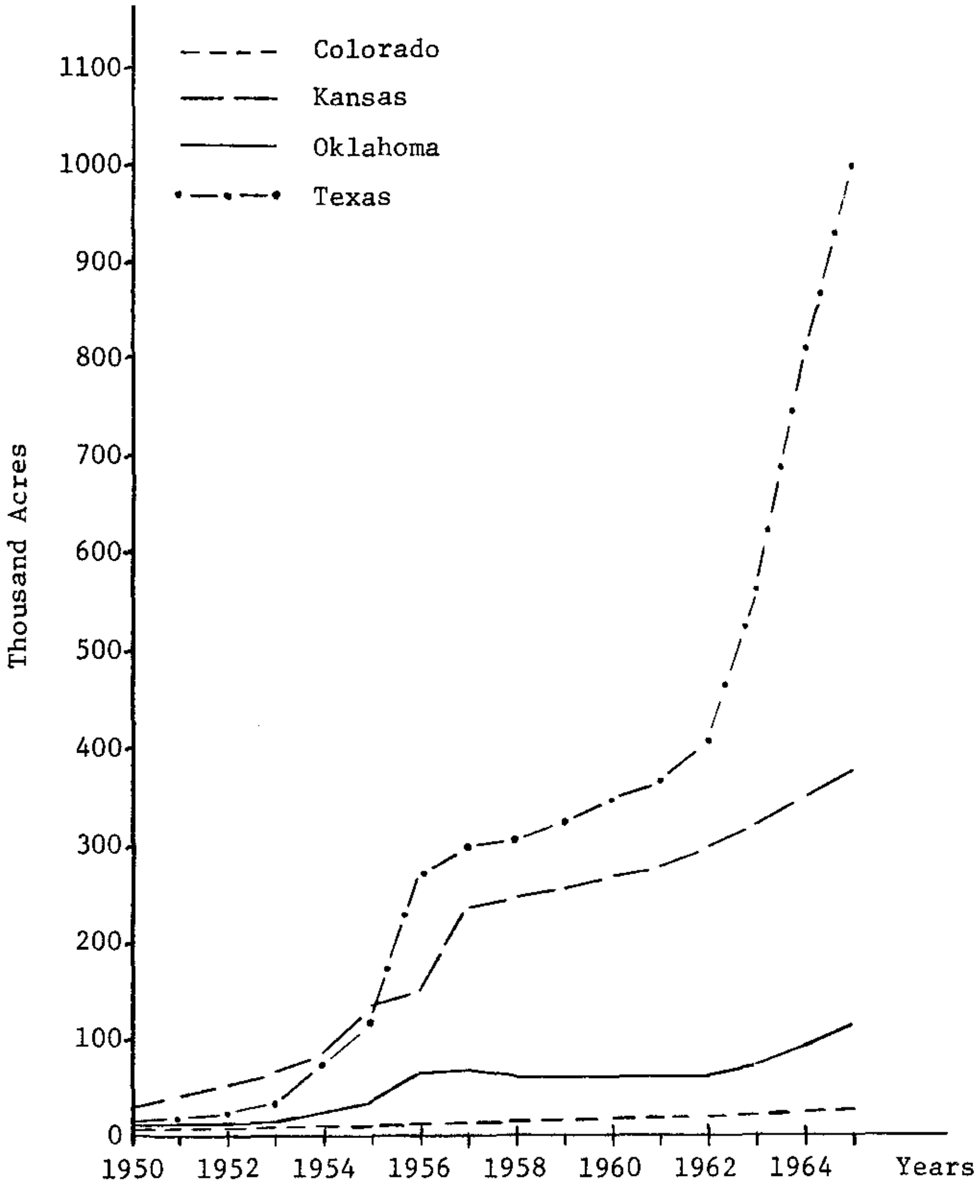

Figure 2. Growth of Irrigation in the Study Area by States 1950-1965

irrigation is expected to increase annually during the next several years. This implies the rate of annual overdraft will be even greater in the future.

The consequence of continued overdraft of the aquifer is a reduction in thickness of the water-saturated material and an increase in the pump lift, thereby increasing the per-unit cost of recovering water from the aquifer. Several studies of the average annual decline in the static water level have been reported in various areas of the Central Basin of the Ogallala Formation [7, pp. 41-49 and 26, pp. 14-32]. These studies indicate the average annual decline in most counties has exceeded two feet since 1963. Continued overdraft is expected to result in significant declines throughout the entire study area.

THE PROBLEM AND OBJECTIVES OF THE PROJECT

Developments of the past decade clearly indicate that further decline in the water table and the quantity of water in storage in the Central Ogallala Formation will occur over time. As the water table declines the unit cost of pumping water increases. *Ceteris paribus*, net returns per acre irrigated will decrease as time proceeds. Eventually it will be uneconomical to pump water for irrigation purposes in some parts of the study area. This implies resources once committed to irrigated production will have to revert to dryland farming. The adjustment from irrigation to dryland farming will result in serious primary and secondary reductions of income in the study area. Primary reduction of income entails the higher net returns per acre of production foregone and some of the resources abandoned in switching to dryland farming. The secondary reduction involves the losses attributed to reduced land prices, and the economic slump created through the multiplier effect by the reduction of demand for inputs and services that compliment irrigated crop production in the study area. How severe the adjustments to the declining water table will be is, in part, determined by how fast the ground water is depleted and, in part, by the actions taken to lessen its adverse affects. Consequently, the questions of concern to land owners, farm operators, businessmen and policy makers alike are (1) what is the economic life of the water supply? (2) what will the economic adjustments in irrigated crop production entail? and (3) what can be done to mitigate the adverse effects of the declining water supply over time? The purpose of this project is to estimate some of the expected changes so that community leaders and policy makers can address themselves to the measures necessary to ease the adverse economic effects of the ground water depletion.

The objectives listed in the project outline are:

(1) To estimate the irrigation demand for water in the high plains area overlying the Ogallala Formation between the Canadian and Arkansas Rivers,

(2) To evaluate the effect of alternative methods of water use regulation on:

    (a) The representative farm organization,

    (b) The supply of the various agricultural commodities that will be produced, and

    (c) The water availability in future years.

(3) To estimate the primary benefits from irrigation under each method of water use regulation.

DEGREE OF ACHIEVEMENT OF PROJECT OBJECTIVES

The principal investigator working in conjunction with two Ph.D. research assistants fulfilled the stated objectives. An extension of time and the availability of funds provided by the Oklahoma Agricultural Experiment Station provided the resources to expand the objectives and complete additional work not mentioned in the original project outline. The objectives of the expanded project can be stated as follows:

1. To project irrigation development, the rate of water use and aquifer depletion over time for the Central Basin of the Ogallala Formation under current institutional constraints,
2. To test whether the projected rate of basin-wide withdrawals represents a potential misallocation of the water resource over time,
3. To evaluate the effect of three alternative methods of water-use regulation on representative farms in the study area, and
4. To estimate the primary benefits from irrigation under projected use of the stock water supply with current institutional restrictions and determine the effect of each method of water-use regulation on the patterns of primary benefits realized.

Work under the first objective projected the rate of water withdrawal from the Central Ogallala Formation for the period 1970-2000. The number of acres irrigated is the major factor affecting annual water withdrawals. Water withdrawals required by the projected irrigated acreage were combined with the estimated amount of water required for municipal, industrial and other nonirrigation purposes to project total water use for the study area. Work under this objective estimates (1) the growth of irrigation in the study area, (2) the rate of depletion of the aquifer over time, and its effects on the pattern of (a) irrigated crop production and (b) net receipts to crop production over time. Projections were made assuming irrigated acreage in the area is restricted to approximately 1.5 million per year. A second set of projections were completed assuming irrigators are permitted to expand irrigated acreage subject to its profitability and current institutional restrictions.

An objective was added to the project at this point. The second objective of the expanded project, to test whether the projected rate of basin-wide withdrawals represents a potential misallocation of the water resource over time, was not included in the original proposal. A multistage sequential decision model (dynamic programming model) was used to generate the rates of ground water withdrawals for the study area that maximize the present value of net receipts to irrigated crop production over time. This model evaluates the effects of removing alternative quantities of ground water at different points in time on the net output of subsequent periods. The withdrawal rates that maximize the basin's net output for the planning horizon are selected as the optimal intertemporal allocation of the ground water resource.

Work completed under objective 3 of the project evaluated the effect of three alternative means of water use regulation on the amount of water used, production levels and net returns of representative farms in the study area. A model simulating crop yields based on soil moisture conditions over the growing season was developed for the study area. A firm simulation model based on this crop production routine was used to analyze the effect of alternative means of water use regulation at the firm level. The three water-use alternatives analyzed were (1) continued pumping at the present rate with no restrictions on water use, (2) restricting the quantity of water pumped per year to 1.5 acre feet per acre of water rights, and (3) restricting the quantity of water pumped per year to 1.5 acre feet per acre of water rights, but allowing the irrigator to apply additional irrigation water if it is economically feasible to pay a tax of $0.50 per acre inch for each acre inch pumped above the quantity limitation. The effect of the three methods of water-use regulation on net farm income, variability of net farm income, net worth, variability of net worth, quantity of water pumped and availability of water for future periods were compared. The alternative methods of restricting water use were also compared by discounting the streams of net returns and comparing present values of those net income streams.

Estimates of primary economic benefits (objective 4) were developed for the two projected rates of irrigation development discussed under objective 1. The net income of representative farms analyzed under objective 3 were used to analyze the impact of each method of water-use regulation on primary irrigation benefits for the study area.

The remainder of this narrative report is composed of six sections. The first four sections discuss the research procedures and analyses for objectives 1 through 4, respectively. The fifth section lists the significant conclusions of the project and the final section lists publications developed.

PROJECTION OF IRRIGATION DEVELOPMENT

Research Procedures

The analytic model used to project (1) the growth of irrigation in the study area, (2) the rate of underground water withdrawal, and (3) the pattern of irrigated crop production over time is a recursive linear programming (RLP) model. Basically, the RLP model is an adaptation of the static linear programming (LP) model to changing conditions of time that necessitate the revision of parts of the LP model for period t + 1 based upon the solution of period t and conditions that prevail in period t + 1. The revision may involve the objective function, the input-output coefficients, the right-hand side restrictions, or any combination of them. James M. Henderson and Richard H. Day have been instrumental in developing and popularizing the RLP model [35, 15, 17 and 83].

The RLP model is well suited for the projection of irrigation development in the Central Ogallala. First, as the overdraft of the aquifer continues, the decline in the water table causes changes in water costs and water availability from one production period to the next; and secondly, the area's supply of crops and irrigation acreage changes from period to period according to an *a priori* projection, which will be discussed later, all of which necessitates revisions in the production model. Moreover, the solutions of the RLP model (1) constitute an optimum with respect to maximizing net returns, (2) yield the levels of the various crops grown, their irrigated and dryland acreages and (3) give the level of inputs used in the production process. Obtaining all of the results in one package is desirable in achieving the first objective of this project.

One major difficulty in the predictive application of the RLP model to regional supply response studies is the task of aggregating. There are too many farms in the study area to treat each of them as an individual decision making unit in the empirical analysis. Therefore, some level of aggregation is necessary if empirical analysis of the problem is to be of a manageable size. In the past, agricultural economists have been using two approaches. The first approach uses a micro technique of programming representative farms to get optimum solutions and then multiplying them by the number of farms each representative farm represents and sum these products to arrive at the aggregate solution. However, the summing procedure introduces what has been called "aggregation bias" common to such micro techniques. The aggregation bias and the problems of using the micro programming approach have been the subject of wide discussion [28 and 89]. The second approach used is a macro technique in which the region is defined as the unit of inquiry rather than the farm, thus yielding aggregate results directly. Implicitly, the macro programming approach considers the whole region or study area as the decision making unit. While eliminating one type of aggregation bias it creates another type in that problems of resource allocation within the farm are completely bypassed. Fixed resources such as tractors, irrigation wells and equipment belong to individual farms and additional investment in such factors of production may depend on equity positions which the macro approach ignores. In addition, implications to the farm firm cannot be made as easily and directly as in the micro programming approach of the representative farm. The advantage of using the macro programming approach lies largely in the fact that data requirements and the time and cost of analysis are substantially less than the micro programming approach. Furthermore, using budgets whose costs and returns are specified on a per acre basis in an LP model may yield approximately the same aggregate values via both models [89 and 16].

In the light of the advantages and disadvantages of both the micro and macro programming approach, it should be emphasized that the answers sought from the empirical analysis should dictate the approach chosen. Since the analysis under objective one is to investigate the availability of ground water in future years and its implications for the entire study area, the macro programming approach is used. The entire area is regarded as a single producing unit stratified by the various combinations of soil and water resource situations. Each soil and water resource stratum is associated with a set of cost and return parameters in the production of the various irrigated crops. The problem to be solved is the combination and levels of crop enterprises to be produced among the various soil and water resource situation strata at different points in time that will maximize total net returns to the study area.

An Overall Description

The RLP production model shown in the flow diagram of Figure 3, has two computational aspects. The first part is a linear programming model that maximizes net returns above total costs subject to a set of restrictions specified for period t. The second part is an updating process in which changes external to the first part are computed and employed in revising the parameters of the linear programming model for the next

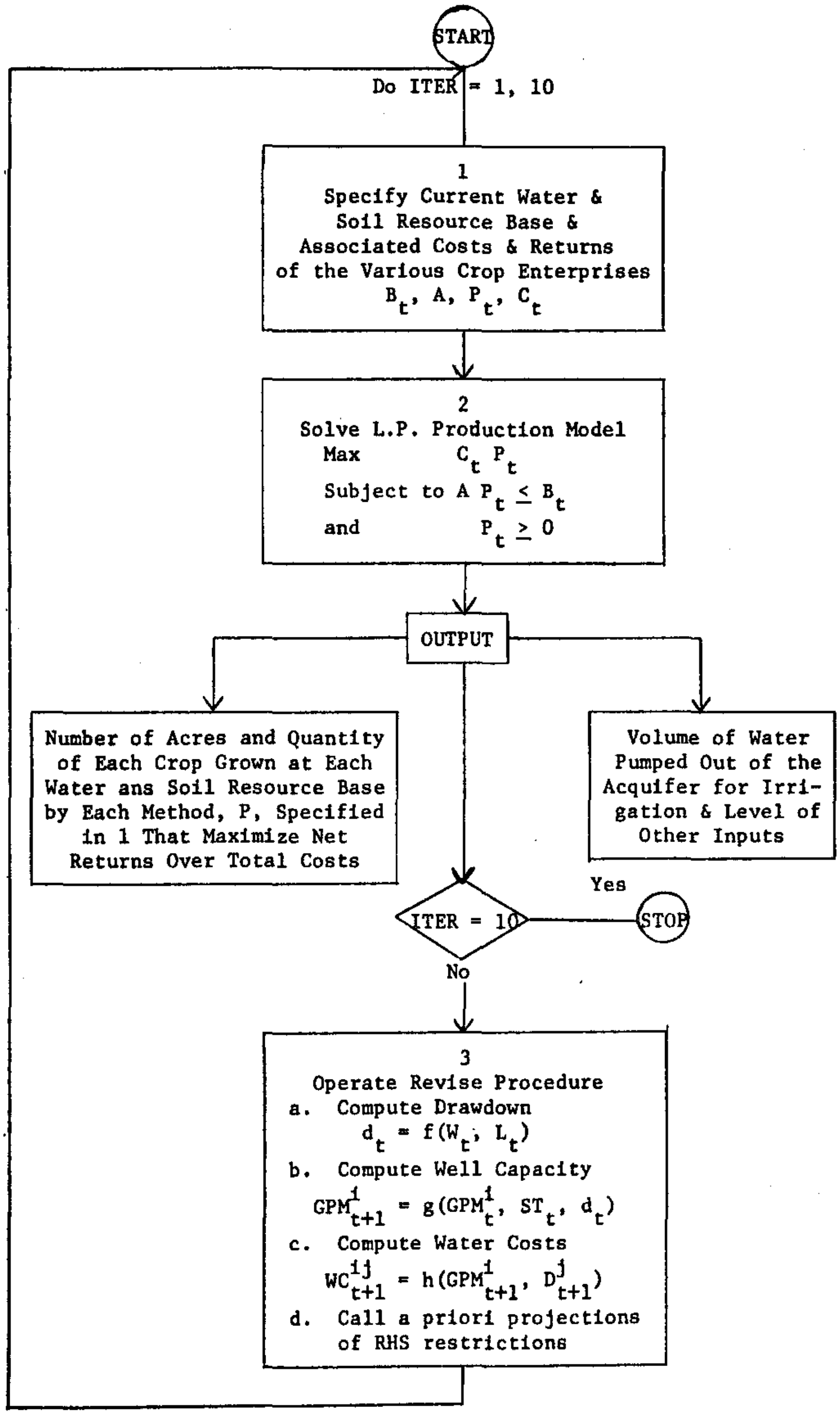

Figure 3. The Recursive Linear Programming Model

period, t + 1. At any production period t the inputs to the model are (1) the soil and water resource base and the appropriate set of production restrictions represented by vector B_t, (2) the various crop enterprises, selling and buying activities represented by matrix P_t, (3) the associated input-output coefficients of the activities in P_t represented by matrix A and (4) the net returns accruing from the activities in P_t represented by vector C_t as shown in Figure 3. The outputs of the model are (1) the number of acres and amount of the various crops grown on each soil and water stratum under different levels of water application, viz. high level, low level and dryland, (2) the volume of water pumped out of the aquifer, (3) the level of other inputs used, and (4) the total net return from all enterprises.

In the second part of the model, several calculations are made to update and specify the parameters of the linear programming model for period t + 1. First, the volume of water used for irrigation in period t is added to an _a priori_ projected water demand for industrial and municipal purposes in that period and adjusted for mean annual recharge. This quantity is denoted as W_t. Then the decline, d_t, in the static water table level at the end of period t is calculated as a function of the net volume of water extracted from the aquifer and the appropriate surface (land) area L, i.e., $d_t = f(W_t, L_t)$. Based on the change in the static water table level, saturated thickness, ST_t^i, and well capacity, GPM_t^i, $i = 1, 2, \ldots, 6$, the next period's well capacity of the six saturated thickness classes are computed.[2] Implicitly we have:

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[2] The saturated thickness classes are defined in the Data Development section which follows.

$$GMP^{i}_{t+1} = g(GPM^{i}_{t}, ST_{t}, d_{t}). \qquad (1)$$

Once these capacities are known and the pump lift, D^{j}_{t}, is updated to D^{j}_{t+1}, (j = 1, 2, ..., 8) representing the eight pump lift classes, the water cost, WC^{ij}_{t+1}, can be calculated.[3] Implicitly we have:

$$WC^{ij}_{t+1} = h(GPM^{i}_{t+1}, D^{j}_{t+1}). \qquad (2)$$

These water costs are used to update the cost of the water buying activities in P_t by revising the appropriate elements of vector C_t. Finally the set of *a priori* projections are used to revise vector B_t. When this process is completed the inputs of the production model are updated and the model is ready to generate the production pattern for period t + 1.

The model was run once for 1965 to specify the initial conditions for 1970. Then t = 1 was made to represent the ten-year period 1970-1979, t = 2 was made to represent the period 1980-1989, etc.

The Underlying Assumptions

Projecting the long term rates of ground water use from the Central Ogallala Formation involves a complex interaction of physical, economic, social and political factors. It is impossible to accurately predict the changes that will occur in all of the relevant variables. A specified level or pattern of change had to be assumed in each case.

On the physical side, all input-output coefficients were held constant at the level achieved by "efficient operators" in 1970. Efficient operators were defined as the upper twenty-five percent of farm operators. Irrigation

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[3] The depth to water classes are defined in the Data Development section which follows.

water requirements per acre of crop, cash inputs, labor requirements and yield level were estimated and held constant over time in the model. Only currently available technology in pumping and distribution systems was used in estimating development and operating costs for irrigation systems over time. Technological breakthroughs in plant breeding, fertilizer use, irrigation systems and other areas were not considered.

It was also assumed that net returns to land and management for a given crop and irrigation level were constant over time, except for the reduction in net returns caused by increased pumping costs on the water table declines. If one assumes that all input and yield levels are constant, this implies the price of all inputs and crops produced are also constant. This may seem to be a very restrictive assumption. However, it is important to note that offsetting changes in the cost of inputs and value of output would not affect the net returns per acre and hence the solution obtained.

The model assumes the general social and political forces operating in the study area will continue to prevail, with one exception--federal farm programs. Changes in the federal farm programs for wheat, feed grain, cotton and sugar beet producers are difficult to predict. The trend in wheat and feed grain programs in recent years has been to reduce the level of support farmers receive. Thus no benefits for wheat and feed grains are included in the net returns for these two crops. However, current program benefits were included for cotton and sugar beets. In each case the level of benefits assumed was held constant over time.

The recursive linear programming model was used to project the rate of irrigation development in the study area during the period 1965 through 2000. Frequently such projections are made assuming the area maintains its historic share of national supply but does not exceed it. One set of projections was developed under this general assumption to facilitate comparisons with other water resource studies and as a comparison to the projected rate of growth. A second set of projections was made under the assumption irrigation development is restricted only by the land, water, labor, credit, social and institutional restrictions currently affecting the rate of development in the Central Ogallala. The formulations of the RLP model for the first and second set of projections are referred to as Model I and Model II, respectively.

Model I

Model I represents a situation in which future agricultural production in the study area and the U. S. is assumed to be in balance with estimated future demands. For this purpose the U. S. Department of Agriculture's national projections, which are based on such an assumption, were disaggregated to derive the study area's future supply. The national projections

> ...are based upon examination of current relationships and evaluation of foreseeable developments. The major forces considered in the projections are population growth, shifts in consumer demands, industrial and other uses of agricultural commodities; livestock feeding efficiencies and feed ration composition; foreign demand for agricultural products and the advance of technology in the production of crops and livestock [104, p. 1].

Since the projections represent an economy where agricultural production is in balance with estimated future demands, the projected national supply of the agricultural commodities can alternatively be viewed as the demand for them. The national projections were made for the years 1980 and 2000 taking 1959-61 as the base period.

A simple shift share technique was used in disaggregating the national projections to that of the study area. The study area's historic proportional share of the national supply was applied to the projected national production figures. The use of such a simple shift share disaggregation procedure as discussed above assumes that regional competition will remain the same and the study area will maintain its share of the national supply at the 1965-67 level. Any interpretation of the projected production will have to take into account the significance of this assumption.

These projections were incorporated in the production model as upper limits in the right-hand side, vector B_t. Thus Model I maximized net returns subject to meeting the specified a priori production goal projected for the period in question.

Model II

Model II represents a situation in which the study area is allowed to produce more than its historic share of the projected U.S. production, subject to an upper constraintimposed by the maximum rate of irrigation growth possible. If such a restriction was not imposed, the model would irrigate every irrigable acre in the entire study area. The maximum rate of growth of irrigation was computed on the basis of the rate at which the maximum physical limit was being approached in the recent past. An exponential growth model, discussed in the data section below, was employed in projecting the number of acres to be irrigated.

Data Development

The input data used to specify the two RLP production models and the specific assumptions used in developing the data are presented in this section. The computations made in updating the parameters related to the water variable are also discussed. For any given production period the basic data for Model I and Model II are the same. The activities represented by matrix P_t and the input-output coefficients denoted by matrix A are identical in both models. The objective function given by C_tP_t is identical in both models only for the initial 1965-69 production period. As the two models withdraw water from the aquifer at different rates, water costs change in different magnitudes in the two models from one production period to the next. Thus some elements of C_t will have different values in Model I and Model II for any production period other than the 1965-69 period. The elements of the right-hand side vector, B_t, are composed of two types of restrictions. The restrictions that delimit the water and soil resource base available for crop production have the same value in both models as they are fixed quantities to the study area. The rest of the elements in vector B_t are different in the two production models. The following discussion indicates differences in the parameters used in the two models.

The Soil Classification Scheme

It was necessary to classify the soils of the study area into homogeneous groups in order to relate the distribution of irrigable and non-irrigable soils to the water resources throughout the study area. The Soil Conservation Service (SCS) county soil surveys provided the basic data for this purpose. Such surveys were not available in published form for the counties of Baca in Colorado, Grant, Haskell, Gray and Meade in Kansas and all the counties in Texas except Hansford. The data for these counties were obtained from field work sheets in the county SCS offices. County SCS personnel were also consulted to verify the classification scheme.

The soils of each county were first divided into irrigable and non-irrigable groups using the irrigated capability units as the criterion of classification. The irrigable soils were further subdivided into clay A, clay B, sand A, and sand B groups. The clay groups include all silty loam, clay loam and silty clay loam soils, while the sand category includes all fine sandy loam and loamy fine sand soils.

The clay A soils are deep, nearly level (zero to three percent slope), moderately fine to medium textured and well drained soils. Soils in the clay B subdivision are moderately fine to medium textured that are characterized by management limitations such as poor drainage, erosion and moderate to steep slopes.

The sand A group consists of deep, well drained and moderately coarse textured soils which are nearly level to gently sloping. Soils in the sand B subdivision are deep, coarse textured with moderately fine to moderately coarse subsoils. They have the same type of management limitations as the clay B group, and range in slope from nearly level to moderately steep.

These soil groups were identified and plotted on the map of each county. Table II summarizes the distribution of each soil group in the study area. Due to their slope and management characteristics all A soils are assumed to be suitable for furrow irrigation and all B soils are assumed to be best suited for sprinkler irrigation.

TABLE II

DISTRIBUTION OF SOIL TYPES IN THE STUDY AREA

Soil Type	No. of Acres	No. of Acres	Percent of Total	Percent of Irrigable Soils
Non-irrigable	3,108,423		27.88	
Irrigable	8,040,915		72.12	100.00
Clay A		5,366,204		66.74
Clay B		801,296		9.96
Sand A		897,170		11.16
Sand B		976,245		12.14
TOTAL	11,149,338		100.00	

The Soil and Water Resource Situation Strata

Hydrologic maps of each county in the study area were used to inventory the water resource. Two maps for each county provided the information required. The saturated-thickness map indicated the number of feet of water-saturated material in the aquifer. The depth-to-water map indicated the vertical distance from the ground surface to the static water table. By superimposing the depth-to-water map on the saturated-thickness map and delineating the area between two adjacent depth-to-water contour lines that lies between two adjacent saturated-thickness contours, a stratified water resource inventory was generated. The stratification involved (1) six classes of saturated thickness ranging from zero to 600 feet by 100-foot class intervals and (2) eight classes of depth to water ranging from less than 50 feet to 400 feet by 50-foot class intervals. Thus a matrix of six by eight water resource strata or 48 water resource situations were defined. The class intervals chosen for the saturated thickness and depth to water were the least common intervals of the contours plotted on the hydrologic maps available for the area. The stratified water resource map was superimposed on the soil inventory map. The area occupied by the various soil groupings falling in the different water resource strata was planimetered to generate the entire soil and water resource base inventory. Table III presents the measured acreage of each soil group by water stratum.

Excluding the non-irrigable soils, which are mostly composed of roughs and breaks along the streams and used for rangeland, there are 32 soil and water resource situations in each of the six saturated thickness classes. This implies a total of 192 soil and water resource situations for the entire study area. The number of acres in each of the 192 soil and water resource situations constitute the land base on which the entire crop production activities take place in the study area. These 192 acreages are entered as right-hand-side restrictions in the B_t vector of both Model I and Model II defining the maximum supply of the land resource available for crop production in the study area.

The Volume of Water in Storage

The quantity of ground water theoretically available for pumping is determined by the hydrologic properties of the aquifer. This quantity can be computed using (3):

$$V = \sum_{k=1}^{6} A_k \cdot h_k \cdot S \quad , \tag{3}$$

where

TABLE III

INVENTORY OF THE SOIL AND WATER RESOURCE BASE

Saturated Thickness	Depth to Water	Clay A Acres	Clay B Acres	Sand A Acres	Sand B Acres	Non Irrigable Acres	Total Acres	Percent
Under 100 ft.								
	Under 50 ft.	109,307	37,799	24,694	38,339	279,716	489,855	4.39
	51-100 ft.	327,401	58,149	58,066	86,754	289,497	819,867	7,35
	101-150 ft.	295,892	14,071	38,564	64,021	46,470	459,018	4.12
	151-200 ft.	294,876	13,284	46,803	48,844	133,981	537.788	4.82
	201-250 ft.	78,469	12,262	14,355	18,606	28,016	151,708	1.36
	251-300 ft.	47,142	12,473	8,423	11,614	20,855	100,507	0.90
	301-350 ft.	17,223	6,923	5,725	8,354	11,014	49,239	0.44
	Over 350 ft.	9,916	4,088	6,382	9,313	7,733	37,432	0.35
	Subtotal	1,180,226	159,049	203,012	285,845	817,282	2,645,414	23.73
101-200 ft.								
	Under 50 ft.	64,404	23,923	20,742	42,163	159,732	310,964	2.79
	51-100 ft.	124,530	49,654	32,080	107,839	223,056	537,159	4.82
	101-150 ft.	182,723	35,932	22,416	25,511	92,988	359,570	3.22
	151-200 ft.	306,505	22,995	27,836	34,708	215,069	607,113	5.54
	201-250 ft.	240,957	22,653	16,355	19,432	58,982	358,379	3.21
	251-300 ft.	140,565	17,526	20,576	32,716	43,936	255,319	2.29
	301-350 ft.	40,830	5,576	6,012	10,398	31,531	94,347	0.85
	Over 350 ft.	4,475	1,066	1,827	3,167	15,168	25,703	0.23
	Sub total	1,104,989	179,325	147,844	275,934	840,462	2,548,554	22.86
201-250 ft.								
	Under 50 ft.	49,217	29,403	4,630	11,575	120,868	215,693	1.94
	51-100 ft.	155,084	68,119	39,663	56,401	313,447	632,714	5.68
	101-150 ft.	194,715	56,813	39,481	11,181	121,663	423,853	3.80
	151-200 ft.	550,915	42,688	40,368	17,154	148,625	799,750	7.17
	201-250 ft.	240,708	22,483	12,690	3,979	49,480	329,340	2.95
	251-300 ft.	122,604	9,471	24,803	7,875	50,066	214,759	1.93
	301-350 ft.	98,096	7,029	17,978	2,082	85,335	210,520	1.89
	Over 350 ft.	22,639	1,768	2,037	---	16,399	42,843	0.38
	Subtotal	1,433,978	237,774	181,650	110,247	905,883	2,869,472	25.74

TABLE III (Continued)

Saturated Thickness	Depth to Water	Clay A Acres	Clay B Acres	Sand A Acres	Sand B Acres	Non Irrigable Acres	Total Acres	Percent
301-400 ft.								
	Under 50 ft.	69,487	14,252	13,239	1,483	47,201	145,662	1.31
	51-100 ft.	174,692	35,460	28,269	23,687	76,678	338,786	3.04
	101-150 ft.	218,434	28,251	29,058	22,252	70,049	368,044	3.30
	151-200 ft.	562,963	76,836	71,451	35,406	132,675	879,331	7.89
	201-250 ft.	89,954	15,060	4,218	2,096	21,686	133,014	1.19
	251-300 ft.	29,873	4,901	141	---	965	35,880	0.32
	301-350 ft.	30,132	5,419	2,573	512	12,824	51,460	0.46
	Over 350 ft.	8,902	1,803	380	78	2,114	13,277	0.12
	Subtotal	1,184,437	181,982	149,329	85,514	364,192	1,965,454	17.63
401-500 ft.								
	Under 50 ft.	2,236	1,799	---	418	25,559	30,012	0.27
	51-100 ft.	54,314	13,491	16,962	13,456	19,263	117,486	1.05
	101-150 ft.	117,557	5,927	58,480	72,732	44,233	298,929	2.68
	151-200 ft.	101,764	4,433	27,866	39,693	29,677	203,433	1.82
	201-250 ft.	13,052	3,540	---	---	1,883	18,475	0.17
	251-300 ft.	14,010	2,791	---	---	1,584	18,385	0.17
	301-350 ft.	15,287	3,052	---	---	1,730	20,069	0.18
	Over 350 ft.	5,497	1,516	---	---	801	7,814	0.07
	Subtotal	323,717	36,549	103,308	126,299	124,730	714,603	6.41
Over 500 ft.								
	Under 50 ft.	15,945	289	7,458	3,694	5,504	32,890	0.30
	51-100 ft.	33,509	---	22,306	14,037	9,496	79,348	0.71
	101-150 ft.	70,493	55	50,966	68,512	26,120	216,646	1.94
	151-200 ft.	17,608	1,742	31,297	6,163	13,898	70,708	0.63
	201-250 ft.	294	910	---	---	206	1,410	0.01
	251-300 ft.	223	690	---	---	157	1,070	0.01
	301-350 ft.	785	2,431	---	---	533	3,769	0.03
	Over 350 ft.	---	---	---	---	---	---	---
	Subtotal	138,857	6,617	112,027	92,406	55,934	405,841	3.63
TOTAL						3,108,423	11,149,338	100.00

$k = 1, 2, \ldots, 6$ represents the k^{th} saturated thickness indicated in Table III,

A_k = surface area associated with the k^{th} saturated-thickness class in acres,

h_k = the midpoint of the k^{th} saturated-thickness class in feet, and

S = the coefficient of storage.

Following Beck, et.al., in Colorado, Gutentag, et.al., in Kansas, Sapik in Oklahoma and Buchanan in Texas, a coefficient of storage of 0.15 was employed in computing the volume of water available for pumping in the Central Ogallala Formation [3, p. 23, 82 and 7, p. 10]. This quantity was estimated to be 369,663,804 acre feet in 1965. Table IV summarizes the distribution of the available water in storage by saturated-thickness and depth-to-water strata in both absolute and relative terms. The distribution of water in the aquifer is skewed in favor of the saturated-thickness classes with less irrigable surface area. While the first two saturated thickness classes (0-100 feet and 101-200 feet) comprise 39.24 percent of the total land area, and 43.98 percent of the total irrigable land, they have only 20.88 percent of the total water supply in storage. On the other hand, while the two deepest saturated thickness classes (401-500 feet and >500 feet) comprise 10.04 percent of the total land area, and 11.69 percent of the total irrigable acres, they have 22.11 percent of the total water supply in storage. The skewness of the water supply distribution implies that about 44 percent of the total irrigable acres will experience rapidly increasing costs of obtaining water from the aquifer as the water table declines. The third and fourth saturated-thickness classes (201-300 feet and 301-400 feet) constitute 43.37 percent of the

TABLE IV

WATER RESOURCES INVENTORY OF STUDY AREA, 1965

Initial Saturated Thickness	Item	Depth to Water (Pump Lift)								
		<50'	51'-100'	101'-150'	151'-200'	201'-250'	251'-300'	301'-350'	>350'	Total
0'-100'	No. of Acres	489,855	819,867	459,018	537,788	151,708	100,507	49,239	37,432	2,645,414
	% of Total	4.39	7.35	4.12	4.82	1.36	0.90	0.44	0.35	23.73
	Ac. Ft. in Storage	3,673,912	6,150,352	3,442,635	4,033,411	1,137,810	753,803	369,291	280,740	19,841,954
	% of Total	.99	1.66	0.94	1.09	0.31	0.20	0.10	0.08	5.37
101'-200'	No. of Acres	310,964	537,159	359,570	607,113	358,379	255,319	94,347	25,703	2,548,554
	% of Total	2.79	4.82	3.22	5.45	3.21	2.29	0.85	0.23	22.86
	Ac. Ft. in Storage	6,996,690	12,086,079	8,090,324	13,660,044	8,063,527	5,744,678	2,122,807	578,318	57,342,467
	% of Total	1.89	3.27	2.19	3.70	2.18	1.55	0.57	0.16	15.51
201'-300'	No. of Acres	215,693	632,714	423,583	799,750	329,340	214,759	210,520	42,843	2,869,472
	% of Total	1.94	5.68	3.80	7.17	2.95	1.93	1.89	0.38	25.74
	Ac. Ft. in Storage	8,088,488	23,726,776	17,599,724	28,954,987	12,350,250	8,053,463	7,894,500	1,606,612	108,274,800
	% of Total	2.19	6.42	4.76	7.83	3.34	2.18	2.14	0.43	29.29
301'-400'	No. of Acres	145,662	338,786	368,044	879,331	133,014	35,880	51,460	13,277	1,965,454
	% of Total	1.31	3.04	3.30	7.89	1.19	0.32	0.46	0.12	17.63
	Ac. Ft. in Storage	7,647,255	17,786,266	18,622,760	46,165,088	6,983,235	1,883,700	2,701,650	697,042	102,486,996
	% of Total	2.07	4.81	5.04	12.49	1.88	0.51	0.73	0.19	27.72
401'-500'	No. of Acres	30,012	117,466	298,929	203,433	18,475	18,385	20,069	7,814	714,603
	% of Total	0.27	1.05	2.68	1.82	0.17	0.17	0.18	0.07	6.41
	Ac. Ft. in Storage	2,025,810	7,930,305	20,177,708	13,731,727	1,247,063	1,240,988	1,354,658	527,445	48,235,704
	% of Total	0.55	2.14	5.46	3.71	0.34	0.34	0.37	0.14	13.05
>500'	No. of Acres	32,890	79,348	216,646	70,708	1,410	1,070	3,769	—	405,841
	% of Total	0.30	0.71	1.94	0.63	0.01	0.01	.03	—	3.63
	Ac. Ft. in Storage	2,713,425	6,546,210	17,873,295	5,833,410	116,325	88,275	310,943	—	33,481,883
	% of Total	0.73	1.77	4.84	1.58	0.03	0.02	0.08	—	9.06
Total	No. of Acres	1,225,076	2,525,360	2,126,060	3,098,123	992,326	625,920	429,404	127,069	11,149,338
	% of Total	11.00	22.65	19.06	27.78	8.89	5.62	3.85	1.15	100.00
	Ac. Ft. in Storage	31,145,580	74,225,988	85,806,446	112,378,667	29,898,210	17,764,907	14,753,849	3,690,157	369,663,804
	% of Total	8.42	20.08	23.22	30.40	8.09	4.80	3.99	1.00	100.00

total land area, and 44.33 percent of the total irrigable acres and have 57.01 percent of the total water supply in storage. This situation coupled with the fact that of the 57.01 percent figure the 45.61 portion lies at an initial depth of 200 feet or less implies that irrigated activity may be sustained for a prolonged period of time in areas where these saturated-thicknesses dominate.

Crop Enterprise Activities

Both models consider only those crops that are currently being produced in significant quantities in the study area. As noted in the introduction, it appears these crops will continue to be the principal irrigated and dryland crops in the study area in future years. Since the production of barley, oats and native pasture are almost exclusively dryland activities, they do not affect water use and, therefore, are left out of the model in order to reduce the size of the programming matrix. However, as the total cropland available includes land resources on which such activities take place, they were represented by a single opportunity cost activity using the dryland net return onbarley as the best alternative.

The irrigated crop enterprises selected for production are grain sorghum, wheat, corn grain, corn silage, alfalfa, sugar beets, cotton and soybeans. Census data on the production of crops in the study area indicate cotton is a minor crop produced only in the Texas portion of the study area. Sugar beets are produced in Colorado and Kansas. Soybeans are not grown in the Colorado and Oklahoma portions. All of the other crops are produced throughout the study area [4, p. 185]. Corn grain, soybeans, corn silage and sugar beets are assumed to be produced only with irrigation, while alfalfa hay, sorghum grain, wheat and cotton are assumed to be produced both under dryland and irrigation.

The resource requirements, costs and returns for the alternative dryland and irrigated crops were developed with the aid of farm management specialists working in the study area. Two levels of water application are provided for all crops except soybeans and sugar beets which have only one rate of water application. The levels of irrigation for each crop, the corresponding yields, costs and returns are shown in the enterprise budgets used for the linear programming production model [4, pp. 196-205].

The Quantity of Crops Produced: Model I

Production of crops is limited to the study area's historic share of the projected national supply in Model I. Supply of crops projected by the USDA for the years 1980 and 2000 is based on the 1959-61 average supply. In applying the shift-share technique, first the study area's proportional share of the national supply in the base period, 1959-61, was computed for the eight irrigated crops. Then its proportional share for 1965-67, the period of most recent complete observation, was computed. The comparison revealed that the study area has made a slight gain in its share of national supply in the feed and feed grain commodities and that it has lost in the production of wheat, reflecting the recent shift to increased cattle production and commercial feedlot operations in the area. Table V summarizes the magnitude and direction of the shifts. Grain sorghum gained about 3.5 percentage points in the study area's share of the national supply, silage gained about one-tenth of a percent and wheat lost about 1.2 percentage points. All other crops made a slight gain. The 1965-67 average study

TABLE V

STUDY AREA'S SHARE OF THE U.S. NATIONAL SUPPLY OF SELECTED IRRIGATED CROPS, AVERAGE 1959-61 AND 1965-67

		1959-61 Av.			1965-67 Av.[1]			
		(1)	(2)	(3)	(4)	(5)	(6)	(7)
Crop	Unit	Study Area Supply	U.S. Supply	(1)÷(2)	Study Area Supply	U.S. Supply	(4)÷(5)	(6)-(3)
Grain Sorghum	bu.	32,314,096	551,609,000	0.058558	66,861,059	717,769,000	0.093151	+0.034593
Wheat	bu.	52,732,092	1,271,086,000	0.041480	40,902,310	1,383,888,000	0.029556	-0.011924
Corn Grain	bu.	1,642,135	3,743,597,000	0.000445	4,712,654	4,307,964,000	0.000953	+0.000659
Silage	tons	433,429	75,785,990	0.005867	697,654	100,558,000	0.006964	+0.001097
Alfalfa	tons	45,235	65,730,000	0.000688	70,432	73,947,667	0.000953	+0.000265
Sugar Beets	tons	48,338	17,046,660	0.002836	111,542	20,208,667	0.005519	+0.002683
Cotton	bales	1,013	14,382,666	0.000070	1,846	10,667,667	0.000173	+0.000103
Soybeans	bu.	16,330	589,257,300	0.000028	293,254	915,596,666	0.000320	+0.000292

[1] Column (7) shows the change in area study's share of the national supply between the two periods.

Source: Computed from U.S. Department of Agriculture, Agricultural Statistics, 1959, 1960, 1961, 1965, 1966, 1967, U.S. Government Printing Office (Washington) and U.S. Department SRS (Colorado, Kansas, Oklahoma, and Texas) Crop and Livestock Reporting Service, reports for 1959, 1960, 1961, 1965, 1966, and 1967.

area's share of the national supply was used to disaggregate the national projections in order to reflect these changes. Table VI presents the supply projection both for the U. S. and the study area. All projections except those for 1980 and 2000 are linear interpolations. Since one iteration of the production model represents the annual production of a period of ten years the projected supply restrictions employed in Model I are those of the midpoint years, i.e., the 1975 projection is used for the period 1970-1979, the 1985 projection is used for the period 1980-1989, etc.

The Distribution of Production in the Study Area: Model I

Since the production model's objective is to maximize net returns it is conceivable that it will attempt to produce the crops on the clay loam A type soils that lie in deep saturated-thickness and shallow depth-to-water resource situations. In order to prevent such a happening in Model I it is assumed that irrigated crop production is distributed among the 48 water resource situations according to the weight each one carries with respect to the total number of irrigable acres in the study area. These weights were calculated using (4):

$$W_{km} = \frac{a_{km}}{A} \tag{4}$$

where

$k = 1, 2, \ldots, 6$, represents the k^{th} saturated-thickness class,

$m = 1, 2, \ldots, 8$, represents the m^{th} depth-to-water class,

W_{km} represents the weight for water resource situation (k, m),

a_{km} represents the number of irrigable acres in water resource situation (k, m), and

TABLE VI

PROJECTED PRODUCTION OF THE PRINCIPAL IRRIGATED CROPS IN THE STUDY AREA AND THE U.S. 1970-2000[1]

Years	Grain Sorghum		Wheat		Corn Grain		Silage		Alfalfa		Sugar Beets[2]		Cotton		Soy Beans	
	Study Area	U.S.	Study Area	U.S.	Study Area	U.S.	Study Area	U.S.	Study Area	U.S.	Study Area	U.S.	Study Area	U.S.	Study Area	U.S.
	1,000 bu.		1,000 bu.		1,000 bu.		1,000 tons		1,000 tons		1,000 tons		1,000 bales		1,000 bu.	
1965-67 (average)	66,861	717,769	40,902	1,383,888	4,711	4,307,964	698	100,558	70	73,948	112	20,209	1.8	10,668	293	915,597
1970	72,963	783,275	45,727	1,547,125	5,022	4,550,500	723	104,174	78	81,506	122	22,076	2.2	12,725	331	1,033,364
1975	79,065	848,780	50,551	1,710,362	5,290	4,793,036	748	107,790	85	89,065	132	23,943	2.6	14,782	369	1,151,132
1980	85,167	914,286	55,376	1,873,600	5,558	5,035,572	773	111,405	92	96,623	142	25,810	2.9	16,840	406	1,268,900
1985	95,230	1,022,322	57,252	1,937,075	5,926	5,369,036	831	119,742	98	103,853	153	27,677	3.0	17,528	427	1,334,400
1990	105,294	1,130,357	59,128	2,000,550	6,294	5,702,501	889	128,078	106	111,084	163	29,545	3.2	18,217	448	1,399,900
1995	115,358	1,238,393	61,004	2,064,025	6,662	6,035,965	946	136,415	113	118,314	173	31,412	3.3	18,905	469	1,465,400
2000	125,421	1,346,429	62,880	2,127,500	7,032	6,371,429	1,004	144,751	120	125,544	184	33,279	3.4	19,594	491	1,531,900

[1] Projections are based on "Preliminary Projections of Economic Activity in the Agricultural, Forestry and Related Economic Sectors of the U. S. and Its Water Resource Regions 1980, 2000, and 2020." ERS and Forest Service, USDA, August, 1967, P. iii, p.2.

[2] Sugar beets production is estimated on the basis of the projection of raw sugar production.

A = 8,040,915 (the total number of irrigable acres in the study area).

The computed weights are given in Table VII. Since the number of irrigable acres in the 48 water resource situations sum to A, the weights must sum to one. Hence we have:

$$\sum_{k=1}^{6} \sum_{m=1}^{8} W_{km} = 1.0 \qquad (5)$$

The production of any one crop is distributed among the 48 water resource situations by multiplying these weights by the appropriate *a priori* projected production for the period in question given in Table VI. For any period t let X_{nt}, n = 1, 2, ..., 8, represent the *a priori* projection of the study area's total production (in hundredweights, bushels, tons, or bales) for the eight irrigated crops in the model. The distribution of production among each water resource situation is given by:

$$X_{nkmt} = W_{km} \cdot X_{nt}$$

where X_{nkmt} is the upper limit for production of the n^{th} crop in water resource situation (k, m) in period t. These 48 upper limits for each crop are entered in the B_t vector of Model I as right-hand-side restrictions.

It is recalled that each of the 48 water resource situations have four types of soils. Given X_{nkmt}, its production among these four soils is allowed to be distributed on the basis of net returns. In each water resource situation, the soils that give the highest net returns are irrigated first. In this way marginal soils do not come into irrigated production unless the production goal for that water resource situation cannot be met, which is consistent with economic rationale. For those crops with dryland alternatives, whenever water costs on any water resource situation

TABLE VII

WEIGHTS USED IN DISTRIBUTING PRODUCTION OF IRRIGATED CROPS AMONG THE FORTY-EIGHT WATER RESOURCE STRATA

Saturated Thickness Class in feet	Depth-to-Water Class in feet	0-50	51-100	101-150	151-200	201-250	251-300	301-400	> 400
	m k	1	2	3	4	5	6	7	8
0-100	1	.02613	.06596	.05131	.05022	.01538	.00991	.00475	.00369
101-200	2	.01881	.03908	.03315	.04876	.03723	.02629	.00781	.00131
201-300	3	.01179	.03970	.03758	.08098	.03480	.02049	.01557	.00329
301-400	4	.01224	.03261	.03706	.09286	.01384	.00434	.00480	.00139
401-500	5	.00055	.01222	.03168	.02161	.00206	.00209	.00228	.00087
> 500	6	.00341	.00869	.02369	.00706	.00015	.00011	.00040	.00000

become so high that dryland production yields higher net returns, irrigated production ceases. However, since dryland crop yields are less than irrigated crop yields, there may not be sufficient acres in that water resource situation to meet the production goal specified by the above procedure, and an infeasible solution may be encountered. To avoid such infeasibilities, dryland production in other water resource situations is allowed to pick up the slack. Since net returns on dryland activities are independent of water costs, an intertransfer of quotas between water resource situations is made possible.

Quantity of Crops Produced: Model II

The potential short-run profitability of expanding irrigation indicates the study area will increase its share of U. S. production for the major irrigated enterprises. Model II represents a situation in which the study area is allowed to produce more than its historic share of the projected U. S. production subject to a limit on the maximum rate of irrigation growth possible. The maximum number of irrigated acres at the various production periods were projected by an exponential growth model of the form developed by George A. Pavelis [73, p. 55]. The model is given by

$$A_t = L - [(L - A_{15})\, e^{\beta(t-15)}], \quad t \geq 15 \qquad (6)$$

where

t = calendar year minus 1950,

A_t = acres irrigated in year $(1950 + t)$,

L = 8,040,915, the maximum physical potential of irrigable acres in the indefinite future,

β = the continuous constant percentage decline in remaining potential as observed for the period 1958-1965, and

A_{15} = 1,554,898, the number of irrigated acres in 1965, the most recent year for which data were available.

Equation (6) indicates that acreage irrigated at time t is the difference between L, the maximum potential physical limit and that part of the limit not reached at time t. In other words $[(L - A_{15})\ e^{\beta(t-15)}]$ represents that portion of irrigable land that has not been irrigated (t-15) years after 1965. When t is equal to 15, (t-15) becomes zero and equation (6) reduces to A_{15}.

First equation (6) was solved for β

$$A_t = L - [(L - A_{t-1})\ e^{\beta}]$$

$$e^{\beta} = \frac{L - A_t}{L - A_{t-1}}$$

$$\beta = \ln\left[\frac{L - A_t}{L - A_{t-1}}\right] \qquad (7)$$

then relation (7) was applied to the latter half of the observed data from 1950 to 1965 in order to give weight to the recent trend of growth in irrigated acres [4, p. 62]. The average β calculated for the eight-year period 1958-1965 (-0.01900) was used to generate the number of acres irrigated in future years. The results are shown in Table VIII.

The model assumes that the maximum physical potential will not be attained in the indefinite future. Growth will be asymptotic to the maximum limit. Since β is computed from observed growth in the past, like any predictive model it assumes past conditions that governed the increase in irrigated acres will prevail and there will be an adequate supply of water.

TABLE VIII

PROJECTIONS OF IRRIGATED ACRES BY THE EXPONENTIAL GROWTH MODEL

Year	Irrigated Acres	Periodic Change
1965	1,554,898	
		587,735
1970	2,142,633	
		534,477
1975	2,677,110	
		486,045
1980	3,163,155	
		442,002
1985	3,605,157	
		401,949
1990	4,007,106	
		365,527
1995	4,372,633	
		332,404
2000	4,705,037	

The first assumption is intuitively valid. If ground water in the future becomes economically and/or physically limiting, increments in irrigated acres may in fact be negative and, therefore, decline in the future. If such be the case, the model's projection will be upward biased after a certain time. The second assumption has a limitation in that future conditions will not be the same as they have been in the past. As irrigation continues to develop and the water table declines, prospective irrigators will have to consider the amount of water available for future use. They will be discouraged if they find volume is low and per unit cost of water is high. If the additional costs are not offset by higher product prices, again the projections will be upward baised. On the other hand, if a technological or an institutional breakthrough occurs that decreases the per unit cost and/or augments the water supply of the region, the converse would be true. However, these will not be serious limitations for the purpose these projected acreages are to serve in the production model. Since the projection is set as an upper limit to the number of irrigated acres, the production model compares the profitability of irrigating the various crops at each soil and water stratum, and this upper limit will be met, if and only if, net returns on the last acre irrigated are higher than that of the corresponding dryland activity.

The Distribution of Production in the Study Area: Model II

The method used to distribute crop production among the 48 water resources of the study area in Model II is to apply the weights, W_{km}, of Table VII to the total projected irrigated acreage.

For any period t let C_{nt}, n = 1, 2, ..., 6, represent the a priori projected upper limit of irrigated acres in the production of the n^{th} crop in the model. The production of each crop is distributed among each water resource situation according to:

$$c_{nkmt} = W_{km} \cdot C_{nt} \tag{8}$$

where c_{nkmt} is the upper limit to the number of irrigated acres in the production of the n^{th} crop in water resource situation (k, m) in period t. Two assumptions are made in the derivation of C_{nt} from the a priori projected irrigated acres of the exponential growth model. The first assumption involves the production of cotton and sugar beets. The length of the growing season limits the production of cotton to only some of the counties in the Texas portion of the study area. Because of this geographic limitation, the declining importance of cotton in the textile industry, and the burgeoning surplus of the CCC, it is assumed that the expansion of cotton production will be at the levels projected for Model I. Likewise sugar beet production is held at the levels projected for Model I due to the limited capacity of growth for its market. The second assumption involves the distribution of the irrigated acreage of the crops used in the model given the declining water table condition of the aquifer as time progresses. For this purpose the results of Model I are analyzed and the average distribution of irrigated acres among the six irrigated crops (excluding sugar beets and cotton) up to and including 1990, the period in which irrigation expansion reaches its peak, was taken as an index. The proportion of irrigated acreage among the six crops in Model I's solutions, Z_n, n = 1, 2, ..., 6, is given in Table IX. Let TC_t be the total number of irrigated acres projected for period t and let SC_t be the sum of irrigated acres devoted to the production of sugar beets and cotton for the corresponding period t in the solution of Model I. Then C_{nt} is derived from the following formula:

$$C_{nt} = Z_n[TC_t - SC_t] \qquad (9)$$ and,

$\left[\sum_{n=1}^{6} C_{nt} = TC_t - SC_t\right] \cdot C_{nt}$ is used in relation (8) above to calculate C_{nkmt}, which is entered in vector B_t of Model II as an upper limit to the number of irrigated acres in the production of the n^{th} crop in water resource situation (k, m) in period t.

Capital and Labor

There are no restrictions in the two production models to limit the use of capital and labor. It is assumed that all the capital necessary can be borrowed at a seven percent simple interest rate and that the labor necessary for all operations can be hired at a wage rate of $1.50 per hour. However, there are two accounting restrictions to sum the total amount of capital and labor required for all production activities in the model.

TABLE IX

DISTRIBUTION OF IRRIGATED ACREAGE AMONG SELECTED CROPS ACCORDING TO THE SOLUTION OF MODEL I

Crop	Proportion (Z_n)
Wheat	0.5493
Grain Sorghum	0.3801
Corn Grain	0.0287
Silage	0.0281
Soybeans	0.0072
Alfalfa	0.0066

Prices

The prices used in the crop enterprise budgets were the "adjusted normalized prices" issued by the Water Resource Council [111]. These are prices adjusted to minimize the direct price support effects or payments under government programs and are consistent with the supply and demand model used to project the national supply of agricultural commodities [111, p. 2].

Irrigation Systems and Water Costs

Two types of irrigation systems are used in the production model. A surface system is employed for those soils with a slope of less than three percent, i.e., soils classified as clay A and sand A. A self-propelled sprinkler system is used for soils with a steeper slope and management problems such as drainage and erosion, i.e., soils classified as clay B and sand B. The cost structures of these two irrigation systems were generated from the models developed by Shaffer and Eidman for the area around the Panhandle of Oklahoma [88]. The assumptions of these models and the costs of the different parts of the irrigation systems are given in [4, pp. 206-210]. The fixed, variable and total costs per acre inch were computed for both irrigation systems for well capacities ranging from 50-1,000 g.p.m. and well depths ranging from 79-925 feet. The estimated costs per acre inch are tabulated in [4, pp. 211-215].

As the availability of water during the summer months is crucial it is assumed that the decision to drill new wells is made on the basis of providing an adequate water supply for irrigating the summer crops. Hence this decision will be made only if the returns from the summer crops will be high enough to recover the investment costs over the life of the well. Total costs of water per acre inch are charged to all summer crop enterprises to reflect this assumption. Wheat enterprises are charged only the variable cost of water per acre inch.

The absolute amount of water available for irrigation has not been restricted in Model I and Model II as this is the variable to be observed as time progresses. At any production period the models make decisions of water application based upon current pumping and distribution costs and the profitability of alternative uses of water among the different crop enterprises. However, a water accounting restriction is included in both models to sum the volume of water used in each production period.

The Relationship Between Declining Water Table, Well Yield and Pumping Costs

A decline in the static water table is directly proportional to the net volume of water removed from the aquifer. Fader and his colleagues in studying the geohydrology of Grant and Stanton counties, southwest Kansas, computed an "aerial drawdown coefficient" for the purpose of estimating future water level declines [26, p. 49]. The aerial drawdown coefficient is given by relation (10).

$$d = \frac{V}{AD} \tag{10}$$

where

d = the aerial drawdown coefficient,

V = the acre feet of water withdrawn from the aquifer,

A = the number of acres overlying the aquifer, and

D = the decline of the static water table in feet.

Using the volume of water withdrawn and changes in the water level for 1939-42 and subsequent years to 1963, they calculated the aerial drawdown coefficient to be 0.20. Assuming this coefficient is representative of the study area the decline in the static water table can be computed from relation (10) by rearranging the terms:

$$D = \frac{V}{Ad} \tag{11}$$

In using relation (11) for estimating water level changes in the future all quantities on the right-hand side of the equation are known. A and d are constants. The net volume of water withdrawn from the aquifer, V, is computed by adjusting the total amount of water used for irrigation and municipal purposes for recharge.

It should be noted that such an approach yields an average decline in the water table throughout the study area. It assumes that water moves in relatively uniform manner from areas of high pressure to areas of low pressure throughout the aquifer. This may not be the case in reality as there will be pockets of heavy concentration of water pumpage and water may not move in sufficient velocity from areas of low pumpage to those of high pumpage to result in a uniform decline of the static water table in the short run. The use of an aerial drawdown coefficient equal to 0.20, which is greater than the coefficient of storage, equal to 0.15, in relation (11) is to reflect such an assumption. However, this may introduce a downward bias in the drawdown computed. If d = 0.15 instead of d = 0.20 were used in relation (11), the drawdowns calculated in each period would be greater. This means that the saturated-thickness would diminish at a faster rate and according to relation (12) well capacities would also decrease at a faster rate. The net effect would be a shorter economic life of the various water resource situations and a higher volume of water left in storage at the terminal period. However, a value of d = 0.20 was used in this study as the best available estimate.

The effect of a declining water table is two-fold. First it increases the pump lift (total dynamic head) by the amount it has declined, thereby increasing per unit pumping costs. Second, a decline in the water table is tantamount to a decrease in the saturated-thickness of the water-bearing material, which affects well capacity. As the saturated-thickness decreases the new well capacity is computed from relation (12).[4]

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[4] This relation, developed for the Ogallala Formation in the Southern High Plains of Texas, was obtained by correspondence with Mr. Frank A. Rayner, Manager of the High Plains Underground Water Conservation District, Lubbock, Texas, and Mr. Frank Hughes, ERS, USDA, Texas A & M University, who reports that the "capacity of wells in the High Plains area decreases in remarkable agreement with the equation."

$$Q_{t+1} = \frac{H^2_{t+1}}{H_t} Q_t \tag{12}$$

where

Q_t = the original well capacity at period t,

Q_{t+1} = the subsequent well capacity at period t+1,

H_t = the original saturated-thickness at period t, and

H_{t+1} = the remaining saturated-thickness at period t+1.

Using the relations (11) and (12) the appropriate pump lift and well capacity are computed for the 48 water resource strata. These results are used to select revised costs per acre inch for the water buying activities in the linear programming models for the next production period.

Results of the Recursive Linear Programming Production Models

The changes projected for the study area by Models I and II are presented in this section. The depletion of the aquifer as projected by the two models and their effect on acres irrigated, quantities of crops produced under irrigation, the underground water storage levels, well capacities, and the aggregate annual income for the 1965 to 2000 period are presented and analyzed.

Testing Model I

Elements of the input-output matrix and the right-hand side vector were specified using Model I assumptions. The solution for 1965 conditions was obtained by using the Mathematical Programming System 360 (MPS 360) simplex algorithm on the IBM-360 computer. The key solution variables were compared with reported values of those variables in 1965 to test the validity of the production model. Criterion variables of the test were the quantities of the various irrigated crops, the total irrigated acreage and the total volume of water pumped during 1965. The model's solution showed that the study area's production was met exactly as specified and that the production process utilized 1,359,730 irrigated acres, 905,894 dryland acres and 2,347,744 acre feet of water. Since the production goals were set as right-hand side equalities, the fact that they were met is not surprising. It merely asserts that the model is functional. Comparison of the model's irrigated acreage with that reported for 1965 shows a slight discrepancy. While Model I used 1,359,730 irrigated acres, the figure reported for 1965 in Table I is 1,528,789. The model solution included 169,059 or 12.43 percent fewer acres than the reported figure. Comparison of the water applied to irrigated acres reveals a striking closeness of the model's solution to that reported in 1965. The model's production used 2,347,744 acre feet whereas the reported amount of water applied to irrigated acres in 1965 is 2,323,759 acre feet. The difference is 23,985 acre feet or 1.03 percent of the reported figure. Thus it was concluded that the model's solution on water use is accurate, but that the solution on the number of irrigated acres requires additional evaluation.

In maximizing net returns Model I's solution selects the high rate of water application wherever a choice of high or low rate is available. In practice all irrigators may not apply the high rate as suggested by the model. The method of reporting irrigated acreage does not reflect these differences. For instance, an acre of grain sorghum on which 16 inches of water have been applied and an acre of grain sorghum on which 24 acre inches have been applied are both reported as one irrigated acre. To the extent that this situation occurs in actual practice, farmers will have to irrigate more acres than the model indicates in order to meet the same production goal because yields per acre are smaller at low rates of water application than on higher rates. Another consideration that must not be overlooked is that farmers by design or accident overreport the number of acres they irrigate. For example, farmers tend to report the entire acreage in a tract is irrigated even though some land is used for turn rows, is not cultivated because it is poorly drained or is not cultivated for other reasons. Speculation on the imposition of controls of water use has encouraged farmers to overreport their irrigated acres as a contingency for higher appropriations in the event of strict control. These considerations support and lend credence to the hypothesis that the model's solution of irrigated acres is a close approximation of the actual number irrigated acres in 1965.

Projected Changes in Irrigated Acres and Water in Storage: Model I

The empirical results of Model I project that as the study area produces its national share of the eight irrigated crops in the future the number of acres irrigated annually increases from 1.36 million in 1965 to a peak of 1.63 million in 1990. Then it declines to 1.46 million in 2000. Although serious questions must be raised with the validity of projections for later years, such projections indicate continued decline after 2000. The decrease in irrigated acres is due to the decline in the water table as the water is mined from the aquifer. The projected irrigated acres and their period-to-period changes are given in columns (1) and (2), respectively, of Table X.

TABLE X

CHANGES IN IRRIGATED ACRES AND GROUND WATER IN STORAGE AS PROJECTED BY MODEL I (1965-2000)

Year	No. Acres Irrigated Annually (1)	Period to Period Change in (1) (2)	Acre Feet Used Annually: For Irrigation (3)	Acre Feet Used Annually: For Municipal & Industrial (4)	Acre Feet Used Annually: For All Purposes (3) + (4) (5)	Period to Period Changes in (5) (6)	Total Ac. Ft. in Under Ground Storage at the Beginning of Period[1] (7)	Water in Storage as a Percent of 1965 (8)
1965	1,359,730	2,680	2,347,744	70,382	2,418,126	57,864	369,663,804	100.00
1970	1,362,410	190,536	2,346,337	129,653	2,475,990	296,446	358,949,554	97.10
1980	1,552,946	72,426	2,623,269	149,167	2,772,436	258,280	336,890,434	91.14
1990	1,625,372	-169,411	2,865,379	162,436	3,030,716	-55,749	311,866,854	84.36
2000	1,455,961		2,797,019	177,948	2,974,967		284,260,474	76.90

[1] For any year after 1965 column (7) is obtained by subtracting n[column (5) = 270,078] from column (7)'s entry in the previous period. n is the number of years in the previous period and 270,078 is the mean annual recharge in acre feet.

The total quantity of water used annually follows the same periodic trend as the number of acres irrigated. It increases from 2.4 million acre feet in 1965 to its highest level, 3.03 million acre feet, in 1990. The quantity gradually declines in subsequent years. The projected annual withdrawal of water for irrigation, municipal and industrial purposes is given in columns (3) and (4), respectively, of Table X. Column (5) shows the total annual withdrawal for all purposes and column (6) indicates the period-to-period change in this total. The estimated stock reserve of water in the aquifer at the beginning of the year is presented in column (7). The projection of the annual water use for municipal and industrial purposes steadily increases through time. The decline in the total annual water use after 1990 is caused by the decline in irrigated acres. As mining of the ground water continues through time, the stock reserve of water in the aquifer decreases steadily from an estimated 369.66 million to a projected 284.26 million acre feet by 2000. In relative terms about 23 percent of the 369.66 million acre feet will be removed from the aquifer by the year 2000. Column (8) of Table X shows the amount of water remaining in the aquifer as a percent of the 1965 level (369.66 million acre feet).

The rate of decline in the water table is directly proportional to the amount of net water withdrawal per year as calculated by relation (5) discussed in the previous section. It increases steadily from 1.3 feet per year in the 1965-69 period to 1.72 feet per year in the 1990-99 period. The decrease in the rate of net water withdrawal results in smaller water table declines during subsequent periods. By 2000 the average cumulative water table decline over the 35-year period (1965-2000) is 54 feet.

The effect of the decline in the water table is two-fold. First, as the saturated-thickness of the aquifer decreases the well capacity declines according to relation (6). Thus wells have to pump more hours in order to deliver the same volume of water as before. Secondly, the depth-to-water increases by the same amount as the decline in the water table increasing the lift. The combined effect of both is to increase the per unit pumping cost of water. The projected annual decline in the water table and the periodic decline in the well capacity for each of the six initial saturated-thickness classes is given in Table XI. The results of Model I indicate that the well capacity of the smallest saturated-thickness class (0-100 feet) declines rapidly and irrigation ceases on all soils of this saturated-thickness class by 1990. The area involved is some 1.8 million acres and accounts for about 22.74 percent of the total irrigable land in the study area. In the second saturated-thickness class (101-200 feet) well capacity declines from 1,000 g.p.m. in 1965 to 431 g.p.m. in 2000 which severely limits the profitability of irrigation on another 1,708,092 acres, 21.24 percent, of the total irrigable land. The rate of decline in well capacity is much slower on the other saturated-thickness classes. The results of Model I indicate that wells in these saturated-thickness classes have a long physical life extending beyond the year 2000.

At this point a word of caution in the interpretation of the results is in order. As the model is run once for a discrete 10-year period, computations of the decline in the water table, well capacities and, hence, water costs for any period t are made on the basis of the results of period t - 1. Therefore, the values of these variables reflect the water situation at the beginning of period t. This implies that (1) water costs toward the

TABLE XI

DECLINE OF THE WATER TABLE AND THE RESULTING WELL CAPACITIES OF THE SIX SATURATED THICKNESS CLASSES AS PROJECTED BY MODEL I (1965-2000)

Year	Water Table Decline in Feet		Well Capacities in GPM[a]					
			Saturated Thickness Class[b]					
	Annual	Cumulative	<100'	101-200'	201-300'	301-400'	401-500'	>500'
1965	1.30	--	500	1,000	1,000	1,000	1,000	1,000
1970	1.41	6.48	365	905	952	966	974	978
1980	1.59	20.58	171	779	847	891	913	925
1990	1.72	36.46	34	599	734	807	847	870
2000	1.69	53.66		431	620	721	778	813

[a] Indicate values at the beginning of period.

[b] Initial 1965 conditions.

end of period t are biased downwards and, hence, the model encourages more water use and tends to bias net returns to irrigation upwards and (2) in the process of declining well capacities, some of the water resource situations may reach well capacities around 50 g.p.m. (which is considered to be too low to maintain irrigation systems) towards the middle of period t instead of the end of period t which again tends to make the model encourage more water use and, hence, bias the net returns to irrigation upwards. The alternative to eliminate this bias would have been to obtain solutions on an annual basis which would be too costly.

Projected Crop Acreages: Model I

The basic assumption in Model I is that the study area will produce its share of the projected national supply of the eight irrigated crops from 1965 through 2000. In general, as the model fulfills these production goals irrigated acreages of each crop increase to a peak about 1980 or 1990 and then decline, whereas the dryland acreages on grain sorghum, wheat, alfalfa and cotton increase as time progresses. This general trend results from the combined effect of an increased production goal the model must meet from one production period to another and the declining water table. On one hand the increased production goals tend to increase irrigated acreage of each crop. On the other hand, the rising water costs on some of the water resource situations tend to decrease irrigated acreage by diverting production to dryland for those crops that have dryland alternatives and by terminating production altogether for those crops produced only on irrigation when their net returns per acre fall to zero or less. In the early production periods the former tendency prevails as water is comparatively cheap. In the latter periods water pumping costs increase resulting in a decrease in irrigated acreage and an increase in dryland acreage. While this is the general trend there are a few fluctuations of irrigated acreage on some crops, especially wheat, caused by adjustments to changes in water costs and increased production goals. As the well capacity on some water resource situations falls from above 750 g.p.m. to some level below 750 g.p.m. the pipes and engines designed initially for 1,000 g.p.m. are changed and total water costs of sprinkler systems may drop as much as $2.52 per acre foot. As well capacity declines further water costs start to rise again until a transition to less than 350 g.p.m. is made at which time smaller pipes and smaller engines are installed reducing total water costs by as much as $2.53 per acre foot and variable costs by as much as $1.87 per acre foot. As well capacity declines further water costs begin to rise again. Such changes in well capacities do not occur simultaneously on all water resource situations. The model responds to these cost changes by increasing or decreasing irrigated acres on some of the crops.

The projected annual irrigated and dryland acreages of all crops for the entire study area are presented in Table XII. The annual irrigated acreage of grain sorghum increases from 520,000 acres in the 1965-69 period to about 891,000 acres in 2000, an increase of 71 percent. During the same span of time the dryland grain sorghum acreage changed from an annual 434,006 acres to 1.54 million acres, an increase of 255 percent.

Net return per acre of irrigated wheat is low compared to the other irrigated crops. Thus the annual irrigated acreage for wheat increases at a very slow rate from 743,436 acres in 1965 to a peak of 837,553 acres in 1980. This is an increase of only 13 percent over the initial level. In the same period the dryland acreage of wheat increased from 470,036 to

TABLE XII

MODEL I's PROJECTIONS OF ANNUAL IRRIGATED AND DRYLAND ACREAGES OF THE VARIOUS CROPS (1965-2000)

Year	Grain Sorghum		Wheat		Corn Grain	Silage	Alfalfa		Sugar Beets	Cotton		Soybeans	Total Acres	
	Irrigated	Dryland	Irrigated	Dryland	Irrigated	Irrigated	Irrigated	Dryland	Irrigated	Irrigated	Dryland	Irrigated	Irrigated	Dryland
1965	520,103	434,006	743,636	470,036	36,236	35,506	9,074	1,358	5,577	1,223	494	8,379	1,359,730	905,894
1970	505,267	1,001,787	750,592	1,130,211	40,543	37,547	8,811	10,717	6,608	1,213	1,726	10,495	1,361,076	1,503,753
1980	592,527	1,275,703	837,553	1,320,899	45,583	45,542	10,204	12,794	7,638	1,439	2,046	11,803	1,552,286	2,611,441
1990	721,490	1,529,263	769,491	1,808,385	51,245	47,901	11,611	14,634	8,669	1,552	2,208	13,410	1,625,372	3,354,488
2000	890,599	1,542,242	426,256	3,193,494	44,372	42,047	13,180	14,683	7,890	1,640	2,379	11,413	1,437,395	4,752,797

1.32 million acres, a growth of 181 percent over the initial period. The annual irrigated acreage of wheat declines to 426,000 acres by 2000 resulting in a large increase in the acreage of dryland wheat to meet the production goals.

The annual irrigated acreage of corn for grain and silage crops reach a peak about 1990. Corn in the study area increases from 36,236 in 1965 to 51,245 in 1990, an increase of 14 percent and then declines. The annual irrigated acreage of silage increases from 35,506 in 1965 to 47,901 in 1990.

The annual irrigated acreage of alfalfa, sugar beets and cotton increases throughout the 1965-2000 period. Soybeans reach a maximum of 13,400 acres in 1990 and decrease to 11,413 in 2000.

The model adjusts the irrigated acreage of each of the eight crops on each of the 48 water resource situations in response to changes in production goals and changes in the water costs over time. The results of Model I show in general the smaller the saturated-thickness and the greater the depth-to-water the more sensitive the resource is to changes in water costs. The changing pattern of production among resource situations is presented elsewhere [4, pp. 89-98].

Projected Annual Production of Crops and Their Aggregate Annual Gross and Net Returns: Model I

The projected annual irrigated and dryland crop production and the associated aggregate gross and net returns are presented in Table XIII. Since the analysis assumes yields per acre are held constant over time, the trend of irrigated and dryland production of each crop follows the same pattern as the projected annual irrigated and dryland acreage of the crop. The annual aggregate gross return from irrigated production of

TABLE XIII

MODEL I's PROJECTION OF ANNUAL PRODUCTION OF CROPS AND THEIR AGGREGATE GROSS AND NET RETURNS (1965–2000)

Year	Grain Sorghum Irrigated 1,000 Cwt	Grain Sorghum Dryland 1,000 Cwt	Wheat Irrigated 1,000 Bu.	Wheat Dryland 1,000 Bu.	Corn Grain Irrigated 1,000 Bu.	Silage Irrigated 1,000 Tons	Alfalfa Irrigated 1,000 Tons	Alfalfa Dryland 1,000 Tons	Sugar Beets Irrigated 1,000 Tons	Cotton Irrigated 1,000 Bales	Cotton Dryland 1,000 Bales	Soybeans Irrigated 1,000 Bu.	Annual Gross Returns From Irrigated 1,000 Dollars	Annual Gross Returns From Dryland 1,000 Dollars	Annual Net Returns From Irrigated 1,000 Dollars	Annual Net Returns From Dryland 1,000 Dollars	Total Annual Net Returns 1,000 Dollars	Estimated Primary Irrigation Benefits 1,000 Dollars
1965	32,098	5,946	34,417	6,486	4,711	698	68	2.4	112	1.6	0.2	293	116,066	19,179	40,639	9,743	47,382	18,418
1970	31,263	13,724	34,954	15,597	5,271	734	66	18.8	132	1.7	0.8	367	116,651	45,475	40,662	15,587	56,248	18,443
1980	36,709	17,477	39,024	18,228	5,926	828	77	22.4	153	2.0	1.0	413	133,437	55,530	45,622	19,049	64,672	20,316
1990	44,687	20,951	36,048	24,956	6,662	946	87	25.6	173	2.2	1.1	469	144,325	70,990	48,197	24,333	72,530	21,869
2000	55,004	21,129	20,274	44,070	5,768	831	99	25.7	158	2.3	1.2	399	136,531	99,015	42,263	33,722	75,985	19,275

crops increase from \$116.1 million in 1965 to \$144.3 million in 1990, an increase of about 25 percent. Annual aggregate net returns from irrigation reaches its highest level of \$48.2 million in 1990, an increase of about 19 percent over the initial level, and then declines. Annual gross returns decline \$7.8 million from 1990 to 2000, while annual net returns decline \$5.9 million. The rising water costs could be expected to result in a greater reduction in annual net returns. However, a large increase in irrigated grain sorghum (having a relatively high gross and net return per acre), coupled with a large decrease in irrigated wheat acreage (having lower gross and net return per acre) masked the effect in the aggregate estimates.

Annual aggregate gross returns from dryland crop production increase from \$19.2 million in the initial period to \$99.00 million in 2000. Annual aggregate net returns from dryland production of crops follows the same trend as the annual aggregate dryland gross return. It increases from \$9.7 million in the initial period to a peak of \$33.7 million in 2000. The total annual aggregate net return from both dryland and irrigation increases from \$47.3 million in 1965 to \$76.0 million in 2000, an increase of 63 percent. It is important to note that total annual aggregate net return is maintained at a high level by an increase of more than 400 percent in dryland acres over the 1965 level. The source of this increase is part of the 8.0 million acres of cropland that was being used for the production of other crops like barley, oats, hay, etc.

The annual primary irrigation benefits, which are defined here as the net income added to the aggregate net farm income of the study area by irrigation, were estimated based on the Model I solutions according to the following relation:

$$\overline{PB}_t = \overline{NR}_t - [(\sum_{j=1}^{4} \sum_{i=1}^{4} N_i C_{it} + N_j S_{jt}) + 1.25 I_t] \qquad (13)$$

where

$\overline{PB}_t$ = the annual primary irrigation benefit in period t,

$\overline{NR}_t$ = the annual net return from irrigation in period t,

$i = j$ = 1, 2, 3, 4 is grain sorghum, wheat, alfalfa, and cotton, respectively,

N_i = the dryland net return per acre of the i^{th} crop on clay loam soils,

N_j = the dryland net return per acre of the j^{th} crop on sandy loam soils,

C_{it} = the amount of clay loam acres on which the i^{th} crop would be produced without irrigation in period t,

S_{jt} = the amount of sandy loam soils on which the j^{th} crop would be produced without irrigation in period t,

I_t = the total number of acres on which corn grain, corn silage, sugar beets and soybeans are produced in period t, and

1.25 = the dryland opportunity cost of I_t.

The estimates shown in the last column of Table XIII indicate that the annual primary irrigation benefits increase from $18.42 million in 1965 to a peak of $21.87 million in 1990, when irrigation development reaches its zenith. Annual primary irrigation benefits decline $2.6 million by 2000 and can be expected to decline even more in subsequent years.

Testing Model II

The assumption of Model I, that the study area will produce a maximum of its historic share of the projected national supply of the eight irrigated crops is relaxed in Model II. In this model irrigation was permitted to increase to the full irrigated acreage projected by the exponential growth model developed above. In addition, the study area's historic share of the projected national supply of the eight irrigated crops was posited as a minimum production goal for the 1965 to 2000 period. The minimum and the maximum on sugar beet and cotton production were the same. All other crops had no maximum production goals.

Elements of the input-output matrix and the right-hand side vector were specified according to the assumptions above for conditions prevailing in 1965. The Model II computer solutions were obtained and the key variables compared to those reported in 1965 to test whether the model was operational. The test variables were the total number of irrigated acres, the total volume of water used and the quantities of the various irrigated crops. Model II's solution showed that the irrigated acres of all crops totaled 1,554,898 acres, exactly the number reported in 1965. Sugar beets and cotton were produced in exactly the quantities specified for 1965. The quantities of the remaining six crops produced were well above the minimum set as the production goal for 1965. The quantity of water pumped by Model II was 2.7 million acre feet versus 2.3 million reported in 1965. This is an excess of 0.4 million acre feet or about 17 percent more than that reported. In view of these comparisons, one can conclude that the model is operational in as far as it met the basic assumptions it incorporated. The Model II solutions show that, except for sugar beets and cotton, the study area produced more than its historic share of the national supply of irrigated crops. The fact that more water is used than reported in 1965 is concomitant to this excess production. It was shown by the results of Model I that when production goals were met exactly as reported for 1965 the amount of water pumped was within one percent of that reported in 1965. In general the results of Model II indicated that if irrigation develops in the study area at the rates projected, the use of resources and the ensuing production of crops will be more intensive and that ground water will be mined at a faster rate than was indicated by Model I. The results of Model II are presented in the following section and a comparison of the key variables in the results of Model I and Model II follows.

Projected Changes in Irrigated Acres and Water in Storage: Model II

The projected number of annual irrigated acres, the rate of ground water withdrawal and the amount of ground water in storage at the beginning of the year are presented in Table XIV. The results of Model II indicate that the number of annual irrigated acres increases from 1.6 million in the initial period to a peak of 3.4 million in 1990. Then it drops to 2.8 million acres in 2000. It is interesting to note that in both models the peak in the number of irrigated acres, though different in magnitude (1.6 million acres in Model I), is reached in the same time period, 1990.

While the minimum production goal is more than met throughout the 1965 to 2000 period, the irrigated acreage projected by the exponential growth model is met as a maximum only in 1965. In 1970, Model II's irrigated acreage is short of the projected maximum by 6,145 acres. In the subsequent years this shortage increases progressively as the rapidly declining water table makes some of the water resource situations uneconomical for irrigated production. Since development of the exponential growth model is based on the total number of initially profitable irrigable acres, the growth in irrigated acres projected also includes the acres that have now been forced out of irrigation due to low well capacity and

TABLE XIV

CHANGES IN IRRIGATED ACRES AND GROUND WATER IN STORAGE AS PROJECTED BY MODEL II (1965-2000)

			Acre Feet Used Annually					
Year	No. Acres Irrigated Annually (1)	Period to Period Change in (1) (2)	For Irrigation (3)	For Municipal & Industrial Purposes (4)	For All Purposes (3) + (4) (5)	Period to Period Change in (5) (6)	Total Ac. Ft. in Under Ground Storage at the Beginning of Period[a] (7)	Water in Storage as a Percent of 1965 (8)
1965	1,554,898		2,685,296	70,382	2,755,678		369,663,804	100.00
		1,116,067				1,984,185		
1970	2,670,965		4,610,210	129,653	4,739,863		357,235,804	96.64
		107,983				207,765		
1980	2,778,948		4,798,461	149,167	4,947,628		312,537,954	84.55
		584,973				1,021,826		
1990	3,363,921		5,807,018	162,436	5,969,454		265,762,454	71.89
		-573,460				-978,537		
2000	2,790,461		4,812,969	177,948	4,990,917		208,768,694	56.48

[a] For any year after 1965 column (7) is obtained by subtracting n[column (5) - 270,078] from column (7)'s entry in the previous period. n is the number of years in the previous period and 270,078 is the mean annual recharge in acre feet.

rising water costs. When irrigated production on these resources is set to zero, part of the projected irrigated acres that were to contribute towards the area maximum are automatically eliminated. Consequently, the model can irrigate only those portions of the projected maximum that are profitable during the period in question. Columns (1) and (2) of Table XIV show Model II's projected irrigated acres and their period-to-period changes, respectively.

The total quantity of water withdrawn from the aquifer annually is primarily a function of the number of acres irrigated per year and, therefore, follows the same periodic trend as irrigated acres. It increases from 2.8 million acre feet in the initial period to a peak of about 6.0 million acre feet in 1990, an increase of about 117 percent over the initial period. It decreases to 5.0 million by the year 2000. The projected total annual withdrawal of ground water and its period-to-period changes are given in columns (5) and (6) of Table XIV, respectively. Columns (7) and (8), respectively, show the quantity of water in storage at the beginning of each year in absolute and relative terms. It declines precipitously from its initial level of about 370 million acre feet to about 209 million acre feet in 2000.

In Model II, the rate of decline in the water table increases from 1.55 feet per year in 1965 to a high of 3.55 feet per year and then, as the annual rate of ground water withdrawal decreases, it declines to 2.94 feet per year in 2000. The total water table decline by 2000 for the deeper saturated-thickness classes would be more than 100 feet, as shown in Table XV. As indicated earlier, the use of an aerial drawdown coefficient which is higher than the coefficient of storage may bias the annual drawdown

TABLE XV

DECLINE OF THE WATER TABLE AND THE RESULTING WELL CAPACITIES OF THE SIX SATURATED THICKNESS CLASSES AS PROJECTED BY MODEL II (1965-2000)

	Water Table Decline in feet		Well Capacities in GPM[a]					
			Saturated Thickness Class[b]					
Year	Annual	Cumulative	<100'	101-200'	201-300'	301-400'	401-500'	>500'
1965	1.55	-	500	1,000	1,000	1,000	1,000	1,000
1970	2.78	7.73	358	900	939	956	966	972
1980	2.91	35.52	42*	582*	736*	807	848	830
1990	3.55	64.62		324*	550	665*	733*	739*
2000	2.94	100.13		111	359	510	604	635

[a] Indicates values at the beginning of period.

[b] Initial 1965 conditions.

*Indicate well capacities at which water costs for sprinkler systems decrease.

downwards and thus tend to encourage a high rate of water use as shown in Table XIV.

Concomitant to the decline in the water table, the well capacities in the six saturated-thicknesses classes also decline. The results of Model II, shown in Table XV, indicate that well capacity in the first saturated-thickness class (0-100 feet) diminishes from 500 g.p.m. in 1965 to 42 g.p.m. by 1980. Consequently irrigated production terminates on the 1.8 million acres or involved about 23 percent of the total irrigable land. In Model I the economic life of these water resource situations is approximately ten years longer. In the second saturated-thickness class (101-200 feet) well capacity declines from 1,000 g.p.m. in the initial period to 111 g.p.m. by 2000. Some 1.7 million acres or 21 percent of the total irrigable land overlie the second saturated-thickness class (101-200 feet). This implies that by 2000 about 44 percent of the initial total irrigable land will not have an adequate water resource for profitable irrigation enterprises. Model I requires an additional forty years to reach this situation. In the third saturated-thickness class (201-300 feet) well capacity declines to 359 g.p.m. by 2000, greatly reducing the profitability of irrigation on another 2.0 million acres. The well capacity decline in the remaining three saturated-thicknesses progresses at a slower rate and, consequently, the economic life of the aquifer in those water resource situations extends well beyond 2000. Unfortunately, the number of irrigable acres in these resource groups is only 2.5 million acres, which is about 32 percent of the total. Furthermore, as pump lifts increase and well capacities diminish irrigating low net return crops on some of the water situations will become uneconomical despite their continued high level of well capacity. Consequently, the water resource base on which irrigation of some crops is profitable decreases even more. The combined effect of the changes discussed results in the low level of irrigated acres in later periods in spite of the higher number of total acres in the three deeper saturated-thickness classes.

Projected Annual Irrigated and Dryland Acreages, Production, and the Associated Aggregate Gross and Net Returns: Model II

In general, as Model II maximizes net returns subject to the conditions specified, irrigated acreage of each crop increases in the early periods and reaches a maximum about 1990. As the depletion of the ground water has its effect, the irrigated acreage declines. The process is reversed for dryland acreages. They decline in the early periods as irrigation develops and then start to increase as some water resource situations are forced out of production due to rising water costs brought about by the depletion of the aquifer. As the model adjusts its production of crops in response to changing water costs, slight fluctuations in the irrigated acreage of grain sorghum, wheat, silage and alfalfa are manifested. The annual irrigated and dryland acreages of all crops as projected by Model II for all periods are presented in Table XVI. Table XVII presents the irrigated and dryland production of each crop and the associated aggregate gross and net returns.

The annual irrigated acreages and production of grain sorghum, wheat, corn and alfalfa grow to about 215 percent of their 1965 level by 1990. The annual irrigated acreages and production of silage, sugar beets, cotton and soybeans grow to 205, 156, 195 and 224 percent of the initial 1965 level by 1990, respectively. In 2000, the annual irrigated acreage and production of grain sorghum, wheat and silage drops to 180 percent. It declines to

TABLE XVI

MODEL II's PROJECTIONS OF ANNUAL IRRIGATED AND DRYLAND ACREAGES OF THE VARIOUS CROPS (1965-2000)

	Grain Sorghum		Wheat		Corn Grain	Silage	Alfalfa		Sugar Beets	Cotton		Soybeans	Total Acres	
Year	Irrigated	Dryland	Irrigated	Dryland	Irrigated	Irrigated	Irrigated	Dryland	Irrigated	Irrigated	Dryland	Irrigated	Irrigated	Dryland
1965	589,569	1,091,799	849,754	5,393,038	44,516	43,584	10,270	---	5,577	876	1,179	10,792	1,554,898	6,486,016
1970	1,014,558	587,403	1,462,804	4,781,767	73,605	75,003	17,616	---	6,608	1,554	778	19,217	2,670,965	5,369,948
1980	1,055,931	787,459	1,518,704	4,472,148	79,740	77,151	18,253	---	7,638	1,656	1,351	19,875	2,778,948	5,260,958
1990	1,279,596	698,652	1,841,527	3,971,126	96,735	89,357	22,111	---	8,669	1,704	1,680	24,222	3,363,921	4,671,458
2000	1,056,003	1,018,059	1,526,322	4,221,194	83,049	78,673	18,110	---	7,890	432	5,519	19,982	2,790,461	5,244,772

TABLE XVII

MODEL II's PROJECTION OF ANNUAL PRODUCTION OF CROPS AND THEIR AGGREGATE GROSS AND NET RETURNS (1965-2000)

Year	Grain Sorghum		Wheat		Corn Grain	Silage	Alfalfa		Sugar Beets	Cotton		Soybeans	Annual Gross Returns From		Annual Net Returns From		Total Annual Net Returns 1,000 Dollars	Estimated Primary Irrigation Benefits 1,000 Dollars
	Irrigated 1,000 Cwt	Dryland	Irrigated 1,000 Bu.	Dryland	Irrigated 1,000 Bu.	Irrigated 1,000 Tons	Irrigated 1,000 Tons	Dryland	Irrigated 1,000 Tons	Irrigated 1,000 Bales	Dryland	Irrigated 1,000 Bu.	Irrigated 1,000 Dollars	Dryland	Irrigated 1,000 Dollars	Dryland		
1965	36,553	14,958	42,035	74,424	5,787	863	77	---	112	1.2	0.6	378	136,586	131,491	49,725	44,820	94,545	23,345
1970	62,902	8,047	72,129	65,988	9,569	1,491	132	---	132	2.2	0.4	673	233,683	107,979	79,273	36,716	115,990	49,257
1980	65,468	10,788	73,629	61,716	10,366	1,534	137	---	153	2.3	0.7	696	241,826	106,379	77,726	36,246	113,972	47,710
1990	79,335	9,572	88,581	54,802	12,576	1,781	166	---	173	2.4	0.9	850	291,696	94,458	81,020	32,197	113,217	51,004
2000	65,412	13,947	72,711	58,252	10,796	1,563	136	---	158	0.6	2.9	699	241,236	106,578	57,555	36,532	94,087	27,539

187 percent on corn, 185 percent on soybeans, 176 percent on alfalfa, 141 percent on sugar beets, and 49 percent on cotton. The irrigated cropping pattern of Model II among the 48 water resource situations is presented by Bekure [4, pp. 111-116].

The aggregate annual gross returns of the study area from irrigated production increases from \$136.6 million in 1965 to \$291.7 million in 1990, an increase of about 114 percent. During the same period of time aggregate annual net income for the study area from irrigated production increases from \$49.7 million to \$81.0 million, an increase of about 63 percent. Aggregate annual gross returns from irrigation decreases to \$57.6 million in 2000. This is a very substantial decrease of about \$220.0 million, and suggests a rapid rate of decline in net returns from irrigation will result beyond the year 2000. Aggregate annual net returns from dryland enterprises decrease from the initial period until 1990 as an increasing number of dryland acres are converted to irrigated production. After 1990, irrigated acres are progressively shifted to dryland production and aggregate annual net return from dryland enterprises increases as shown in Table XVII.

The primary irrigation benefits were estimated from the Model II solutions using relation (1). The assumptions of Model II permit the production of grain sorghum, wheat, alfalfa and cotton, (the crops with dryland alternatives), to such high levels that the 8.0 million cropland acres in the study area would not be adequate to produce them without irrigation after 1970. An accurate estimate of primary irrigation benefits after 1970 should be based on the maximum net returns that accrue to the dryland production of these crops on the 8.0 million cropland acres. The level of production of grain sorghum, wheat, alfalfa and cotton for 2020, which requires 7.9 million acres of cropland, (a close approximation to 8.0 million acres), is used to estimate the primary irrigation benefits in Model II. That is, after 1970 the t in relation (13) refers to 2020 in all cases except $\overline{NB}_t$ and $\overline{NR}_t$, where t refers to the period in question. The last column of Table XVII indicates that the estimated annual primary irrigation benefits increase from $23.3 million in 1965 to a peak of $51.0 million in 1990, when irrigation development is at its highest level. Annual primary benefits decline to $27.5 million in 2000, reflecting the effect of the rapid depletion of the aquifer in the Model II solutions.

Comparison of the Results of Model I and Model II

The results of Model I and Model II exhibit similar trends over time. In both cases growth of irrigation in the study area occurs from 1965 to 1990. The extent of irrigation in both models declines during the closing years of the twentieth century. In both cases irrigated production of crops and their associated aggregate gross and net receipts follow the same periodic trend as the growth and decline of irrigation. In both models the direction of changes in the level of underground water storage and well capacities is the same. The results differ only in magnitude and timing, which arise from differences in the basic assumptions of the two models. Model I's basic assumption, that the production of the study area will not surpass its historic share of the projected national supply of the eight crops, effectively restricts a rapid growth of irrigation. Consequently, the ground water is depleted at a slower rate and most of the water resources have a longer economic life than in Model II, which assumes irrigation in the study area will grow at a somewhat slower rate than in the recent past. With no upper restriction on production of irrigated crops (except for sugar beets and cotton) this assumption enables Model II to increase the irrigated acreage of crops at a more accelerated rate than that of Model I (compare columns (2) and (3) of Table X with those of Table XIV), which results in a faster depletion of the water resources.

Because the solution to Model I includes irrigation on only 1.4 to 1.6 million acres, the water withdrawals, net income from irrigation and primary economic benefits can be used to indicate the effect of limiting irrigation development in the area to approximately the 1965 level--1.5 million acres. A comparison of Model I and II results indicates the trade-off between water saved for the future and economic benefits to be derived during the remainder of the twentieth century.

The results for objective one have been derived, as mentioned previously, under the assumption that irrigators acting individually will attempt to maximize their net returns to the water resource in the short run. The analysis completed for objective 2 investigates whether the resultant annual rate of depletion of the aquifer is suboptimal as compared to the rate of depletion which maximizes the study area's net income over the entire planning horizon.

TESTING PROJECTED RATES FOR POTENTIAL MISALLOCATION

The underground water supply in the Central Ogallala is a stock resource, which possesses the property of commonality since proprietors of the overlying land obtain their water from a common reservoir. The optimum allocation of this stock resource among different production periods requires that the rate at which it is used should be such that the present value of the stream of future incomes is a maximum. Proprietors, acting individually in their self interest, may tend to misallocate the intertemporal use of the underground water resource. That is, the firm tends to maximize net returns to the quantity of water it removes from year to year without reference to its complete planning horizon. Given the option of making a group decision, irrigating firms may wish to ascertain a more dependable future supply at lower costs by reducing the current rate of use.

The purpose of the analysis reported in this section is to determine if the projected rate of ground water withdrawal over time developed under Objective 1 represents a potential misallocation of the ground water in the Central Ogallala. The initial part develops a sequential decision model that is used to select the water use strategy that maximizes the study area's net income from irrigation under various levels of underground water storage for a given planning horizon. The results of the sequential decision model are compared with the rate of ground water withdrawal projected by the recursive linear programming model to determine if the projected water withdrawals appear to exceed the optimal rate.

Research Procedures

In a closed aquifer where natural recharge is extremely low and ground water mining is practiced, the problem of optimal allocation of ground water over time is essentially a problem of choice among the various quantities of water to leave stored in the aquifer at different points in time. The decision of how much water to withdraw in any period t has a direct bearing on how much will be left in storage for the following period t + 1. More important, the decision to withdraw a certain quantity of water not only determines the net income for period t but also influences the per unit cost of water in subsequent periods. The hydrologic relationships dictate that when a quantity of water is pumped out of the aquifer in period t, the water table declines thereby increasing pump lift and decreasing well capacity for period t + 1, a phenomenon which translates to a higher per unit water cost and hence, _ceteris paribus_, lower net return per unit of water used in period t + 1. Thus the net return at any future period is a function of the storage level at that period (which incidentally is a function of the initial storage level and the cumulated withdrawals in the interim periods) and the decision to withdraw W_t quantity of water in that period. The problem is to find the optimal decisions, for all periods in a given planning horizon, which may be defined as those decisions of the rate of ground water withdrawal that will maximize the study area's net income over the entire planning horizon. Since there is an interdependence between decisions and storage levels from period to period, a sequential multi-period decision model is required to map the decision strategy in all the intermediate periods to attain the goal of maximum net return from all withdrawals in the planning horizon. An optimizing technique that is capable of accommodating such a model has been developed by Richard Bellman

[5]. The technique is commonly referred to as dynamic programming.

The Multi-Stage Sequential Decision Process

The decision process or economic activity is performed in time periods or intervals which are referred to as *stages*. The multi-stage sequential decision process consists of a series of these stages joined together so that the output of one stage becomes the input to the next [70, p. 26].

A stage may represent any span of time suitable for the particular problem under study. For our purposes the economic decision to extract ground water at a certain rate per year will be made for a planning horizon subdivided into 10-year intervals. Thus a stage refers to a period ten years in length.

An important concept of the model is the *state* of the process, which describes the condition of the system at the beginning and end of each stage. In this study the level of underground water at the beginning of a stage is referred to as the *input state*. The *output state* is the level of underground water in storage at the end of a stage.

At each stage there exists a set of relevant *alternative decisions* among which one will be selected as the optimal policy to be carried out in that stage. Here the decision variable is the annual rate of groundwater withdrawal. The set of alternative decisions contains various quantities of water to be withdrawn annually.

The decision to execute one alternative from the set transforms the condition of the system from an input state at the beginning of the stage into an output state at the end of the stage. In other words, the alternative selected at a particular stage and state of the process dictates the state which the system will occupy in the following stages. This leads to the concept of *transition* and *transition probabilities*.

Transition refers to the transformation of a given input state to any output state via a given alternative decision. For any given input state and an alternative decision the output state depends upon the magnitude of the alternative selected and the nature of other variables that affect the state of the system. Considering the nature of these other variables, each alternative decision may transform the given input state to a given output state with certainty or with some degree of uncertainty. If the transformation is known to occur with certainty, the process is said to be deterministic, i.e., a given alternative taken in a certain input state has a unique outcome. The transition probability, defined as the probability that a given input state will end up in a certain output state via a selected alternative decision, in the deterministic case is either zero or one. If the process is stochastic, any alternative selected has no unique outcome and the transition probabilities take on values from one to zero. (One may find stochastic processes in which some of the transition probabilities are one or zero.) The multi-stage decision process analyzed in this study is assumed deterministic because natural annual recharge is small in relation to the magnitude of the decision variable, the rate of groundwater withdrawal, and little is known about its variability.

Associated with each transformation is a stage return that accrues to the execution of the policy selected in that stage. The stage return in this study is the total net returns derived from applying the selected quantity of water to irrigated crops in the study area.

Consider the following schematic and mathematical representation of the multi-stage decision model. First define M discrete underground water storage levels S_i, i = 1, 2, ..., M, each level representing a state, and

K discrete alternative rates of ground water withdrawal, W_k, k = 1, 2, ..., K. Define P^k_{ij} as the transition probability of the system in transforming from input state i to output state j via alternative decision k. Define R^k_{ij} as the net return accruing from alternative decision k being carried out and the system transiting from input state i to output state j. In reference to a particular stage n, n = 1, 2, ..., N, of the N stage system we have

$S_i(n)$ = input state of the system in the n^{th} stage,

$S_j(n)$ = output state of the system in the n^{th} stage,

$W^k_i(n)$ = k^{th} alternative decision selected as optimal in the n^{th} stage, and

$R^k_{ij}(n)$ = the net return accruing to $W^k_i(n)$.

The Stage Transformation

In general the n^{th} stage transformation of the state of the process may be represented by:

$$S_i(n) = T_{(n-1)}[S_i(n-1),\ W^k_i(n-1)] \tag{14}$$

Relation (14) indicates that the input state in the n^{th} stage, or alternatively the output state of the $n-1^{th}$ stage, is a function T of the input state in the preceding stage, $S_i(n-1)$, and the optimal decision taken in that stage. In the underground water situation the transformation function can be expressed explicitly by the recursive relation:

$$S_i(n) = S_i(n-1) + A(n-1) - W^k_i(n-1) \tag{15}$$

where

$S_i(n-1)$ = level of ground water storage at the beginning of stage (n-1),

$A(n-1)$ = addition to storage by natural recharge during stage $(n-1)$, and

$W_i^k(n-1)$ = quantity of water withdrawn in stage $(n-1)$.

Thus $S_i(n)$ is independent of the latter stages $n + 1$ through N; it depends only on the decisions made prior to stage n in the stages 1 through $n - 1$. The series of transformations can be carried back to the initial stage as shown by:

$$S_n = T_{n-1}(S_{n-1}, W_{n-1}^k)^{13}$$

$$= T_{n-1}\ [T_{n-2}(S_{n-2}, W_{n-2}^k), W_{n-1}^k]$$

$$= T_{n-1}\{T_{n-2}[T_{n-3}(S_{n-3}, W_{n-3}^k)], W_{n-2}^k, W_{n-1}^k\}$$

$$= T_{n-1}\ \{T_{n-2}[T_{n-3}, \ldots\ [T_1(S_1, W_1^k)], W_2^k], \ldots, W_{n-1}\} \qquad (16)$$

The Stage Return and the Optimization Principle

The stage return at any stage n is given by:

$$R_i^k(n) = f_n[S_i(n), W_i^k(n)] \qquad (17)$$

which says that the stage return, $R_i^k(n)$, is a function of the input state S_i at the beginning of the stage and the k^{th} alternative decisions selected in that stage. In general, for the N stage system we have a sequence of stage returns given by the criterion function:

$$F[S_i(1), S_i(2), \ldots, S_i(N); W_i^k(1), W_i^k(2), \ldots, W_i^k(N)] \qquad (18)$$

The optimization problem is one of choosing the $W_i^k(n)$ at each stage n so as to maximize the criterion function F over all stages one through N for the entire planning horizon. An optimal policy function $W(W_i^k(1), W_i^k(2), \ldots, W_i^k(N))$ specifies the optimal decision for all combinations of input states, S_i, and stages n which will result in the optimization of the criterion function F.

In order to apply the dynamic programming technique to solve the optimization problem, the criterion function F has to display the property of a Markovian dependence which states that:

> After any number of decisions say k, we wish the effect of the remaining N-k stages of the decision process upon the total returns to depend only upon the state of the system at the end of the kth decision and the subsequent decisions [6, p. 54].

In other words, once the input state is specified at each stage, the decision taken in that stage is independent of the decisions taken in earlier stages so that only the decision in the current and subsequent remaining stages affect the total net returns. In the case of the optimum intertemporal allocation of groundwater the storage level at the beginning of each stage must absorb all the influences of the decisions taken heretofore. Henceforth, the total net return is affected by the choice of the rate of ground water withdrawal in the current and subsequent stages.

This property of a Markovian dependence enables one to decompose the criterion function F into a sum of separate individual stage returns and we have:

$$f_1[S_i(1), W_i^k(1)] + f_2[S_i(2), W_i^k(2)] + \ldots + f_N[S_i(N), W_i^k(N)].$$

Given the initial state of the system as S_1 and the number of stages as N, the maximization problem can now be stated as

$$f_N(S_1) = \underset{W_n^k}{\text{Max}} \; [f_1(S_1, W_1^k) + f_2(S_2, W_2^k) + \ldots + f_N(S_N, W_N^K)] \qquad (19)$$

Due to the separability of the criterion function relation (19) can be restated as

$$f_N(S_1) = \underset{W_1^k}{\text{Max}} \, [f_1(S_1,\, W_1^k) + \underset{W_2^k}{\text{Max}} \, \underset{W_3^k}{\text{Max}} \ldots \underset{W_N^k}{\text{Max}} \, f_2(S_2,\, W_2^k + f_3(S_3,\, W_3^k)$$

$$+ \ldots + f_N(S_N,\, W_N^k)] \tag{20}$$

Note the expression:

$$\underset{W_2^k}{\text{Max}} \, \underset{W_3^k}{\text{Max}} \ldots \underset{W_N^k}{\text{Max}} \, [f_2(S_2,\, W_2^k) + f_3(S_3,\, W_3^k) + \ldots + f_N(S_N,\, W_N^k)]$$

represents the total net return from an (N-1)-stage decision process as S_2 the initial state of the system. Hence we can write it as

$$f_{N-1}(S_2) = \underset{W_2^k}{\text{Max}} \, \underset{W_3^k}{\text{Max}} \ldots \underset{W_N^k}{\text{Max}} \, [f_2(S_2,\, W_2^k) + f_3(S_3,\, W_3^k) + \ldots$$

$$+ f_N(S_N,\, W_N^k)] \qquad N \geq 2 \tag{21}$$

Substituting this result in relation (20) it simplifies to:

$$f_N(S_1) = \underset{W_1^k}{\text{Max}} \, [f_1(S_1,\, W_1^k) + f_{N-1}(S_2)] \tag{22}$$

using the transformation equation (16), $S_2 = T_1(S_1,\, W_1^k)$, and we can write (22) as

$$f_N(S_1) = \underset{W_i^k}{\text{Max}} \, \{f_1(S_1,\, W_1^k) + f_{N-1}[T_1(S_1,\, W_1^k)]\}. \tag{23}$$

Relation (14) gives the recursive optimization equation connecting all members of the sequence $f_N(S_1)$. It can also be derived from Bellman's principle of optimality which states that:

> An optimal policy has the property that whatever the initial state and the initial decision, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision [6, p. 57].

The first optimal decision W_1^k yields the return $f_1(S_1, W_1^k)$ transforming the state of the process from S_1 into $T_1(S_1, W_1^k)$ and also reducing the number of stages from N to N-1. The optimal decision for the remaining N-1 stages and the state resulting from the first decision is

$$\underset{W_1^k}{\text{Max}} \{f_{N-1}[T_1(S_1, W_1^k)]\}.$$

Combining this with the first decision we have

$$f_N(S_1) = \underset{W_1^k}{\text{Max}} \{f_1(S_1, W_1^k) + f_{N-1}[T_1(S_1, W_1^k)]\} \tag{24}$$

which is exactly the same as relation (22).

The above formulation is centered around S_1 as the only input state or initial condition of the system. Since there are S_i, $i = 1, 2, \ldots, M$, possible input states at each stage relation (22) should be written to include all the possible inputs of the system. Doing so results in

$$f_N(S_i) = \underset{W_i^k}{\text{Max}} \{f_1(S_i(1), W_i^k(1)) + f_{N-1}[T_1(S_i(1), W_i^k(1))]\}. \tag{25}$$

At this point we shall introduce into the recursive optimization equation (a) the interest rate, r, in order to discount to a comparable level the net return streams generated by the sequential decisions at different time periods as the system moves from stage to stage, (b) the transition probabilities to reflect the deterministic or stochastic nature of the process and (c) the explicit transformation function to replace its implicit counterpart. Thus, we have for any stage n and any input state i

$$f_n(S_i) = \underset{W_i^k(n)}{\text{Max}} \{f_n[S_i(n), W_i^k(n)] + (1 + r)^{-1} \cdot \sum_{j=1}^{M} P_{ij}^k \cdot$$

$$f_{n-1}[S_i(n-2) + A(n) - W_i^k(n-2)]\}, \tag{26}$$

$i = 1, 2, \ldots, M$; $n = 0, 1, 2, \ldots, N$. Substituting the stage return $R_i^k(n)$ for $f_n[S_i(n), W_i^k(n)]$, β for $(1 + r)^{-1}$, S_j for $[S_i(n-2) + A(n) - W_i^k(n-2)]$ and $\underset{k}{\max}$ for $\underset{W_i^k(n)}{\text{Max}}$ we have

$$f_n(S_i) = \underset{k}{\text{Max}} [R_i^k(n) + \beta \cdot \sum_{j=1}^{M} P_{ij} \cdot f_{n-1}S_j], \tag{27}$$

$i = 1, 2, \ldots, M$; $n = 0, 1, 2, \ldots, N$.

Relation (27) may be interpreted to say that the expected present value of an N-n stage process under an optimal policy is the maximum sum of the expected net return accruing to the decision in stage n and the discounted expected net returns from the remaining n-1 stages, provided an optimal policy will be carried out in the remaining n-1 stages.

The recursive solution of equation (27) starting from n = 1 and continuing through n = N yields the optimal N-stage returns. Since we are dealing in the future and we wish to maximize the expected discounted total net return, such an optimization can be carried only from the future to the present [50, p. 57]. Computationally, the procedure involves starting with the last stage of the planning horizon treated as a one-stage process and carrying out the maximization on k for all S_i. Then the second stage from the last is taken up as a two-stage process using the results of the one-stage process to carry out the maximization on k for all S_i. Then a three-stage, four-stage, ..., n-stage process is treated yielding the maximum expected discounted net return. Stated formally, the backward iterative solution, carried from the future to the present, for each state i = 1, 2, ..., M becomes:

$$
\begin{aligned}
f_i(N) &= \underset{k}{\text{Max}}\ [R_i^k(N) + \beta \cdot \sum_{j=1}^{M} P_{ij} \cdot f_j(N+1)] \\
f_i(N-1) &= \underset{k}{\text{Max}}\ [R_i^k(N-1) + \beta \cdot \sum_{j=1}^{M} P_{ij} \cdot f_j(N)] \\
&\ \vdots \\
f_i(n) &= \underset{k}{\text{Max}}\ [R_i^k(n) + \beta \cdot \sum_{j=1}^{M} P_{ij} \cdot f_j(n+1)]^5 \\
&\ \vdots \\
f_i(2) &= \underset{k}{\text{Max}}\ [R_i^k(2) + \beta \cdot \sum_{j=1}^{M} P_{ij} \cdot f_j(3)] \\
f_i(1) &= \underset{k}{\text{Max}}\ [R_i^k(1) + \beta \cdot \sum_{j=1}^{M} P_{ij} \cdot f_j(2)] \qquad (28)
\end{aligned}
$$

i = 1, 2, ..., M; k = 1, 2, ..., K.

Since N + 1 is outside the system, a terminal value of zero is assigned to $f_j(N+1)$ so that the recursive solution can be stated as:

$$f_i(N) = \underset{k}{\text{Max}}\ R_i^k(N). \qquad (29)$$

For a solution to each $f_i(N)$ for the next iteration, $f_j(n-1)$, n = 0, 1, 2, ..., N, is supplied by the solution for $f_i(n)$ in the current iteration. For any period having n-stages remaining in the planning horizon, the optimal strategy for each of possible M initial input states is given by the function

---

[5] Note that n refers to the number of stages remaining in the planning horizon. Hence, n = 1 refers to the last stage N, n = 2 refers to the second last stage N-1, etc.

$W[W_i^k(n), W_i^k(n-1), \ldots, W_i^k(2), W_i^k(1)]$ which is a set of those alternative ground water withdrawal rates that maximize the sum of discounted expected net returns at each remaining stage in the planning horizon. Associated with the function W is the function $F[f_i(n), f_i(n-1), \ldots, f_i(2), f_i(1)]$ which gives the corresponding sum of maximum discounted expected net return.

In Figure 4 [70, p. 17], given the initial, 1970, level of ground water storage as state one (S_1), stage one represents the first ten-year period of the planning horizon in which the decision on the optimal rate of ground water withdrawal, $W_i^k(1)$, is made. $R_i^k(1)$ is the net return that accrues to the optimal decision, $W_i^k(1)$, in stage one. The execution of $W_i^k(1)$ transforms the state of the system into an output state, $S_j(1)$, which becomes the input state, $S_i(2)$, in the second stage (1980-89), where the optimal decision, $W_i^k(2)$, is selected and the system is transformed to an output state, $S_i(3)$. The process continues in a similar fashion from stage to stage until the final stage, N = 10, is reached.

Data Development

This section discusses the development of the input data that are necessary to determine the optimal intertemporal allocation of ground water from the Central Ogallala Formation as a multi-stage sequential decision model capable of solution by the dynamic programming technique. The first step in setting up the multi-stage sequential model for optimization by the dynamic programming technique is to define the component parts and specify their values as input data. These component parts, described in the Research Procedures section, are (a) the possible input and output states of the system at all stages, (b) the sets of alternative decisions in each

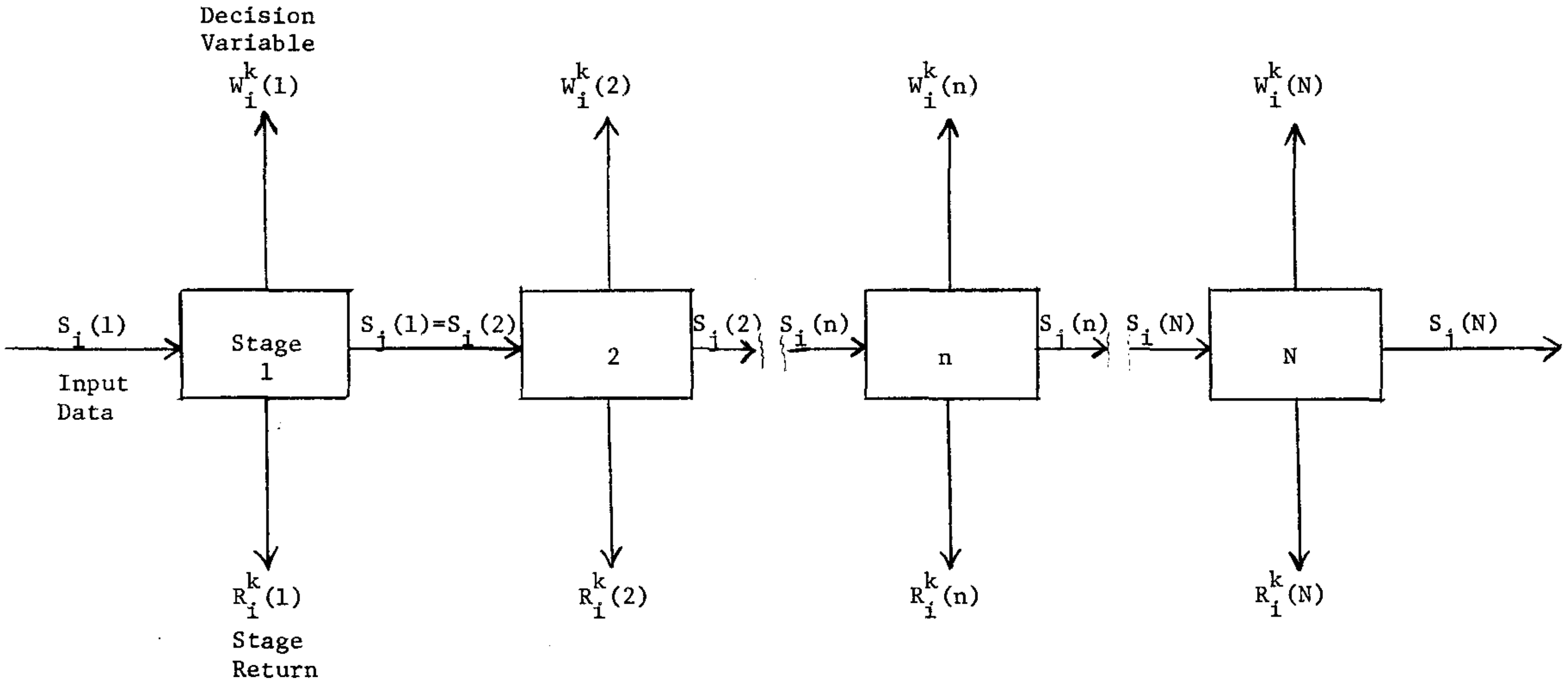

Figure 4. The Multistage Sequential Decision Model

state, (c) the transition probabilities associated with each alternative in the set for each state and (d) the net returns that accrue to each alternative in each state. The number of stages in the planning horizon and the appropriate discount rate to be used also need to be determined.

It has been mentioned that the net returns associated with alternative withdrawal rates play a very important role in determining the optimal rate of ground water withdrawal at each stage of the planning horizon. Any chosen alternative rate will have different net returns depending on how much water is pumped from each of the 48 water resource situations as each of them have unique water costs at any time. Ideally, the optimum rate of withdrawal for each water resource situation should be determined and aggregated for the study area. This could be accomplished in the following manner. First, the two linear programming production models (with a matrix of 840 rows by 4116 columns) would be modified to generate the net returns associated with each alternative rate of water withdrawal for each of the 48 water resource situations. Secondly, the optimum rate of withdrawal would be found for each water resource situation. This implies that 48 sequential decision models have to be constructed and run. This procedure was judged to be too cumbersome and taxing in the amount of personnel and computer time required to process the voluminous input and output data. Some simplifying assumptions were made to circumvent this difficulty and at the same time provide a reasonable approximation to the ideal procedure.

The first simplification is to stratify the water resource situations by the six saturated-thickness classes and one weighted average depth-to-water class (see columns (1) and (2) of Table XIX) instead of the eight depth-to-water classes. This reduces the water resource situation from 48 to six. The choice not to reduce the saturated-thickness classes was made because they determine well capacity and hence, are the principal determinants of water cost and availability. Since water costs due to lift are linear in nature, the weighted average depth introduces little or no cost bias.

The six water resource situations were programmed individually using variable resource programming (parametric programming) to generate the net returns associated with each level of water made available. Two sets of models for each of the six water resource situations were used to reflect the assumptions of Model I and Model II. These parametric programming models were not run over time as they are designed to yield conditional answers of the "if ... then" type. In other words, given various storage levels of ground water in each saturated-thickness class the models generate a set of net returns corresponding to alternative rates of ground water withdrawal irrespective of the time dimension.

The number of irrigated acres and/or the production restrictions change over time in the two RLP production models, thus influencing the optimal solutions. To avoid the expense of programming numerous acreage-storage level combinations, the parametric programming models assume that the number of irrigated acres and the production of crops in each saturated-thickness class will not exceed the maximum reached in 1990 with the corresponding production models I and II. That is, Model I's levels of production of the various crops for 1990 in each saturated-thickness class were made upper limits in the right-hand side of the parametric models that correspond to Model I. Model II's number of irrigated acres for 1990 in each saturated-thickness class was made an upper limit in the right-hand side of the parametric models that correspond to Model II. After the net returns associated with the different storage levels in each of the six saturated-thickness classes were obtained for the two sets of assumptions corresponding to Model I and Model II, the data for each saturated-thickness class was used by the sequential decision model. The name Model A is designated to refer to the sequential decision models that use data generated by the parametric programming models reflecting the assumptions of Model I. Model B refers to the sequential decision models using data generated by the parametric programming models reflecting the assumptions of Model II.

The Discrete Input and Output States

The total amount of water available in storage in 1970 in each of the six saturated-thickness classes is subdivided into a convenient set of discrete intervals which are designated as input states S_i and output states S_j. Table XVIII shows the number of these states in each saturated-thickness class and the range of ground water storage level they represent in both Models A and B. Note that the size of the class interval for the states is not the same in all saturated-thickness classes. It was chosen on the basis of the maximum use rate of water permitted by the upper limit on crop production and/or irrigated acreage restrictions imposed on the two types of parametric programming models and the magnitude of changes in water costs from one state to the next. Since the rate of water use is more conservative in Model A, the states in the first saturated-thickness class (0-100 feet) have an interval of only one million acre feet. In the next four saturated-thickness classes the class interval of the states is a wider 5 million acre feet as water costs change at a slower rate and use rates of water are high. The sixth saturated-thickness class (>500 feet)

TABLE XVIII

DISCRETE INTERVALS OF GROUND WATER STORAGE LEVELS DESIGNATING THE VARIOUS STATES OF EACH SATURATED THICKNESS CLASS IN THE SEQUENTIAL DECISION MODELS A AND B[1]

State	Saturated Thickness Class					
S_i or S_j	0-100'	101-200'	201-300'	301-400'	401-500'	>500'
	Storage of Ground Water in Million Ac. Ft.					
1	19.1-20.0	53.0-57.0	104.0-108.0	98.0-102.0	44.0-48.0	31.1-33.0
2	18.1-19.0	47.0-52.0	99.0-103.0	93.0-97.0	39.0-43.0	29.1-31.0
3	17.1-18.0	43.0-46.0	94.0-98.0	88.0-92.0	34.0-38.0	27.1-29.0
4	16.1-17.0	38.0-42.0	89.0-93.0	83.0-87.0	29.0-33.0	25.1-27.0
5	15.1-16.0	33.0-37.0	84.0-88.0	78.0-82.0	24.0-28.0	23.1-15.0
6	14.1-15.0	28.0-32.0	79.0-83.0	73.0-77.0	19.0-23.0	21.1-23.0
7	13.1-14.0	23.0-27.0	74.0-78.0	68.0-72.0	14.0-18.0	19.1-21.0
8	12.1-13.0	18.0-22.0	69.0-73.0	63.0-67.0	9.0-13.0	17.1-19.0
9	11.1-12.0	13.0-17.0	64.0-68.0	58.0-62.0	0.0-8.0	15.1-17.0
10	10.1-11.0	8.0-12.0	59.0-63.0	53.0-57.0		13.1-15.0
11	9.1-10.0	0.0-7.0	54.0-58.0	48.0-52.0		11.1-13.0
12	0.0-9.0		49.0-43.0	43.0-47.0		9.1-11.0
13			44.0-48.0	38.0-42.0		7.1-9.0
14			39.0-43.0	33.0-37.0		5.1-7.0
15			34.0-38.0	28.0-32.0		0.0-5.0
16			29.0-33.0	23.0-27.0		
17			24.0-28.0	18.0-22.0		
18			19.0-23.0	13.0-17.0		
19			0.0-19.0	0.0-12.0		

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[1] In Model B saturated thickness classes 0-100' and >500' have a smaller number of states. Since water use is higher in these WRS's under the optimal solutions of Model II, the class interval of their states is made 5 million acre feet (vs. 1 and 2 million acre feet for saturated thiskness class 0-100' and >500' in Model I) resulting in a smaller number of states.

possesses less than five percent of the total irrigable area limiting the extent of irrigated activity. The maximum use rate of water in this water resource situation can be only 2.5 million acre feet per year in Model I. The class interval of its states is two million acre feet. The number of states in each water situation was determined by the storage level that limits well capacities to such a level that irrigation systems cannot be sustained. The storage levels of the last state in each saturated-thickness class represents such a situation.

In Model B the number of states and their class intervals are the same as in Model A for all saturated-thickness classes except 0-100 feet and >500 feet. Water use is high in these water resource situations under the optimal solutions of Model II and the class interval of their states is increased to five million acre feet for Model B. According to this classification there are only four states in saturated-thickness class 0-100 feet --namely,

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \end{bmatrix} = \begin{bmatrix} 16\text{-}20 \\ 11\text{-}16 \\ 6\text{-}10 \\ 0\text{-}5 \end{bmatrix} \text{ million acre feet}$$

and seven states in saturated-thickness class >500 feet--namely,

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ S_7 \end{bmatrix} = \begin{bmatrix} 29\text{-}33 \\ 24\text{-}28 \\ 19\text{-}23 \\ 14\text{-}18 \\ 9\text{-}13 \\ 4\text{-}8 \\ 0\text{-}3 \end{bmatrix} \text{ million acre feet.}$$

The Discrete Alternative Rates of Ground Water Withdrawal

The number of discrete annual rates of ground water withdrawal, their range of values in the states to which they belong and the amount by which successive rates are incremented are shown for each water resource situation in Table XIX for both Model A and Model B. The annual rates of withdrawal were determined by experimentation in conjunction with the class size of the states and the maximum ground water use permitted by the crop production and/or acreage restrictions imposed on the corresponding parametric programming models. The information in the first row of Table XIX indicates the first saturated-thickness class has ten alternative annual rates of ground water withdrawal for each state of Model A ranging from zero to 0.45 million acre feet by successive increments of 0.05 million acre feet. In Model B the first saturated-thickness class has 11 alternative annual rates of ground water withdrawal for each state ranging from zero to one million acre feet by successive increments of 0.1 million acre feet. The rest of the data in Table XIX for the remaining water resource situations can be interpreted in a similar fashion. Note that in Model B the ranges permit higher rates of ground water withdrawal reflecting the extensive use of water in the second production model.

The Transition Probabilities

The elements of the transition probabilities matrix for each set of alternative rates of ground water withdrawal in each input state define the probability that the state will occupy a certain output state at the end of a stage via each alternative decision taken in that stage. In this study the multi-stage decision process is formulated as a deterministic

TABLE XIX

ALTERNATIVE ANNUAL RATES OF GROUND WATER WITHDRAWAL IN EACH STATE OF THE SIX SATURATED THICKNESS CLASSES IN THE SEQUENTIAL DECISION MODELS A AND B

Saturated Thickness Class (1)	Weighted Average Depth (2)	Range of Alternative Annual Rates of Ground Water Withdrawal (3)		Increment in Successive Rates of (3) (4)		Number of Alternative Annual Rates of Ground Water Withdrawal (5)	
in ft.	in ft.	in mill. ac. ft.		in mill. ac. ft.		in mill. ac. ft.	
		Model A	Model B	Model A	Model B	Model A	Model B
0-100	119	0-0.45	0-1.00	0.05	0.10	10	11
101-200	159	0-0.90	0.1.50	0.10	0.10	10	16
201-300	161	0-0.90	0-2.10	0.10	0.10	10	22
301-400	148	0-0.90	0-1.20	0.10	0.10	10	13
401-500	142	0-0.30	0-0.70	0.10	0.10	4	8
Over 500	118	0-0.25	0-0.40	0.05	0.10	6	5

case and, therefore, the elements of the probability matrix are either one or zero.

The multi-stage decision process was treated as nonstochastic because of the relative smallness of the relevant random variable, annual recharge, with respect to the magnitude of the class interval of the states. The average annual recharge for the entire area is about 0.27 million acre feet per year while the smallest class interval of the states in the six water resource situation aggregates to 23 million acre feet. This implies that recharge would have to be 85 times the estimated amount to increase the ground water storage level one state. The highest total annual precipitation recorded during the 25-year period of 1941-1965 was about 31 inches. Annual recharge from this amount of yearly precipitation will amount to only 0.46 million acre feet, which is not sufficient to affect the status of any state. The minimum value recharge can have is zero and the absence of recharge will not transfer any given state to a lower one. In the absence of large streams recharging the Central Ogallala Formation the range of variation in precipitation will not significantly alter the ground water storage level within the framework of the classification of the input and output states of the system. Because the estimated demand of ground water for municipal and industrial purposes averages about 0.21 million acre feet per year, it can be assumed that the average annual recharge will satisfy this demand. Consequently, both the recharge component and the industrial and municipal ground water demand component can be omitted in the multi-stage sequential decision models.

With these assumptions the input states at any stage of the system are transformed to other output states only by the magnitude of the rate of ground water withdrawal considered to be optimal for that stage. Since these decisions are known with certainty and the transformations are performed *ex post* for the next stage, the multi-stage sequential decision model can be formulated as a nonstochastic process. In other words, given the state of the process at the beginning of any stage and the optimal decision on the rate of withdrawal, the output state of the system for the next stage is unique. Since there are 12 formulations of the system, two formulations each representing Model A and Model B for the six water resource situations, presenting all of the transition probability matrices is a lengthy process. Therefore, saturated-thickness class 401-500 feet for Model B is chosen to serve as an illustration of how the transition probabilities were constructed in each of the 12 formulations. Table XX shows nine sets of transition probabilities, one set for each state. An interpretation of the data presented is as follows.

Suppose the system is in input state S_1 at the beginning of a given stage, there are eight alternative rates of ground water withdrawal ranging from zero to seven million acre feet per stage to choose from. If any one of the alternatives W_1 through W_5 is chosen as the optimal policy, the output of the system will still be in S_1 at the end of the stage because the probability associated with each of them is one. If one the other hand any of the alternatives W_6 through W_8 is chosen as the optimal policy, the state of the system will transit to output state S_2 at the end of that stage. The rest of the probabilities for the remaining states are interpreted in a similar fashion. Using Tables XVIII and XIX as a guide one can visualize how the probability matrices for each of the water resource situations are constructed.

TABLE XX

THE TRANSITION PROBABILITY MATRIX FOR SATURATED THICKNESS CLASS 401-500 FT. IN MODEL B

Input State at the Beginning of a Stage S_i	Storage Level mill. ac. ft.	Alternative Rate of Withdrawal per Stage W_k	mill. ac. ft.	Output State at the End of a Stage S_j								
				1	2	3	4	5	6	7	8	9
				Transition Probabilities P_{ij}^k								
1	44-48	1	0	1	0	0	0	0	0	0	0	0
		2	1	1	0	0	0	0	0	0	0	0
		3	2	1	0	0	0	0	0	0	0	0
		4	3	1	0	0	0	0	0	0	0	0
		5	4	1	0	0	0	0	0	0	0	0
		6	5	0	1	0	0	0	0	0	0	0
		7	6	0	1	0	0	0	0	0	0	0
		8	7	0	1	0	0	0	0	0	0	0
2	39-43	1	0	0	1	0	0	0	0	0	0	0
		2	1	0	1	0	0	0	0	0	0	0
		3	2	0	1	0	0	0	0	0	0	0
		4	3	0	1	0	0	0	0	0	0	0
		5	4	0	1	0	0	0	0	0	0	0
		6	5	0	0	1	0	0	0	0	0	0
		7	6	0	0	1	0	0	0	0	0	0
		8	7	0	0	1	0	0	0	0	0	0
3	34-38	1	0	0	0	1	0	0	0	0	0	0
		2	1	0	0	1	0	0	0	0	0	0
		3	2	0	0	1	0	0	0	0	0	0
		4	3	0	0	1	0	0	0	0	0	0
		5	4	0	0	1	0	0	0	0	0	0
		6	5	0	0	0	1	0	0	0	0	0
		7	6	0	0	0	1	0	0	0	0	0
		8	7	0	0	0	1	0	0	0	0	0

TABLE XX (Continued)

Input State at the Beginning of a Stage S_i	Storage Level mill. ac. ft.	Alternative Rate of Withdrawal per Stage W_k	mill. ac. ft.	Output State at the End of a Stage S_j								
				1	2	3	4	5	6	7	8	9
				Transition Probabilities P_{ij}^k								
4	29-33	1	0	0	0	0	1	0	0	0	0	0
		2	1	0	0	0	1	0	0	0	0	0
		3	2	0	0	0	1	0	0	0	0	0
		4	3	0	0	0	1	0	0	0	0	0
		5	4	0	0	0	1	0	0	0	0	0
		6	5	0	0	0	0	1	0	0	0	0
		7	6	0	0	0	0	1	0	0	0	0
		8	7	0	0	0	0	1	0	0	0	0
5	24-28	1	0	0	0	0	0	1	0	0	0	0
		2	1	0	0	0	0	1	0	0	0	0
		3	2	0	0	0	0	1	0	0	0	0
		4	3	0	0	0	0	1	0	0	0	0
		5	4	0	0	0	0	1	0	0	0	0
		6	5	0	0	0	0	0	1	0	0	0
		7	6	0	0	0	0	0	1	0	0	0
		8	7	0	0	0	0	0	1	0	0	0
6	19-23	1	0	0	0	0	0	0	1	0	0	0
		2	1	0	0	0	0	0	1	0	0	0
		3	2	0	0	0	0	0	1	0	0	0
		4	3	0	0	0	0	0	1	0	0	0
		5	4	0	0	0	0	0	1	0	0	0
		6	5	0	0	0	0	0	0	1	0	0
		7	6	0	0	0	0	0	0	1	0	0
		8	7	0	0	0	0	0	0	1	0	0

TABLE XX (Continued)

Input State at the Beginning of a Stage S_i	Storage Level mill. ac. ft.	Alternative Rate of Withdrawal per Stage		Output State at the End of a Stage S_j								
		W_k	mill. ac. ft.	1	2	3	4	5	6	7	8	9
				Transition Probabilities P_{ij}^k								
7	14-18	1	0	0	0	0	0	0	0	1	0	0
		2	1	0	0	0	0	0	0	1	0	0
		3	2	0	0	0	0	0	0	1	0	0
		4	3	0	0	0	0	0	0	1	0	0
		5	4	0	0	0	0	0	0	1	0	0
		6	5	0	0	0	0	0	0	0	1	0
		7	6	0	0	0	0	0	0	0	1	0
		8	7	0	0	0	0	0	0	0	1	0
8	9-13	1	0	0	0	0	0	0	0	0	1	0
		2	1	0	0	0	0	0	0	0	1	0
		3	2	0	0	0	0	0	0	0	1	0
		4	3	0	0	0	0	0	0	0	1	0
		5	4	0	0	0	0	0	0	0	1	0
		6	5	0	0	0	0	0	0	0	0	1
		7	6	0	0	0	0	0	0	0	0	1
		8	7	0	0	0	0	0	0	0	0	1
9	0-8	1	0	0	0	0	0	0	0	0	0	1
		2	1	0	0	0	0	0	0	0	0	1
		3	2	0	0	0	0	0	0	0	0	1
		4	3	0	0	0	0	0	0	0	0	1
		5	4	0	0	0	0	0	0	0	0	1
		6	5	0	0	0	0	0	0	0	0	1
		7	6	0	0	0	0	0	0	0	0	1
		8	7	0	0	0	0	0	0	0	0	1

The Stage Returns

The stage return constitutes the criterion by which the multi-stage decision model selects the optimal policy. The rate of withdrawal that contributes the most to the expected discounted net returns for each rate of ground water withdrawal in each state was generated from the results of the parametric programming models discussed earlier in this section. Presentation of the net returns data from each of the 12 formulations would be a lengthy process. To conserve space, input data for the one saturated-thickness class, 401-500 feet, for Model B are presented in Table XXI. This set of net returns is associated with the matrices of transition probabilities given in Table XX. When no ground water is withdrawn, alternative W_1, the dryland net return of $4.13 million is common to all storage levels. For any storage level greater than W_1, reading down Table XXI column-wise, note that the net returns decrease reflecting the effects of increasing water costs from one input state to the next. For any given input state, reading across Table XXI row-wise, note that the differences between net returns of successive alternatives becomes smaller. In other words, while the rate of increase in the ground water withdrawn is the same between successive alternatives, the rate of increase in the corresponding net returns is decreasing indicating diminishing returns exist in applying larger and larger quantities of ground water. In input state S_9 any withdrawal of ground water results in negative net returns. The net returns generated from the parametric programming models for the remaining water resource situations are given by Bekure [4, pp. 216-227].

TABLE XXI

UNDISCOUNTED ANNUAL STAGE RETURN MATRIX FOR SATURATED THICKNESS CLASS 401-500 FT. IN MODEL B

Input State S_i	Storage Level mill.ac.ft.	Alternative Rate of Withdrawal Per Stage in million ac. ft.							
		0	1	2	3	4	5	6	7
		Undiscounted Annual Net Returns in Million Dollars							
1	44-48	4.13	6.81	9.18	11.50	13.55	15.38	16.91	18.31
2	39-43	4.13	6.79	9.15	11.44	13.48	15.31	16.83	18.21
3	34-38	4.13	6.78	9.11	11.39	13.41	15.23	16.73	18.10
4	29-33	4.13	6.76	9.07	11.33	13.34	15.04	16.51	17.85
5	24-28	4.13	6.72	8.99	11.19	13.15	14.89	16.33	17.65
6	19-23	4.13	6.68	8.92	11.08	13.01	14.54	15.91	17.17
7	14-18	4.13	6.60	8.76	10.81	12.66	14.13	15.43	16.60
8	9-13	4.13	6.50	8.59	10.51	12.27	13.77	14.99	16.11
9	0-8	4.13	-[a]	-	-	-	-	-	-

[a] (-) entries indicate that net returns will be negative if those decisions are made in the corresponding state.

The Planning Horizon and the Discount Rate

Projections were developed under objective 1 for the period 1965 through 2000. The primary concern is to determine if the projected rates of withdrawal represent an intertemporal misallocation. The multi-stage sequential decision model assigns a value of zero to any water remaining in storage at the end of the planning horizon. A 100-year period from 1970-2069 was selected as the planning horizon. An infinite planning horizon could have been selected, but the present value of income received after 2070 is so small it would not affect the optimal withdrawal rates selected for the 1970-2000 period.

In the formulation of the sequential decision model the planning horizon is divided into ten ten-year intervals defining the ten stages of the system. Stage one represents the production period 1970-79, stage two represents 1980-89, etc., to stage ten which represents the production period 2060-69.

The rate of ground water withdrawal selected as optimal for each state in a given stage represents the sum of ten equal annual rates. However, the same cannot be said of the associated stage returns. As the stages represent a ten-year interval at different points in the planning horizon, the net return attributed to the first year of a given stage is not of the same value as that attributed to the tenth year. To make them comparable, the net returns of each of the ten years are discounted to their present values at the beginning of the stage. More important is making the net returns of the tenth stage comparable to those of the first stage. The use of a discount factor implies that net returns expected to accrue in time periods near to the present are of greater consequence in decision making than net returns of equal magnitude in distant time periods. The net returns in all time periods of the planning horizon are made comparable by applying an appropriate interest rate and discounting procedure.

The selection of an appropriate discount rate is important. Since ground water is developed by the private capital of farmers, the relevant discount rate may be narrowed to the selection from the interest rates f farmers face. These rates range from those on production credit in agriculture to rates on personal savings of farmers or the rate of return on fixed capital (asset equity) in farm production. It is clear that there is no one single value for the discount rate. In this study, three discount rates, $r = 0.00$, $r = 0.04$, and $r = 0.08$, are used to test the sensitivity of the optimal solution. The procedure used to discount the various net returns within a stage and between stages is given by the following relation:

$$\begin{aligned} PV = {} & R_1 \frac{1 - (1 + r)^{-n}}{r} + R_2 \frac{1 - (1 + r)^{-n}}{r} (1 + r)^{-n} \\ & + R_3 \frac{1 - (1 + r)^{-n}}{r} (1 + r)^{-2n} + \ldots \\ & + R_m \frac{1 - (1 + r)^{-n}}{r} (1 + r)^{-n(m-1)} \\ & + \ldots + R_{10} \frac{1 - (1 + r)^{-n}}{r} (1 + r)^{-9n} \end{aligned} \qquad (30)$$

where

PV = the expected present value of the stream of net returns from all stages in the planning horizon,

$m = 1, 2, \ldots, 10$ is the m^{th} stage,

$n = 10$ is the number of years in each stage,

R_m = the annual net return attributed to the optimal annual rate of ground water withdrawal in stage m,

r = the discount rate used,

$\frac{1 - (1 + r)^{-n}}{r}$ = the annuity of a net return of \$1.00 for n years at a discount rate of r, and

$(1 + r)^{-n(m-1)}$ = the present value formula for the mth stage.

There are three factors in each term of the series in relation (30), namely, R_m, the annuity formula, and the present value formula. The annuity formula discounts the stream of equal annual net returns, R_m, to the beginning of the stage and sums the ten years of each stage. The present value formula discounts the expected total net return of each stage back to the beginning of the planning horizon. In the first term of relation (30), the present value formula is implicit because it reduces to one as m-1 is equal to zero.

Solutions of the Multi-Stage Sequential Decision Models

The multi-stage decision models for each of the six saturated-thickness classes under the two assumptions of Model A and Model B were optimized using the dynamic programming technique. The computer algorithm developed for the technique follows Howard's "value iterative" method [41, pp. 26-31]. Three sets of solutions were obtained for each water resource situation under Model A and Model B. For the sake of brevity, only one solution using the 0.04 discount rate for saturated-thickness class 401-500 feet under the assumptions of Model B is shown in Tables XXII and XXIII. Note that this solution corresponds to the transition probabilities and net returns given in Tables XX and XXI, respectively. The optimal policies and their corresponding maximum expected discounted net returns are conditional for every

TABLE XXII

SOLUTION OF THE MULTISTAGE SEQUENTIAL DECISION MODEL: OPTIMAL RATES OF GROUND WATER WITHDRAWAL FOR SATURATED THICKNESS 401-500 FT. MODEL B, r = 0.04

State of the System S_i	Storage Level mill. ac. ft.	Stage in the Planning Horizon Optimal Rates of Ground Water Withdrawal in million ac. ft. 1	2	3	4	5	6	7	8	9	10
1	44-48	7*	7	7	7	7	7	7	7	7	7
2	39-43	7	7*	7	7	7	7	7	7	7	7
3	34-38	7	7	7	7	7	7	7	7	7	7
4	29-33	7	7	7*	7	7	7	7	7	7	7
5	24-28	7	7	7	7*	7	7	7	7	7	7
6	19-23	7	7	7	7	7*	7	7	7	7	7
7	14-18	7	7	7	7	7	7	7	7	7	7
8	9-13	4	4	4	4	4	4*	4	4	4	7
9	0-8	0	0	0	0	0	0	0*	0*	0*	0*

TABLE XXIII

SOLUTION OF THE MULTISTAGE SEQUENTIAL DECISION MODEL MAXIMUM EXPECTED DISCOUNTED NET RETURNS CORRESPONDING TO THE OPTIMAL DECISIONS FOR SATURATED THICKNESS

401-500 FT. MODEL B, $r = 0.04$[a]

State of the System S_i	Storage Level mill. ac. ft.	Stage in the Planning Horizon									
		1	2	3	4	5	6	7	8	9	10
		Total Expected Discounted Net Returns in Mill. Dollars									
1	44-48	436.6*	434.2	430.5	422.1	409.3	389.7	359.9	315.2	248.2	148.5
2	39-43	429.0	426.5*	422.9	417.4	405.0	386.0	357.0	312.9	246.8	147.7
3	34-38	418.9	416.4	412.8	407.3	399.3	380.9	352.8	309.9	244.6	146.8
4	29-33	405.3	402.8	399.1*	393.7	385.7	373.7	346.5	305.0	241.4	144.7
5	24-28	388.1	385.6	382.0	376.5*	368.5	356.6	339.0	299.0	237.2	143.1
6	19-23	365.1	362.6	358.9	353.5	345.5*	333.6	316.0	290.0	230.2	139.2
7	14-18	336.8	334.3	330.6	325.2	317.2	305.3	287.6	261.6	223.0	134.7
8	9-13	301.6	299.1	295.5	290.0	282.0	270.1*	252.5	226.4	187.8	130.7
9	0-8	101.3	100.3	98.9	96.7	93.5	88.8	81.8*	71.5*	56.2*	33.5*

[a] The maximum expected net returns refers to the entire planning horizon provided that optimal policies are followed in the remaining stages.

possible input state of the system in each stage. To map the optimal policy to be followed through the ten stages of the planning horizon, one must start at the beginning of stage one and proceed step by step to stage ten. In saturated-thickness class 401-500 feet, the input state in stage one is 47 million acre feet which falls in input state S_1. The optimal policy, as shown in Table XXII, is to withdraw seven million acre feet of water during the first ten years. This reduces the supply of ground water in storage to 40 million acre feet which transforms the system to output state S_2 at the end of stage one. This implies that the input state of the system at the beginning of stage two is S_2. The optimal policy in stage two when the system is in S_2 is again to withdraw seven million acre feet which reduces the supply in storage to 33 million acre feet thus transforming the system to output state S_4 at the end of the second stage. At the beginning of stage three the input state of the system is S_4. Following this procedure one can trace the movement of the system from stage to stage and map the optimum strategy for allocating the ground water over the planning horizon. The asterisks in Table XXII indicate the input state of the system, the optimal rate of ground water withdrawal and the resulting output of the system for the ten stages of the planning horizon. Table XXIII shows the maximum discounted net returns that can be expected from the current and remaining stages during the 100-year planning horizon provided that an optimal policy is followed at each subsequent stage. The values with asterisks are the maximum expected discounted net returns corresponding to the optimal policies also shown by asterisks in Table XXII.

The optimal rates of ground water withdrawal at each stage for the six water resource situations using each of the three discount rates were traced by the procedure described above. The optimal policies and their corresponding maximum expected net returns for the current and remaining stages are presented in Table XXIV for both Model A and Model B. The results are also aggregated for the study area.

At any given stage of the planning horizon, there are economic forces working in opposite directions. Increased costs of pumping and distributing water for the remaining stages in the planning horizon tend to discourage high rates of ground water withdrawal in the current stage. Diminishing marginal net returns to water set in at high rates of water use, particularly if the storage level is low, which again tends to reduce the optimal rate of ground water withdrawal per stage. On the other hand, higher preference for income in the early stages of the planning horizon as reflected by the discount factor and increasing marginal returns to additional rates of water, particularly when storage levels are high and withdrawal rates are low, tend to increase the optimal rates of withdrawal per stage. The fact that high storage levels are encountered at the stages toward the beginning of the planning horizon tend to reinforce the time preference effect and thus intensify the optimal ground water withdrawal rates in the early periods. The results tabulated in Table XXIV are the net effects of the interplay of these forces. Examining the optimal policies from stage to stage reveals that the optimal rates of ground water withdrawal are higher at the beginning of the planning horizon and progressively diminish towards the end.

In general solutions of the optimal rates of ground water withdrawal under the assumptions of Model A indicate that, except in a few borderline cases due to the discreteness of the states, the results are the same

TABLE XXIV

OPTIMAL POLICIES OF GROUND WATER WITHDRAWAL AND THEIR EXPECTED DISCOUNTED NET RETURNS ACCORDING TO THE SOLUTIONS OF THE MULTISTAGE SEQUENTIAL DECISION MODELS A AND B AT THREE DISCOUNT RATES

				Study Area	Model A							Study Area	Model B						
					Saturated Thickness in Ft.								Saturated Thickness in Ft.						
Stage	Discount at r =	Item	Unit	Level of Storage	0-100	101-200	201-300	301-400	401-500	>500	Total Study Area	Level of Storage	0-100	101-200	201-300	301-400	401-500	>500	Total Study Area
1 (1970-79)	0.00	Rate of Withdrawal	mill.Ac.Ft.	358.0	3.5	4.0	9.0	8.0	3.0	1.5	29.0	357.0	4.0	4.0	9.0	4.0	4.0	4.0	29.0
		Expected Discounted Net Income	mill. dol.		2,326.510	2,159.260	2,531.730	2,067.29	789.580	514.360	10,388.73		2,223.549	3,000.479	3,996.594	2,439.528	1,671.023	1,176.799	14,507.972
	0.04	Rate of Withdrawal	mill.Ac.Ft.	358.0	3.5	9.0	9.0	8.0	3.0	2.0	34.5	357.0	9.0	14.0	19.0	12.0	7.0	4.0	65.0
		Expected Discounted Net Income	mill. dol.		398.946	551.554	626.648	512.054	193.504	126.699	2,409.405		557.285	1,886.455	2,542.246	1,473.613	436.647	643.975	7,540.221
	0.08	Rate of Withdrawal	mill.Ac.Ft	358.0	4.5	9.0	9.0	8.0	3.0	2.0	35.5	357.0	9.0	14.0	21.0	12.0	7.0	4.0	67.0
		Expected Discounted Net Income	mill. dol.		234.527	284.841	320.410	261.939	98.635	64.871	1,265.223		294.482	1,710.961	2,276.122	1,276.472	227.050	543.889	6,328.976
		RLP Rate of Withdrawal	mill.Ac.Ft.	358.0	2.3	5.56	9.41	6.55	2.5	2.18	26.43	357.0	10.44	9.82	11.29	9.15	3.39	2.01	46.10
2 (1980-89	0.00	Rate of Withdrawal	mill.Ac.Ft.	329.0	3.5	4.0	9.0	8.0	3.0	1.5	29.0	328.0	4.0	4.0	9.0	4.0	4.0	4.0	29.0
		Expected Discounted Net Income	mill. dol.		2,104.660	1,928.820	2,262.170	1,848.030	710.620	463.220	9,317.520		2,005.717	2,670.156	3,543.732	2,224.593	1,486.413	1,059.119	12,989.730
	0.04	Rate of Withdrawal	mill.Ac.Ft.	323.5	3.5	9.0	9.0	8.0	3.0	2.0	34.5	292.0	9.0	14.0	19.0	12.0	7.0	4.0	65.0
		Expected Discounted Net Income	mill. dol.		370.862	533.168	616.534	502.698	191.626	124.999	2,341.287		528.512	1,700.634	2,371.897	1,434.740	426.520	637.726	7,100.029
	0.08	Rate of Withdrawal	mill.Ac.Ft.	323.5	3.5	9.0	9.0	8.0	3.0	2.0	34.5	290.0	9.0	14.0	19.0	12.0	7.0	4.0	65.0
		Expected Discounted Net Income	mill. dol.		200.723	280.001	318.677	260.233	98.583	64.605	1,222.822		278.147	1,597.770	2,197.119	1,263.653	224.964	543.602	6,105.255
		RLP Rate of Withdrawal	mill.Ac.Ft.	331.6	0.40	6.19	8.80	7.63	2.80	2.45	27.91	310.90	0.004	13.20	15.19	12.27	4.56	2.71	47.934

TABLE XXIV (Continued)

				Study Area	Model A							Study Area	Model B						
				Level of	Saturated Thickness in Ft.						Total	Level of	Saturated Thickness in Ft.						Total
Stage	Discount at r =	Item	Unit	Storage	0-100	101-200	201-300	301-400	401-500	>500	Study Area	Storage	0-100	101-200	201-300	301-400	401-500	>500	Study Area
3 (1990-99)	0.00	Rate of Withdrawal	mill.Ac.Ft.	300.0	0.5	4.0	9.0	8.0	3.0	1.5	26.0	299.0	4.0	4.0	9.0	4.0	4.0	4.0	29.0
		Expected Discounted Net Income	mill. dol.		988.400	1,699.410	1,989.410	1,638.990	627.82	410.51	7,354.54		1,742.907	2,340.833	3,092.395	1,998.776	1,351.609	930.477	11,456.997
	0.04	Rate of Withdrawal	mill.Ac.Ft.	289.0	1.5	9.0	9.0	8.0	3.0	2.0	32.5	227.0	0.0	9.0	19.0	12.0	7.0	4.0	51.0
		Expected Discounted Net Income	mill. dol.		315.340	505.200	602.690	494.599	187.697	122.692	2,228.218		282.381	1,407.962	2,116.662	1,378.100	399.122	621.157	6,205.384
	0.08	Rate of Withdrawal	mill.Ac.Ft.	289.0	1.5	9.0	9.0	8.0	3.0	2.0	32.5	225.0	0.0	9.0	19.0	12.0	7.0	4.0	51.0
		Expected Discounted Net Income	mill. dol.		154.082	271.388	315.904	259.053	97.871	64.282	1,162.580		147.272	1,315.854	2,033.828	1,239.978	217.234	536.660	5,490.826
		RLP Rate of Withdrawal	mill.Ac.Ft.	303.7	0.43	6.13	8.60	8.64	3.30	2.24	29.34	263.0	0.0	15.95	18.44	14.86	5.54	3.29	58.08
4 (2000-09	0.00	Rate of Withdrawal	mill.Ac.Ft.	274.0	0.5	4.0	9.0	8.0	3.0	1.5	26.0	270.0	4.0	4.0	9.0	12.0	4.0	4.0	37.0
		Expected Discounted Net Income	mill. dol.		864.85	1,502.730	1,719.800	1,419.920	549.340	358.180	6,414.820		1,467.228	2,120.632	2,642.508	1,837.241	1,116.857	806.495	9,990.961
	0.04	Rate of Withdrawal	mill.Ac.Ft.	256.5	0.0	9.0	9.0	8.0	3.0	2.0	31.0	176.0	0.0	9.0	14.0	12.0	7.0	4.0	46.0
		Expected Discounted Net Income	mill. dol.		252.137	479.798	582.789	478.846	183.608	119.517	2,096.695		276.229	1,259.907	1,751.600	1,307.130	376.522	601.898	5,573.286
	0.08	Rate of Withdrawal	mill.Ac.Ft.	256.5	0.0	9.0	9.0	8.0	3.0	2.0	31.0	174.0	0.0	9.0	14.0	12.0	7.0	4.0	46.0
		Expected Discounted Net Income	mill. dol.		134.000	262.595	311.710	255.745	97.631	63.875	1,125.887		146.909	1,193.440	1,793.134	1,212.381	210.465	530.295	5,016.624
		RLP Rate of Withdrawal	mill.Ac.Ft.	274.4	0.0	3.27	6.99	9.41	3.75	1.43	24.85	204.9	0.0	0.01	21.05	16.87	6.34	3.76	48.03

TABLE XXIV (Continued)

Stage	Discount at r =	Item	Unit	Study Area Level of Storage	Model A							Study Area Level of Storage	Model B						
					Saturated Thickness in Ft.						Total Study Area		Saturated Thickness in Ft.						Total Study Area
					0-100	101-200	201-300	301-400	401-500	>500			0-100	101-200	201-300	301-400	401-500	>500	
5 (2010-19)	0.00	Rate of Withdrawal	mill.Ac.Ft.	248.0	0.0	4.0	9.0	8.0	3.0	1.5	25.5	233.0	0.0	4.0	9.0	12.0	4.0	4.0	33.0
		Expected Discounted Net Income	mill. dol.		646.61	1,276.130	1,467.400	1,213.440	468.220	306.000	5,377.800		703.942	1,793.434	2,291.160	1,509.361	987.131	681.469	7,966.497
	0.04	Rate of Withdrawal	mill.Ac.Ft.	225.5	0.0	9.0	9.0	8.0	3.0	2.0	31.0	130.0	0.0	0.0	14.0	12.0	7.0	4.0	37.0
		Expected Discounted Net Income	mill. dol.		243.825	433.737	559.612	460.893	176.556	114.940	1,989.563		267.123	604.566	1,478.545	1,177.572	345.467	573.795	4,447.068
	0.08	Rate of Withdrawal	mill.Ac.Ft.	225.5	0.0	9.0	9.0	8.0	3.0	2.0	31.0	128.0	0.0	0.0	14.0	12.0	7.0	4.0	37.0
		Expected Discounted Net Income	mill. dol.		133.356	241.720	307.950	253.080	96.565	63.142	1,095.813		146.127	547.868	1,282.532	1,142.872	198.741	519.983	3,838.123
6 (2020-29)	0.00	Rate of Withdrawal	mill.Ac.Ft.	222.5	0.0	0.0	9.0	8.0	3.0	1.5	21.5	200.0	0.0	4.0	14.0	12.0	4.0	4.0	38.0
		Expected Discounted Net Income	mill. dol.		538.840	1,049.470	1,200.370	997.32	387.80	255.30	4,429.100		590.329	1,468.279	1,841.480	1,236.042	806.857	547.480	6,490.467
	0.04	Rate of Withdrawal	mill.Ac.Ft.	194.5	0.0	0.0	9.0	8.0	3.0	2.0	22.0	93.0	0.0	0.0	4.0	12.0	4.0	4.0	24.0
		Expected Discounted Net Income	mill. dol.		231.522	216.329	521.363	432.334	166.626	108.555	1,676.729		253.645	574.061	1,090.766	1,037.486	270.110	525.260	3,751.328
	0.08	Rate of Withdrawal	mill.Ac.Ft.	194.5	0.0	0.0	9.0	8.0	3.0	2.0	22.0	91.0	0.0	0.0	4.0	12.0	5.0	4.0	25.0
		Expected Discounted Net Income	mill. dol.		131.816	123.166	299.084	247.473	94.868	62.030	958.437		144.438	541.535	1,028.963	1,057.053	151.302	495.499	3,418.790

TABLE XXIV (Continued)

Stage	Discount at r =	Item	Unit	Study Area Level of Storage	Model A							Study Area Level of Storage	Model B						
					Saturated Thickness in Ft. 0-190	101-200	201-300	301-400	401-500	>500	Total Study Area		Saturated Thickness in Ft. 0-100	101-200	201-300	301-400	401-500	>500	Total Study Area
7 (2030-39)	0.00	Rate of Withdrawal	mill.Ac.Ft.	201.0	0.0	0.0	9.0	8.0	3.0	1.5	21.5	162.0	0.0	4.0	9.0	9.0	4.0	4.0	30.0
		Expected Discounted Net Income	mill. dol.		431.070	828.490	938.990	782.003	310.240	203.530	7,494.323		472.629	1,144.804	1,315.458	965.220	628.951	437.984	4,965.046
	0.04	Rate of Withdrawal	mill.Ac.Ft.	172.5	0.0	0.0	9.0	8.0	3.0	1.5	21.5	69.0	0.0	0.0	0.0	12.0	0.0	4.0	16.0
		Expected Discounted Net Income	mill. dol.		213.312	199.3'4	467.502	390.081	153.520	99.441	1,541.170		233.695	528.909	608.136	851.470	81.838	483.947	2,787.995
	0.08	Rate of Withdrawal	mill.Ac.Ft.	172.5	0.0	0.0	9.0	8.0	3.0	2.0	22.0	66.0	0.0	0.0	4.0	12.0	0.0	4.0	20.0
		Expected Discounted Net Income	mill. dol.		128.490	120.058	282.818	236.425	92.474	60.036	920.301		140.791	527.862	1,002.983	914.427	49.304	482.988	3,118.355
8 (2040-49)	0.00	Rate of Withdrawal	mill.Ac.Ft.	179.5	0.0	0.0	9.0	8.0	3.0	1.5	21.5	132.0	0.0	4.0	4.0	9.0	7.0	4.0	28.0
		Expected Discounted Net Income	mill. dol.		323.300	604.760	690.420	581.413	229.280	151.970	2,581.143		354.197	825.154	880.309	697.502	498.861	310.094	3,566.117
	0.04	Rate of Withdrawal	mill.Ac.Ft.	151.0	0.0	0.0	9.0	8.0	3.0	2.0	22.0	53.0	0.0	0.0	0.0	4.0	0.0	4.0	8.0
		Expected Discounted Net Income	mill. dol.		186.358	174.128	399.000	336.568	132.159	87.034	1,315.247		204.165	462.077	531.293	592.260	71.497	399.121	2,260.413
	0.08	Rate of Withdrawal	mill.Ac.Ft.	150.5	0.0	0.0	9.0	8.0	3.0	2.0	22.0	46.0	0.0	0.0	0.0	4.0	0.0	4.0	8.0
		Expected Discounted Net Income	mill. dol.		121.308	113.348	260.302	219.884	86.028	56.278	857.148		132.917	498.342	572.990	638.743	46.545	430.446	2,319.983

TABLE XXIV (Continued)

				Model A								Model B							
				Study Area Level of Storage	Saturated Thickness in Ft.						Total	Study Area Level of Storage	Saturated Thickness in Ft.						Total
Stage	Discount at r =	Item	Unit		0-100	101-200	201-300	301-400	401-500	>500	Study Area		0-100	101-200	201-300	301-400	401-500	>500	Study Area
9 (2050-59)	0.00	Rate of Withdrawal	mill.Ac.Ft.	158.0	0.0	0.0	4.0	8.0	3.0	1.5	16.5	104.0	0.0	4.0	4.0	9.0	4.0	0.0	21.0
		Expected Discounted Net Income	mill. dol.		215.540	413.300	436.410	379.087	152.856	100.900	1,698.087		236.132	513.338	552.819	437.270	283.859	49.051	2,072.469
	0.04	Rate of Withdrawal	mill.Ac.Ft.	129.0	0.0	0.0	4.0	8.0	3.0	2.0	17.0	45.0	0.0	0.0	0.0	4.0	0.0	0.0	4.0
		Expected Discounted Net Income	mill. dol.		146.462	136.850	293.972	257.926	103.866	68.040	1,007.116		160.457	363.153	417.551	465.466	56.190	74.265	1,537.082
	0.08	Rate of Withdrawal	mill.Ac.Ft.	128.5	0.0	0.0	4.0	8.0	3.0	1.5	16.5	38.0	0.0	0.0	0.0	4.0	0.0	0.0	4.0
		Expected Discounted Net Income	mill. dol.		105.800	98.857	210.688	186.531	75.030	48.693	725.599		115.920	434.614	499.716	557.059	40.594	89.072	1,736.975
10 (2060-69)	0.00	Rate of Withdrawal	mill.Ac.Ft.	141.5	0.0	0.0	0.0	8.0	3.0	1.5	12.5	83.0	0.0	3.0	4.0	9.0	0.0	0.0	16.0
		Expected Discounted Net Income	mill. dol.		107.770	184.620	115.760	188.298	75.540	50.63	722.618		118.066	208.546	227.382	249.210	41.345	24.526	869.075
	0.04	Rate of Withdrawal	mill.Ac.Ft.	112.0	0.0	0.0	0.0	8.0	3.0	2.0	13.0	37.0	0.0	0.0	0.0	4.0	0.0	0.0	4.0
		Expected Discounted Net Income	mill. dol.		87.409	81.672	93.887	152.725	61.271	40.366	517.330		95.761	216.730	249.195	277.791	33.534	44.418	917.429
	0.08	Rate of Withdrawal	mill.Ac.Ft.	112.0	0.0	0.0	0.0	8.0	3.0	2.0	13.0	34.0	0.0	0.0	0.0	4.0	0.0	0.0	4.0
		Expected Discounted Net Income	mill. dol.		72.312	67.567	77.672	126.348	50.689	33.394	427.982		79.223	297.030	341.522	380.713	27.743	60.875	1,187.106

for the two discount rates of four and eight percent. The results for a zero discount rate (i.e., for not discounting) indicate that the optimal rate of withdrawal is substantially reduced. This implies that the optimal policy is sensitive only to discount rates close to zero. If future returns are discounted at very low interest rates, the results show that it is advantageous to use a low rate of ground water withdrawal so that there will be an adequate supply for future years. Discounting at rates equal to or higher than four percent requires high rates of ground water withdrawal to maximize the present value of the net return stream for the ten stages of the planning horizon.

In Model B there is no difference between the optimal policies obtained by discounting at a rate of four percent and those obtained by using a rate of eight percent except in three stages. In stages one, six and seven, discounting by eight percent results in higher rates of ground water withdrawal. The difference in the rates are two, one and four million acre feet in the respective stages. Note that the difference between the optimal rate of ground water withdrawal with no discounting and discounting by using either four or eight percent is substantially higher than that in Model A. This difference in stages one and two of Model B is about 36 million acre feet compared to 5.5 million acre feet in Model A, again reflecting the effect of the restrictive assumptions of product supply in Model I. As discounting at interest rates of four and eight percent encourage optimal policies of high rates of ground water withdrawal, the water in saturated-thickness class 0-100 feet becomes uneconomical for irrigation purposes, thus resulting in optimal policies of no water withdrawal in stage three. Diminishing net returns to water in saturated-thickness class 101-200 feet also cause optimal policies of reduced rates of ground water withdrawal in stage three. This means that the aggregate optimal policy for the study area is decreased to a lower rate of ground water withdrawal. Therefore, the gap in the aggregate optimal policy between discounting and not discounting diminishes in stages three and four. Beginning with stage five, the optimal rate of withdrawal in saturated-thickness class 101-200 feet with discounting becomes zero, narrowing the gap further. In stage six diminishing marginal net returns in saturated-thickness class 201-300 feet force a reduced optimal rate of withdrawal. Notice that the optimal rate of ground water withdrawal is higher with no discounting than with discounting. This illustrates that a slower rate of mining the aquifer over time is optimal when the time preference for money income is ignored. The slower withdrawal rate contributes to diminishing net returns of future years through increased pumping and distribution costs at a very gradual rate.

Policy Implications of the Results

It is interesting to compare the rate at which water is withdrawn from the aquifer by production Models I and II for the period 1970-2000 and the optimal policy suggested by the corresponding multi-stage sequential decision Models A and B. In order to make the comparison it may be necessary to reiterate the assumption that the results of the two linear programming models will be regarded as a close approximation of how irrigators will perform if decisions of allocating ground water are left for them to make on an individual basis. On the other hand, the solutions of the multi-stage sequential decision models are assumed to represent decisions on the intertemporal allocation of ground water being taken by all irrigators acting in concert through a public agency or through one or more water districts.

The last entries of each stage in Table XXIV give the corresponding linear programming rates of ground water withdrawal. Comparing the total rates of ground water withdrawal for the study area, Model I's rates are less than the rates suggested optimal by Model A in stages one, two and four for all three discount rates. In stage three they are somewhat higher than the optimal rate with no discounting, but less than the optimal rates with discount rates of four and eight percent. However, looking at the column in Table XXIV indicating the study area's level of ground water storage, one finds that Model I's storage levels are higher than those of Model A using four and eight percent discount rates. These results suggest that depleting the study area's water supply according to the solutions of Model I will not result in general uneconomic mining of the Central Ogallala Formation. If production is limited to the 1965 level (about 1.5 million acres), the only control that can be justified economically is well spacing to avoid interference between neighboring wells.

A comparison of Model II's rate of ground water withdrawal with that suggested optimal by Model B shows that in stages one and two, Model II's rate is substantially lower (17 to 21 million acre feet) for discount rates using four and eight percent. The rate of ground water withdrawal in Model II is 7 million acre feet greater than Model B in stage 3 and 2 million greater in stage 4 for both discount rates. However, looking at the study area level of ground water storage, Model II has a higher level of supply than those indicated for both discount rates in all four stages (see Table XXIV). A more accurate comparison of Model II and Model B solutions can be made by using the Model B aggregate conditional optimal rates of ground water withdrawal for 15 input states of the system shown in Table XXV. Let

TABLE XXV

AGGREGATE CONDITIONAL OPTIMAL POLICIES OF GROUND WATER WITHDRAWAL FOR POSSIBLE INPUT STATES OF THE SYSTEM ACCORDING TO MODEL B

State of the System	Ground Water Storage Level	Stage					Stage			
		1	2	3	4	5-10	1	2	3	4-10
		Stages Remaining in Planning Horizon					Stages Remaining in Planning Horizon			
		10	9	8	7	6-1	10	9	8	7-1
		r = 0.04					r = 0.08			
S_i	Mill. ac. ft.	Optimal Rates of Ground Water Withdrawal Mill. ac. ft.					Optimal Rates of Ground Water Withdrawal Mill. ac. ft.			
1	346-370	68	68	67	65	65	68	68	67	67
2	321-345	68	67	65	65	65	68	67	65	65
3	301-320	63	62	60	60	60	63	62	62	62
4	276-300	59	58	56	56	56	59	58	58	58
5	251-275	57	56	56	56	56	57	56	56	56
6	231-250	57	56	51	51	51	57	56	56	56
7	211-230	52	47	47	47	47	52	47	47	47
8	186-210	47	47	47	47	47	47	47	47	47
9	161-185	41	41	41	41	41	41	41	41	41
10	136-160	32	32	31	31	31	32	32	31	31
11	111-135	31	31	26	26	26	31	31	31	31
12	86-110	26	26	26	26	26	26	26	26	26
13	66-85	18	18	18	18	18	18	18	18	18
14	46-65	8	8	8	8	8	8	8	8	8
15	0-45	0	0	0	0	0	0	0	0	0

the system be in state one where the ground water storage is between 346 and 370 million acre feet and let the discount rate be four percent. Then, Table XXV indicates that the optimal policy to follow in stage one, where there are ten stages remaining in the planning horizon, is 68 million acre feet (6.8 million acre feet annually). If the system were in stage two the optimal policy would again be 68 million acre feet; but it would be 67 million acre feet if the system were in stage three. Notice that for stages four through ten, the optimal policy converges to a single value. For a discount rate of eight percent, convergence of the optimal policy occurs in stage three.

A comparison of the study area level of storage at each stage for Model II and Model B in Table XXIV indicates the quantity of water remaining in storage under the optimal withdrawal policy with a discount rate of either four or eight percent is less than the remaining storage projected by Model II. For instance the amount of water remaining in storage at the start of stage 4 (the year 2000) following withdrawals required by Model II is 204.9 million acre feet. The study area levels of storage following withdrawal policies prescribed by Model B are 176.0 and 174.0 million acre feet for four and eight percent discount rates, respectively. The conclusion that can be drawn from the comparison of the results of Model II and Model B is that the projected rates of withdrawal do not exceed the rate required to maximize the present value of primary irrigation benefits prior to 1990. However, the rates of annual ground water withdrawal exceed the amount which will maximize the study area's net returns starting from stage 3. This implies that measures other than the spacing of wells may be necessary to regulate the extraction of ground water from the Central Ogallala

Formation to conform to those rates which will maximize the study area's net returns over a longer period of time.

ANALYZING THE IMPACT OF ALTERNATIVE METHODS OF WATER USE REGULATION

The projections of irrigation development completed under objective 1 of this study indicate the static water level can be expected to continue declining in the Central Ogallala. Using the criterion of maximizing the present value of net returns to the stock water supply, the analysis under the second objective indicates the projected rates of withdrawal for the next twenty years do not exceed the optimum withdrawal rates. However, many area residents and policy makers place a high value on the conservation of water and other natural resources. Furthermore, one may question whether the procedure used in objective 2 underestimates the value of the stock water supply in future years.[6] To the extent future values are underestimated, the withdrawal policy selected will tend to recommend using larger quantities in the near future and smaller quantities in the distant future than are actually optimal. For these reasons, interest prevails in the effect of alternative methods of water-use regulation in the Central Ogallala. The purpose of this portion of the study is to investigate the effect of three alternative methods of water use regulation on representative farms and the entire study area. More specifically the subobjectives of

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[6] In general, any change in technology which either reduces the water pumping and distribution costs or increases production per acre foot of water used (e.g., improved varieties or reduced evapotranspiration) would tend to increase the value of water in the future. Changes in demand and supply conditions resulting in higher prices for the products would also increase the value of water in the future. Given the current federal farm programs designed to reduce production of feed and food grains, it can also be argued that the current value of water used in production of irrigated crops in the study area is much lower--perhaps zero--when considered from a national standpoint.

this portion of the analysis are:

(1) To construct a model of a representative farm firm capable of simulating the effects of soil moisture and atmospheric stress during critical stages of plant development on yields of the major irrigated and dryland crops of the Central Ogallala.

(2) To simulate, for poor and adequate water resource situations, over a 20-year period, several alternative methods of regulating water-use including

   (a) Continued pumping at the present rate with no restraints on water use;

   (b) Restricting the quantity of water pumped per crop year to 1.5 acre feet per acre of water rights; and

   (c) Restricting the quantity of water pumped per crop year to 1.5 acre feet per acre of water rights, but allowing the irrigator to apply additional irrigation water if it is economically feasible to pay a graduated tax of \$.50 per acre inch for each acre inch pumped above the quantity limitation.

(3) To compare the effects of the three methods of water-use regulation on net farm income, variability of net farm income, net worth, variability of net worth, quantity of water pumped and availability of water for future periods.

(4) To evaluate the alternative methods of restraining water use by discounting the streams of net returns and comparing present values of those net income streams.

Research Procedures

Rationale for the Regulatory Means Analyzed

Previous discussion has established that the water supply in the Central Ogallala is a stock resource having the property of commonality. The problem of commonality of water use leads to "spill-over" costs. That is, all the costs of pumping are not borne by the individual irrigator, but fall upon other pumpers in the basin and society in general [59, pp. 428-429].

These "spill-over" costs result in a divergence of private and social costs. The difference in optimal water allocations caused by the divergence of private and social costs is illustrated in Figure 5. The marginal social cost curve (MSC) lies above the marginal private cost curve (MPC). The marginal value product curve (MVP) represents the value of water in use. The individual irrigator in seeking to optimally allocate his water resources considers only marginal private costs. Thus, the optimal allocation of water resources for the individual occurs where the MPC of pumping the incremental unit of water equals the MVP of that unit of water, or at point D in Figure 5. Each individual pumps ob acre feet of irrigation water.

The socially optimal allocation of water results only when marginal social costs are considered in the allocative process. Each producer should equate MSC and MVP (point C in Figure 5) with the socially optimal allocation of water being oa acre feet. Thus, if the individual producer does not consider the full social and private cost of irrigation water used in production, his decisions tend to push water use beyond socially optimum levels by an amount equal to ab.

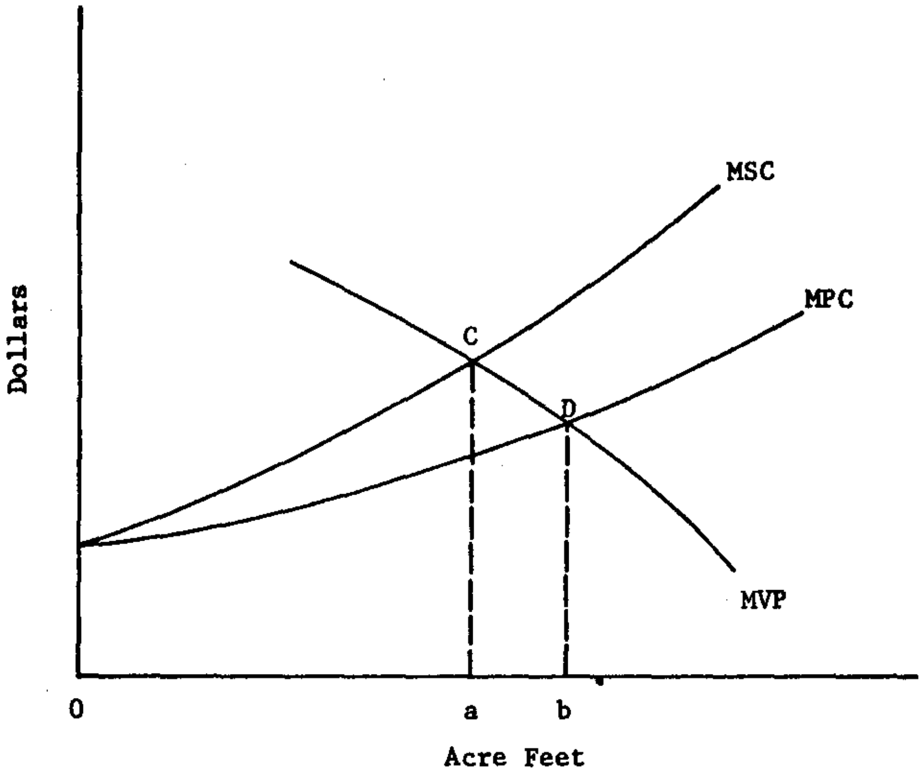

Figure 5. Illustration of the Divergence of Private and Social Costs and the Resulting Resource Allocations

Even though rights in water exist through the Doctrine of Prior Appropriation, Water Resource Boards maintain a measure of control over water use. For example, the Oklahoma Water Resources Board has the power to order proper spacing of wells to insure an orderly withdrawal of water in relation to average annual recharge. It can also require metering of wells to record amounts pumped and can require persons to cease excessive withdrawals in reverse order of their water rights. It is empowered to restrict the rate of water use to one cubic foot of water per second for each seventy acres, or equivalent thereof, delivered on the land, for a specified time in each year [81, p. 15]. By not indicating the intended length of "a specified time in each year," water use may be restricted to any amount desired by the Water Resources Board.

The existence of regulatory power and exercising this power are two different matters. Many questions concerning the effect of each alternative control measure on water use, private versus social costs, the pattern of regional production and the impact on regional income must be evaluated before policy makers can recommend a course of action.

Two institutional alternatives appear capable of more closely aligning marginal private and marginal social costs. The first of these is limiting the quantity of water each irrigator is allowed to pump per year. The socially optimal limitation, as depicted in Figure 6, is oa acre feet per individual. By limiting individual pumpers to oa acre feet, the objective of forcing alignment of MSC and MVP is achieved and a socially optimal allocation of water resources results.

Theoretically, limiting water use to socially optimal levels through the use of a quantity limitation is sound. From a practical standpoint,

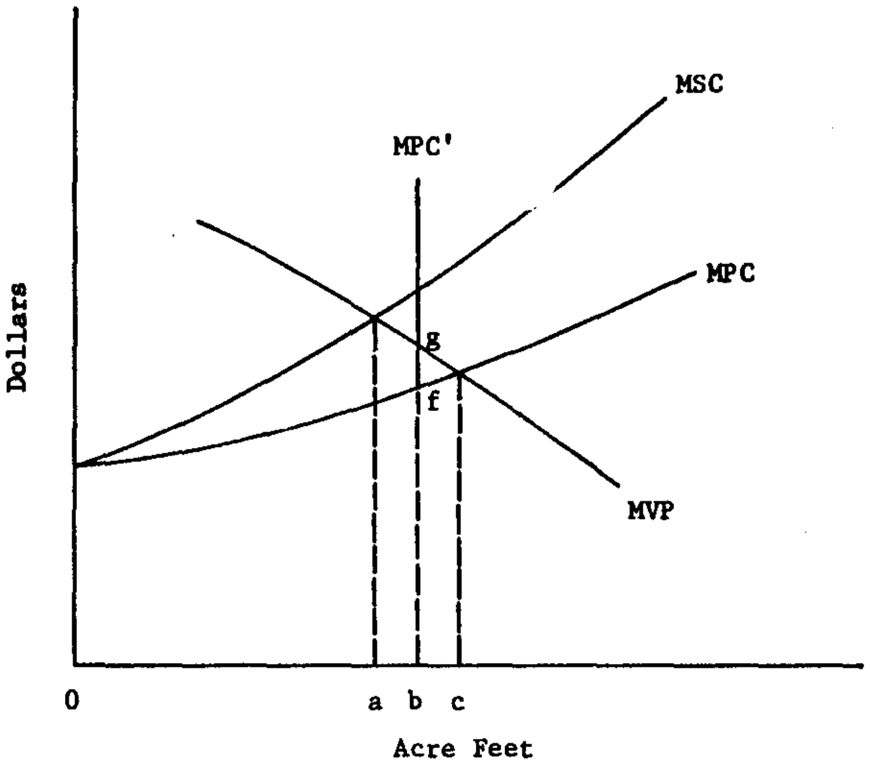

Figure 6. The Effect of a Quantity Limitation on Divergence of Private and Social Costs and Resource Allocation

several problems arise. First, a quantity limitation works best when annual recharge is large relative to water use. The limitation can be set to a "safe yield" for the aquifer and socially optimal resource allocations achieved. However, if recharge is negligible relative to current water usage, and such is the case in the study area, limitation of water use to a safe yield, or to the amount of average annual recharge, would not be economic. A realistic quantity limitation might be ob acre feet per year in Figure 6. If the irrigator is forced to observe the quantity restriction, with the alternative being a severe penalty in the form of a fine or assessment, his marginal private cost curve is MPC out to ob acre feet of irrigation water per year at which point the marginal private cost curve becomes vertical. A fine or assessment equal to or greater than fg will provide sufficient incentive for the irrigator to consider marginal private cost curve MPC' and restrict pumping to ob acre feet per year. Water use is greater than the socially optimal level of oa acre feet per year, but less than oc acre feet per year under unrestricted pumping.

A second institutional alternative is for the Water Resource Board to place a tax on each acre inch or acre foot of irrigation water pumped during the crop year. The effect on the optimal allocation of irrigation water by an individual producer is shown in Figure 7. Since the analysis is static, the MVP curve remains constant. A per unit tax on each acre foot of irrigation water pumped shifts the marginal private cost (MPC) curve upward. If the tax is a constant rate per unit equal to hk in Figure 7, the new marginal private cost curve (MPC') is parallel to and above the old MPC curve. Rather than pumping oc acre feet per year, the individual irrigator equates MPV and MPC', reducing the number of acre feet pumped to ob.

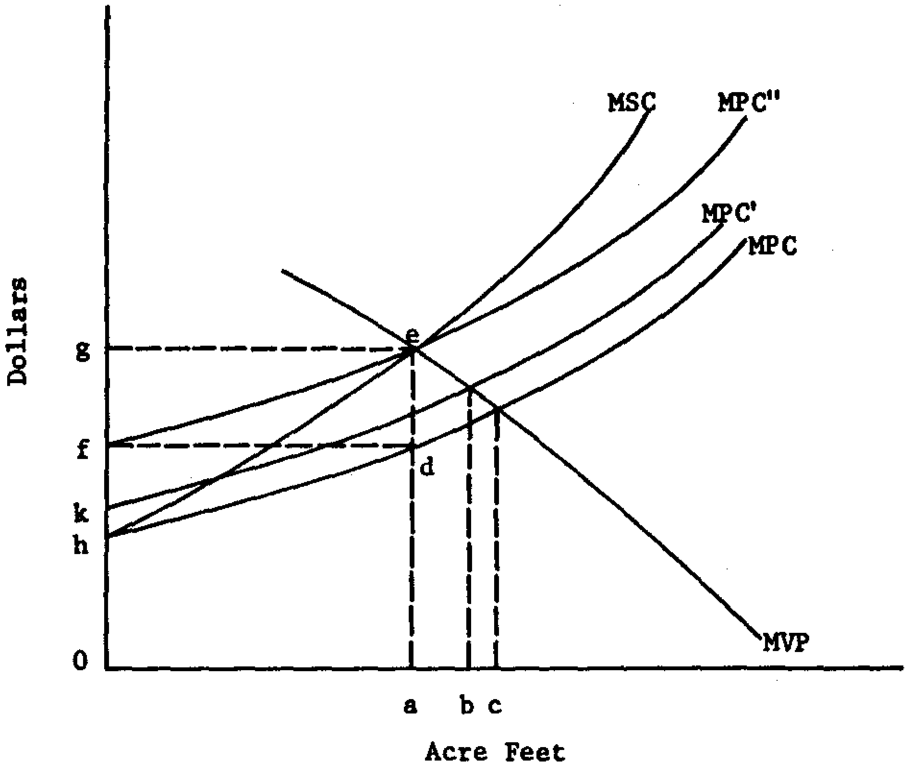

Figure 7. Illustration of the Effects of Alternative Tax Measures on the Divergence of Private and Social Costs and Resource Allocation

However, ob acre feet exceeds the socially optimal oa acre feet by an amount equal to ab. By raising the constant tax rate to de dollars per acre foot, the producer considers the full private and social costs of pumping irrigation water. The tax rate de per unit shifts the MPC curve upward to MPC". This tax rate induces the producer to optimally allocate water by equating MVP and MPC", resulting in the socially optimal oa acre feet of irrigation water being pumped. A per unit tax of de would generate revenue for the controlling agency equal to the rectangle fged. The excess of social over private cost is only hed. Clearly revenue generated exceeds the divergence of private and social costs when the tax rate is de per unit. Several alternatives exist to utilize the revenue. One is to return a portion of the revenue collected to pumpers as a bonus unrelated to the quantity of water pumped. This approach would involve an income transfer from the larger to the smaller pumpers. A second alternative is to return a portion of the revenue to pumpers with payments being inversely related to the quantity pumped. This method of payments provides an incentive to reduce pumping.

The optimal per unit tax for all water users is not the constant de per unit of water pumped. This tax rate is optimal only for the marginal unit at oa acre feet. For units less than oa, the optimal rate would be a graduated tax which, for any point between o and a, equates MPC and MSC [59, p. 434].

A slightly different approach to taxing water use is taken in this study. No attempt was made to impose a tax of sufficient magnitude to align MPC and MVP at the socially optimal level of water use. Instead, the individual irrigator is allowed to pump without taxation until a quantity limitation, such as the limitation discussed in Figure 6, is reached. Once the quantity limitation is attained, additional water is pumped only if the irrigator is willing to pay a substantial tax on each unit of water pumped above the quantity limitation. This situation is presented graphically in Figure 8. Quantity oa represents the socially optimal allocation of the water resource at the point where MVP equals MSC. Quantity od represents the optimal allocation of water by the individual producer who considers only private costs in equating MVP and MPC. Quantity ob represents the number of units of water pumped by an individual irrigator under the quantity restriction depicted in Figure 6. Assume that once ob units have been pumped, the irrigator must pay a per unit tax equal to fg on the marginal unit pumped above ob units. In effect the irrigator must now consider marginal private cost curve MPC'. At ob units of water pumped, MPC' is less than MVP. The economically rational producer will expand water use to oc units where MPC' equals MVP.

Both ob and oc are less than quantity od pumped with restrictions, but both exceed the socially optimal rate of oa acre feet per year. Thus, neither the quantity restriction nor graduated per unit tax considered here will successfully force a socially optimal allocation of irrigation water. However, from society's standpoint, both are to be preferred over unrestricted pumping because both reduce the divergence of private and social costs.

The institutional alternatives by no means exhaust the possibilities. Additional restraints might include (1) a lump sum tax or well tax on each irrigation well; (2) a limit on the number of wells per section or per farm; (3) a limit on well spacing, etc. Available time and funding did not permit

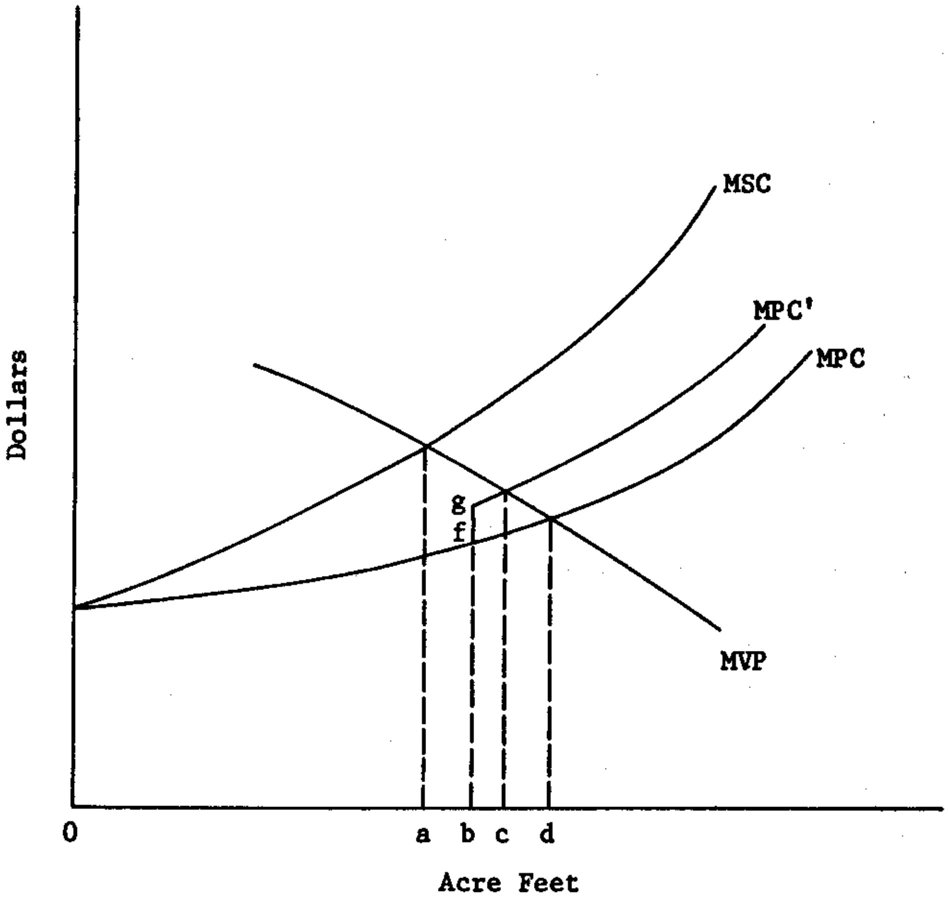

Figure 8. The Effect of a Graduated Tax per Unit Pumped Above a Quantity Limitation on Divergence of Private and Social Costs and Resource Allocation

evaluation of every possible alternative. However, one might say that those alternatives which do not force the irrigator to consider marginal social costs as well as marginal private costs will do little to eliminate the divergence of private and social costs.

This section treats problems of resource allocation and institutional alternatives from the standpoint of static economic theory. It should be emphasized that weather uncertainty adds a degree of complexity to the analysis. The actual situation is dynamic rather than static. That is, the marginal value product curve for the water resource has an expected value and variance. Nevertheless, considering the conceptual issues in a static framework is useful in suggesting means of limiting water use to the socially desirable amount. A dynamic MVP curve complicates specification of the optimal allocation of water under the various water-use regulatory alternatives, but does not change the relevant means of solving the problem. Thus no attempt is made here to incorporate dynamics into the conceptual analysis.

Aside from society's interests, how can the irrigator evaluate various water-use regulatory devices? The economic problem facing the irrigator is one of factor-factor substitution. Water represents both factors. However, water in the current time period is considered a different factor than the same water in a later time period [51, p. 1118]. The allocation problem is to determine in which time interval the marginal value product of water is the greatest. The decision is made by comparing the present value of discounted streams of net returns resulting from alternative water application rates. The alternative with the greatest present value of net returns is selected over all other alternatives.

The Simulation Model

A firm-level simulation model is used as the basic model in the analysis of the three alternative means of water-use regulation. The General Agricultural Firm Simulator developed by Hutton and Hinman [44] is modified to simulate a representative farm for the study area. The major modification made to the General Agricultural Firm Simulator in this study is the development of a new production subset. The Production Subset is designed to overcome some of the shortcomings of the General Agricultural Firm Simulator while adding a dimension of sophistication and realism to the production process not previously obtained in simulation models designed to solve economic problems. Some of the general characteristics of the General Agricultural Firm Simulator are discussed below followed by a detailed discussion of the development and structure of the Production Subset used in the study.

The General Agricultural Firm Simulator, a computer simulation routine useful to solve a variety of farm firm simulation models, consists of a master program and a series of subroutines. The model is designed to utilize information on the production, financial and institutional resources available to the firm, as well as crop production, livestock production and marketing alternatives. The precise nature of organizing the data, the logic of the operation of the General Agricultural Firm Simulator and the printout of the Simulator results are discussed elsewhere [54, pp. 43-48].

The General Agricultural Firm Simulator is, as the name implies, quite general in nature. Many types of agricultural firms may be simulated and many types of problematic situations investigated by modifying the input data to reflect the desired situation. For this study a model is needed that will permit evaluation of the effects on the farm of various water-use regulatory alternatives. It is essential to simulate the firm in a framework that considers variable rainfall, evapotranspiration and the effects of soil moisture stress during critical stages of plant development on final crop yield. The assumptions of the General Agricultural Firm Simulator that yields are normally and independently distributed with given mean and standard deviation is inappropriate. Likewise, the assumption that the amount of irrigation water required by season of the year is constant for a given type of agricultural production is also inappropriate when studying water-use regulation alternatives. Thus the method of computing yields for both irrigated and dryland crops as well as the amount of irrigation water required by irrigated crops in the General Agricultural Firm Simulator is replaced with the Production Subset.

The Production Subset

The basic idea embodied in the Production Subset is that crop yields can be estimated as a function of soil and atmospheric conditions, or soil moisture stress and atmospheric stress, during critical stages of plant development. If soil moisture and atmospheric conditions are ideal throughout the growing season, some potential yield is achieved for each crop. When sufficient water is not maintained in the plant root system, soil moisture stress occurs and the result is a reduction in crop yield. The amount of yield reduction depends upon the length and severity of moisture and atmospheric stress in relation to the stage of plant development. Even when soil moisture is adequate, severe atmospheric conditions can cause plant stress and reductions in crop yield. A combination of high temperature, low relative humidity and high wind movement creates a demand for more moisture than the plant is able to transpire. The resulting plant stress causes a reduction in final crop yield. Thus, yield reduction (YR_{ij}) for a crop is a function of daily soil moisture and atmospheric stress as they relate to the critical stages of plant development. In implicit form, this relationship may be expressed as

$$YR_{ij} = f(SM_{ij}, AS_{ij}) \tag{31}$$

where SM represents soil moisture stress, AS represents atmospheric stress and i and j represent the day and stage of plant development, respectively.

Soil moisture at any point in time is a function of daily rainfall (RN_{ij}); evapotranspiration (EV_{ij}), which represents evaporative losses of moisture to plants and the atmosphere; and, additions of moisture to the profile through irrigation applications (I_{ij}), or

$$SM_{ij} = h(RN_{ij}, EV_{ij}, I_{ij}). \tag{32}$$

Atmospheric demand for soil moisture is a function of pan evaporation (PE_{ij}), or

$$AS_{ij} = g(PE_{ij}). \tag{33}$$

Thus, crop yield reduction on day i of stage j is a function of the random variables rainfall, evapotranspiration, irrigation application rate and pan evaporation. Irrigation is considered a random variable since applications are governed by the other random variables mentioned above. The implicit function for crop yield reduction is derived by substituting (32) and (33) into (31) to get

$$YR_{ij} = f(RN_{ij}, EV_{ij}, I_{ij}, PE_{ij}). \tag{34}$$

The implicit production function for yield of crop k (Y^k) is obtained by summing m daily yield reductions across n critical stages of plant development and subtracting the result from a potential yield under adequate moisture conditions (PY^k) as follows:

$$Y^k = PY^k - \sum_{j=1}^{n} \sum_{i=1}^{m} f(RN_{ij}, EV_{ij}^{k}, I_{ij}^{k}, PE_{ij}). \qquad (35)$$

A series of k such equations are required to fully describe k individual crops or crop blocks. By summing across the k crops or crop blocks, a net returns equation for the farm operation can easily be derived.

Prediction of crop yields based on available soil moisture at critical stages of plant development can be accomplished in at least two ways. One approach is to estimate a predictive equation in which crop yield is the dependent variable and the explanatory variables include rainfall, irrigation application, pan evaporation, some measure of evapotranspiration, temperature, wind movement and relative humidity during each critical stage of plant development for each crop being considered. This approach has definite appeal because regression analysis is a comparatively simple technique to use and the results can be evaluated in terms of significance level of regression coefficients, predictive ability of the equation and R^2. Though appealing, the approach is not without problems. The primary problem is that little research has been done to establish the relationships between soil moisture and atmospheric stress at critical stages of plant development for the major crops of the study area. Compounding the significant data problems are the difficulties of formulating appropriate functional forms for the equations, a lack of independence among the explanatory variables and the existence of a large random component not readily explainable through the use of measurable weather variables.

A second approach to estimating the effects of moisture stress on crop yield is to make independent studies of soil moisture and the yield effects of moisture stress during critical stages of plant development. Soil moisture may be studied within the context of a daily soil moisture balance system. In a separate analysis, the critical stages of plant development for each individual crop may be identified and the effects of moisture and atmospheric stress on yield during that stage evaluated. Then the two may be combined into a dynamic soil moisture-crop yield system capable of simulating soil moisture throughout the growing season, and determining final yield for each crop as a function of the level of moisture and atmospheric stress occurring during the critical stages of plant development. The latter approach is utilized in this study.

The Soil Moisture Balance

The soil moisture balance for this study is based upon the findings and ideas presented by Van Bavel [106], Thornthwaite [101], Thornthwaite and Mather [102], Holmes and Robinson [38 and 39], Denmead and Shaw [18] and Ligon, et.al. [53]. The balance provides daily adjustments to soil moisture to reflect additions through rainfall and subtractions through estimates of evapotranspiration. Daily net additions to soil moisture occur when rainfall exceeds actual evapotranspiration and depletions occur when the opposite is true.

A 51-inch soil profile is utilized in constructing the daily moisture balance. Based on experimental moisture release data for Richfield clay loam soil at Goodwell, Oklahoma, field capacity and permanent wilting point are estimated to be 16.32 and 8.69 inches of soil moisture, respectively.[7] The 51-inch profile is divided into an upper and lower layer. The upper layer consists of the top nine inches of soil which contains moisture most readily available for plant use. The upper layer holds 2.88 inches of soil moisture at field capacity and 1.53 inches at permanent wilting point. The lower 42 inches of the profile (from nine down to 51 inches) retains 13.44 inches of soil moisture at field capacity and 7.16 inches at permanent wilting point.

When rainfall occurs, water is added to the upper nine inches of the soil profile. It is assumed that water percolates from the upper profile to the lower profile at a rate proportional to the amount of moisture in the upper zone.[8] Specifically, it is assumed that five percent of the water in the upper zone percolates to the lower zone each day until soil moisture in the upper zone reaches 1.53 inches of moisture (permanent wilting point). Then water movement to the lower zone ceases.

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[7] Richfield clay loam soil was selected as the soil for which the moisture balance would be constructed for several reasons. The data on field capacity and permanent wilting point were readily available. It is the predominant irrigable clay loam soil in the study area. Irrigable clay and clay loam soils compose 6,167,500 acres (76.7 percent) of the 8,040,915 irrigable acres in the study area.

[8] In a study by Winton Covey and M. E. Bloodworth, "Mathematical Study of the Flow of Water to Plant Roots," Texas Agricultural Experiment Station, MP-599 (College Station, 1962), empirical evidence indicates that moisture diffusitivity of a soil may be assumed an exponential function of soil moisture content. The exponential relationship may be expressed as $D = re^{\theta\beta}$, where D is diffusitivity, θ is volumetric water content and both r and β are constants. A serious drawback to the use of an exponential function to approximate water movement within the soil profile is that the constants r and β must be estimated empirically for each soil and have not been estimated for the soils of the study area.

Water is withdrawn from the soil profile as a result of evapotranspiration. Two concepts of evapotranspiration are considered. The first, potential evapotranspiration, refers to the quantity of water which would be evaporated and transpired from a particular crop under conditions of ample water supply in the soil. Daily amounts of potential evapotranspiration are estimated from daily pan evaporation readings [77]. The second, actual evapotranspiration, indicates the amount of evapotranspiration which actually occurs during a given day. It is a function of potential evapotranspiration and soil moisture conditions. Actual evapotranspiration is always equal to or less than potential evapotranspiration. The two are assumed equal only when soil moisture is at field capacity in the upper layer of the soil profile. Once soil moisture falls below field capacity in the upper zone, actual evapotranspiration is assumed proportional to the amount of moisture remaining in the upper zone. All actual evapotranspiration occurs from the upper zone (0 to 9 inches) until soil moisture reaches permanent wilting point of 1.53 inches. Then moisture is drawn from the lower layer with actual evapotranspiration being proportional to the amount of soil moisture remaining in the lower zone (9 to 51 inches) of the profile. Once soil moisture in the lower zone of the profile reaches permanent wilting point of 7.16 inches, actual evapotranspiration is assumed to cease.

The following series of equations describes, in mathematical notation, the system used to calculate actual evapotranspiration on a daily basis.

$$AE_i = EP_i \frac{SMU_i}{2.88}, \quad 1.53 \leq SMU_i \leq 2.88 \tag{36}$$

$$AE_i = EP_i \frac{SML_i}{13.44}, \quad SMU_i = 1.53; \quad 7.16 \leq SML \leq 13.44 \tag{37}$$

$$AE_i = 0, \quad SMU_i = 1.53, \quad SML_i = 7.16 \tag{38}$$

where AE_i equals actual evapotranspiration, day i; EP_i equals potential evapotranspiration, day i; SMU_i equals inches of soil moisture, upper (0-9 inch) layer, day i; SML_i equals inches of soil moisture, lower (9-51 inch) layer, day i.

Equation (36) states that if moisture in the upper layer of the soil profile is between field capacity and the permanent wilting point of 1.53 inches, then actual evapotranspiration from the upper layer is a function of potential evapotranspiration and is proportional to the amount of water remaining in the upper layer. Equation (37) indicates that once soil moisture in the upper layer of the soil profile has been depleted to the minimum 1.53-inch level, actual evapotranspiration is a function of potential evapotranspiration and occurs from the lower profile at a rate proportional to the amount of soil moisture in the lower layer. Equation (38) indicates that evapotranspiration ceases when moisture in both layers of the soil profile reaches permanent wilting point.

Except for the variation in potential evapotranspiration for different crops at different stages of plant development, the primary variables composing the moisture balance are rainfall and pan evaporation. To simulate daily values of soil moisture throughout the growing season, daily values of rainfall and pan evaporation are required. Generating daily values for these two variables is considered in turn.

Rainfall Probability Distribution

Rainfall throughout the study area is characterized by two predominate features. First, yearly average rainfall is very low. It ranges from 15 inches in the western portion of the study area to 19 inches in the eastern part of the Oklahoma Panhandle. Second, daily and yearly rainfall are quite variable. During the 29 years from 1941 through 1969, daily rainfall at the U.S. National Weather Service, Goodwell, Oklahoma (approximately the geographical center of the study area), ranges from zero to 5.38 inches. The long-term average number of days per year with zero rainfall is approximately 275.

To simulate soil moisture throughout the crop year, a means is needed to accurately represent the rainfall pattern which might be expected based on historical rainfall patterns. This study used discrete, empirical probability distributions based on actual daily observations of rainfall for the past 29 years. The growing season is divided into seven monthly periods, beginning on April 1 and ending on October 31. Each month is further divided into two periods. The first period of each month is 15 days long. The second period of each month is either 15 or 16 days long depending upon whether the month has 30 or 31 days. The discrete empirical probability distributions estimated for each of the 14 periods of the growing season are presented in Table XXVI. Each distribution is independent of the other distributions. Generating daily rainfall events from a different distribution every two weeks takes into account differences in the actual distribution of rainfall during the growing season.

Generating daily rainfall values from a discrete probability distribution can present a problem because of the computer storage and time required. However, a very fast procedure developed by Marsaglia was utilized to generate random variates from each discrete probability density function [56, pp. 37-38].

TABLE XXVI

DISCRETE DAILY RAINFALL PROBABILITIES BY PERIOD OF THE CROP YEAR

Inches of Rainfall	Apr. 1-15	Apr. 16-30	May 1-15	May 16-31	June 1-15	June 16-30	July 1-15	July 16-31	Aug. 1-15	Aug. 16-31	Sept. 1-15	Sept. 16-30	Oct. 1-15	Oct. 16-31
.00	.851	.871	.782	.746	.733	.786	.743	.776	.759	.800	.846	.844	.878	.862
.01-.05	.041	.023	.071	.058	.051	.051	.044	.034	.039	.062	.034	.039	.030	.030
.06-.10	.039	.023	.018	.022	.051	.039	.021	.032	.037	.022	.032	.025	.014	.026
.11-.15	.023	.016	.011	.024	.011	.021	.025	.026	.021	.015	.018	.018	.014	.009
.16-.20	.007	.007	.018	.022	.021	.007	.014	.017	.016	.015	.011	.011	.005	.017
.21-.25	.005	.005	.009	.017	.018	.021	.016	.013	.007	.004	.007	.009	.011	.002
.26-.30	.007	.011	.002	.011	.011	.009	.002	.009	.018	.013	.007	.005	.002	.011
.31-.35	.002	.002	.009	.011	.011	.009	.002	.004	.007	.009	.007	.007	.002	.006
.36-.40	.002	.002	.007	.009	.009	.007	.009	.009	.007	.007	.007	.002	.002	.004
.41-.45	.007	.005	.005	.011	.011	.005	.023	.011	.014	.007	.002		.005	.006
.46-.50	.005	.007	.007	.011	.009	.005		.002	.009	.004		.007	.005	.002
.51-.55		.007	.018	.011	.009	.002	.018	.004	.005	.002		.002	.005	
.56-.60	.005		.005	.015	.007	.002	.005	.004	.002	.004			.002	.004
.61-.65	.002	.005		.002	.005		.005	.009	.005	.002				.002
.66-.70		.002		.002			.005		.007	.002		.002		
.71-.75			.007	.002			.007	.002	.002			.005	.002	.002
.76-.80	.002	.005	.002		.005	.007		.002	.005	.002			.005	.002
.81-.85			.002	.002	.007		.005	.002	.002		.002	.005	.005	
.86-.90			.002	.002	.002		.002	.006	.002	.002	.002	.002	.002	
.91-.95	.002				.002	.009	.005	.006	.005	.002	.002		.002	.002
.96-1.00			.002	.002		.002	.007	.004	.002	.002				
1.01-1.05		.002	.005	.002	.005		.005	.002	.002	.002	.002	.002		
1.06-1.10			.002		.002		.005	.009			.002			
1.11-1.15	.002		.002	.004		.002	.002					.005		
1.16-1.20		.002	.002		.002		.005		.007		.005	.002		.002

TABLE XXVI (Continued)

Inches of Rainfall	Apr. 1-15	Apr. 16-30	May 1-15	May 16-31	June 1-15	June 16-30	July 1-15	July 16-31	Aug. 1-15	Aug. 16-31	Sept. 1-15	Sept. 16-30	Oct. 1-15	Oct. 16-31
1.21-1.25							.005			.002	.002	.005		
1.26-1.30		.002							.002					
1.31-1.35		.002		.006			.005	.002	.005					
1.36-1.40			.002			.002				.007				.002
1.41-1.45			.002		.005	.005	.005						.002	
1.46-1.50						.002							.002	
1.51-1.55			.002				.002	.002	.002					
1.56-1.60									.002					
1.61-1.65							.002							
1.66-1.70								.002		.004	.002			.002
1.71-1.75			.002				.002			.002			.002	
1.76-1.80														
1.81-1.85					.002			.004		.002	.002			
1.86-1.90				.002	.002									
1.91-1.95				.002	.002				.002					
1.96-2.00				.004							.002			
>2.00					.002	.007	.005	.004	.007	.004	.002	.002	.002	.004

Pan Evaporation Probability Distributions

Pan evaporation, like rainfall, is an integral component of the soil moisture balance system. To simulate soil moisture throughout the growing season, daily pan evaporation values must be generated for each period of the growing season.

Pan evaporation measurements were taken from a Class A weather pan located at the U.S. National Weather Service, Goodwell, Oklahoma. Sufficient information is available to estimate pan evaporation probability density functions for 12 periods, the first beginning on May 1 and the last ending on October 31. These periods correspond exactly to the rainfall periods, except that no pan evaporation distributions were estimated for April.

Daily pan evaporation values are generally small during the early portion of the growing season, increase to a peak level during July and August and decline to a low level in October. Plottings of daily pan evaporation observations for each period of the growing season reveal several outstanding characteristics. First, the sample data indicates that the pan evaporation distributions are positively skewed. Second, all observations are equal to or greater than zero. Third, the symmetry or skewness of the distribution changes from period to period during the growing season.

The lognormal distribution is used to describe pan evaporation in this study. It is a continuous positively skewed probability density function having all values equal to or greater than zero. It is easily derived, being completely defined by the mean and variance and is easy to manipulate in the analysis.

Aitchinson and Brown discuss alternative methods of estimating the parameters of a lognormal distribution. Parameters of each distribution are estimated by the method of maximum likelihool [2, p. 39]. Estimates of the mean, variance and standard deviation for each of the pan evaporation distributions are given in Table XXVII.

Equation (39) may be used to generate a series of n random pan evaporation observations from a lognormal distribution with mean m_l and standard deviation s_l.

$$x_i = e^{m_l + s_l Z_i} \qquad (39)$$

where m_l and s_l are the mean and standard deviation of the lognormally distributed transformed variable and Z_i represents a series of n random normal deviates. Generating pan evaporation values from a different distribution for each two-week period accounts for the changing distribution of pan evaporation throughout the growing season.

Simulating Soil Moisture During the Crop Year

Utilizing the rainfall and pan evaporation distributions, daily values for each are generated throughout the growing season. The absence of pan evaporation data for the November through April period necessitates estimation of soil moisture at the beginning of May based on available weather data for the previous month or months. Equation (40), estimated by multiple linear regression, adequately predicts soil moisture at the beginning of May based upon rainfall during the month of April.

$$SM_{bm} = 8.69 + \underset{(0.26)}{0.22R_{ma}} + \underset{(1.05)}{2.33R_{lwa}} \qquad (40)$$

TABLE XXVII

SUMMARY OF MEAN, VARIANCE AND STANDARD DEVIATION FOR LOGARITHMICALLY TRANSFORMED PAN EVAPORATION DATA BY PERIODS OF THE YEAR

	x is Distributed Lognormally			y=log x is Distributed Normally		
	Mean	Variance	Std. Dev.	Mean	Variance	Std. Dev.
May 1-15	.38023	.06025	.24546	-1.11687	.31021	.55696
May 16-31	.34863	.04668	.21606	-1.21614	.44774	.66913
June 1-15	.40382	.06009	.24513	-1.02709	.31102	.55769
June 16-30	.46678	.06091	.24680	-.83398	.22946	.47902
July 1-15	.45500	.07547	.27472	-.95027	.49978	.70695
July 16-31	.46152	.06323	.25145	-.89505	.36145	.60121
Aug. 1-15	.39789	.04926	.22194	-1.22882	.25953	.50944
Aug. 16-31	.37178	.04750	.21795	-1.10846	.30757	.55459
Sept. 1-15	.32364	.04720	.21725	-1.27964	.40251	.63444
Sept. 16-30	.27510	.03548	.18835	-1.43233	.35790	.59825
Oct. 1-15	.28648	.05066	.22508	-1.33889	.37783	.61468
Oct. 16-31	.20776	.02673	.16350	-1.71473	.33835	.58168

where SM_{bm} represents the soil moisture at the beginning of May, in inches; R_{ma} represents the rainfall during the month of April, in inches; and R_{lwa} represents the rainfall during the last week in April, in inches. Standard errors of the regression coefficients appear in parentheses below the equation. The R^2 for Equation (40) is 0.90.

The soil moisture balance works as follows: Given beginning soil moisture on May 1, the soil moisture balance generates daily rainfall and pan evaporation values. Potential evapotranspiration is calculated based on pan evaporation and the particular stage of plant development for each crop. Actual evapotranspiration is calculated based upon potential evapotranspiration and soil moisture in the upper profile as long as soil moisture in that layer exceeds permanent wilting point, and then from the lower profile until soil moisture in that layer reaches permanent wilting point. Next, rainfall is compared with actual evapotranspiration. If rainfall exceeds actual evapotranspiration, the difference between the two is added to the upper layer of the soil profile, with five percent of the upper layer moisture percolating to the lower profile. If the upper profile reaches field capacity, additions of soil moisture are made to the lower profile. If both layers reach field capacity, excess water is considered runoff. If, when rainfall is compared with actual evapotranspiration, the latter exceeds the former, soil moisture is reduced by the amount of the difference between the two. Soil moisture declines in the upper profile, with soil moisture also percolating from the upper to lower profile, until permanent wilting point in the upper profile is reached. Then, soil moisture is drawn from the lower profile until soil moisture in that layer reaches permanent wilting point. Once both layers of the profile have reached permanent wilting point, depletion of moisture ceases. Each day of the growing season, a similar set of computations is made based on soil moisture, rainfall and evapotranspiration [54, p. 63].

Testing the Soil Moisture Balance

Prior to using the soil moisture balance to maintain a record of soil moisture throughout the growing season, a statistical test is made to insure that it is performing satisfactorily. To perform satisfactorily, the moisture balance must utilize probabilistic rainfall and pan evaporation readings and generate a distribution of soil moisture values that does not differ significantly from the actual distribution of soil moisture observed for the study area.

Soil moisture, which is a function of heavily skewed rainfall and lognormally distributed pan evaporation, is not normally distributed over the growing season. Thus, the frequently used parametric "t" test is inappropriate for testing the soil moisture distribution.

Fortunately, nonparametric statistical tests exist which may be used to test for statistical differences between two distributions without requiring assumptions about those distributions. The Mann-Whitney U test may be used to test whether two independent groups, A and B, come from the same population; that is, whether A and B have the same distribution. The null hypothesis, H_o, is that A and B have the same distribution. The alternative hypothesis is that A is larger than B [94, pp. 116-127]. The actual and simulated soil moisture values serve as the two groups, A and B, for the test. The procedures required to use the Mann-Whitney U tests, details of the requisite computations and an explanation of the results are presented in [54, pp. 272-275]. The results of the test are stated here in probability terms. The computed value of the test statistic, Z, is 0.802, where Z is approximately normally distributed with zero mean and unit variance. The probability of a value of Z as extreme as 0.802 under the null hypothesis is 0.412. There is no statistical basis for rejecting the null hypothesis of no difference between the actual and simulated soil moisture distributions. Thus, the soil moisture balance system is judged satisfactory from a statistical standpoint. The next steps are to estimate the effects on final crop yield of soil moisture stress during each stage of plant development for each relevant crop. Then the moisture balance and stress-yield relationships are integrated into a dynamic moisture-yield system.

Crop Yields as a Function of Soil Moisture and Atmospheric Stress During Critical Stages of Plant Development

Considerable research has been undertaken to study the effects of various factors, including row spacing, planting rates, seeding date, fertilizer levels, and irrigation rates, on the major crops of the study area, such as grain sorghum [1, 46, 61, 62, 63, 65, 66, 67, 68, 69, 75, 92, 96, 98, 113], wheat [47, 48, 79, 87, 98] and corn [18, 20, 40, 90], as well as on a few minor crops, including alfalfa and sugar beets [71, 86]. However, relatively few studies attempted to establish empirical relationships between timing of water application and crop yield, and between various levels of moisture stress at different stages of plant development and the corresponding yield reductions. Those that have are limited to the major irrigated crops--grain sorghum, wheat and corn [12, 19, 64, 65, 80, 93].

Several general conclusions may be drawn from the results of these research efforts. First, reductions in crop yield may occur as a result of either soil moisture conditions or severe atmospheric conditions. Soil moisture deficiency may subject plants to soil moisture stress resulting in growth retardation and yield reduction regardless of atmospheric conditions. Similarly, even if soil moisture is adequate for normal plant development, severe atmospheric conditions may demand more water than the plant is capable of transpiring and the result is growth retardation and yield reduction. The second general conclusion is that each crop has a unique set of critical stages of plant development which must be identified and studied. Third, the daily effects of moisture and atmospheric stress vary from stage to stage for a single crop and differ from crop to crop. These three general conclusions are used in specifying the general form of the yield-moisture relationship.

Integration of the Soil Moisture Balance With Crop Yield Reductions

Calculation of soil moisture on a daily basis as a function of rainfall and evapotranspiration permits consideration of the effects of soil moisture and atmospheric demands on crop yields on a daily basis. If on day i of stage j of crop k development, soil moisture is inadequate, the plant is subjected to moisture stress and final yield is reduced. Also, if on the same day atmospheric demands for moisture are greater than the plant's ability to transpire moisture to the atmosphere, plant stress occurs and final yield is further reduced. The combined effects of soil moisture and atmospheric stress acting to reduce yield is assumed to be additive and can be expressed as

$$YR_{ij}^{k} = \theta_{j}^{k} \, SMD_{ij} + b_{j}^{k} \, (P_{ij} - P_{A}) \tag{41}$$

where YR_{ij}^{k} represents the yield reduction, day i, stage j, crop k; θ_{j}^{k}

represents the coefficient reflecting yield reduction, in units per day, resulting from adverse soil moisture conditions, stage j, crop k; SMD_{ij} represents the soil moisture depletion in inches, day i, stage j; b_j^k represents the coefficient reflecting yield reduction in units per day due to severe atmospheric demands upon the plant, stage j, crop k; P_{ij} represents the pan evaporation in inches, day i, stage j; and P_A represents a critical pan evaporation level at or below which no yield reductions occur that are directly attributable to severe atmospheric conditions.

Equation (41) indicates that crop yield reductions for a given day and stage of plant development are the sum of soil moisture and atmospheric components. The coefficient θ_j^k must be estimated for j critical stages of plant development for each crop. The variable SMD_{ij} is assumed to have the form shown in (42) for Richfield clay loam soil.

$$SMD_{ij} = (13.8 - SMT_{ij})/5.11 \;, \quad SMT_{ij} < 13.8 \qquad (42)$$

where 13.8 represents the inches of soil moisture for Richfield clay loam soil below which plants begin to suffer moisture stress and yield begins to be reduced; SMT_{ij} represents the inches of soil moisture which exist in the entire profile on day i of stage j; and 5.11 represents the difference between the critical moisture level of 13.8 inches and permanent wilting point of 8.69 inches.

Equation (42) states that as long as the soil moisture level is less than 13.8 inches, SMD_{ij} increases as soil moisture decreases, reaching 1.0 when soil moisture reaches the permanent wilting point of 8.69 inches. Thus, the daily reduction in crop yield due to soil moisture conditions is assumed to be a linear function of the level of soil moisture between the critical moisture point and permanent wilting point.

The second term on the right-hand side of equation (41) represents the effect of atmospheric stress upon crop yield. The coefficient b_j^k must be estimated for each of j stages for k crops included in the model. Values of P_{ij} are generated daily (as part of the soil moisture balance) from log-normal distributions of pan evaporation. The value of P_A emphasizes the importance of excessive atmospheric demands upon the plant even though soil moisture may be above the permanent wilting point. If atmospheric demands exceed the plant's ability to transpire moisture to the atmosphere, plant stress occurs and yields are reduced. A value of 0.40 inches per day is used for P_A in this study.[9] It is assumed no yield reduction due to excessive atmospheric demand occurs unless pan evaporation for a given day exceeds 0.40 inches.

Equations (41) and (42) and the soil moisture balance complete the link between daily moisture readings and crop yield reductions due to moisture and atmospheric stress. The following sections develop critical stages of plant development, water-use rates and yield reduction coefficients for each crop.

Stages of Plant Development and Yield Reduction Coefficients for Grain Sorghum

The growing season for grain sorghum in the study area is divided into three stages defined as preboot, boot-heading and grain-filling. The actual dates on which these critical stages begin and end is variable. Factors

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[9] The criterion for selecting the value of P_A, established in consultation with agronomists and agricultural engineers familiar with the area, is that the critical value of P_A would occur approximately 20 percent of the time during the vegetative stage of plant development for each crop. Study of pan evaporation patterns during the vegetative stages of plant development for each crop reveals that the value of P_A satisfying the criterion is approximately 0.40 inches per day.

that affect plant growth and the time at which each stage is reached include date of planting, moisture conditions at planting, fertilization level, the amount of stress which occurs at each stage of development, and timing and amounts of rainfall and irrigation received. However, in simulating crop yield as a function of soil moisture during these critical stages, it is necessary to assume a specific beginning and ending date for each stage. Otherwise soil moisture and atmospheric stress coefficients would vary, not only from stage to stage and crop to crop, but from year to year as well. Data to estimate such varying relationships are not available. Consequently, fixed length stages are assumed.

Grain sorghum is a summer crop. Farm operators begin preplant irrigations during May, often plant about June 1 and expect emergence by June 7. From June 7 until about mid-July, soil moisture and atmospheric stress have little effect on final yield if soil moisture is adequate during the succeeding stages of development. The preboot stage occurs between the 12-inch stage and boot stage. Preboot stage is assumed to begin on July 16 and end on August 4, lasting 21 days. The boot-heading stage is assumed to begin on August 5 and end on September 1, lasting 28 days. The grain-filling stage is assumed to begin on September 2 and end on September 22, lasting 21 days. From September 23 until maturity and harvest, moisture and atmospheric stress are assumed to have no effect on final crop yield.

In attempting to approximate the relationship between evapotranspiration and stages of grain sorghum development in the study area, it is assumed that pan evaporation, which is positively correlated with temperature and solar radiation, follows essentially the same pattern throughout the growing season as the concept of mean potential evapotranspiration plotted by Jensen and Sletten [46, p. 8]. A measure of daily potential evapotranspiration for grain sorghum is calculated as a function of pan evaporation values generated in the soil moisture balance. It is assumed that potential evapotranspiration equals 25 percent of pan evaporation from the beginning of the growing season on May 1 until plant emergence on June 7. From plant emergence until July 15, when approximately 80 percent ground cover has been reached, potential evapotranspiration is assumed to increase linearly from 25 percent to 55 percent of pan evaporation. Pan evaporation increases during this period also, and daily values of potential evapotranspiration increase rapidly. From July 15 until September 1, potential evapotranspiration remains a constant 55 percent of pan evaporation, however, both decline during this period. From September 1 until the end of the growing season, potential evapotranspiration is assumed to equal 50 percent of pan evaporation, with both values reaching low levels in late September and early October.

Dryland grain sorghum and irrigated grain sorghum are handled somewhat differently within the model. Water-use curves for irrigated grain sorghum are predicated upon the assumption that adequate soil moisture conditions exist throughout the growing season [46, p. 8]. Under adequate moisture conditions, potential evapotranspiration is much higher than under dryland conditions. Thus, approximation of water-use rates and potential evapotranspiration utilizing the curves developed for irrigated grain sorghum is inappropriate. Still, potential evapotranspiration changes during the growing season as grain sorghum develops from emergence to 80 percent of ground cover. Research to establish realistic values for dryland grain sorghum is sparce. It is assumed that potential evapotranspiration equals point, nevertheless represent the best available estimates until more data are available.

Equation (43) presents soil moisture and atmospheric stress coefficients for the preboot stage of grain sorghum development. Superscripts designating the crop have been eliminated since each crop is discussed individually.

$$YR_{ip} = 0.30\ SMD_{ip} + 1.30\ (P_{ip} - 0.40) \qquad (43)$$

A soil moisture stress coefficient of 0.30 for the preboot stage of grain sorghum development denotes that as soil moisture approaches wilting point, yield reduction approaches 0.30 bushels per day. Thus, if soil moisture remains near wilting point for the entire preboot stage, the potential yield reduction is approximately 6.3 bushels (0.30 x 21 days) per acre. Total yield reduction during the preboot stage is obtained by summing the 21 daily soil moisture and atmospheric reductions as indicated in (44)

$$YR_p = \sum_{i=1}^{21} 0.30\ \left(\frac{13.8 - SMT_{ip}}{5.11}\right) + 1.30\ (P_{ip} - 0.40). \qquad (44)$$

Coefficients for the boot-heading stage are presented in equation (45). Boot-heading is the most critical stage of grain sorghum development as reflected in the larger θ_j and b_j values. Potential yield reduction due to soil moisture stress increases to 57.12 bushels per acre.

$$YR_{ib} = 2.04\ SMD_{ib} + 1.65\ (P_{ib} - 0.40) \qquad (45)$$

Coefficients for the grain-filling stage of grain sorghum development, shown in equation (46), indicate that adequate moisture during grain-filling is more critical to plant development and final yield than during the preboot stage, but less critical than during the boot-heading stage. Maximum

25 percent of pan evaporation from the beginning of the growing season until the beginning of boot-heading stage of dryland grain sorghum development. From boot-heading stage to the end of grain-filling stage, potential evapotranspiration is assumed to equal 75 percent of pan evaporation. While the potential for evapotranspiration may be high, actual evapotranspiration is likely to be low because of low soil moisture on dryland grain sorghum. Considering the lack of empirical work on dryland grain sorghum water-use rates, one can say in defense of these values that they were judged realistic by experts in the field, and generated realistic dryland grain sorghum yields when used in the model.

Soil moisture and atmospheric yield reduction coefficients were developed for each of the three critical stages of grain sorghum. The study conducted by Musick and Grimes [64] at Garden City, Kansas, just north of the study area, provided valuable insights regarding the relative importance of each stage of development and the percentage reduction in yield that might be expected when grain sorghum was subjected to moisture stress for different lengths of time during each critical stage of development. The relationships developed by Musick and Grimes were refined and adjusted in consultation with agronomists, agricultural engineers, farm management agents and irrigation specialists to fit the study area.

Coefficients were synthesized and tested rather than being estimated by sophisticated mathematical procedures. While it might be argued that mathematical estimation is preferable, the almost complete lack of adequate data for the study area effectively eliminates that alternative. In addition, it is emphasized that the coefficients, while probably not as accurate as implied by the use of two places to the right of the decimal potential yield reduction due to soil moisture stress is 26.67 bushels per acre.

$$YR_{ig} = 1.27\ SMD_{ig} + 1.50\ (P_{ig} - 0.40) \qquad (46)$$

Determination of the final yield reduction for grain sorghum is accomplished by summing N daily yield reductions for each of three stages of plant development, or

$$YR = \sum_{j=1}^{3} \sum_{i=1}^{N} YR_{ij}\ . \qquad (47)$$

Final yield is then computed by subtracting the grain sorghum yield reduction from the yield that would be expected if adequate moisture conditions existed throughout the entire growing season. Under adequate moisture conditions, a potential irrigated yield of 145.0 bushels per acre (8,120 pounds) was assumed.

Farm operators raising dryland grain sorghum plant a different genotype. The dryland genotype is well suited to dryland production, but has a potential yield under adequate moisture conditions of about 100 bushels per acre (5,600 pounds). The same equations used to compute irrigated grain sorghum yield reductions are used to compute dryland yield reductions. However, one constraint is placed upon production of dryland grain sorghum. Since it receives no irrigation water, dryland acreage must have sufficient soil moisture stored in the root zone, or receive sufficient rainfall during May or June if a stand is to be achieved. It is assumed that if between May 15 and June 25 soil moisture in the upper nine inches fails to reach one-half of its capacity (2.21 inches) or daily rainfall fails to reach 0.68 inches (that amount which will raise soil moisture in the upper profile from permanent wilting point to 2.21 inches), no stand is established and dryland grain sorghum yield is zero for the year. Such dryland grain sorghum crop failures occur about 20 percent of the time in the study area, or about one year in five.

Stages of Plant Development and Yield Reduction Coefficients for Wheat

Procedures similar to those for grain sorghum were utilized to synthesize soil moisture and atmospheric stress coefficients for the critical stages of wheat development. The basic source used to develop the relationships was a study conducted by Musick, Grimes and Herron in southwestern Kansas [65]. The growing season for wheat was divided into four critical periods or stages of plant development: preboot, boot, flower and milk.

The preboot stage is assumed to begin on May 1 and end on May 15, lasting 15 days. Moisture stress is relatively unimportant during preboot if adequate moisture exists during subsequent stages. Equation (48) specifies the soil moisture depletion and atmospheric stress parameters for the preboot stages of wheat development. The atmospheric parameter of zero indicates that wheat yield is resistant to atmospheric stress during the preboot stage. Potential yield reduction due to soil moisture stress is 6.75 bushels per acre.

$$YR_{ip} = 0.45\ SMD_{ij} + 0.00\ (P_{ip} - 0.40) \qquad (48)$$

The boot stage is assumed to last from May 16 to May 28, or 13 days. Moisture stress is critical during the boot stage with potential yield reduction due to soil moisture stress increasing to 13.26 bushels per year. The boot stage daily yield reduction relationships are given in equation (49).

$$YR_{ib} = 1.02\ SMD_{ib} + 1.10\ (P_{ib} - 0.40) \tag{49}$$

The flower stage of wheat development is assumed to commence about May 29 and last until June 6; only 8 days. Soil moisture stress is less critical that during boot stage, but more critical than during either preboot or milk stages of development, as indicated by (50). Potential yield reduction due to soil moisture stress during flower stage is 12.40 bushels per acre.

$$YR_{if} = 1.55\ SMD_{if} + 1.20\ (P_{if} - 0.40) \tag{50}$$

The milk stage of wheat development is assumed to begin on June 7 and end on June 13, lasting 7 days. Soil moisture stress is less critical than during boot or flower, but more critical than during preboot stage. The potential yield reduction due to soil moisture stress during milk stage is 11.62 bushels per acre. Atmospheric demands are a more significant source of yield reduction during the milk stage than during any other stage of development. Equation (51) represents the daily yield reduction relationships for milk stage.

$$YR_{im} = 1.66\ SMD_{im} + 1.50\ (P_{im} - 0.40) \tag{51}$$

Under adequate soil moisture conditions, a potential irrigated wheat yield of 75.0 bushels per acre is assumed. Wheat planted for dryland production is a different genotype--one which achieves a potential yield of approximately 55.0 bushels per acre under adequate moisture and atmospheric conditions.

As with dryland grain sorghum, an additional assumption is made to account for wheat crop failure. It is assumed that if on any day from

September 1 to October 31, soil moisture in the upper profile fails to reach one-half of capacity, or rainfall fails to equal 0.68 inches, a wheat stand is not achieved and zero yield is indicated.

Stages of Plant Development and Yield Reduction Coefficients for Corn

The basic ideas and research results from which the corn coefficients are synthesized are presented in studies conducted by Dale and Shaw [12], Denmead and Shaw [19, 20] and Robins and Domingo [80].

The growing season for corn is divided into five critical growth stages: first vegetative, second vegetative, silking, milk, and dough. Planting is assumed to occur on May 1 with emergence on May 7. The first vegetative stage begins at emergence and ends on June 5, lasting 30 days. The effects of moisture stress are small during this initial stage if sufficient moisture exists during subsequent stages of development. Equation (52) presents the soil moisture and atmospheric relationships for the first vegetative stage of corn development. Potential yield reduction due to moisture stress in this stage is 6 bushels per acre.

$$YR_{iv_1} = 0.20\ SMD_{iv_1} + 0.10\ (P_{iv_1} - 0.40) \qquad (52)$$

The second vegetative stage of corn development is assumed to begin about June 6 and last 27 days, ending on July 2. The importance of soil moisture stress increases significantly with potential yield reduction reaching 31.05 bushels per acre. The coefficients are shown in (53).

$$YR_{iv_2} = 1.15\ SMD_{iv_2} + 0.60\ (P_{iv_2} - 0.40) \qquad (53)$$

The silking stage of corn development is assumed to last from about July 3 to July 18, a total of 16 days. The increased importance of moisture stress during silking stage is reflected in a potential yield reduction of 48.8 bushels per acre.

$$YR_{is} = 3.05\ SMD_{is} + 1.60\ (P_{is} - 0.40) \tag{54}$$

The milk stage of corn development is assumed to begin on July 19 and, lasting 22 days, end on August 9. Milk stage is slightly more important than the early and late vegetative stages. Yield reduction coefficients for milk stage are expressed in (55). Potential yield reduction is 25.08 bushels per acre.

$$YR_{im} = 1.14\ SMD_{im} + 0.40\ (P_{im} - 0.40) \tag{55}$$

Finally, the dough stage of corn development is assumed to commence around August 10 and end on August 24, lasting 15 days. Moisture stress is slightly less important during the dough stage as reflected in (56). Potential yield reduction due to soil moisture stress is 23.55 bushels per acre.

$$YR_{id} = 1.57\ SMD_{id} + 0.10\ (P_{id} - 0.40) \tag{56}$$

Potential yield for irrigated corn under adequate moisture and atmospheric conditions is assumed to equal 150.0 bushels per acre.

Stages of Plant Development and Yield Reduction Coefficients for Corn Silage

Little agronomic research relating soil moisture stress and severe atmospheric demands to corn silage yield is available for the study area. Agronomists and area agents in the study area indicate cattle feeders are demanding "grain-type" corn for silage and producers are responding to market demand. Thus, it is assumed that corn grown for silage is a "grain-type" corn and has the same critical stages of plant development and stress coefficients as corn grown for grain. Corn silage yields are estimated as a function of corn for grain yields. A corn silage yield comparable to the 150.0-bushel corn grain yield under adequate moisture conditions is 27.0 tons per acre. A coefficient relating corn grain and corn silage yields is obtained by dividing 27.0 tons by 150.0 bushels to get 0.18. Then corn silage yield (CSY) is computed as a linear function of corn grain yield (CGY) from the relation CSY = 0.18 CGY.

Stages of Plant Development and Yield Reduction Coefficients for Small Grain Grazing and Native Pasture Yields

Lack of empirical data makes it even more difficult to estimate soil moisture and atmospheric stress coefficients for small grain grazing and native pasture than for the crops discussed previously. Small grain grazing yields on dryland are positively correlated with dryland wheat yields because both are winter crops grown under dryland conditions. Consequently, a linear relationship is assumed between dryland wheat yield in bushels per acre and dryland small grain grazing yield in animal unit months (AUM). A 14.0-bushel per acre dryland wheat yield is assumed equivalent to 1.8 AUM of small grain grazing [31, pp. 9-10]. A coefficient relating dryland wheat yield and small grain grazing yield is derived by dividing 1.8 by 14.0 to get 0.129. Then, small grain grazing yield in AUM (SGPY) is computed as a linear function of dryland wheat yield (DWY) in the relation SGPY = 0.129 DWY.

The relationships between native pasture yield and either dryland wheat or small grain grazing yield have not been established. Therefore, native pasture yield is assumed constant at one AUM per acre.

Defining Typical Resource Situations

The primary basis for selecting typical resource situations is the saturated thickness of the Ogallala Formation. Saturated thickness is a critical determinant of both the quantity of water in storage and the yield of an irrigation well or system in gallons per minute. The land area and amount of water in storage is summarized by saturated thickness interval in Table IV. The number of acres overlying each saturated thickness interval and the percent of the total study area represented by each saturated thickness interval are presented.

Although the range in saturated thickness in Table IV suggests it would be desirable to define several resource situations for analysis, the available resources required limiting the analysis to two basic resource situations, designed to represent "poor" and "adequate" water positions for this study. The saturated thickness intervals ≤100 and 101-200 feet are combined to represent the poor water situation. The remaining four saturated thickness intervals are combined to represent the adequate water situation. The two basic resource situations are defined in Table XXVIII.

Resource Situation 1 represents 46.59 percent of the total land area, however, the underlying formation contains only 20.88 percent of the available water. Resource Situation 2 represents 53.41 percent of the surface area, however, overlies 79.12 percent of the available water. The weighted average saturated thickness of underground formation for Resource Situation 1 is approximately 100 feet and for Resource Situation 2 is approximately 325 feet. Each resource situation is characterized by a representative farm firm and the effects of continued pumping on saturated thickness and well yield are simulated through time.

TABLE XXVIII

DEFINITION OF TWO BASIC RESOURCE SITUATIONS FOR THE STUDY AREA

Resource Situation	Weighted Ave. Feet of Sat. Thickness	Acres Within Each Resource Situation	Percent of Study Area Acres	Acre Feet of Water Within Each Resource Situation	Percent of Study Area Water
1	100	5,193,968	46.59	77,184,421	20.88
2	325	5,955,370	53.41	292,479,383	79.12

Over time, the incidence and distribution of benefits and costs of irrigating from the Central Ogallala Formation will not be uniform. Irrigation wells in Resource Situation 1 will not yield 1,000 g.p.m. when pumped from 100 feet of saturated thickness of Ogallala Formation, assuming average permeability. As saturated thickness declines, well yields decline and irrigators are forced to drill additional irrigation wells to maintain their historic production pattern. The irrigator eventually is forced to reduce irrigated acreage and return to dryland farming. The return to dryland farming comes not as a result of physical exhaustion of the aquifer, but as a direct result of rapidly rising irrigation costs.

Irrigation operators pumping with 325 feet of saturated thickness do not experience the immediate decline in well yields and rising pumping costs of irrigators in Resource Situation 1. Properly designed irrigation wells yield 1,000 g.p.m. until the saturated thickness declines from 325 feet to approximately 125 feet. Assuming an average rate of decline of five feet or less per year, suggests irrigators in Resource Situation 2 will experience 40 or more years of adequate water before well yields decline appreciably and pumping costs rise rapidly.

A Representative Farm for the Study Area

Time, human resources and computer problems act as significant constraints when defining a manageable number of representative farms or resource situations to be programmed. In the previous section two basic resource situations are defined. Since each resource situation must be subjected to three institutional alternatives with respect to water use, one modal representative irrigated farm operation is defined for the study area. This modal operation is synthesized from individual farm surveys taken from a random sample of 78 irrigation operators in the study area during the summer of 1970.[10]

The distribution of farm sizes for the 78 operations reveals that the modal farm size is between 500 and 1,000 acres and that the farm sizes representing the greatest number of farms tend to be associated with intervals containing multiples of 640 acres--full sections. Closer examination reveals that the largest number of farms range in size from 601 to 700 acres. Since farms have a tendancy to be even sections in size, a modal representative farm of 640 acres, or one section, is defined for this study.

Organization of Production for the Representative Farm

Surveys from the 78 randomly sampled farm operations were utilized to develop an organization for the representative farm. The organization of production is presented in Table XXIX. A total of 315 acres of cropland are irrigated. Grain sorghum and corn compose 230 acres of irrigated summer

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[10] The random sample of 78 irrigated operators was a portion of a more extensive survey in 1970 taken by Wyatte L. Harmon and Roy E. Hatch, Agricultural Economists, Farm Production Economics Division, Economic Research Service, U.S. Department of Agriculture, in connection with a study for essentially the same study area.

TABLE XXIX

THE ORGANIZATION, WHEAT AND FEED GRAIN ALLOTMENTS AND CONSERVING BASE FOR REPRESENTATIVE CASH GRAIN FARM, CENTRAL OGALLALA FORMATION

Cropland	(Acres)
Irrigated Grain Sorghum	170
Block G1 (80)	
Block G2 (40)	
Block G3 (30)	
Block G4 (20)	
Irrigated Wheat	85
Block W1 (65)	
Block W2 (20)	
Irrigated Corn	60
Block C1 (40)	
Block C2 (20)	
Dryland Grain Sorghum	30
Block G5 (30)	
Dryland Wheat	85
Block W3 (85)	
Idle or Fallow	66
Diverted	84
Lost to Turnrows	15
Total Cropland	595
Pastureland	
Dryland Non-Tillable Pasture	40
Total Pastureland	40
Other Land	
Home, Buildings and Roads	5
Total Other Land	5
Total Land in Farm	640
Allotments	
Wheat	185
Feed Grain	120
Conserving Base	55

crops and the remaining 85 irrigated acres are planted in winter wheat. There are 30 acres of dryland grain sorghum and 85 acres of dryland wheat.

Each of the above crops is divided into one or more crop blocks. For example, each dryland crop is planted in a single crop block. Irrigated wheat and corn are each planted in two crop blocks. Irrigated grain sorghum is planted in four crop blocks. The acreage in each block appears in parentheses in Table XXIX. Each crop block has its own soil moisture balance to maintain a daily record of stress conditions. The farm operator is assumed to irrigate each crop block by block. Thus, if pumping capacity is insufficient to irrigate an entire crop, perhaps only one block suffers severe moisture stress rather than the entire crop suffering moderate stress.

All grain sorghum is assumed harvested for grain. Two-thirds of the corn is harvested for grain and one-third for silage. The remaining 165 acres of cropland is divided among three land use categories--66 acres are idle or fallow, 84 acres are diverted and 15 acres are assumed lost due to turnrows, etc. Graze-out small grain is assumed planted on the diverted acres and may be grazed from November 1 until May 15 without penalty. The representative farm also contains 40 acres of native pasture. The homestead, buildings and roads are assumed to occupy the remaining five acres.

The analytical models employed in this study make no attempt to determine an optimum organization of production. Thus, the organization of production developed from the random sample of farms is adopted as the starting point for simulation of both resource situations and each institutional alternative.

Assumptions concerning the machinery complement, overhead costs and labor requirements are considered representative of the study area. The listing of the machinery complement is given by Mapp [54, pp. 260-261]. Annual overhead costs for the 640-acre cash grain farm total $3,380 [54, p. 93]. Family labor is assumed available at the rate of 200 hours per month and additional labor may be hired in eight-hour increments at $2.00 per hour.

Price Assumptions

Prices used in the analysis for this objective are the same as those assumed for previous objectives--"adjusted normalized prices" issued by the Water Resources Council [45]. The price estimates are considered "normalized" since the use on long-term, nonlinear trend lines removes many of the abnormalities caused by weather and other short-term chance events. The normalized prices are then adjusted to reduce the influence of Government price support programs. Adjusted normalized prices for commodities are further adjusted to the State level through the use of a ratio of State to U.S. normalized pirces received by farmers.

U.S. adjusted normalized prices are $1.30 per bushel for wheat, $0.95 per bushel for grain sorghum and $1.05 per bushel for corn. The average ratio of State to U.S. prices for the study area is 0.995, 0.985 and 1.06 for wheat, grain sorghum and corn, respectively. The adjusted normalized prices computed for use in this study are $1.20 per bushel for wheat, $0.94 per bushel for grain sorghum and $1.11 per bushel for corn. A price of $5.50 per ton is assumed for corn silage in the field. That is, the buyer performs the harvesting operation. Small grain pasture is assumed sold at $8.00 per AUM and native pasture at $3.00 per AUM.

Government Programs

Full participation in the 1971 Wheat and Feed Grain Programs is assumed for each of the resource situations. Of the 185-acre wheat allotment, 60 acres must be set aside in addition to the 55-acre conserving base, to qualify for wheat certificate payments. The face value of the wheat certificate, based on a $1.29 per bushel wheat price and $2.90 per bushel parity price, is $1.61 per bushel. Payments are made based on the domestic allotment (80 acres), face value of the wheat certificate and the projected yield per acre for the farm.

Of the 120-acre feed grain base, 24 acres in addition to the conserving base must be set aside to qualify for feed grain payments. Payment rates of $0.32 per bushel for corn and $0.29 per bushel for grain sorghum are assumed. Feed grain payments are received on 50 percent of the base, or 60 acres. Grain sorghum payments are received on 46 acres and corn payments on 14 acres of the feed grain base. Payments are based upon the number of acres, payments rate and projected yield for the total acres planted. Projected yields for grain sorghum, corn and wheat are based on a five-year moving average of yields for all acres of each crop planted on the representative farm. The five-year moving average reduces the influence of yearly variations in yield, but permits yields and government payments to increase as irrigation pumping capacity is expanded.

Once compliance with the set-aside and conserving base features of the 1971 Wheat and Feed Grain Programs has been established, the remaining cropland may be planted in any crop. Free substitution between wheat and feed grains is permitted. Thus, when simulating the representative farm through time, planting a total of 295 acres (the total of wheat and feed grain allotments) to either wheat, grain sorghum or corn is sufficient to maintain government program history on the farm.

Irrigation Wells and Pumping Costs

Representative farm firms for both Resource Situations 1 and 2 are assumed to have one irrigation well at the beginning of all simulation runs.[11] The adequate-water farm firms in Resource Situation 2 are assumed to have an irrigation well capable of producing 1,000 g.p.m. over the 20-year span of each simulation run. However, firms in Resource Situation 1, with 100 feet of saturated thickness, are assumed to begin each 20-year run with a single irrigation well, pump, motor and distribution system, capable of pumping 780 g.p.m. during the initial year of the simulation run. With the pump bowls located on the redbed underlying the Ogallala Formation, each year's pumping has several effects. First, the saturated thickness of the formation is reduced. Second, the reduction in saturated thickness leads to a reduction in pump yield. Third, the reduced capacity increases the per unit cost of delivering each acre inch of water to the plants. Fourth, the reduced capacity also alters the operator's irrigation schedule by making it more difficult to achieve timely water applications.

The relationship between declining saturated thickness and reduced well capacity is expressed in equation (57).[12]

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[11] See Harry P. Mapp [54, pp. 96-100] for a discussion of the method used to compute the initial well yields for each resource situation and the amount of saturated thickness required to sustain a 1,000 g.p.m. yield.

[12] Equation (57) was developed in the Southern High Plains of Texas for irrigation wells pumping from the Ogallala Formation. The relation was obtained by correspondence with Mr. Frank A. Rayner, Manager of the High Plains Underground Water Conservation District, Lubbock, Texas, and Mr. Frank Hughes, ERS, USDA, Texas A & M University, College Station, Texas.

$$Q_t = (\frac{H_t^{2}}{H_{t-1}}) Q_{t-1} \qquad (57)$$

where Q_t represents the well capacity in the current period t; Q_{t-1} represents the well capacity in the preceding period t-1; H_t represents the remaining feet of saturated thickness in the current period t; and H_{t-1} represents the feet of saturated thickness in the preceding period t - 1.

Equation (57) is used to compute current pumping capacity at the beginning of each crop year within the Production Subset of the model. Experimentation with the model reveals that at least 700-g.p.m. well capacity is required to adequately irrigate the original production organization on the representative farm. Thus, a decision rule is built into the Production Subset which allows the irrigator to drill an additional well if pumping capacity falls below 750 g.p.m. during a crop year. The new well is assumed drilled during the non-irrigation season and pumping capacity the following year is increased by the capacity of the existing well. For example, if the yield of irrigation well 1 declines below 750 g.p.m. during the current season to, say, 700 g.p.m. by the end of the crop year, the producer is assumed to drill a second well and connect it to the original distribution system which increases the system capacity to 1,400 g.p.m. for the following crop year. Yields for both wells then decline as the saturated thickness diminishes until system capacity falls below 750 g.p.m. again. Then the irrigator is assumed to drill a third well, designed to deliver the average yield of the other two wells, raising system pumping capacity by 50 percent. Three irrigation wells is the maximum assumed for the one-section representative farm firm.

Detailed information regarding investment, ownership and pumping costs for irrigation wells of Resource Situations 1 and 2 are presented in [54, pp. 318-324]. All irrigation systems utilized in the model are furrow or surface systems suited to Richfield clay loam soils.

Development of Irrigation Strategies

It is not difficult to prescribe an optimum irrigation strategy for the farm operator under static conditions. Static economic theory indicates the rational operator should utilize each unit of irrigation water in its highest value use so that the marginal value product of the last unit applied just equals its marginal resource cost.

The optimal strategy prescribed under static conditions is difficult to apply under the dynamic conditions faced by the irrigator in the field. Static theory implies the ability to change water applications instantaneously from one crop to another. In practice, once the operator begins to irrigate, he finds it economic to add from 1.0 to 3.0 inches of water to the soil profile of a crop before changing the irrigation set to another crop or another field. Thus, even though water is the type of resource that appears to be infinitely divisible, problems of indivisibilities exist. However, these indivisibilities do not invalidate the economic concept of applying water to its highest valued use. Each irrigation operator has an idea of the critical water-use periods for each crop and which of the several crops requiring water during a specific period has the highest use value for the irrigation water available. He applies water during a specific period first to the crop which has the highest use value (marginal value product) for that unit of irrigation water. Once that crop has received an irrigation application, the crop or crop block having the highest marginal value product for the next unit of irrigation water receives the next irrigation application. The operator may switch crop priorities from one part of the growing season to another in response to changes in the value of irrigation water among crops.

Delineation of Irrigation Periods

This line of reasoning leads to the development of a series of irrigation strategies for the growing season. Table XXX presents a crop calendar covering the period May 1 through September 30. The crop calendar shows the critical stages of plant development for grain sorghum, wheat and corn and indicates the periods when two or more crops are in direct competition for irrigation water.

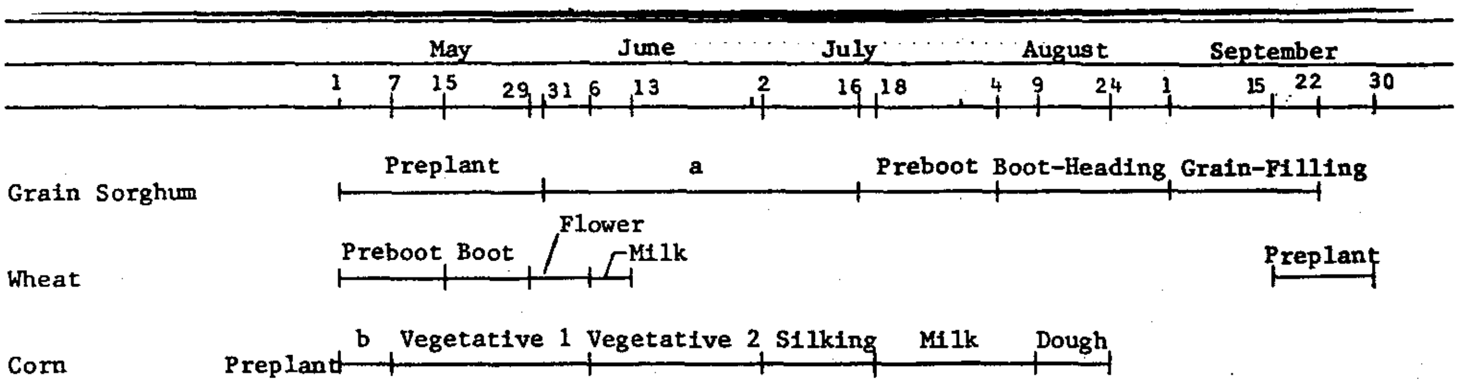
The entire period covered by the crop calendar is divided into five irrigation periods. The basis for selecting the beginning point of each period is the beginning of a critical stage of plant development for a crop. For instance, irrigation period 2 begins on May 16, when wheat reaches boot stage, and lasts until June 6 when the late vegetative stage for corn begins. Irrigation priorities established during this period are wheat first, corn second and grain sorghum last. These priorities are based on the marginal value product of irrigation water on the crops during this 20-day period of the growing season. The information presented in Table XXX for periods 1, 3, 4 and 5 can be interpreted in a similar manner.

Irrigation Strategies by Periods

Application of irrigation water depends upon the level of soil moisture existing in the soil profile of a crop. The model assumes that the decision to irrigate is made when the level of soil moisture falls

TABLE XXX

DELINEATION OF CRITICAL STAGES OF PLANT DEVELOPMENT, IRRIGATION PRIORITIES AND IRRIGATION STRATEGIES



Critical Periods	(1) May 1- May 15	(2) May 16- June 5	(3) June 6 - August 4	(4) August 5 - September 15	(5) Sept. 16-30	
Irrigation Priorities[c]	G,W,C	W,C,G	C, G	G, C	G,W	
Pumping Days	14	20	56	39	14	

[a] No stage name is given to grain sorghum between preplatn irrigation applications and preboot stage. Moisture stress during this period has little effect if moisture is adequate during subsequent stages of development.

[b] Plant emergence occurs between May 1 and May 7.

[c] Irrigation priorities G, W and C represent grain sorghum, wheat and corn, respectively. All blocks of the crop listed first in a critical period are irrigated before any block of the second or third priority crops.

below 50 percent of available soil moisture, or 12.5 inches. If soil moisture in the entire profile for a crop equals or exceeds 12.5 inches, no irrigation water is applied. If available soil moisture falls below the 50 percent available level during a critical stage of development, additional water is applied based on the priorities discussed above and available pumping capacity. If sufficient water is available, the entire crop receives a 3.0-inch addition to the soil profile. However, if plants on the part of the field already irrigated begin to show signs of plant stress before the entire application can be completed, irrigators are assumed to reduce the application rate on the remaining acres, and return to the original portion of the crop to begin a new application. These assumptions appear reasonable based on the actions of irrigators in the area.

Varying irrigation rates on shifting numbers of acres during different stages of plant development is extremely difficult to handle from a modelling standpoint. Therefore, as indicated in Table XXIX, total acreage of each irrigated crop is divided into several blocks. The 170.0 acres of irrigated grain sorghum are divided into four blocks of 80.0 acres, 40.0 acres, 30.0 acres and 20.0 acres. Similarly, the irrigated wheat and the irrigated corn are each divided into two blocks. Block 1 of any crop is always irrigated first, followed by block 2, etc. If, using grain sorghum as an example, block 4 is being irrigated and block 1 begins to suffer moisture stress, the irrigation application rate is reduced on block 4 and block 1 is the next block to be irrigated.

The general procedure for scheduling and executing irrigation applications is the same for every period and may be discussed in general terms. Each period has a set of crop irrigation priorities as outlined in Table XXXI. The priorities determine the order in which soil moisture values are checked against the critical value (usually 50 percent available soil moisture or 12.5 inches). Assume the order of priorities is (1) grain sorghum, (2) wheat and (3) corn, as it is for period 1. On the first day of the period, soil moisture for the first block of grain sorghum, G1, is checked against 12.5 inches of soil moisture. If soil moisture for G1 equals or exceeds 12.5 inches, no irrigation application is scheduled for G1 and soil moisture for G2 is checked against 12.5 inches, etc. If all four grain sorghum blocks have soil moisture in excess of 12.5 inches, then soil moisture for the first block of wheat (W1), the second priority crop, is checked against 12.5 inches. This process continues as long as soil moisture for each block exceeds 12.5 inches. After soil moisture for both blocks of the third priority crop, corn, have been checked against 12.5 inches, and soil moisture for all blocks is found to exceed 12.5 inches, the day is incremented to day 2 of the period and soil moisture under the first block of the first priority crop is again checked against 12.5 inches. In the above example, no irrigation applications would be scheduled during day 1 of period 1.

Now consider the usual situation where an irrigation application is required. Assume that on day 1 of the period, soil moisture under G1 is less than 12.5 inches. The farm operator schedules an irrigation application for G1. Ideally, once an application has begun, he would like to add 3.0 inches of soil moisture to the G1 profile. Due to evapotransporation and water losses from leakage and seepage, only about two-thirds of the water pumped from the aquifer enters the soil profile of the irrigated crop. Therefore, 4.5 inches must be drawn from the aquifer to insure a

TABLE XXXI

MOISTURE LEVELS AT WHICH IRRIGATIONS ARE SCHEDULED AND PRIORITIES ESTABLISHED BY IRRIGATION PERIODS

	Irrigation Period											
	1			2			3		4		5	
Irrigation Priority Order	GS	W	C	W	C	GS	C	GS	GS	C	GS	W
Inches of Soil Moisture at which Irrigations are Scheduled	12.50	10.98	10.98	12.50	12.50	12.50	12.50	10.98	12.50	12.50	10.98	12.50
Inches of Soil Moisture at which Priority on Water is Established	9.45	10.98	10.98	10.98	10.98	10.98	10.98	9.45	10.98	10.98	9.45	9.45

3.0-inch addition to the soil profile. Based on the requirement of 4.5 acre inches per acre, the irrigation water requirement is computed from (58):

$$WR_{ij} = 4.5\ AC_{ij} \tag{58}$$

where WR_{ij} equals the water requirement, block i, crop j; and AC_{ij} equals the acres planted in block i, crop j.

Then the water requirement is compared with the pumping capacity for the period. Pumping capacity is computed based on gallons per minute delivered by the irrigation system as follows:

$$BPC_i = (GPM \times 1440.0 \times DAYS_i)/27{,}155.0 \tag{59}$$

where BPC_i equals the beginning pumping capacity for period i in acre inches; GPM equals the irrigation system pumping capacity in gallons per minute; 1440.0 equals the number of minutes per day; $DAYS_i$ equals the number of days in period i; and 27,155.0 equals the gallons per acre inch.

Assuming that pumping capacity for the period equals or exceeds the water requirement for G1, the irrigation application is initiated. The number of days required to apply WR_{ij} acre inches is computed and no other crops can be irrigated until the application of G1 has been completed. The total application is divided by the number of days required to apply it, and the appropriate proportion is added to soil moisture each day. Once the application on G1 is complete, the remaining pumping capacity for the period is computed and soil moisture under the second block of the top priority crop, G2, is checked against 12.5 inches. If soil moisture exceeds 12.5 inches, soil moisture under G3 is checked, etc. If, however, G2 soil moisture is less than 12.5 inches, its water requirement is computed using (58) and is then compared to the remaining pumping capacity for the period. If sufficient capacity exists, the irrigation is scheduled, the number of days required computed and the appropriate amount of moisture per day added to the soil profile. No other crop may be irrigated until the application on G2 has been completed. The G2 water requirement is deducted from pumping capacity for the period, and then soil moisture for G3 is checked against 12.5 inches. This procedure continues unaltered until one of four following events occurs. (a) The water requirement for any block of a crop exceeds the remaining pumping capacity for the period. (b) The number of days remaining in the period is insufficient to allow a full irrigation. (c) A block of higher priority reaches a low soil moisture level while a low priority crop is being irrigated. (d) The period comes to an end. These events will be considered in turn.

(a) If the water requirement for a block of a crop exceeds the remaining pumping capacity for the period, based on a 4.5-inch application per acre, the number of acre inches which can be applied per acre is computed. If that number equals or exceeds 1.5 acre inches per acre, the irrigation is scheduled and the application made. If at least 1.5 acre inches per acre cannot be applied, no irrigation application is made to the block in question.

(b) If the number of days remaining in the period is insufficient to allow a full irrigation, water is applied at the computed rate per day until the period ends.

(c) If a block of higher priority reaches a low soil moisture level while a lower priority crop or block is being scheduled for irrigation, the irrigation application on that block is reduced to 1.5 acre inches per acre. Then the higher priority crop moisture is checked, and a full 4.5-inch irrigation application is made, assuming time and pumping capacity exist to complete the application.

(d) When the period comes to an end, no further irrigations are scheduled based on crop priorities for the current period. Soil moisture under block 1 of the highest priority crop in the next period is checked against 12.5 inches of soil moisture.

The same procedure continues through all five of the irrigation periods. At the end of the crop year, crop yields on each block of each crop are computed based on soil moisture and atmospheric stress suffered during the critical stages of development and accumulated throughout the growing season.

The coefficients used in applying the irrigation strategies are not intended to imply that the operator is capable of distinguishing between levels of available soil moisture to two decimal places. The decision rules are merely an attempt to simulate the decisions operators make based on feel of the soil and appearance of plants. Since these actions must be computerized, the rules are quite specific in nature.

The next sections of this report outline procedures utilized in simulating institutional alternatives to water-use regulation. The first alternative is no regulation or restraint on water use. The second alternative is an absolute limit on the number of acre inches pumped per year. The third alternative allows irrigators to pump more than the quantity limit if they pay a graduated tax per unit of water pumped above the limit. These are considered in turn.

Simulation of Representative Farm Firms Without Institutional Restraints on Water Use

The initial institutional alternative considered is to allow unrestricted pumping from the Central Basin of the Ogallala Formation by firms in both Resource Situations 1 and 2. This alternative coincides with a continuation of current policy in accordance with present interpretations of ground water law in the study area.

For the unrestricted water-use alternative, the decision rules followed by irrigators are based upon the level of available soil moisture during critical stages of plant development as outlined in previous sections. Irrigators in Resource Situation 1 face declining well yields over the 20-year simulated time period. When capacity of the irrigation system falls below 750 g.p.m. in a given year, the irrigator is assumed to drill a new well at the end of that year. When the operator has three irrigation wells, his response to declining well yields and rising pumping costs is to reduce the number of irrigated acres. The decision rule used to reduce irrigated acres is based on a comparison of net returns per acre above variable costs for each irrigated block and opportunity cost net returns per acre for the best dryland alternative--dryland wheat. Opportunity cost net returns on dryland wheat, considered as returns to land, overhead, risk and management, are \$5.24 per acre [54, p. 115]. Every year after the third well has been added the net return per acre above variable costs in each block is compared to the \$5.24 opportunity cost for dryland wheat. If the opportunity cost dryland net return is greater, the block is planted to dryland wheat the following year.

Simulation of Representative Farm Firms With a Limit on the Quantity of Irrigation Water an Operator May Pump During the Growing Season

The second institutional alternative restricts the quantity of irrigation water the individual operator is allowed to pump during the crop year. The authority of Water Resources Boards to restrict the quantity of water pumped is documented above. It is assumed that each irrigator is restricted to pumping 1.5 acre feet of irrigation water per acre of water rights per crop year. For the representative farm firms of this study, water rights to irrigate 315 acres are assumed. At 1.5 acre feet per acre of water right, the irrigator is limited to pumping 472.5 acre feet per year or 5,670 acre inches per year.

The controlling agency is assumed to say nothing about the allocation or distribution of this water among periods of the crop year. The irrigator is free to pump his system at capacity from the beginning of the irrigation season until he has arrived at the quantity limit, or limit pumping in the early periods due to uncertainty about future moisture conditions. The rational irrigator is assumed to hedge current pumping due to uncertainty about future water needs during later stages of plant development. He is assumed to pump according to soil moisture depletion levels and crop priorities established for the unconstrained simulation runs discussed previously, however, establishes maximum amounts of water to be added to each crop during each stage of plant development. The maximum levels by crops and irrigation periods are reflected in Table XXXII.

These figures indicate, for example, that no more than 4.5 acre inches of irrigation water will be applied to each acre of grain sorghum during irrigation period 1. With an irrigation efficiency of two-thirds, a 3.0-inch real addition to the soil profile is implied by a 4.5 acre inch per acre water application. These self-imposed irrigation guidelines provide enough flexibility to allow sufficient water to be applied during very dry years, yet induce the irrigator to conserve water for subsequent periods to meet unexpected demands. During a year of high and timely rainfall, the irrigator will likely not pump 5,670 acre inches of water. However, during a year characterized by either untimely or low rainfall, the irrigator may easily reach the quantity limit during irrigation period 4 and be unable to complete grain sorghum irrigations or to prewater wheat during September.

TABLE XXXII

MAXIMUM INCHES OF WATER APPLIED PER ACRE BY CROPS AND PERIODS OF THE GROWING SEASON IN RESPONSE TO A QUANTITY LIMITATION

Period	Grain Sorghum	Wheat	Corn
April	0.0	0.0	6.0
Period 1	4.5	4.5	0.0
Period 2	4.5	9.0	4.5
Period 3	9.0	0.0	18.0
Period 4	13.5	0.0	4.5
Period 5	0.0	9.0	0.0
Total	31.5	22.5	33.0

No change in production organization is assumed. It might be argued that the rational irrigator would respond to a quantity limitation by reducing irrigated acres to the maximum number he can fully irrigate. While this course of action makes sense from an economic standpoint, it is not being followed by the operators experiencing declining well yields and water supplies. The tendency is to protect the historic production organization by applying less water per acre while maintaining the same number of acres [54, p. 119]. Once it becomes unprofitable to irrigate a crop block, however, producers naturally respond by reducing irrigated acreage. The net returns per acre above total variable costs for each crop block is compared with dryland wheat opportunity cost net returns per acre. Crop blocks whose net returns per acre fail to exceed opportunity cost net returns per acre are converted to dryland wheat the following year in a multi-period run.

Simulation of Representative Farm Firms With a Graduated Tax on Each Acre Inch of Water Pumped Above the Quantity Limitation

The third institutional alternative considered assumes that each irrigator is restricted to pumping 1.5 acre feet per acre of water rights, or 5,670 acre inches of water per year. However, the irrigator is permitted to pump in excess of 5,670 acre inches per year if he is willing to pay a tax on each acre inch of water pumped above the quantity limitation.

The tax rate of $0.50 per acre inch is based upon tax rates which have been utilized in irrigation districts in California. At $0.50 per acre inch, the tax rate is $6.00 per acre foot. The magnitude of the graduated tax may seem excessive, however, it should be emphasized that the tax is applied to the additional or marginal unit of irrigation water. The irrigator would not find it economical to pay a $0.50 per acre inch tax on every unit pumped. However, the marginal value product of irrigation water during a critical stage of plant development, given inadequate soil moisture conditions, is quite high. The model assumes decision rules for simulation of the quantity limitation, specified in the previous section, are followed until the quantity limitation is reached. Thereafter, the irrigator is assumed to decide whether or not to irrigate based upon the potential loss in yield which will occur if the irrigation is not applied.

The critical decisions involve whether or not to continue irrigating grain sorghum during irrigation period 4 and whether or not to apply a preplant irrigation on wheat during irrigation period 5. The preplant irrigation on wheat is quite often of critical importance if a good stand is to be achieved. In the Production Subset of the model, failure to preplant irrigated wheat is assumed to reduce the potential yield by 15 bushels. Fifteen bushels of wheat at $1.29 per bushel returns gross revenue of $19.35. The variable cost of the additional irrigation is approximately $8.70.[13] The value of the marginal product resulting from an additional irrigation on wheat clearly exceeds the marginal resource cost. Thus, the irrigator is assumed to apply a preplant irrigation on wheat during irrigation period 5 every year.

The decision whether or not to irrigate grain sorghum during irrigation period 4 is a function of soil moisture and days of potential yield reduction remaining in irrigation period 4. If soil moisture is low enough that that the potential yield reduction is equal to or greater than ten bushels, the decision is to irrigate.[14] All wells are metered and the irrigator pays

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[13] Variable costs of $8.70 include variable pumping costs of $1.00 per acre inch for a 4.5-inch application, additional labor costs of $0.75, added harvesting and hauling costs of $1.20 and water taxes of $2.25.

[14] Gross revenues from nine and ten bushels of grain sorghum at $0.94 per bushel are $8.46 and $9.40, respectively. The cost of the additional irrigation, assuming variable pumping cost per acre inch is $1.00, additional labor cost is $0.75, tax payments are $2.25 and added harvesting and hauling costs are either $0.99 or $1.10, total $8.49 and $8.60 for nine and ten bushels potential yield reduction, respectively. The added costs exceed added revenues for a nine-bushel potential yield reduction, however, added revenues exceed added costs and an additional irrigation is justified if potential yield reduction is equal to or greater than ten bushels.

a tax of $0.50 per acre inch on each acre inch in excess of the 5,670 acre inches pumped during the crop year.

Results of Simulating Alternative Methods of Water-Use Regulation

The initial portion of this section summarizes the effects of each of the three methods of water-use regulation on the representative firm and the water supply in Resource Situation 1. The subsequent part concentrates on Resource Situation 2.

Effects of Unrestricted Water Use on Resource Situation 1

Resource Situation 1 represents the poor water situation for the study area. Average saturated thickness of the underground aquifer is 100 feet. This amount of saturated thickness will support a well yield of approximately 780 g.p.m. Assumptions concerning the number of acres irrigated, acreage planted to each crop, and the decision rules followed to drill additional wells and revert acreage to dryland are discussed in the previous section. Operation of the representative firm under each method of regulation was simulated over a 20-year time horizon and each method of regulation was replicated 15 times. The results of the simulation analysis of three water-use regulatory alternatives are presented below.

Effects on Acre Inches Pumped

The effect of unrestricted water use on the quantity of water pumped through time is shown in Table XXXIII. The mean, standard deviation, maximum, minimum and range have been computed using the 15 replications for each year of the 20-year planning horizon [54, p. 130].

TABLE XXXIII

SUMMARY OF TOTAL ACRE INCHES PUMPED, NET FARM INCOME AND NET WORTH FOR RESOURCE SITUATION 1 WITH NO RESTRICTIONS ON WATER USE*

	Year																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
									Total Acre Inches Pumped											
Mean	5805	5550	6911	7385	7061	6585	6446	5935	6006	6218	5898	5361	4610	4028	3280	2859	2486	2208	1963	1324
Std. Dev.	972	500	1632	1265	1094	686	323	426	754	846	503	561	539	555	472	575	377	406	565	982
Maximum	6699	6039	8305	9265	8229	7607	7092	6704	7568	7492	6506	6195	5161	4949	4143	3827	3419	2739	2361	2202
Minimum	2852	4225	1923	4704	4454	5369	5605	5167	4981	4490	4646	4136	3261	2858	2561	1760	1819	903	0	0
Range	3847	1815	6382	4561	3776	2238	1486	1542	2587	3002	1861	2059	1901	2091	1592	2066	1600	1837	2361	2202
										Net Farm Income										
Mean	9019	9809	13546	13839	15045	14624	13593	11454	10870	11324	8780	6405	7502	6838	7719	5714	7503	4351	1031	2183
Std. Dev.	4151	5470	3462	4452	3957	5840	4700	4489	5051	5775	5761	4851	7620	5666	4591	4797	4930	5453	6503	5639
Maximum	15567	22925	20868	21891	23876	24824	23607	19976	21111	20037	19987	16314	22603	14697	18372	11547	15699	10238	11113	11924
Minimum	3397	2714	8850	7947	9571	4581	3261	4568	3300	2996	902	-3534	-8629	-309	2466	-3668	571	-8407	-9883	-8109
Range	12170	20211	12018	13944	14305	20243	20346	15408	17811	17041	19085	19848	31232	15006	15906	15215	15128	18645	20996	20033
Coef.of Var.	0.46	0.56	0.26	0.32	0.26	0.40	0.35	0.39	0.46	0.51	0.66	0.76	1.02	0.83	0.59	0.84	0.66	1.25	6.31	2.58
										Net Worth										
Mean	120792	121923	126075	130517	135829	141397	145683	148152	151717	155568	156182	154778	154237	153286	152979	150822	150268	146852	140219	135555
Std. Dev.	3334	5673	7259	9347	10140	13568	15926	17393	17780	21031	23995	27241	30219	32832	34975	38402	41421	44527	48768	52346
Maximum	126009	135524	143124	154176	160580	167980	171284	172769	175885	182686	181628	186824	198840	198111	201896	203823	208574	210251	213980	214774
Minimum	116195	113631	115735	119604	125187	122641	123589	122553	125468	122706	118326	111788	96592	88783	84606	74377	67422	59749	47057	36537
Range	9814	21893	27389	34572	35393	45339	47695	50216	50417	59980	63302	75036	102248	109328	117290	129446	141152	150502	166923	178237

*The values in this table are based on 15 replications. The values for each replication are reported by Mapp [54, pp. 130, 134 and 183].

The mean values in Table XXXIII highlight several interesting phenomena. The second irrigation well is usually added at the end of the second or third crop year, and its effect on pumping capacity for the irrigation system is apparent. Average acre inches pumped increases from 5,550 in year 2 to 6,911 and 7,385 acre inches, respectively, during years 3 and 4. The third irrigation well is usually drilled at the end of either crop year 8 or 9. Increased pumping capacity is reflected through an increase in pumping from 5,935 acre inches in year 8 to 6,006 and 6,218 acre inches during crop years 9 and 10, respectively. After the third irrigation well is drilled, declines in acre inches pumped result from (1) declining well yields; (2) increasing pumping costs; and (3) the resulting reduction in irrigated acreage. Mean values decline steadily from 6,218 acre inches in year 10 to 1,324 acre inches in year 20.

The maximum number of acre inches pumped during any replication of any year is 9,265 during the 14th replication of crop year 4. A combination of excess pumping capacity after the addition of well 2 and extremely dry weather conditions during the year are primary causal factors. The minimum number of acre inches pumped during any replication of crop year 4 is 4,704, which occurs in replication 11.

During five of the fifteen replications all irrigated crops were converted to dryland wheat by crop year 20 and zero pumping occurred. In one of the replications conversion to total dryland farming occurred by crop year 19. Thus one-third of the replications simulated result in a return to dryland farming by the 20th year. Variable pumping costs per acre inch during the final year in which irrigated crops are raised ranged from \$1.42 to \$1.68 for the five replications reverting all land to dryland crop production.

Saturated thickness of the underground aquifer at the end of the 20-year simulation runs ranges from 33.42 to 37.53 feet and averages 35.84 feet. Transforming these figures into feet of decline in saturated thickness results in an average decline of 64.16 feet over the 20-year period, an average decline of 3.21 feet per year. The original 100 feet of saturated thickness underlying Resource Situation 1 contained approximately 9,600 acre feet of water which could be withdrawn for irrigation purposes.[15] The decline in saturated thickness to 35.84 acre feet leaves approximately 3,440 acre feet of water that is uneconomical to pump for irrigation purposes. Thus, of the original volume, only 35.84 percent remains at the end of the 20-year unrestricted simulation of Resource Situation 1.

Effects on Net Farm Income

Effects of water-use regulation on net farm income are of great importance to individual farm operators and to the economy of the Central Ogallala Formation. Net farm income is computed in the General Agricultural Firm Simulator as the difference between gross farm income and gross farm expense. As used in the context of the simulation model, it represents net returns to land, operator labor, management and risk. Net farm income is computed each year of a multi-period simulation run. The simulation runs are sequential and firm financial changes are updated each year to reflect the current status of the firm.

Table XXXIII contains a summary of net farm income resulting from the 15 replications of a 20-year simulation of Resource Situation 1 without

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[15] The figure 9,600 acre feet is computed assuming 640 acres overlie the 100 feet of saturated thickness and that the specific yield of the Ogallala Formation is 0.15. Then 640 acres x 100 feet x 0.15 = 9,600 acre feet.

water-use regulation. The mean, standard deviation, maximum, minimum and range have been computed for each year of the planning horizon.

Net farm income for farms in Resource Situation 1 increases rapidly during the initial years of irrigation system expansion. From year 1 to year 5, mean net farm income increases from $9,019 to $15,045, the maximum mean value for any year of the run. The rise in net farm income over a five-year period is primarily due to increased pumping capacity which increases irrigated crop yields. Increased yields result in greater government payments, which are computed on the basis of a five-year moving average of yields for wheat and feed grains. After year 5, mean net farm income declines gradually to $10,870 in year 9, rises to $11,324 in year 10 with additional irrigation expansion, and then follows an erratic, but declining trend through year 18. Mean net farm incomes the final two years are very low reflecting several adverse conditions. (1) Declining well yields and rising pumping costs contribute to declining profitability of the irrigated operation. (2) Conversion of an increaseing number of acres to dryland production reduces the mean net farm income and increases variability of income. Effects of adverse weather conditions contribute to years of very low and even negative net farm income.

During the initial five years, mean net farm income rises while variability of income, as measured by the standard deviation, declines. The income stability contributed by government payments is obvious throughout the initial and intermediate periods of the analysis. Income variability remains relatively stable across the 20-year simulation run. However, as mean net income declines in years 11 through 20, the coefficient of variation rises. The coefficient of variation is expressed as

$$cv = s/\bar{x} \qquad (60)$$

where cv represents the coefficient of variation; s represents standard deviation; and $\bar{x}$ represents the mean. The coefficient of variation affords a valid comparison of the variation among large values, such as income in initial periods, and variation among small values such as income in later periods [72, p. 64]. The lowest coefficient of variation is 0.26 in years 3 and 5 of the 20-year simulation of net farm income. In years 18, 19 and 20, the coefficient of variation is 1.25, 6.31 and 2.58, respectively.

The maximum net farm income for any replication of any year, $24,824, occurs early in the period, year 6. The minimum net farm income of -$9,883 occurs during the later part of the period, year 19. The maximum range in net farm income of $31,232 occurs during year 13. These figures emphasize the tremendous variability in net farm income that exists within the study area. Irrigation and government programs are definite stabilizing influences on net farm income. However, as the water supply is depleted, crop yields decline and dependence on dryland production increases. As the importance of government programs continue to decline, variable weather conditions significantly affect variability of net farm income in the poor water situation.

Effects on Net Worth

The Farm Firm Simulation Model computes net worth of the representative firm after each year of a multi-period simulation run. Net worth is, of course, computed as the difference between total assets and total debts. Over time, assets and debts are constantly changing. Real estate and chattle debt payments are made each year until the beginning levels have been reduced to zero. An initial real estate debt of $42,000 and an initial chattle debt of $5,234 are assumed. The chattle debt is paid off in five years and the real estate debt is retired during year 15. No further real estate or chattle debts are accumulated during the 20-year simulation runs. However, other short-term loans are required periodically to maintain the cash balance required for operation of the business.

Table XXXIII presents a summary of net worth for representative farms in Resource Situation 1 based on 15 replications of a 20-year simulation of the firm. The mean, standard deviation, maximum, minimum and range of net worth values are given for each year of the simulation run. Mean values of net worth exhibit several characteristics. (1) There is a definite trend in net worth through time. (2) The trend in net worth is not linear, but tends to follow a sigmoid pattern. (3) Net worth reaches a maximum in year 11. This maximum lags behind full irrigation development by one or two years. (4) After reaching a maximum in year 11, mean net worth for Resource Situation 1 declines steadily to year 20. Mean net worth at the end of year 1 is $120,792, increases steadily to $156,182 in year 11 and declines to $135,555 at the end of year 20. The standard deviation of net worth increases steadily from $3,334 in year 1 to $52,346 in year 20. Relative variability, as measured by the coefficient of variation, increases steadily over time from 0.03 in year 1 to 0.15 in year 11 to 0.39 in year 20. Increasing variability is again a function of several interelated factors. (1) Declining well yields over time result in less reliance on irrigation water to stabilize crop yields. (2) The shift of crop acres from irrigated production to dryland production tends to increase variability in yields, net returns and net worth over time. (3) Despite the completely random nature of rainfall and pan evaporation events in the Production Subset, series of "wet crop years" and of "dry crop years" years appear in the simulation runs. This phenomenon has been observed and documented for a study area which encompasses a portion of the Central Ogallala Formation [32, pp. 20-24]. The existence of series of good years contribute to a high ending net worth during replications 5 and 7 ($214,744 and $199,225, respectively). Series of dry years contribute to low ending net worth during replications 6 and 14 ($40,527 and $36,537, respectively).

The maximum and minimum net worth figures both occur during year 20. A range of $178,237 exists between the maximum of $214,774 and the minimum of $36,537.

Effects of a Quantity Restriction on Resource Situation 1

The second water-use regulatory alternative simulated is a limit on the quantity of irrigation water an individual is allowed to pump during a crop year. The irrigator is limited to pumping 1.5 acre feet per acre of water rights established for the representative farm firm. Water rights are assumed for 315 acres, resulting in a maximum allowable pumping of 472.5 acre feet or 5,670 acre inches per year. The model assumes the irrigator can continue pumping until the end of the day during which the quantity restriction is reached. Thus, there is some variation in pumping levels above 5,670 acre inches, despite the quantity limitation.

Effects on Acre Inches Pumped

Resource Situation 1 was simulated over a 20-year period and replicated 15 times. The mean, standard deviation, maximum, minimum and range of acre inches pumped are given in Table XXXIV for each year of the simulation runs.

TABLE XXXIV

SUMMARY OF TOTAL ACRE INCHES PUMPED, NET FARM INCOME AND NET WORTH FOR RESOURCE SITUATION 1 WITH A QUANTITY RESTRICTION ON WATER USE*

	Year																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
										Total Acre Inches Pumped										
Mean	5387	5466	5704	5661	5599	5656	5687	5674	5566	5339	5601	5644	5244	4817	3990	3561	3116	2559	2489	1791
Std. Dev.	768	450	26	121	391	159	13	79	305	534	288	114	615	817	638	720	624	634	393	925
Maximum	5710	5708	5741	5739	5728	5730	5714	5716	5715	5694	5717	5712	5709	5701	5097	4442	4000	3621	3173	2975
Minimum	2851	4227	5636	5257	4188	5084	5673	5394	4600	4024	4583	5295	3623	2970	2939	1802	1988	1157	1796	0
Range	2859	1481	105	482	1540	646	41	322	1115	1570	1134	417	2086	2731	2158	2640	2012	2464	1377	2975
										Net Farm Income										
Mean	8791	9715	11250	11131	12270	12707	11417	9913	10234	11899	7815	5613	6204	5621	5586	5335	7581	4591	1253	2447
Std. Dev.	4703	5548	4941	5815	5412	6926	5519	5099	5650	6164	6780	5557	7314	5567	5329	5189	5056	6295	6719	6400
Maximum	15567	22923	21290	22023	23559	23952	21864	18723	20746	22296	20585	16739	20396	13841	15133	12322	15020	12777	13187	13910
Minimum	2280	2102	3186	2476	4518	1056	-1160	2614	2398	2153	-3292	-2921	-7607	-2724	-2909	-4262	-132	-11727	-10800	-10007
Range	13287	20821	18104	19547	19041	22896	23024	16109	18348	20143	23877	19660	28003	16565	18042	16584	15152	24504	23987	23917
Coef.of Var.	0.53	0.57	0.44	0.52	0.44	0.55	0.48	0.51	0.55	0.52	0.86	0.99	1.18	0.99	0.95	0.97	0.67	1.37	5.36	2.62
										Net Worth										
Mean	120575	121620	123890	126086	129165	132506	134916	136115	137592	140430	142714	140513	138615	136320	133937	131294	130695	127367	120903	115617
Std. Dev.	3825	6043	8455	11314	12275	16483	19805	22033	23178	25921	29252	33201	35892	39565	42733	46457	49403	53403	57903	61904
Maximum	126009	135524	142473	153603	156761	163285	164465	165447	167270	172615	174443	176468	168443	186491	190856	192574	197294	201406	206441	205332
Minimum	115128	112423	112779	109463	114607	111754	104866	99873	99156	98546	90879	82845	79670	69812	61469	52776	49074	29851	15686	2198
Range	10881	23101	29694	44140	42154	51531	59599	65574	68114	74069	83564	93623	106773	116679	129387	139798	148220	171555	190755	203134

*The values in this table are based on 15 replications. The values for each replication are reported by Mapp [54, pp. 141, 143, 147].

Mean values of total acre inches pumped are relatively constant from year 1 through year 12. Slightly higher values in year 3 and in years 11 and 12 reflect the increased pumping capacity created by addition of irrigation wells 2 and 3. Irrigation well 2 is added at the end of crop year 2 and well 3 is added at the end of year 10 or 11, depending on when total system capacity falls below 750 g.p.m. Beginning with year 13, mean values of acre inches pumped decline steadily from 5,244 to 1,791 acre inches in year 20. Maximum mean acre inches pumped of 5,704 occurs during year 3 when pumping capacity of the irrigation system is greatest. Minimum pumping occurs during year 20, as expected, reflecting declining well yields and conversion of irrigated acreage to dryland wheat production. Complete conversion to dryland farming during the 20-year simulation occurs during 2 of 15 replications, or only about 13.3 percent of the time.

Maximum range in acre inches pumped for a single year is 2,975 acre inches in year 3. A total of 2,975 acre inches were pumped during replication 5 and a minimum of zero acre inches during replications 6 and 15.

Remaining saturated thickness of the underground aquifer at the end of the 20-year simulation run ranges from 36.08 to 41.57 feet, averaging 38.37 feet. With a beginning saturated thickness of 100 feet, an average remaining saturated thickness of 38.37 feet indicates a 61.33-foot decline in the water table. Over the 20-year period, the rate of decline averages 3.07 feet per year. Thus, even with a quantity limitation of 1.5 acre feet per acre of water rights, significant reductions in saturated thickness occur over a 20-year period. The distribution of water withdrawals differs from the unrestricted pumping situation. With the quantity limitation, less water is withdrawn in early years and more in late years of the

20-year simulation, but the resulting decline in saturated thickness is very similar in magnitude for both situations.

Effects on Net Farm Income

The effect on net farm income for representative farms in Resource Situation 1 of a limit on the quantity of irrigation water pumped per year also is illustrated in Table XXXIV. The mean, standard deviation, maximum, minimum and range in net farm income are shown for each year of the 20-year run.

Mean values of net farm income generally reflect the development and expansion of irrigation facilities over time, as well as the impact of the declining water level on system pumping capacity, pumping costs per acre inch and the transition from irrigated to dryland production. Mean net farm income increases from $8,791 in year 1 to $11,250 in year 3. The impact of increased pumping capacity caused by the addition of well 2 is reflected in year 3 net farm income. The maximum value of mean net farm income is $12,270 and occurs in year 5. There are at least two plausible explanations for the maximum occurring in year 5. (1) With the quantity restriction on water pumping in effect, the excess pumping capacity created by addition of well 2 in year 3 is not depleted as rapidly as under the unrestricted alternative. Thus, adequate water may be applied with precise timing to insure good to excellent irrigated crop yields. (2) Excellent crop yields over the initial years are translated into substantial wheat and feed grain payments which, of course, contribute directly to net farm income.

Mean net farm income declines from year 5 through year 8, increases during years 9 and 10, reflecting additional irrigation expansion to a three-well system. In most years the third well is added after crop year 9 and mean net farm income in year 10 is $11,899. Mean net farm income declines dramatically to $7,815 in year 11 and to $5,613 in year 12, but stabilizes for years 13 through 16. Year 17 mean net farm income of $7,581, contradicts the trend due primarily to favorable random weather events leading to increased crop yields despite declining well yields. Mean net farm income in years 19 and 20 is $1,253 and $2,447, respectively.

Standard deviation of net farm income has a general upward trend through time. Relative variability, as measured by the coefficient of variation, is virtually stable for years 1 through 10, ranging from a low of 0.44 in year 3 to a high of 0.57 in year 2. The coefficient of variation increases from 0.52 in year 10 to 1.18 in year 13 and remains in the 0.95 to 0.97 interval before declining to 0.67 in year 17. Thereafter, the coefficient rises rapidly to 1.37 in year 18 and 5.36 in year 19 before declining to 2.62 in year 20. The large coefficient of variation in year 19 is attributable to a combination of factors including (1) continued irrigation of acres which were marginally profitable during year 18, and (2) insufficient water to offset lack of natural rainfall during the growing season. The mean net farm income for year 19 is only $1,253, while standard deviation is $6,719. The replications during which the operator continues to irrigate with insufficient pumping capacity results in negative net farm incomes and the resulting increase in magnitude of the coefficient of variation.

In general, variability of net farm income with a quantity limitation exceeds variability of net farm income under conditions of unrestricted pumping. From years 17 through 20, variability of net farm income, as measured by the coefficient of variation, were quite similar for both the unrestricted water-use alternatives.

Effects on Net Worth

Restricting water use to 5,670 acre inches per year has a definite and significant impact on the representative firm's net worth over the 20-year simulation run. Net worth of the firm follows a sigmoid pattern over the 20-year interval, first increasing at an increasing rate, then at a decreasing rate and finally decreasing absolutely.

Tabel XXXIV includes a summary of net worth figures generated from 15 replications of a 20-year simulation of the quantity limitation. Mean, standard deviation, maximum, minimum and range of net worth are shown for each year. Mean net worth increases from \$120,575 at the end of year 1 to \$142,714 in year 11. Thereafter, net worth decreases steadily to \$115,617 in year 20. It should be noted that ending mean net worth in year 20 is less than mean net worth after year 1 of the simulation sequence. If farm managers operating in the poor water resource situation react to the quantity limitation in the manner assumed in this model, indications are that depletion of the water supply coupled with gradual conversion toward dryland farming in years 11 through 20 results in absolute reductions in net worth within a 20-year period.

Standard deviation of net worth increases steadily over the 20-year simulation period. The transition is from a mean and standard deviation of \$120,575 and \$3,825, respectively, in year 1 to a mean and standard deviation of \$115,617 and \$61,094, respectively, in year 20. In terms of relative variability, this transition corresponds to an increase in the coefficient of variation from 0.03 to 0.54. The maximum and minimum values of net worth generated by the General Agricultural Firm Simulator occur in years 19 and 20, respectively. Maximum net worth equals $206,441 and minimum net worth equals $2,198. It might be argued that the rational farm operator would quit farming before reducing net worth to such a low level. The overall implications of simulating a quantity restriction on pumping by individual firms appear clear. Over time profitability and net worth of the firm increase until declining water supplies and rising water costs force the conversion toward dryland farming. From that point on, profitability and net worth decline. It is not unrealistic for net worth at the end of 20 years to be less than it was at the beginning of the period. It is likely that ending net worth is significantly lower than for the irrigator who is not restricted in his pumping over time.

Effects of a Graduated Tax on Resource Situation 1

The third institutional alternative considered is the imposition of a per unit tax on each acre inch of water pumped above the quantity limitation. The irrigator is assumed to follow the same set of decision rules as specified for irrigators facing a quantity restriction, with one exception. The irrigator is allowed to pump as many acre inches above the limitation as he desires so long as he pays a graduated tax of $0.50 for each acre inch pumped above the limit. An economic decision rule is followed by irrigators in deciding whether or not to apply water above the limit. The irrigator evaluates the potential yield reduction which will occur, projecting present moisture conditions, if he does not irrigate. The value of the potential loss for a given crop block is compared with the cost of an additional irrigation, plus added harvesting and hauling costs. If the value of potential yield reduction exceeds the cost of an additional irrigation, the application is made. The decision rules followed are discussed above.

Effects on Acre Inches Pumped

Table XXXV summarizes the effects of a graduated tax per unit above the quantity limit on total acre inches pumped during 15 replicatons of each of 20 crop years. The mean, standard deviation, maximum, minimum and range of acre inches pumped have been computed for each year of the simulation analysis.

Mean values of total acre inches pumped per year reflect the expansion and development of irrigation facilities on the farm firm representing Resource Situation 1. That is, the highest number of acre inches pumped occurs during year 3, reflecting the excess pumping capacity created by addition of a second irrigation well. Mean acre inches pumped fluctuates between 6,040 and 6,144 acre inches to year 7 and then declines until the addition of well 3, which usually occurs at the end of crop year 10. The addition of well 3 results in a pumping increase during year 11. From year 12 through year 20, mean acre inches pumped declines steadily, reaching 1,447 acre inches during year 20.

Simulation of the graduated tax results in complete conversion to dryland production during five of the fifteen replications of the 20-year simulation run. In four of the five replications the final transition comes in year 20. In one replication both years 19 and 20 are simulated with complete dryland production. This pattern of conversion to dryland production exhibits the same timing characteristics as exemplified in the

TABLE XXXV

SUMMARY OF TOTAL ACRE INCHES PUMPED, NET FARM INCOME AND NET WORTH FOR RESOURCE SITUATION 1 WITH A GRADUATED TAX PER UNIT PUMPED ABOVE THE QUANTITY LIMIT*

	Year																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Total Acre Inches Pumped																				
Mean	5549	5429	6371	6045	6144	6040	6115	5878	5514	5580	5954	5659	4836	4266	3533	3274	2854	2508	2483	1447
Std. Dev.	929	451	500	1408	666	463	223	397	472	639	488	573	717	741	497	673	486	522	1113	1085
Maximum	6072	5842	7118	7216	7109	6732	6426	6312	6111	6604	6488	6373	5652	5300	4364	4263	3790	3271	5705	2529
Minimum	2722	4226	5500	1799	4530	5094	5519	5085	4396	4390	4572	4481	3315	2867	2791	1894	2066	1087	0	0
Range	3350	1616	1618	5417	2579	1638	907	1227	1715	2214	1916	1892	2337	2433	1573	2369	1724	2184	5705	2529
Net Farm Income																				
Mean	9473	10461	13595	14042	14966	15346	14333	12995	12842	13368	10477	8018	8865	8288	9270	8065	9956	6944	3867	5056
Std. Dev.	4499	5327	3918	4953	4436	6305	4891	4553	5317	5902	5910	4583	7117	5038	4548	4257	2491	5799	5695	5667
Maximum	16394	23014	21994	23259	24074	25073	23766	20683	22930	23625	21125	16670	23583	16016	20608	12898	17195	14356	11529	17229
Minimum	3493	3246	7280	7179	8610	4556	3115	6225	4105	5560	510	1489	-2778	1463	3036	-275	4764	-6463	-4418	-3920
Range	12901	19768	14714	16080	15464	20517	20651	14448	18825	18065	20615	15181	26361	14553	17572	13173	12431	20819	15942	21149
Coef.of Var.	0.47	0.51	0.29	0.35	0.30	0.41	0.34	0.35	0.41	0.44	0.56	0.57	0.80	0.61	0.49	0.53	0.43	0.84	1.47	1.12
Net Worth																				
Mean	121150	122833	127009	131559	136827	142331	147196	150995	154741	160349	163903	164060	164886	164234	166641	166762	168633	167714	164086	161676
Std. Dev.	3607	5653	7536	9847	10268	13848	16486	18075	18847	20570	24426	27310	29337	34823	33874	36645	38763	41459	44808	48017
Maximum	126622	136305	144574	156630	161736	169614	172764	176163	179803	187048	189066	193850	206338	207107	214434	218592	224886	228764	233555	235767
Minimum	116287	114135	116647	118636	125491	123708	121722	120623	122730	125214	118202	113229	112449	106347	106036	98261	97124	89890	77978	66865
Range	10335	22170	27927	37994	36245	45906	51042	55540	57073	61834	70864	80621	93889	101360	108398	120331	127762	138874	155577	268902

*The values in this table are based on 15 replications. The values for each replication are reported by Mapp [54, pp. 150,153, 155].

unrestricted simulation analysis. The quantity of water pumped under taxation is less than under unrestricted pumping, however, the addition of a per unit tax on each unit above the quantity limitation results in a similar timing of conversion to dryland production.

The maximum number of acre inches pumped during any replication is 7,216 during year 4. The minimum, of course, is zero and occurred during both years 19 and 20. The maximum range within a single year of 5,417 acre inches occurs during year 4, when a maximum of 7,216 acre inches and a minimum of 1,799 acre inches are pumped.

The range in remaining saturated thickness at the end of the 20-year simulation period is from 34.67 to 40.97 feet, averaging 37.72 feet. Translating this into feet decline in saturated thickness results in an average foot-decline of 62.28 feet over the 20-year period, or an average of 3.11 acre feet per year. Of the total volume of water underlying the representative farm, assuming a beginning saturated thickness of 100 feet, only about 38 percent remains at the end of 20 years under the graduated tax alternative.

Effect on Net Farm Income

The effects on net farm income of a graduated tax on each acre inch of irrigation water pumped above the quantity limitation also are illustrated in Table XXXV. The mean, standard deviation, maximum, minimum and range of net farm income have been computed for each year of the 20-year simulation run.

Mean values of net farm income increase steadily from $9,473 in year 1 to $15,346 in year 6. This dramatic rise may be attributed to several interrelated factors. First, expansion of irrigation facilities by the addition of well 2 increases pumping capacity significantly. Second, the additional pumping capacity insures proper timing for the very profitable irrigations of grain sorghum and wheat in irrigation periods 4 and 5. Higher wheat and grain sorghum yields lead not only to increased net returns per acre, but to higher government payments for the farm operator. Mean net farm declines during years 7, 8 and 9, but increases to $13,368 in year 10 with the addition of irrigation well 3. Thereafter, mean net farm income declines steadily except for individual yearly increases due to favorable soil moisture and atmospheric stress conditions in years 15 and 17.

The maximum value of net farm income generated in any year is $25,073 in year 6. The minimum of -$6,463 occurred in year 18. The greatest range occurs during year 13 with the difference being $26,361.

Variability, as measured by the standard deviation, does not follow a definite trend. Generally, it rises when mean net farm income rises and declines as net farm income declines, remaining between 0.29 and 0.61 for the initial seventeen years. Coefficients of variation for years 18, 19 and 20 are 0.84, 1.47 and 1.12, respectively. Stability of net farm income is greater under the graduated tax than under either the unrestricted or quantity restriction alternatives.

Effects on Net Worth

Table XXXV summarizes the effects on net worth for representative firms in Resource Situation 1 of a graduated tax on each acre inch of water pumped above the quantity limitation. The mean, standard deviation, maximum, minimum and range of net worth have been computed across the 15 replications of each year of the simulation run.

Net worth of the representative farm firm increases steadily from year 1 through year 13, dips slightly in year 14 and increases during years 15, 16 and 17, before declining in years 18, 19 and 20. The maximum mean value of $168,633 occurs in year 17. Variability of net worth increases steadily also from 0.03 in 1 to 0.30 in year 20. Maximum and minimum individual values of net worth both occur during year 20. The maximum net worth of $235,767 is generated during replication 5, while the minimum value of net worth of $66,865 is generated in replication 6. Mean value of ending net worth in year 20 is $161,676.

Statistical Comparisons of Unrestricted Pumping, a Quantity Limitation and a Graduated Tax on Resource Situation 1

This section compares the three methods of water-use regulation graphically and statistically, relating the different effects each has on water use; remaining saturated thickness; net farm income; income variability, as measured by the coefficient of variation; and net worth at the end of the 20-year simulation period. Tests are conducted to determine whether mean values of the relevant variables over the 20-year period differ significantly. Implications are drawn regarding differences in results of the three alternatives and their effects on the firm and the region.

Acre Inches Pumped

Figure 9 illustrates the effect on each water-use alternative on mean acre inches pumped through time. From year 1 through year 10, mean values of total acre inches pumped under unrestricted pumping exceed acre inches pumped under the quantity limitation and graduated tax alternatives. During

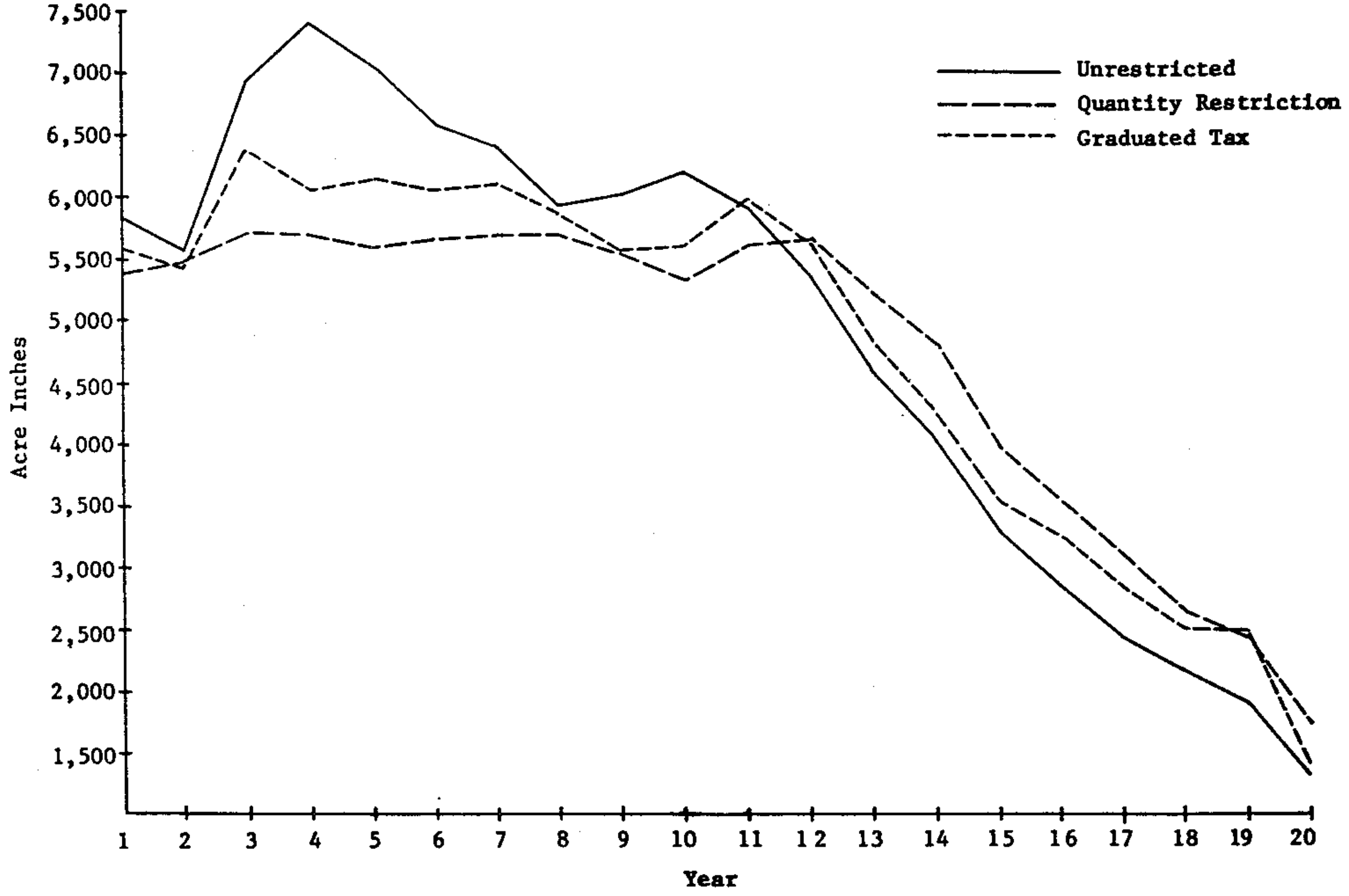

Figure 9. A Comparison of Mean Acre Inches of Irrigation Water Pumped Under Alternative Water-Use Regulation Methods for Resource Situation 1

the same period, the irrigator paying a graduated tax for each acre inch above the quantity limit finds it profitable to pump water in excess of the quantity limitation every year except one. This exception occurred during year 2 when pumping capacity is limited. Irrigation well 3 is usually added by year 10 under the unrestricted alternative; by year 11 under the graduated tax alternative; and by year 12 under the quantity limitation. The lag which develops reflects the different rates of pumping under each alternative in early years of the simulated time period. High early period pumping rates under the unrestricted alternative lead to lower system capacities and earlier additions of well 3. Lower pumping rates under the quantity limitation result in a slower decline in system pumping capacity and thus a lag in the requirement for well 3 until about year 12.

From year 12 to year 20, there is a complete change in the pattern of total acre inches pumped under the three water-use alternatives. Excessive pumping in early periods under the unrestricted alternative reduces irrigation system capacity to such an extent that the lowest mean total acre inches pumped from years 12 through 20 is by the unrestricted irrigator. The second largest rate of water use during the same period occurs under the graduated tax alternative. The largest rate of water use during the period occurs under the quantity limitation simply because the pumping capacity under this alternative is not depleted as rapidly in earlier years of the simulated time period as for the other two alternatives.

All three methods of water-use regulation result in approximately the same mean number of acre inches pumped during year 20. In addition, the feet of saturated thickness remaining at the end of year 20 are 35.84, 38.37 and 37.72 for unrestricted, quantity limitation and graduated tax alternatives, respectively. Thus, though the patterns of water use exhibit considerable variation, particularly during years 1 through 12, the feet of saturated thickness remaining at the end of 20 years is approximately the same for all three alternatives.

Policy makers might ask whether the mean acre inches pumped over the 20-year period under alternative methods of water-use regulation differ significantly. This question can best be answered by testing the difference in means for statistical significance, rather than by making subjective evaluation based on the graphs in Figure 9. The Wilcoxon Matched-Pairs, Signed Ranks Test is a powerful nonparametric test that may be used to test whether two related groups differ significantly [94, pp. 75-83]. A detailed discussion of the Wilcoxon Matched-Pairs, Signed Ranks Test is included in [54, pp. 275-277].

Statistical tests between each set of mean values of total acre inches pumped under the three institutional alternatives reveal no significant differences among any of the distributions at the five percent level. Thus, even though Figure 9 indicates a seemingly large difference in acre inches pumped from year 3 through 7 under the unrestricted and quantity limitation alternatives, the means are not significantly different, from a statistical standpoint.

Since timeliness of application in relation to critical stages of plant development is more important to final yield and net returns than is the total number of acre inches applied, the possibility of significant differences among net farm income and net worth means still exists.

Net Farm Income

Mean values of net farm income over the 20-year period under unrestricted, quantity restriction and graduated tax alternatives are presented graphically in Figure 10. Several outstanding features merit attention. By far the most important is that net farm income under the graduated tax alternative exceeds net farm income under the unrestricted pumping alternative during every year except year 5. From year 1 through year 5, net farm income under both alternatives increases and the level of net farm income is approximately the same for both. Beginning with year 6, net farm income under the graduated tax alternative exceeds net farm income under unrestricted pumping by a wider margin. Several interrelated factors create this phenomenon. First, the unrestricted irrigator tends to operate his irrigation system at its maximum capacity. In responding to soil moisture levels throughout the growing season, the tendency is to apply more water than is profitable. By reducing applications of water during some periods, applying water on grain sorghum during irrigation period 4 only if it is profitable and insuring a preplant irrigation of wheat every year, the irrigator operating under the graduated tax alternative is able to pay the tax and still achieve higher net farm income.

A second factor contributing to higher net farm income under the graduated tax alternative is that less water is pumped during earlier periods thus enabling the taxed irrigator to achieve more timely irrigations in relation to plant needs during later critical periods of development. More timely applications lead to higher final crop yields for the same amount of irrigation water. Since pumping costs rise more slowly, net returns per acre and net farm income are higher. A third related

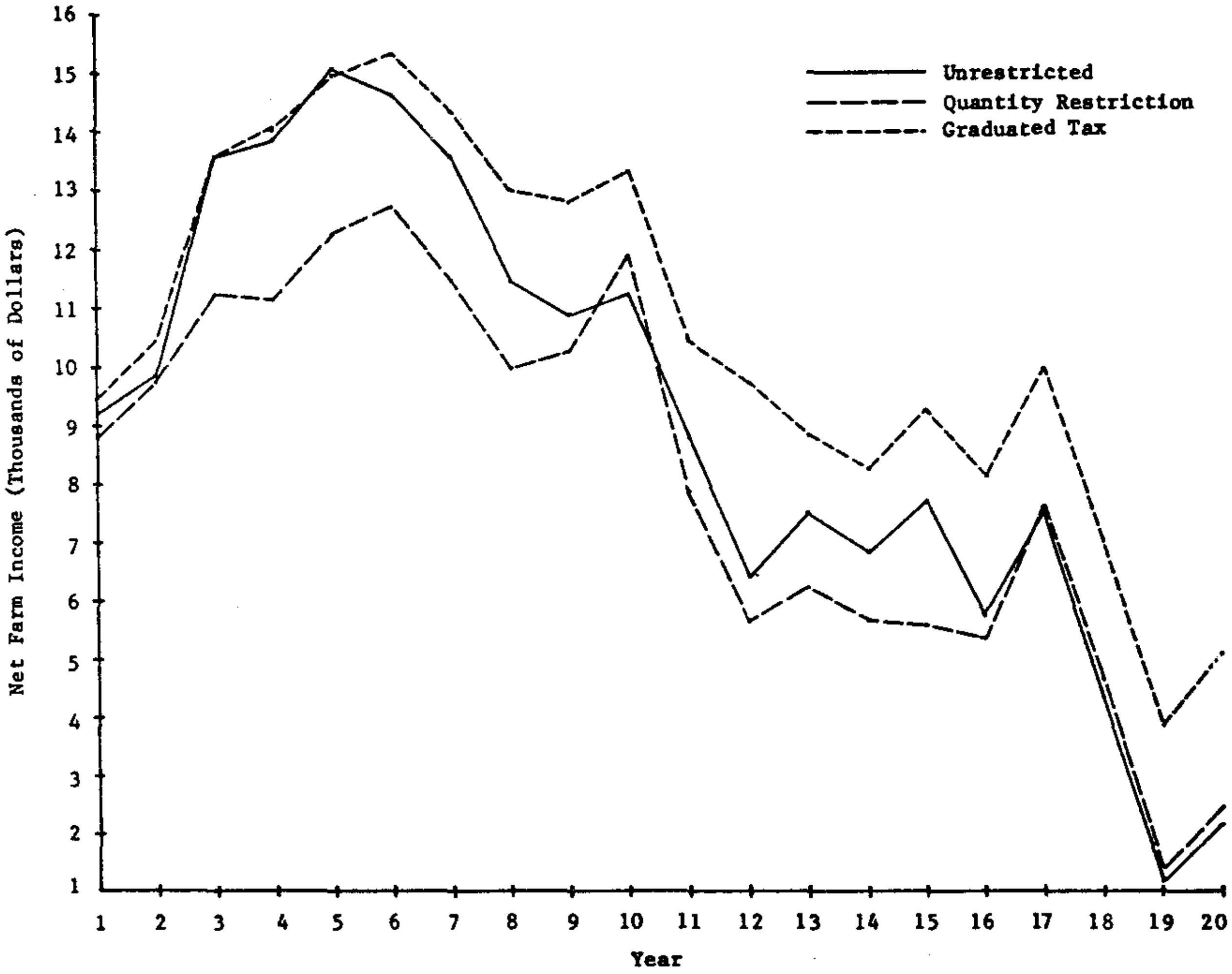

Figure 10. A Comparison of Mean Net Farm Income Under Alternative Water-Use Regulation Methods for Resource Situation 1

factor is that higher yields are reflected in higher government payments, particularly from years 11 through 20, for the irrigator under the graduated tax alternative. Higher government payments contribute directly to higher net farm income.

Net farm income under the quantity restriction is of interest also. It is lower than net farm income under the graduated tax during every year and exceeds net farm income under unrestricted pumping conditions during year 10 and from year 17 through year 20. Net farm income under unrestricted and quantity restriction alternatives are almost identical from year 16 through 20, however, higher remaining pumping capacity enables the quantity restriction alternative to maintain a higher net farm income during this period.

An analysis of Figure 10 suggests that mean net farm income under the graduated tax alternative differs significantly from mean net farm income under a quantity restriction. This hypothesis, among others, was tested through the use of the Wilcoxon Matched-Pairs, Signed Ranks Test. Three tests were conducted on mean values of net farm income. All three allow us to reject the null hypothesis of no difference between the mean values of net farm income at the .01 probability level. Mean net farm income under the graduated tax alternative is above that under either the unrestricted pumping or quantity limitation alternatives. Mean net farm income under unrestricted pumping is above that under the quantity limitation.

Figure 11 illustrates the effects of the three water-use regulatory alternatives on variability of net farm income, as measured by the coefficient of variation. The coefficient of variation resulting from a quantity restriction on water use is consistently higher from year 1 to year 18.

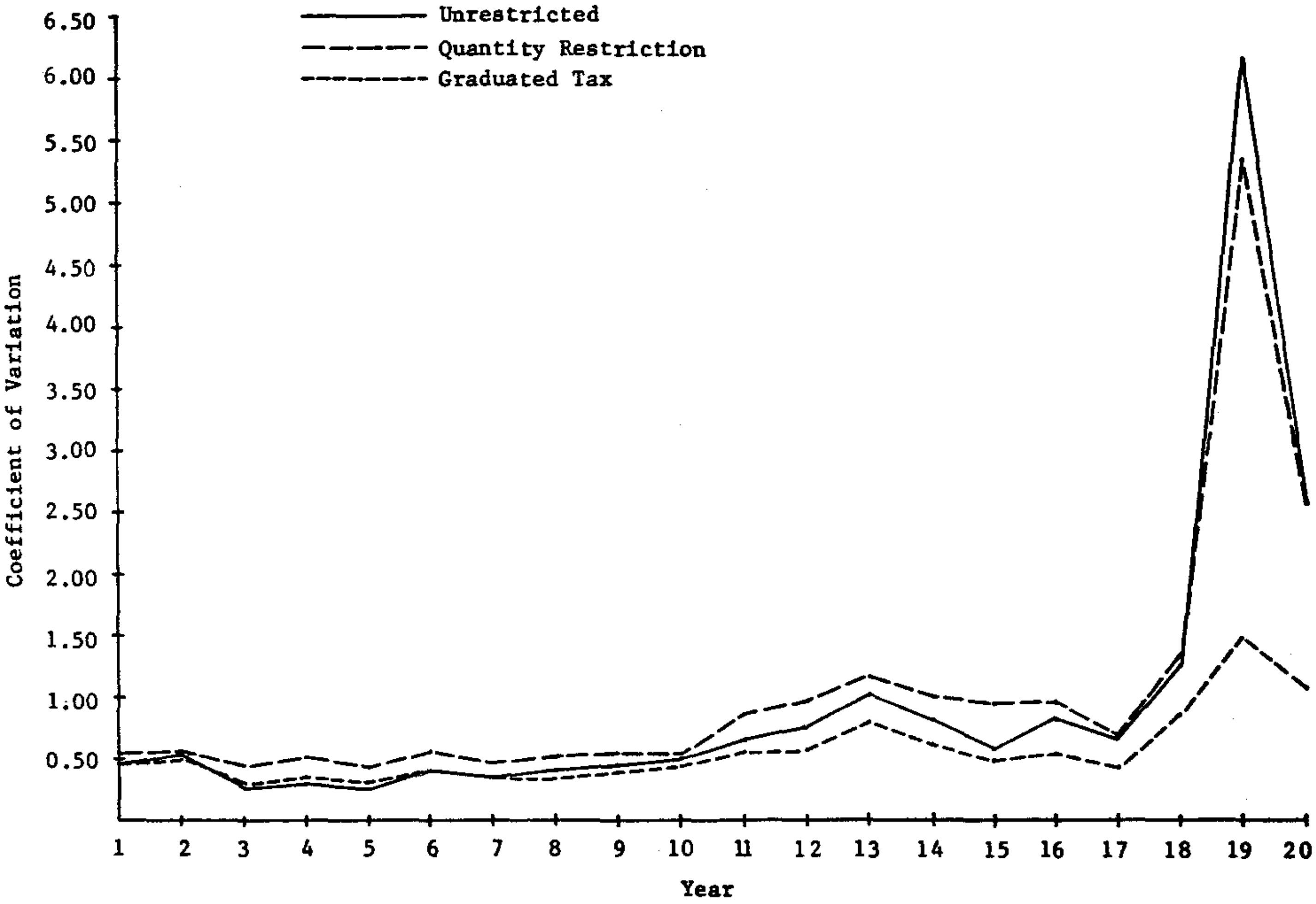

Figure 11. A Comparison of Coefficients of Variation of Net Farm Income Under Alternative Water-Use Regulation Methods for Resource Situation 1

This is not an unexpected result. The quantity restriction is often reached during irrigation period 4 when grain sorghum is in the boot-heading and grain-filling stages of plant development. Failure to apply needed moisture during this period reduces final yield unless natural rainfall is sufficient to compensate for the lack of irrigation water. In addition, when the quantity restriction is reached, preplant irrigations on irrigated wheat are eliminated. The existence of a stand on wheat is then determined by Fall soil moisture conditions. About 20 percent of the time no stand is achieved and wheat yield is assumed to be zero. Both of the above factors combine to increase variability of net farm income relative to mean net farm income under the unrestricted and graduated taxation alternatives.

Coefficients of variation of net farm income under the unrestricted and graduated tax alternatives are approximately the same for the first few years of the simulated time period. Coefficient of variation for unrestricted pumping is larger than that of the graduated tax for year 2, approximately equal during years 6 and 7, and then is larger for years 8 through 20. Thus, after year 7, the coefficient of variation for the graduated tax alternative is lower than for either the unrestricted or quantity restriction alternatives.

The marked increase in coefficients of variation during years 18, 19 and 20 reflects the declining pumping capacity, declining proportion of irrigated acres and increased variability resulting from dryland production. Extreme variability occurring in year 19 relative to years 18 and 20 results from the random occurrence of very dry years across replications of year 19. The reduced variability under the graduated tax alternative results from timely applications of irrigation water during irrigation periods 4 and 5. These applications stabilize wheat and grain sorghum yields, and government payments, thus reducing variability of net farm income.

Net Worth

Mean values of net worth over the 20-year simulated time period under unrestricted, quantity restriction and graduated tax alternatives are presented in Figure 12. Graphs of the three sets of means leave no doubt that net worth under the graduated tax alternative is higher throughout the period. Net worth under the unrestricted alternative is second largest over the 20-year period followed by net worth under the quantity limitation alternative. The differences appear significant, particularly after about year 10. The means were tested for statistical significance using the Wilcoxon Matched-Pairs, Signed Ranks Test. Application of the testing procedure substantiates this intuitive conclusion [54, pp. 169-171].

Effects of Unrestricted Water Use on Resource Situation 2

Resource Situation 2 represents the adequate water situation within the study area. The weighted average saturated thickness of the underground formation is 325 feet. Only about 125 feet of saturated thickness are required to maintain an irrigation system pumping capacity of 1,000 g.p.m. Consequently, irrigation operators represented by Resource Situation 2 may lower the static water level by approximately 200 feet before well yields begin to decline and a significant rise in pumping costs occurs. Thus the well yield remains constant at 1,000 g.p.m. for the 20-year period and no additional wells are required to maintain irrigated production of 315 acres of cropland. No expansions or contractions of

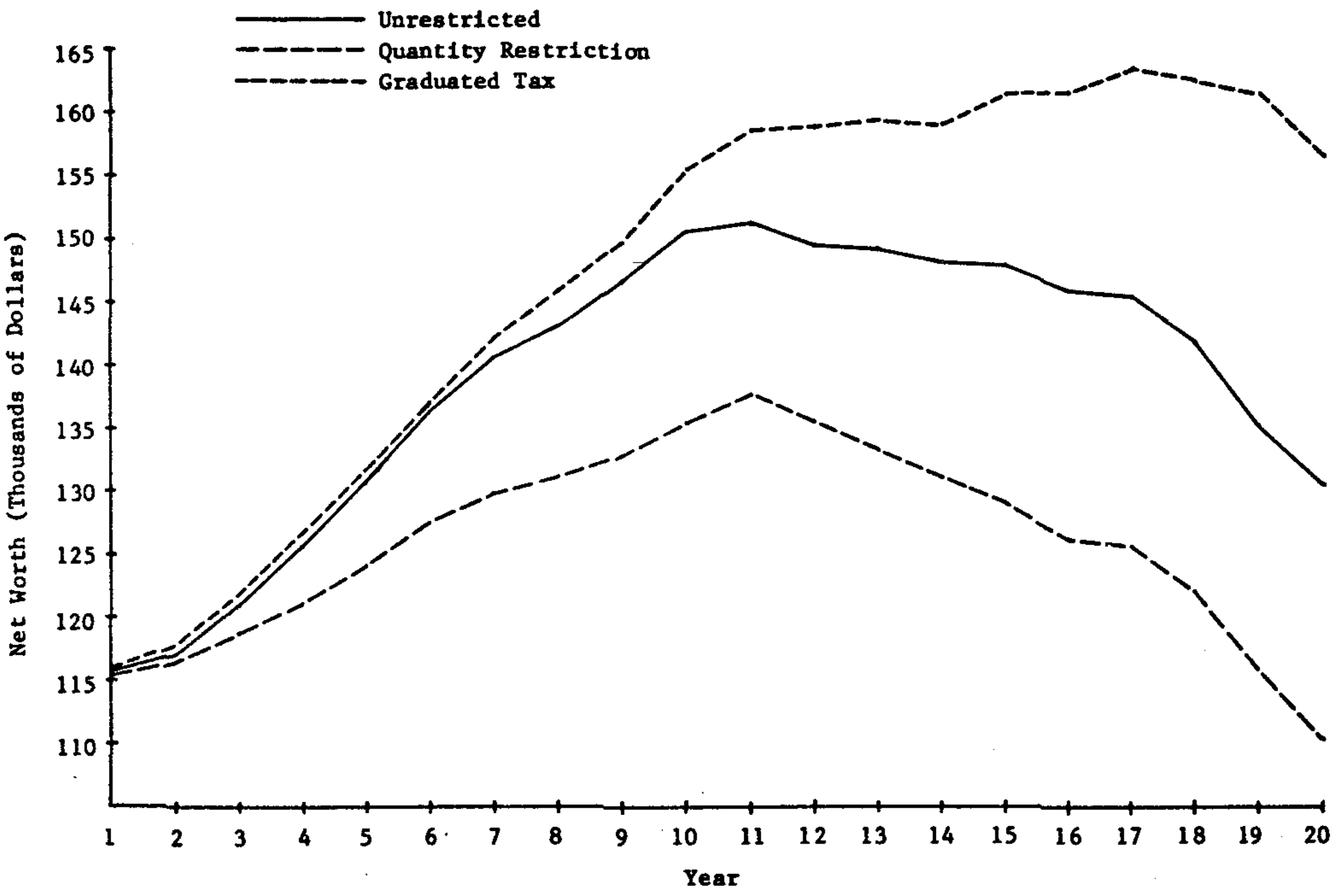

Figure 12. A Comparison of Mean Net Worth Under Alternative Water-Use Regulation Methods for Resource Situation 1

irrigated cropland are assumed for representative farms in Resource Situation 2. Other assumptions are similar to the starting situation for Resource Situation 1.

Effects on Acre Inches Pumped

A summary of total acre inches pumped under the unrestricted water-use alternative is presented in Table XXXVI. The mean, standard deviation, maximum, minimum and range of acre inches pumped are given for each year. Since well capacity remains at 1,000 g.p.m. throughout the 20-year simulated time period, there are no significant changes in system capacity as there were for Resource Situation 1. Variability in quantity of water pumped results from random variation in rainfall and evapotranspiration rather than variations in pumping capacity and number of acres irrigated.

Mean values of total acre inches pumped range from 6,662 in year 10 to 7,233 in year 14. The maximum number of acre inches pumped during any of the simulation runs is 7,925 pumped during year 11, and again during year 18. Minimum quantity of water pumped is 3,007 acre inches in year 1. The greatest range in acre inches pumped is 4,806 in year 1. The considerable variability in total acre inches pumped is one indication of the weather variability existing in the study area and of the ability of the Production Subset to simulate these variable weather conditions.

Saturated thickness at the end of the 20-year period under unrestricted pumping ranges from a minimum of 230.49 feet to a maximum of 240.62 feet, averaging 235.03 feet. In terms of feet of decline in saturated thickness, the mean decline over 15 replications at the end of 20 years is 89.89 feet for an average rate of decline of 4.50 feet per year.

TABLE XXXVI

SUMMARY OF TOTAL ACRE INCHES PUMPED, NET FARM INCOME AND NET WORTH FOR RESOURCE SITUATION 2 WITH NO RESTRICTIONS ON WATER USE*

	Year																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	Total Acre Inches Pumped																			
Mean	6692	6711	6835	6777	6861	6743	7065	7043	6900	6662	6948	7181	6963	7233	6871	7061	6974	6843	6972	6823
Std. Dev.	1249	971	622	910	1134	806	429	739	833	795	866	635	1095	596	916	741	710	1127	846	705
Maximum	7813	7745	7474	7862	7921	7921	7670	7865	7742	7865	7925	7835	7802	7895	7685	7835	7865	7925	7791	7862
Minimum	3007	4297	5602	4770	3911	5325	6142	5878	5051	4740	4950	5681	4005	5947	4567	4791	4860	3352	5130	5227
Range	4806	3448	1872	3092	4010	2596	1528	1987	2691	3125	2975	2154	3797	1948	3118	3044	3005	4573	2661	2635
	Net Farm Income																			
Mean	10598	12434	14413	14767	16754	17192	16421	15353	16601	18563	17420	16172	17506	16974	18548	17794	19644	18908	17364	19293
Std. Dev.	3872	5526	3340	4307	4152	5243	4112	4191	4764	4613	4545	3490	5950	4022	3774	3374	3744	4423	5045	3336
Maximum	16403	24868	21941	22167	26548	26226	24518	23334	25546	26076	26156	22400	31737	23400	27602	22434	27433	26993	24284	25059
Minimum	4330	4443	9930	7454	11030	8516	7454	9988	8612	10998	10232	12213	8665	10124	13451	12118	13455	9660	9324	13491
Range	12073	20425	12011	14713	15518	17710	17064	13346	16934	15078	15924	10187	23072	13276	14151	10316	13978	17333	14960	11568
Coef.of Var.	0.37	0.44	0.23	0.29	0.25	0.30	0.25	0.27	0.29	0.25	0.26	0.22	0.34	0.24	0.20	0.19	0.19	0.23	0.29	0.17
	Net Worth																			
Mean	123260	126540	131612	137033	143829	150914	157618	163414	170317	178998	187035	194557	203181	211904	221762	231107	242128	252870	262853	274723
Std. Dev.	3087	5496	6776	8404	8697	11432	13379	14393	14915	16441	18423	20073	20889	22453	23291	24673	25219	26488	27195	27653
Maximum	127823	139113	146822	158384	164767	172328	178578	183586	190288	199919	206092	215627	230909	238398	251790	263616	276946	290089	305424	318245
Minimum	118215	117570	120917	127959	134297	134381	138461	140193	146150	152712	154236	159291	168613	174833	182991	188138	197833	204098	212925	223685
Range	9608	21543	25905	30425	30500	37947	40117	43393	44138	47207	51856	56336	62296	63565	68799	75478	79113	85991	97499	94560

*The values in this table are based on 15 replications. The values for each replication are reported by Mapp [54, pp. 174, 176, 178].

Effects on Net Farm Income

The effects on net farm income of unrestricted pumping by representative farms in Resource Situation 2 are presented in Table XXXVI. The mean, standard deviation, maximum, minimum, range and coefficient of variation of net farm income are shown by year.

Mean values of net farm income, while fluctuating widely from year to year, have a general upward trend over the 20-year period. The rise is rapid during the first five years as the result of high crop yields per acre and a corresponding rise in government payments. Mean net farm income rises from \$10,598 in year 1 to \$16,754 in year 5. Over the same period, mean values of government payments (wheat certificates plus feed grain payments) rise from \$8,218 to \$13,625. So, of the \$6,156 increase in net farm income, \$5,403 results from an increase in government payments. Government payments, which are computed on the basis of a five-year moving average, stabilize after year 5 and remain in the \$13,200 to \$13,700 range. Mean net farm income continues its upward trend as chattle debts are paid off and the beginning real estate debt is retired. Cash reserves above the \$10,000 minimum specified in the Farm Firm Simulation Model earn interest also. The maximum mean net farm income is \$19,644 in year 17 and mean net farm income in year 20 is \$19,293.

Variability of net farm income fails to follow a definite pattern over the 20-year simulated time period. Relative variability, as measured by the coefficient of variation, ranges from a high of 0.44 during year 2 to a low of 0.17 during year 20. In general, the coefficient of variation is low, and is expected to be lower in this unrestricted simulation than for either the graduated tax or quantity limitation alternatives.

The maximum yearly value of net farm income is $31,737 generated in year 13. The minimum value of net farm income is $4,330 generated in year 1. The greatest range in net farm income levels for a single year occurs during year 13 when $23,072 is the difference between a maximum of $31,737 and a minimum of $8,665. Although variability from year to year is significant, the unrestricted pumping alternative under adequate water conditions leads to relatively stable, increasing net farm income over time.

Effects on Net Worth

Table XXXVI also presents the effects on net worth of unrestricted pumping for representative farms in Resource Situation 2 based on 15 replications of a 20-year simulation of the firm. The mean, standard deviation, maximum, minimum and range in net worth are given for each year of the simulation run.

Mean values of net worth increase steadily from year 1 through year 20 of the simulated time period. The minimum mean net worth is $123,260 in year 1. Maximum mean net worth is the ending net worth of $274,723. Ending net worth has a range of $94,560. This figure is the difference between the maximum ending net worth of $318,245 in replication 5 and the minimum net worth of $223,685 in replication 2. Two factors contribute to rising net worth over the 20-year period. The first is gradual retirement of chattle and real estate debt, which reduces liabilities. The second is gradual accumulation of cash assets.

Effects of a Quantity Restriction on Resource Situation 2

The quantity restriction limits the individual irrigator to pumping 1.5 acre feet per acre of water rights. For the representative farm firm with 315 irrigated acres, the limitation is 5,670 acre inches per crop year, the same quantity limit used in Resource Situation 1. It is assumed the irrigator, rather than pump water with abandon in every critical irrigation period, also follows the same decision rules regarding use of his limited water supply as the irrigator in Resource Situation 1.

Effects on Acre Inches Pumped

The effect of a quantity restriction on acre inches pumped per crop year is reflected in Table XXXVII. The table presents the mean, standard deviation, maximum, minimum, and range of total acre inches pumped per year over the 20-year simulated time period.

Mean values showed little variability, as expected, ranging from a minimum of 5,472 acre inches in year 1 to a maximum of 5,699 acre inches in year 7. Individual yearly observations show considerably more variation. The maximum number of acre inches pumped during any year is 5,730 in year 1. The minimum number of acre inches pumped, 3,008, also occurred during year 1, resulting in a maximum range of 2,722 acre inches during year 1.

Saturated thickness remaining at the end of the 20-year simulation runs varies from a minimum of 250.82 feet to a maximum of 254.26 feet. Mean saturated thickness after 20 years under the quantity restriction is 251.81 feet. Assuming a beginning saturated thickness of 325 feet, this represents an average decline in saturated thickness of 73.19 feet or

TABLE XXXVII

SUMMARY OF TOTAL ACRE INCHES PUMPED, NET FARM INCOME AND NET WORTH FOR RESOURCE SITUATION 2 WITH A QUANTITY RESTRICTION ON WATER USE*

	Year																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
										Total Acre Inches Pumped										
Mean	5472	5560	5679	5599	5636	5643	5699	5696	5642	5597	5638	5691	5537	5673	5590	5627	5637	5545	5659	5665
Std. Dev.	697	397	54	267	536	154	17	19	166	309	192	15	467	52	293	232	216	607	147	141
Maximum	5730	5722	5722	5722	6639	5722	5722	5722	5722	5722	5722	5722	5716	5716	5692	5715	5722	5722	5723	5722
Minimum	3008	4297	5490	4770	3911	5130	5673	5672	5051	4545	4950	5677	4005	5490	4567	4791	4860	3352	5230	5160
Range	2722	1425	232	952	2738	592	49	50	671	1177	772	45	1711	226	1125	924	462	2370	593	562
										Net Farm Income										
Mean	9576	10791	12367	12200	13440	13787	12984	11561	12885	15079	13497	12311	13352	12874	14451	13427	15762	14816	13429	15632
Std. Dev.	4528	6180	4362	5303	5094	6768	5299	5558	5439	5879	6316	4536	6467	5293	4614	4608	4347	5963	6252	3761
Maximum	16468	24380	21365	22255	24923	24587	23574	20881	23940	25220	25314	20194	26891	20889	23891	20475	24905	25061	22218	22632
Minimum	2950	2056	5955	4676	6450	2323	952	3046	5108	6979	4019	7922	2797	4388	8898	6207	8768	757	3955	10572
Range	13518	22324	15410	17579	18473	22264	22622	17835	18832	18241	21295	12272	24094	16501	14993	14268	16137	24304	18263	12060
Coef.of Var.	0.47	0.57	0.35	0.43	0.38	0.49	0.41	0.48	0.42	0.39	0.47	0.37	0.48	0.41	0.32	0.34	0.28	0.40	0.47	0.24
										Net Worth										
Mean	122422	124338	127707	130959	135085	139384	143197	145803	149548	155118	159566	163292	167938	172412	178031	184999	189558	195606	200897	208230
Std. Dev.	3645	6531	8685	11109	12248	16215	19241	21047	21922	24186	26861	29457	30820	33197	34851	37023	38707	41322	43073	44598
Maximum	127874	139167	146925	158560	163443	171039	173098	173151	177219	184875	186774	193090	206940	210573	222820	231064	240426	250272	263798	273540
Minimum	116961	113611	114148	116986	121293	116013	114903	111556	114290	114120	112121	111747	115898	115480	119606	117778	123374	121680	120504	125291
Range	10913	25556	32677	41574	42150	55026	58195	61595	62929	70755	74653	81343	91042	95093	103214	113286	117052	128592	143294	148249

*The values in this table are based on 15 replications. The values for each replication are reported by Mapp [54, pp. 181, 183, 185].

3.66 feet per year. This rate of decline under the quantity restriction compares to the 4.50 feet per year decline for the unrestricted pumping alternative. The implications of various water-use rates for different regulatory alternatives is discussed in detail in a subsequent section.

Effects on Net Farm Income

The mean, standard deviation, coefficient of variation, maximum, minimum and range of net farm income were computed for each crop year and are also shown in Table XXXVII. Net farm income under quantity restriction follows essentially the same pattern as under the unrestricted water-use alternative except that the level of income is considerably lower under the quantity restriction. Mean values of net farm income increase from the minimum level of $9,576 for year 1 to $15,632 in year 20, however, the highest mean net farm income is $15,762 in year 17. A major proportion of the increase results during the first five years and is attributable to increased yields leading to increased government payments. From year 1 to year 5, net farm income increases from $9,576 to $13,440, or by $3,864. During the same period, government payments, composed of wheat certificate and feed grain payments, increase from $7,610 to $11,406, or $3,796. After year 5, total government payments, which are computed on the basis of five-year moving averages for the individual crops concerned, stabilize in the $10,700 to $11,500 range. Net farm income continues to rise, in general, but with considerable variability.

Relative variability, of net farm income, as measured by the coefficient of variation, fluctuates from year to year. The maximum value is 0.57 in year 2 and the minimum value is 0.24 in year 20. Variability of net farm income is related to yield variability. The quantity restriction results in failure to fully irrigate grain sorghum during boot-heading and grain-filling stages of crop development and failure to preplant irrigate all irrigated wheat acreages. During years in which full irrigation applications cannot be completed, final crop yield is more dependent upon highly variable natural rainfall. Thus, restricting the quantity pumped to 5,670 acre inches per year reduces crop yield, increases yield variability and, as a result, increases variability of net farm income.

Effects on Net Worth

The final portion of Table XXXVII summarizes the effects of a quantity restriction on net worth for representative farms in Resource Situation 2. The mean, standard deviation, maximum, minimum and range of net worth for each year are given.

Net worth increases continuously from year 1 through year 20. Beginning net worth at the end of year 1 is $122,422. Ending net worth is $208,230. Between the two points, mean values of net worth increase approximately linearly. The maximum value of net worth generated during any simulated year ($273,540) occurs as expected, during year 20. The minimum net worth value for any year ($113,611) is generated in year 2.

Effects of a Graduated Tax on Resource Situation 2

The third water-use regulatory alternative considered is the imposition of a graduated tax on each unit of irrigation water pumped above the quantity limitation. The irrigator is allowed to pump as much water as he desires, however, a tax of $0.50 per acre inch is charged for each acre inch pumped above the 5,670 acre inch limit. The decision rules followed in allocating water during the growing season are the same as those used under Resource Situation 1.

Effects on Acre Inches Pumped

Table XXXVIII presents a summary of total acre inches pumped under the graduated tax alternative for 15 replications of a 20-year simulation of Resource Situation 2. The mean, standard deviation, maximum, minimum and range of acre inches pumped are shown for each of the 20 years.

Mean values of total acre inches pumped range from a low of 5,875 in year 1 to a high of 6,274 in year 12. Fluctuations between these extremes follow no definite pattern. Variation in acre inches pumped per year exceed that of the quantity restriction, but are not as great as under unrestricted pumping. The maximum number of acre inches pumped is 6,795 and occurs during three different years--years 1, 12 and 19. The minimum number of acre inches pumped is 2,722 in year 1, thus the maximum range in acre inches pumped also occurs in year 1.

Saturated thickness at the end of the 20-year simulation runs ranges from 242.88 to 249.19 feet, averaging 245.61 feet. Assuming a beginning saturated thickness of 325 feet, the average decline in saturated thickness is 79.39 feet, or about 3.97 feet per year. This rate of decline compares with 4.50 feet per year for the unrestricted alternative and 3.66 feet per year for the quantity limitation alternative.

Effects on Net Farm Income

The middle section of Table XXXVIII presents the mean, standard deviation, maximum, minimum, range and coefficient of variation of net farm income under the graduated tax alternative for Resource Situation 2.

TABLE XXXVIII

SUMMARY OF TOTAL ACRE INCHES PUMPED, NET FARM INCOME AND NET WORTH FOR RESOURCE SITUATION 2 WITH A GRADUATED TAX PER UNIT PUMPED ABOVE THE QUANTITY LIMIT*

	Year																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
									Total Acre Inches Pumped											
Mean	5875	6010	6035	6070	5931	6000	6249	6157	6107	5960	6131	6274	6173	6209	6073	6209	6161	6032	6099	6130
Std. Dev.	1046	668	391	651	696	488	225	458	576	645	451	343	765	460	559	436	410	806	511	416
Maximum	6795	6750	6495	6780	6735	6735	6660	6735	6735	6570	6735	6795	6735	6735	6645	6645	6645	6645	6795	6645
Minimum	2722	4297	5265	4695	3911	5130	5850	5402	4699	4320	4950	5535	3915	5310	4477	4791	4860	3352	4950	5160
Range	4073	2453	1230	2085	2824	1605	810	1333	2036	2250	1785	1260	2860	1425	2168	1854	1785	3293	1845	1485
										Net Farm Income										
Mean	10866	12380	14314	14604	16383	16790	61651	14990	16298	18456	16871	15739	16798	16501	18036	17216	19572	18631	16921	19020
Std. Dev.	4294	5722	3917	4933	4557	5966	4582	4761	5225	4984	5215	3985	6265	4698	4269	3871	3945	4923	5827	3730
Maximum	17467	24866	22849	23613	26549	26348	24667	23944	26176	26617	25974	22849	31541	23656	27520	22035	26908	26596	24582	24621
Minimum	4629	4428	8132	7479	10173	6705	5236	8493	8161	11051	9148	11093	7631	7678	11608	10126	12841	7567	7869	12329
Range	12838	20438	14717	16134	16376	19643	19431	15451	18015	15566	16826	11756	23910	15978	15912	11909	14067	19029	16713	12292
Coef.of Var.	0.40	0.46	0.27	0.34	0.28	0.36	0.28	0.32	0.32	0.27	0.31	0.25	0.37	0.28	0.24	0.22	0.20	0.26	0.34	0.20
										Net Worth										
Mean	123468	126713	131705	136975	143491	150247	156733	162247	168920	177506	185138	191906	200203	208430	217774	226535	237395	247674	257181	268714
Std. Dev.	3416	5913	7635	9573	10055	13322	15712	16875	17470	19297	21553	24467	24636	36286	27381	28991	29756	31502	32494	33184
Maximum	128627	139948	148184	160773	166971	175788	180248	184081	190531	200388	205715	217482	233771	241188	255038	266398	280915	294494	310292	323366
Minimum	118464	116832	119772	124094	132114	131737	132641	133200	138895	144216	144810	148885	157447	162519	171583	176140	186929	186838	195235	205603
Range	10163	23116	23412	36679	34857	44051	47607	50881	51636	56172	60905	68597	76324	78669	83455	90258	93986	107657	114967	117763

*The values in this table are based on 15 replications. The values for each replication are reported by Mapp [54, pp. 188, 190, 192].

Mean values of net farm income under the graduated tax alternative increase generally over the 20-year period, though not without yearly fluctuations. The lowest mean net farm income is \$10,866 in year 1 and the highest is \$19,572 in year 17. Mean net farm income in year 20 is \$19,020. A rapid rise in mean net farm income occurs from year 1 (\$10,866) to year 6 (\$16,790), largely because of a rapid increase in government payments (from \$8,217 in year 1 to \$13,296 in year 5). Government payments are relatively stable (between \$12,900 and \$13,300 per year) after year 5, but the mean values of net farm income continue to rise. Relative variability, as measured by the coefficients of variation, is greatest in years 1 and 2 (0.40 and 0.46, respectively) and declines as a larger portion of net farm income is received from government payments. It remains in the 0.20 to 0.37 range after year 5.

The maximum value of net farm income generated is \$31,541 in year 13, while the minimum value is \$4,428 in year 2. The maximum range in net farm income, \$23,910, occurs in year 13.

Effect on Net Worth

The lower portion of Table XXXVIII presents a summary of net worth resulting from 15 replications of a 20-year simulation of Resource Situation 2 under the graduated tax alternative. Mean values of net worth increase steadily from \$123,468 in year 1 to \$268,714 in year 20. The increase is approximately linear. The combination of increased government payments during the initial five years, retirement of chattle and real estate debts over the next ten years and accumulation of excess cash reserves above \$10,000 combine to increase net worth at a relatively constant rate over time.

The maximum value of net worth generated is $323,366 in year 20. The minimum value of $116,832 occurs in year 2. The maximum range in net worth (of $117,763) occurs in year 20.

Statistical Comparisons of Unrestricted Pumping, a Quantity Limitation and a Graduated Tax on Resource Situation 2

Acre Inches Pumped

Figure 13 illustrates the effect on total acre inches pumped for each water-use regulatory alternative. Several features are obvious at first glance. First, the number of acre inches pumped under the unrestricted alternative exceed total acre inches pumped under the graduated tax alternative by a wide margin. Second, acre inches pumped under the graduated tax alternative likewise exceed acre inches pumped under the quantity restriction by a wide margin. Third, there is considerably more variability associated with the unrestricted alternative. Of the three alternatives, the quantity restriction has the smallest variation in total acre inches pumped, as expected.

Of critical importance to policy makers is whether the three water-use regulatory alternatives differ with respect to total acre inches pumped from a statistical standpoint. To answer this question, mean values of total acre inches pumped over the 20-year period are tested for significant differences using the Wilcoxon Matched-Pairs, Signed Ranks Test.

A detailed discussion of the hypothesis tested in each case, the critical level and the computed value of the statistic is reported by Mapp [54, pp. 195-196]. The tests reveal a significant difference between mean values of acre inches pumped for the unrestricted pumping versus

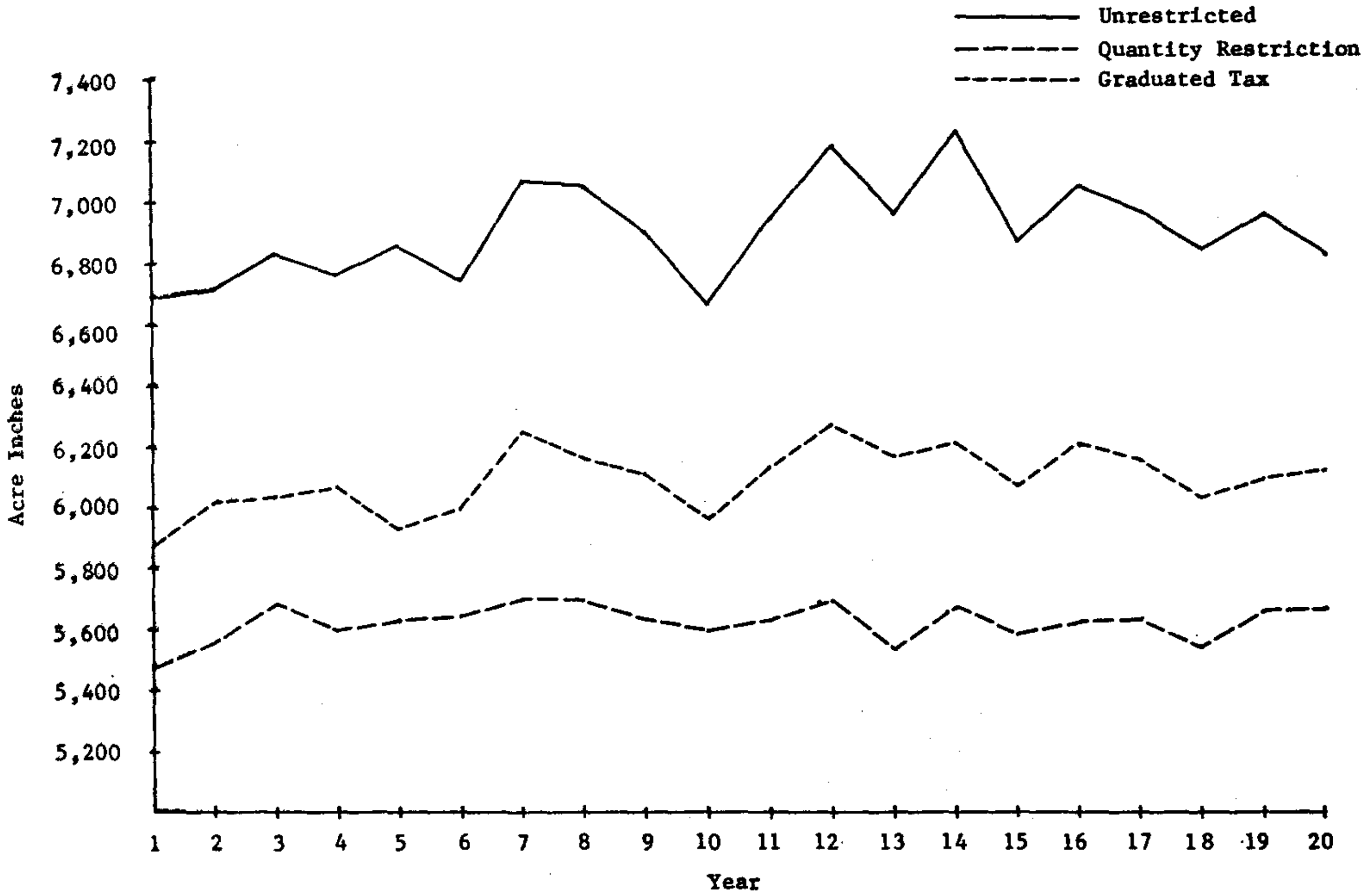

Figure 13. A Comparison of Mean Acre Inches of Irrigation Water Pumped Under Alternative Water-Use Regulation Methods for Resource Situation 2

quantity limitation alternatives, unrestricted pumping versus graduated tax alternatives and graduated tax versus quantity limitation alternatives. Referring to Figure 13, statistical tests reveal that each set of means of total acre inches pumped is above the set or sets of means underlying it.

Net Farm Income

A graphic presentation of mean net farm income over a 20-year period under unrestricted, quantity restriction and graduated taxation alternatives appears in Figure 14. The graph illustrates the effect on net farm income of increased yields and increasing government payments over the initial five years of the simulated time period. From year 5 through year 20, the increase in net farm income is moderate, reflecting gradual retirement of chattle and real estate debts and accumulation of cash in excess of the $10,000 minimum specified at the beginning of the simulation analysis.

The level of farm income under the graduated tax alternative is only slightly less than under unrestricted pumping. Both unrestricted pumping and the graduated tax alternative have levels of net farm income which greatly exceed the level under the quantity restriction. Based on the graphic analysis, three statistical tests are conducted to test three hypotheses. The first test conducted is to determine whether or not significant differences exist between mean net farm income under unrestricted pumping and the quantity restriction. The second test conducted is to determine whether or not a significant difference exists between mean net farm income under the graduated tax alternative and a quantity restriction on pumping. The final statistical test concerning net farm income tested the null hypothesis of no difference between the mean under unrestricted pumping and the mean under graduated taxation.

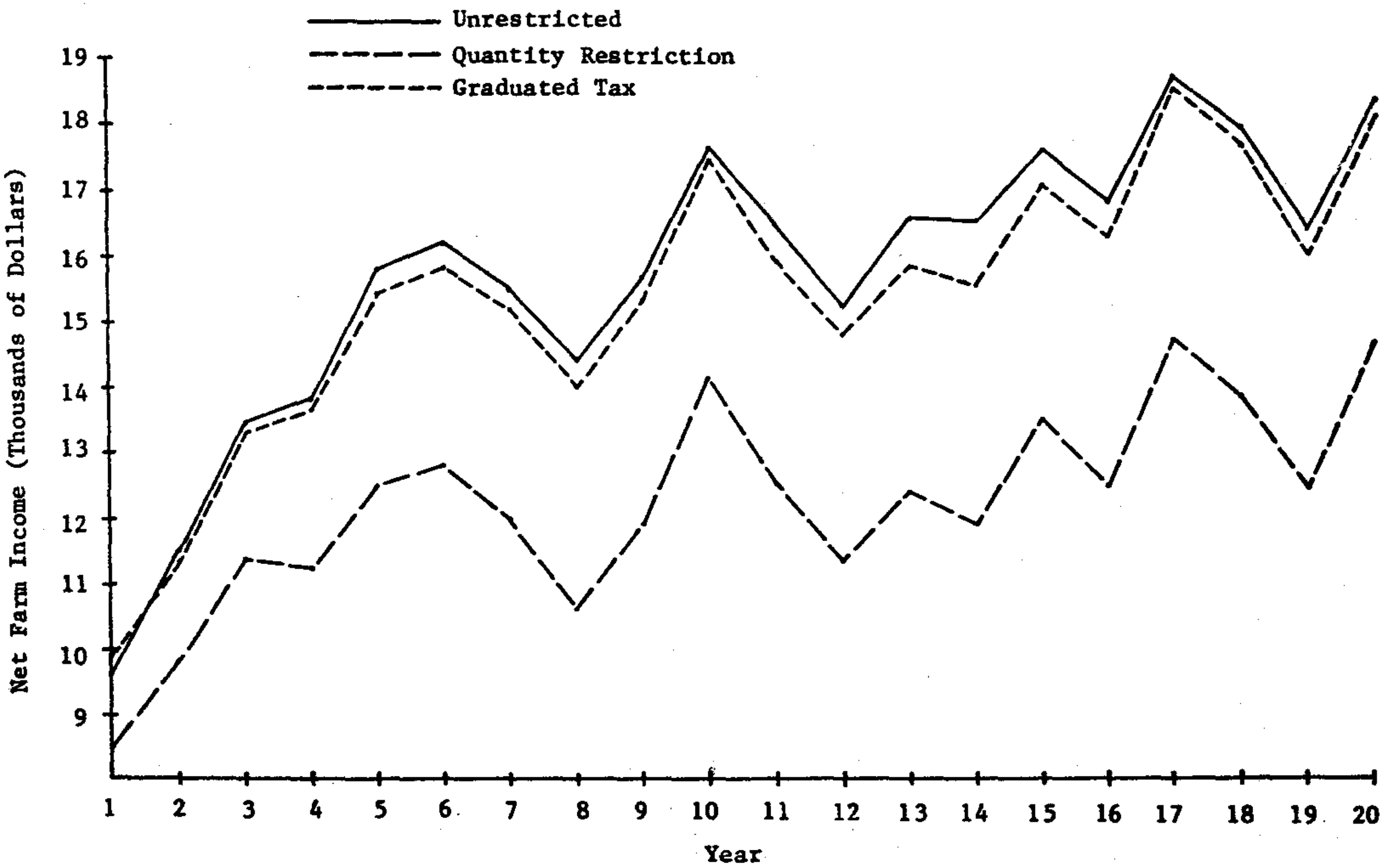

Figure 14. A Comparison of Mean Net Farm Income Under Alternative Water-Use Regulation Methods for Resource Situation 2

For Resource Situation 2, the three statistical tests substantiate that mean net farm income under unrestricted pumping exceeds that under either the graduated tax alternative of the quantity limitation. The mean under a graduated tax is significantly larger than under the quantity limitation.

A comparison of Figures 13 and 14 reveals that the difference between mean acre inches pumped over the 20-year period for unrestricted pumping versus graduated taxation is greater than the difference between corresponding means of net farm income. That is, irrigators pumping without restrictions tend to apply irrigation water to the point where its marginal value product is very low. Thus, the irrigator operating under graduated taxation is able to apply significantly less water, pay the tax on additional water pumped above the quantity limitation and achieve a level of net farm income which appears reasonably close to that achieved under unrestricted pumping. From a policy maker's standpoint, the graduated tax might appear preferable to unrestricted pumping since it reduces pumping significantly while maintaining net farm income at a reasonable level. The farmer would prefer to pump without restrictions, not only because of the additional freedom afforded by that alternative, but because net farm income is larger.

The quantity restriction results in significantly lower total acre inches pumped and lower net farm income than the other two alternatives. Variability of net farm income is much greater than under the other two alternatives. The quantity restriction is likely to be the least preferred alternative by irrigators in the area. Policy makers wishing to pursue this alternative must build their case by evaluating two important factors. (1) The quantity limitation lengthens the life of the aquifer and provides a longer, though lower stream of net income. (2) Unrestricted pumping shortens the economic life of the aquifer and thus provides a shorter, higher stream of net farm income for individual irrigators. By discounting the streams of net returns over the life of the aquifer under alternative policies, a rational economic decision can be made. The life of the aquifer is not projected in this analysis. However, a discounting model is utilized in a subsequent section to compare net income streams under alternative policies over the 20-year span of this analysis.

Figure 15 compares relative variability of net farm income in terms of the coefficient of variation. As expected, coefficients of variation hold the opposite relationships of levels of net farm income. That is, the quantity restriction on water use results in the greatest relative variability of net farm income. The unrestricted water-use alternative results in the lowest relative variability in net farm income, with the graduated tax alternative falling between the two.

Net Worth

Figure 16 presents the mean values of net worth over the 20-year simulation period graphically. Net worth increases almost linearly, but at a slightly increasing rate, for all three water-use alternatives. Net worth levels under unrestricted pumping and graduated taxation are nearly identical and both exceed net worth under the quantity restriction by a large margin. Application of the Wilcoxon Matched-Pairs, Signed Rank Test to the mean values of net worth data indicates mean net worth for both unrestricted pumping and the graduated tax differ significantly from mean net worth under a quantity limitation. Also the two former means differ significantly from one another.

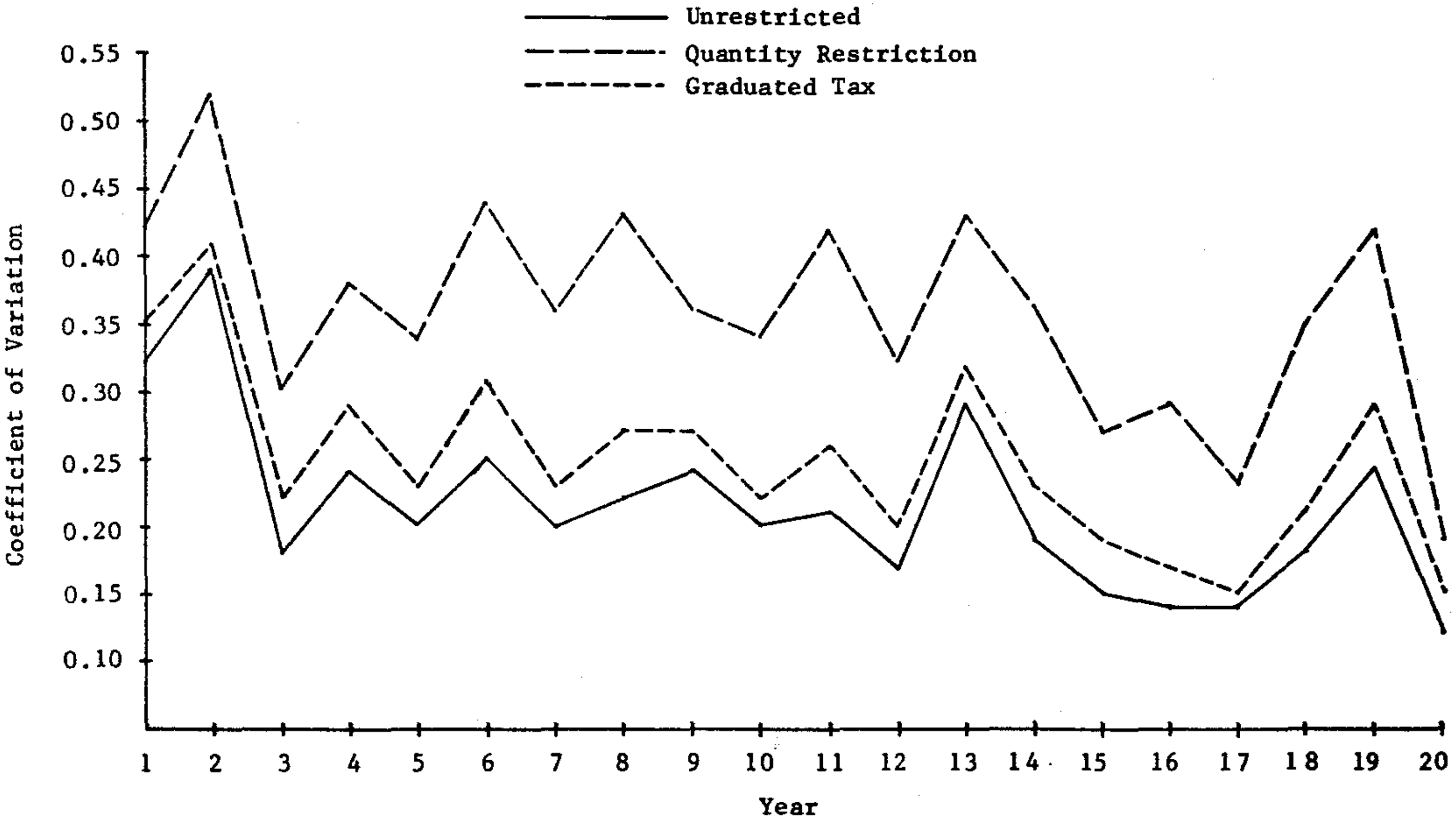

Figure 15. A Comparison of Coefficients of Variation of Mean Net Farm Income Under Alternative Water-Use Regulation Methods for Resource Situation 2

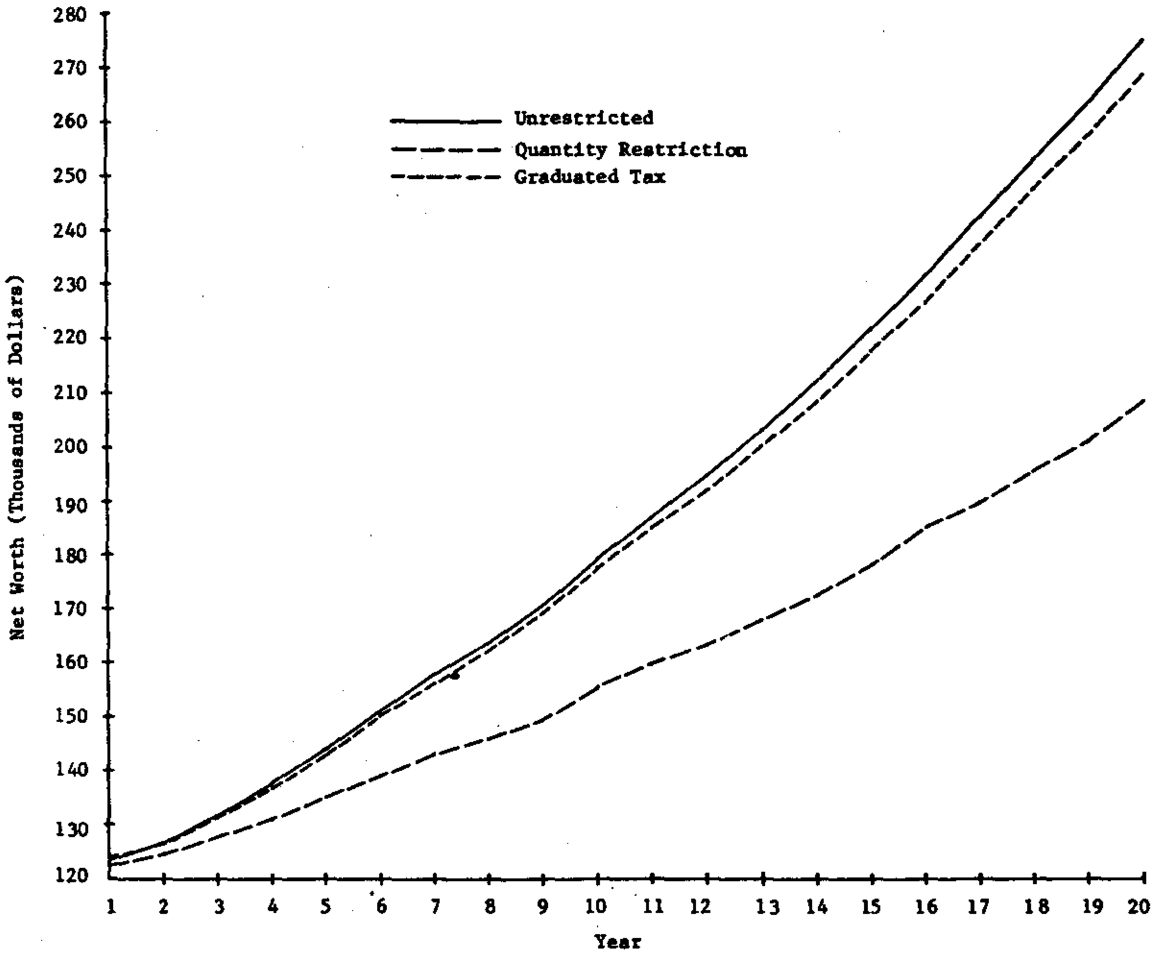

Figure 16. A Comparison of Mean Net Worth Under Alternative Water-Use Regulation Methods for Resource Situation 2

Comparison of Net Farm Income and Government Payments

The importance of government payments as a component of net farm income is mentioned above. This section presents direct comparisons of net farm income and government payments under three water-use alternatives for the two resource situations. The payments shown are the maximum for which the operator would be eligible. Not all operators participate in the price support programs and many of those that do participate do so at less than the maximum level. Thus the amount of payments shown for the representative farm overstates the proportion of net farm income farmers in the area derive from government payments.

Comparisons between mean values of net farm income under unrestricted pumping, a quantity restriction and graduated taxation for Resource Situation 1 are presented in Table XXXIX. Government payments are a significant portion of net farm income under all three water-use alternatives. Under unrestricted pumping, net farm income exceeds government payments from year 1 through year 6. Beginning in year 7, government payments exceed net farm income. That is, without government payments, net farm income would be negative from year 7 through year 20 for the unrestricted alternative.

Comparisons on net farm income and government payments under both the quantity limitation and graduated taxation lead to the same conclusion. The impact of government payments is the difference between positive and negative net farm income. Under the quantity restriction, government payments exceed net farm income beginning in year 7. Under the graduated tax alternative, net farm income exceeds government payments for the first seven years of the simulation run. However, from year 8 through year 20, with the exception of year 10, government payments exceed net farm income.

TABLE XXXIX

COMPARISON OF NET FARM INCOME AND GOVERNMENT PAYMENTS UNDER THREE WATER-USE ALTERNATIVES FOR RESOURCE SITUATION 1

	No Restrictions		Quantity Restriction		Graduated Tax	
Year	Net Farm Income	Government Payments	Net Farm Income	Government Payments	Net Farm Income	Government Payments
1	9,019	8,048	8,791	7,838	9,473	8,084
2	9,809	9,043	9,715	8,808	10,461	9,144
3	13,546	10,565	11,250	9,876	13,595	10,647
4	13,839	12,107	11,131	11,001	14,042	12,189
5	15,045	13,615	12,270	12,086	14,966	13,624
6	14,624	13,761	12,707	12,073	15,346	13,699
7	13,593	13,827	11,417	11,778	14,333	13,737
8	11,454	13,331	9,913	11,534	12,995	13,384
9	10,870	13,048	10,234	11,367	12,842	13,084
10	11,324	13,035	11,899	11,545	13,368	13,105
11	8,780	12,789	7,815	11,393	10,477	12,968
12	6,406	12,477	5,613	11,366	8,018	12,995
13	7,502	12,270	6,204	11,449	8,865	12,638
14	6,838	11,822	5,621	11,512	8,288	12,450
15	7,719	11,051	5,586	11,205	9,270	11,972
16	5,714	10,405	5,335	11,045	8,065	11,639
17	7,503	10,152	7,581	11,112	9,956	11,446
18	4,351	9,792	4,591	10,655	6,944	10,920
19	1,031	9,315	1,253	10,072	3,867	10,370
20	2,183	8,911	2,447	9,634	5,056	9,952

The model used did not afford the irrigation operator an opportunity to expand his operation. It appears that irrigators in Resource Situation 1 are faced with the alternative of expanding the size of operation, or going out of business. Even with substantial government payments, net farm income is quite low by the time the water supply reaches economic exhaustion. These implications hold for Resource Situation 1, regardless of the water-use alternative followed.

Government payments are important to the irrigation operator represented by the adequate water position in Resource Situation 2. A comparison of mean values of net farm income and government payments under the three water-use regulatory alternatives is presented in Table XL.

Under unrestricted pumping, government payments increase from $8,218 in year 1 to $13,648 in year 6 and remain stable for the remainder of the simulated time period. Net farm income exceeds government payments every year. Thus, positive net farm income is possible under unrestricted pumping for irrigators in Resource Situation 2. However, without government payments net farm income would range from less than $2,000 to about $6,000.

Under the quantity restriction, both the level of net farm income and government payments are lower than under unrestricted pumping. Net farm income exceeds government payments during every year of the simulation run. However, the difference between the two is smaller than for the unrestricted alternative. Net farm income exceeds government payments by a minimum of about $300 and a maximum of just under $5,000.

The relationship between net farm income and government payments under the graduated tax alternative compares favorably with the relationship under unrestricted pumping. Net farm income exceeds government payments under

TABLE XL

COMPARISON OF NET FARM INCOME AND GOVERNMENT PAYMENTS UNDER THREE WATER-USE ALTERNATIVES FOR RESOURCE SITUATION 2

Year	No Restrictions		Quantity Restriction		Graduated Tax	
	Net Farm Income	Government Payments	Net Farm Income	Government Payments	Net Farm Income	Government Payments
1	10,598	8,218	9,576	7,610	10,866	8,127
2	12,434	9,431	10,791	8,512	12,380	9,305
3	14,413	10,776	12,367	9,407	14,314	10,571
4	14,767	12,166	12,200	10,382	14,604	11,924
5	16,754	13,621	13,440	11,406	16,383	13,196
6	17,192	13,648	13,787	11,451	16,790	13,293
7	16,421	13,554	12,984	11,332	16,151	13,237
8	15,353	13,456	11,561	11,256	14,990	13,150
9	16,601	13,438	12,885	11,238	16,298	13,129
10	18,563	13,534	15,079	11,320	18,456	13,260
11	17,420	13,446	13,497	11,253	16,871	13,160
12	16,172	13,381	12,311	11,126	15,739	13,094
13	17,506	13,431	13,352	11,075	16,798	13,115
14	16,974	13,400	12,874	10,937	16,501	13,041
15	18,548	13,320	14,451	10,816	18,037	12,925
16	17,794	13,296	13,427	10,737	17,216	12,904
17	19,644	13,449	15,762	10,882	19,572	13,069
18	18,908	13,407	14,816	10,866	18,631	13,047
19	17,364	13,356	13,429	10,907	16,921	13,022
20	19,293	13,395	15,632	10,980	19,020	13,070

every year of the 20-year simulation run. The difference between the two ranges from about $1,800 to over $6,000.

The impact of government payments is of great significance to irrigators in both the poor and adequate water situations for the representative farm defined in this study. Without government payments, many individual operators would be forced to either reduce current consumption or borrow heavily to maintain that consumption level. Their alternatives are to expand the operation or migrate from the farm.

The implications drawn here are based upon the simulation of 640-acre representative farms defined for this study. Assumptions regarding prices, irrigation strategies and expansion of irrigation facilities are quite specific. Extrapolation from these resource situations to others in the study area must be made with caution.

Relative Rates of Water Withdrawal for Each Water-Use Alternative

Table XLI summarizes saturated thickness remaining at the end of the 20-year simulation run. For Resource Situation 1, the mean values of feet of remaining saturated thickness are 35.84, 38.37 and 37.72 for unrestricted pumping, quantity restriction and graduated tax alternatives, respectively. Water is used at different rates for each alternative. That is, unrestricted pumping results in more rapid pumping in early periods and slower withdrawals, due to declining pump capacity, in later periods. The quantity restriction results in lower rates of withdrawal in early periods, but lower rates in later periods because greater pumping capacity remains for the irrigation system. Pumping or withdrawal rates for the graduated tax alternative remain between those for the unrestricted and taxed alternatives. Regardless of the alternative utilized, the ending position is approximately the same. The individual either completely returns to dryland farming or is maintaining

TABLE XLI

REMAINING SATURATED THICKNESS OF OGALLALA FORMATION AT THE END OF 20-YEAR SIMULATION RUNS

	Resource Situation 1			Resource Situation 2		
Replication	Number Restrictions	Quantity Limitation	Graduated Tax	Number Restrictions	Quantity Limitation	Graduated Tax
Mean	35.84	38.37	37.72	235.03	251.81	245.61
Maximum	37.53	41.57	40.97	240.62	254.26	249.19
Minimum	33.42	36.08	34.67	230.49	250.82	242.88
Range	4.11	5.49	6.30	10.13	3.44	6.31
Av. Feet Decline	64.16	61.33	62.28	89.97	73.19	79.39
Av. Decline/Year	3.21	3.07	3.11	4.50	3.66	3.97

about 80 acres of irrigated grain sorghum and attempting to spread fixed costs of the irrigation system over 40 to 65 acres of irrigated wheat during portions of the crop year not devoted to intensive irrigation of summer crops. The decline in saturated thickness is 64.16, 61.33 and 62.28 feet for the unrestricted, quantity restriction and graduated tax alternatives, respectively. The average decline is 3.21, 3.07 and 3.11 feet per year for the three alternatives. From the standpoint of the underground water supply, all alternatives will lead to economic exhaustion within Resource Situation 1 in about 20 years, given the assumptions of the model.

Based on water-use rates in Resource Situation 1, there is little reason for policy makers to restrict water use with a quantity limitation of 1.5 acre feet per acre of water rights. It results in lower levels of net farm income while depleting the water supply at approximately the same point in time as for the other two alternatives. The policy maker might lean toward a graduated tax if water-use regulation is deemed desirable. Higher levels of net farm income are due primarily to individual action to restrict water use in earlier periods of the crop year, and to utilize economic decision rules in allocating water once the quantity limitation has been reached. One might argue against any type of water restriction in the poor water situation on the grounds that rational irrigators merely need to be informed that applying economic decision rules in the allocation of water can lead to higher levels of net farm income. An educational program to encourage voluntary application of rational economic decision rules to allocating the existing water supply would be more palatable to individual operators as well as to policy makers within the study area. The model developed in this study is capable of providing information regarding various irrigation strategies and their impact on net farm income.

Table XLI also presents feet of remaining saturated thickness for each water-use alternative for Resource Situation 2. Mean levels of saturated thickness are 235.08, 251.81 and 245.61 for the unrestricted, quantity restriction and graduated tax alternatives. The feet decline in saturated thickness are 89.97, 73.19 and 79.39 for the three water-use alternatives, respectively.

An 89.97-foot decline in saturated thickness for the unrestricted alternative is an average of about 4.50 feet per year. With approximately 110 feet of saturated thickness before well yields begin to decline, the unrestricted irrigator in Resource Situation 2 may be able to pump for an additional 24 years (a total of 44 years) before encountering significant changes in pumping capacity, and for perhaps an additional 35 years (a total of 55) before facing a reduction in irrigated acres.

The graduated tax alternative results in a 79.39-foot decline in saturated thickness, averaging 3.97 feet per year. At the end of 20 years, approximately 121 feet of saturated thickness remain before well reductions begin to occur. If the water table continues to decline at 1.97 feet per year, an irrigator in Resource Situation 2, operating under the graduated tax alternative, may be able to pump an additional 30 years (a total of 50 years) before well yield declines commence, and for perhaps an additional 41 years (a total of 61 years) before facing a reduction in irrigated acreage.

Pumping under a quantity restriction results in a decline of 73.19 feet in saturated thickness for an average of 3.66 feet per year. Almost 127 feet of saturated thickness remain before yield reductions begin. If the water table continues to decline at a rate of 3.66 feet per year, perhaps 35 years (a total of 55 years) of pumping remain before the irrigator in Resource Situation 2, pumping under a quantity restriction, is faced with declining well yields and rising pumping costs. Perhaps an additional 46 years (a total of 66 years) pumping exists before any reduction in irrigated acreage is necessary.

These statements apply strictly to the individual irrigator with a beginning saturated thickness of 325 feet, depth to water of 125 feet, well depth of 450 feet and pump depth of 400 feet. They also assume the irrigator is pumping from a closed basin one section in size with a given 1,000 g.p.m. well and constant production organization. One must exercise great care when extrapolating from the assumed situation to all irrigators who are classified in Resource Situation 2. Some individuals in Resource Situation 2 have just above 200 feet of saturated thickness and experience an impact on well yield and pumping cost before 20 years have expired, assuming a decline of 4.5 feet per year in saturated thickness. Other individuals in Resource Situation 2 have perhaps 500 feet of saturated thickness and a seemingly endless water supply. At least, barring extraordinary and unforeseen circumstances, their water supply is sufficient for this generation. Thus, statements regarding the water situation for Resource Situation 2 must be viewed as applying to the modal representative farm firm defined for this study. Considerable variation exists among individual operators. Unfortunately, only a limited number of situations could be simulated with the available funds.

Discounting Net Income Streams to Their Present Value

The streams of net farm income resulting under the unrestricted, quantity restriction and graduated tax alternatives are discounted to their present value at several interest rates. Present values of net farm income for each regulatory alternative at four different interest rates for Resource Situations 1 and 2 are presented in Table XLII.

The discounting model is appropriate because income in the current time period is worth more than income in future time periods due to uncertainty about the future and a preference by most individuals for current rather than future income. Through time, the discounting factor, $\frac{1}{(i+1)^n}$, increases. Thus, the value of future net income is reduced relative to the value of current net income. The magnitude of present values increases as interest rates decline because the discounting factor declines with the interest rate. Thus, the value of net income, when discounted, is larger.

Implications to be drawn from the analysis do not vary with interest rates. For Resource Situation 1, present value of net income is greatest for the graduated tax alternative. This finding is not surprising since net farm income under the graduated tax alternative exceeds net farm income under the unrestricted pumping alternative during every year but one. Present value of net farm income under unrestricted pumping exceeds that under the quantity limitation. Net farm income under unrestricted pumping greatly exceeds net farm income under a quantity restriction during early years of the simulated time period. During early years, the discount factor is small, and discounted values of net farm income large. It is only during year 10 and years 17, 18, 19 and 20 that net farm income under a quantity restriction slightly exceeds net farm income under unrestricted pumping. In late periods, the discount factor is large, and contributions to the present value of net farm income by these excesses of income under a quantity restriction over income under unrestricted pumping are small.

TABLE XLII

PRESENT VALUE OF NET FARM INCOME FOR THREE WATER-USE REGULATION ALTERNATIVES AT FOUR INTEREST RATES

	Resource Situation 1			Resource Situation 2		
	Water-Use Regulation Alternative			Water-Use Regulation Alternative		
Interest Rate	No Regulation	Quantity Limitation	Graduated Tax	No Regulation	Quantity Limitation	Graduated Tax
.08	101,264	89,695	112,843	155,056	124,868	151,760
.05	123,421	109,469	139,711	200,776	160,733	196,366
.03	142,643	126,696	163,444	242,817	193,728	236,743
.01	166,761	148,392	193,694	298,321	237,257	291,736

For Resource Situation 2, the present value of net farm income under unrestricted pumping exceeds present values under both graduated taxation and a quantity limitation. This result is expected since the level of net farm income under unrestricted pumping exceeds that under the graduated tax every year except year 1. Since the levels of net farm income remain homologous over time, the present values are nearly the same. Present values of net farm income under both unrestricted pumping and graduated taxation exceed present value of net farm income under the quantity limitation. This finding is consistent with the significant differences found between distributions of net farm income under unrestricted pumping and graduated taxation when tested against the distribution under the quantity restriction.

Based on computation of present values of net farm income over the 20-year simulated time period, one can conclude that the timing aspects of the streams of net farm income do not differ enough for the implications of this analysis to be changed. A more valid basis of comparison would be to compute the present value of the longer, smaller stream of net farm income under the quantity restriction and compare it with a shorter, larger stream resulting under unrestricted pumping. Unfortunately, this study does not lend itself to that type of analysis.

ESTIMATING PRIMARY ECONOMIC BENEFITS UNDER ALTERNATIVE METHODS OF WATER-USE REGULATION

The final objective of this study is to estimate the primary economic benefits from irrigation under projected use of the stock water supply with current restrictions and the effect of each method of water-use regulation on the pattern of primary benefits realized. Primary economic benefits are defined in this study as the increase in net returns to land and management resulting from irrigated crop production. That is, aggregate net return to land and management with irrigation minus aggregate net return to land and management without irrigation is the measure of primary economic benefits that is used.

The primary economic benefits estimated in this study are those accruing to residents of the study area. They were estimated using "adjusted normalized prices" issued by the Water Resources Council for the crops produced [111]. These are prices adjusted to remove the direct price support effects of government programs. While it is argued these are primary economic benefits to the study area, it is certainly admitted they may be offset to some extent by a reduction in primary benefits derived from feed grain, food grain and forage production in other areas of the country. Estimates of the net primary benefits to the U.S. of irrigation development in the Central Ogallala is the subject of another research project. Such a project should also consider the benefits of development in other areas of the country as well as those in the Central Ogallala.

The analysis under the first objective of this study projected the rate of irrigation development, estimated net farm income of crop production and estimated the primary economic benefits of irrigation development in the study area. These estimates were developed using an aggregate model for the study area. The analysis under objective 3 used a micro-level simulation model to determine the effect of two methods of water-use regulation on the amount of water used and net farm income of representative farm firms. The analysis in this section reviews the estimates of primary economic benefits developed under objective 1, and discusses implications of Objective 3 results for estimates of the primary economic benefits.

Primary Economic Benefits With Current Restrictions

The aggregate increase in net crop income (primary economic benefits) was estimated for the projected development of irrigation in the Central Ogallala as part of the analysis under objective 1. The analysis was completed using a recursive linear programming model for the study area. The aggregate model assumed constant technology and used adjusted normalized prices to remove the direct effects of government price support programs. Development of the model, the assumptions, and the input data are given in pages 10 through 43 of this report and are not repeated here. The recursive linear programming model was used to project production levels, water use, income and primary economic benefits over time under two broad assumptions. The projections obtained under the two sets of assumptions are referred to as Model I and Model II results, respectively.

Model I assumes the area is restricted to its historic share of the projected national supply. The supply of crops projected by the USDA for the years 1980 and 2000, and the proportion of national supply produced in the study during the 1965-67 period were used to develop the production targets [Table VI]. Model I projections indicated the study area's historic proportion of U.S. domestic needs can be met over the period with about 1.4 to 1.6 million acres of irrigated production (Table XII). This is approximately the number of acres that were irrigated in 1965. Thus the Model I estimates of primary economic benefits approximate the level of primary benefits that would be projected if restrictions are imposed limiting irrigation development to approximately the 1965 level (1.5 million acres).

Model II assumes the area is permitted to increase its share of U.S. production of feed grains, food grains and forages. The area is restricted to the historic share of national domestic needs for cotton and sugar beets, however. The development of irrigation in each resource situation depends on the profitability of such development. It is also limited to a maximum growth rate per year reflecting the effect of capital rationing, labor availability, availability of well drillers, and sociological factors on the rate of irrigation development. Thus Model II results reflect a projected rate of irrigation development that is constrained by the major factors judged to constrain actual development in the Central Ogallala under the current institutional constraints.

The annual net income from crop production and the estimates of the primary economic benefits of irrigation development for both Models I and II are presented in Table XLIII. Estimates of primary economic benefits derived from Model I projections indicate annual benefits of $18.4 million to $21.9 million will result over the 35-year period if irrigation is limited to approximately 1.5 million acres per year. Production at this

TABLE XLIII

PROJECTED AGGREGATE ANNUAL NET INCOME FROM CROP PRODUCTION AND PRIMARY ECONOMIC BENEFITS OF IRRIGATION

	Model I[a]				Model II[b]			
Year	Annual Net Returns Irrigated	Annual Net Returns Dryland	Total Annual Net Returns (000's of dol.)	Estimated Primary Irrigation Benefits	Annual Net Returns Irrigated	Annual Net Returns Dryland	Total Annual Net Returns (000's of dol.)	Estimated Primary Irrigation Benefits
1965	40,639	9,743	47,382	18,418	49,725	44,820	94,545	23,345
1970	40,662	15,587	56,248	18,443	79,273	36,716	115,990	49,257
1980	45,622	19,049	64,672	20,316	77,726	36,246	113,972	47,710
1990	48,197	24,333	72,530	21,869	81,020	32,197	113,217	51,004
2000	42,263	33,722	75,985	19,275	57,555	36,532	94,087	27,539

[a] The net returns and irrigation benefits for Model I are the same as those shown in Table XIII.

[b] The net returns and irrigation benefits for Model II are the same as those shown in Table XVII.

level will use approximately 23 percent of the stock water supply by 2000 (Table X).

Information on the profitability of irrigation in the study area and the availability of capital, labor and managerial resources indicate stringent institutional restrictions would be required to limit the acreage irrigated in the Central Ogallala Basin to 1.5 million annually. It should be noted in passing that the method of restricting irrigated acreage will significantly affect the distribution of primary economic benefits among residents of the study area. For instance, if irrigation is limited to the acreage developed prior to a specified date, the benefits can be expected to accrue to the owners of land on which irrigation is permitted. To the extent the irrigated area is operated by land owners, the benefits will accrue to the irrigation farmers. Furthermore, when a piece of land carrying the right to irrigate is sold, the increased anticipated income due to irrigation will tend to be capitalized into the sale price. Thus, limiting development by restricting it to specific tracts of land can be expected to result in the current owners of those tracts receiving the major part of the area's primary economic benefits of the stock water supply. Many alternative means of imposing an acreage limitation can be developed. A second possibility is the establishment of a regulatory agency, such as a water district, which is given the authority to sell the right to irrigate for a specified number of years (say 10 or 15) in specified block sizes (say 160 acres) under competitive bidding. A proportionate share of the irrigation rights could be sold each year, with the proceeds of the sale used to support public services for the region.

In this case the distribution of benefits would depend on the price received for the right to irrigate and the type of services provided with the proceeds.

The primary economic benefits estimated from Model II results increase from \$23.3 million for 1.5 million irrigated acres in 1965 to \$49.0 million for 2.7 million acres in 1970. The level of primary benefits remain relatively constant due to the increased irrigation costs as the acreage irrigated increases to 3.4 million in 1990. Projections developed with Model II indicate 43.5 percent of the stock water supply will be exhausted by 2000 (Table XIV), resulting in increased pumping costs, reduced well yields, declining irrigated acreage (2.8 million in 2000) and rapidly declining primary economic benefits (\$27.5 million in 2000).

Both Models I and II assume the same level of prices and technology over time. However, the primary economic benefits estimated from the results of the two models provide an interesting contrast. Model I suggests restricting irrigated production to 1.5 million acres annually will result in a relatively constant stream of annual economic benefits during the remainder of this century. It will also preserve a larger portion of the water supply for use after 2000 and hence a larger portion of the economic benefits also can be expected to be derived after the turn of the century. Model II results indicate the projected rate of development will result in annual primary economic benefits approximately \$30 million greater than those projected by Model I for the 1970-1990 period, with rapidly declining benefits during the last decade of the century. Because a large portion of the stock water supply has been depleted by 2000, relatively little of the primary benefits can be expected to be derived after the turn of the century.

Both Models I and II assume current irrigation technology and hold it constant over the 1965 through 2000 period. Thus estimated primary benefits for development restricted at other levels, such as 2.0 or 2.5 million acres, will fall within the range of benefits bounded by the Model I and II estimates.

The Impact of Alternative Restrictions

Both Models I and II assume no real incentive is provided encouraging farmers to maximize efficiency of water use at the firm level. Model I assumes restrictions are placed on production levels, but as much water can be pumped as desired by simply paying the pumping and distribution costs per acre inch. Model II permits production levels to increase over time, but no incentive is provided to increase production per acre foot of water used. Thus a moot question is, what is the impact of alternative means of water-use regulation on the stream of primary economic benefits that could be derived from the stock water supply? The analysis completed under objective 3 provides some information of interest in answering this question.

Aggregation Problems

A comparison of the results developed under objectives 1 and 3 must be approached with caution. The aggregate recursive linear programming model is an effective means of incorporating regional constraints on production levels and water use in developing the study area projections.

However, the detailed specifications (concerning response to moisture conditions by season, weather uncertainty and irrigation strategies) required to analyze water-use alternatives at the firm level cannot be easily incorporated into an aggregate model. Thus the detailed simulation model was developed for use under objective 3.

Unfortunately, some of the detail included to develop meaningful results at the firm level prevent aggregating the representative firm analyses into regional totals comparable to those estimated under objective 1. Four features are particularly important. First, the available resources permitted developing representative firm results for only two water resource situations under objective 3, instead of the 48 included in the recursive linear programming model. Thus aggregation of figures presented in previous sections is based on data available relating the potential for irrigation development to specific saturated thickness intervals.

Development of irrigation facilities depends upon a great many factors including age of the operator, years of farming experience, years of irrigation experience, financial condition, managerial ability, borrowing capacity, labor availability and others, in addition to the existence of a water supply sufficient for current needs. Thus, it may be argued that irrigators in the less than 200-foot saturated thickness interval are as likely to develop or expand irrigation facilities as irrigators in the greater than 200-foot saturated thickness interval, as long as saturated thickness is sufficient to irrigate the production organization. If this is the case, those portions of the study area represented by Resource Situation 1 may be expected to continue to develop as rapidly as the adequate water situations. It is assumed, based upon the above argument, that irrigation development in each of the resource situations is proportional to the number of irrigable acres in the two resource situations. Weights of .4659 and .5341 for Resource Situations 1 and 2, respectively, were used to develop aggregate results (Table XXVIII).

A second reason that aggregation of the firm simulations will not yield comparable results to those obtained from Models I and II is that the specification of the firm model incorporates some detail that cannot be included in the recursive linear programming model. Most operating irrigated units in the study area have a real estate mortgage. Thus the representative firms simulated started with $42,000 of real estate debt. As a result, the farm incurred interest charges in each year on the unpaid principal, reducing the net farm income figure reported. Government program participation was considered in the firm level model because it represents an important source of income. Inclusion of the program affects both the level of income received and, to some extent, the combination of enterprises. The effect of payments can be removed for comparison with the results under objective 1, but the effect on the combination of enterprises cannot.

A third problem in aggregating the firm level results is that the simulation model incorporates very specific behavioral assumptions concerning the use of irrigation water under each method of water-use regulation. These assumptions appear to represent the way farmers in the area facing a limited supply allocate the available water. Nevertheless, one must question if farmers in the area, given time, more experience and the benefits of educational programs would not develop irrigation strategies resulting in more efficient water use than those simulated under objective 3 for the unrestricted and quantity restriction alternatives.

A particularly important problem in developing regional aggregates from the firm simulation results in that only the modal size of irrigated farm--a 640-acre unit--was simulated. The impact of economies of size is expected to increase the primary economic benefits derived per section of irrigated land as the size of farm increases. The opportunity to spread machinery ownership costs, other overhead costs and make more efficient utilization of the operator's labor indicates farms in the area obtain substantial economies as the size of the farm (measured in acres) is increased. Thus net farm income per section increases as the size of the farm increases. However, the same economies are expected to occur regardless of the method of water-use regulation used. Economies of size are not expected to affect the water used per section, yield level, government payments per section and the difference in net farm income per section under the three water-use alternatives. Thus the estimates of net farm income are expected to underestimate the actual net farm income farmers received. But the estimates are considered a good measure of the difference in net farm income and the amount of water withdrawn for the three water-use alternatives.

The Impact of a Quantity Restriction

Based on the above discussion, the analysis in this section compares the net farm income obtained using a method of water-use regulation with the net farm income derived without restrictions to provide a measure of the difference in net primary benefits obtained annually per section of land. The difference in the amount of water used must also be considered to estimate the difference in the primary economic benefits for a fixed water supply. The results developed under objective 3 indicate net farm income varies over the 20-year planning horizon simulated. Since some firms have been irrigating for many years, while others are just developing irrigation, perhaps the most reasonable measure of difference in net farm income to use is the average over the 20-year period. Average net farm income over the 20-year period simulated is presented in Table XLIV for two resource situations. Resource Situation 1 is considered representative of a "poor" water situation. It has an initial saturated thickness of 100 feet and an initial pumping capacity of 780 g.p.m. Resource Situation 2 represents a "good" water situation. It has an initial saturated thickness of 325 feet and an initial pumping capacity of 1,000 g.p.m. Information in Table XXVIII indicates 46.59 percent of irrigable soils in the study area have 200 feet or less of saturated thickness (Resource Situation 1), with the remaining 53.41 percent having more than 200 feet (Resource Situation 2). These percentages were used to weight Resource Situation 1 and 2 results to obtain the aggregate situation shown in Table XLIV.

The net farm income shown for the resource situations under no restrictions can be used as a basis for comparison or 100 percent. It represents the restrictions farmers face at the current time and can be considered as comparable firm level results for the projections developed under recursive linear programming Model II. As noted in the previous section, payments received under the current government program may be an important source of potential income for a farm of this size, but the period over which the program will remain in effect is uncertain. Furthermore, the estimates of primary economic benefits developed under objective 1 do not include government program payments. Thus net farm income without government program payments is used to estimate the effect of alternative methods of restriction on the primary economic benefits.

TABLE XLIV

AVERAGE NET FARM INCOME SIMULATED FOR A 640-ACRE FARM UNDER ALTERNATIVE WATER-USE SITUATIONS[a]

	Unit	Resource Situation 1	Resource Situation 2	Aggregate[b]
No Restriction				
Net Farm Income w/o Govt. Payments	Dol.	-2,411	3,850	933
Average Annual Water Used	Ac.In.	4,896	6,908	5,971
Quantity Restriction				
Net Farm Income w/o Govt. Payments	Dol.	-2,799	2,536	50
Average Annual Water Used	Ac.In.	4,733	5,624	5,209
Graduated Tax				
Net Farm Income w/o Govt. Payments	Dol.	-1,391	3,845	1,406
Average Annual Water Used	Ac.In.	4,774	6,094	5,479
Average Annual Tax Payment	Dol.	71	212	146

[a] The 640-acre representative farm has 315 acres of irrigated crop production.

[b] The weights used for aggregation are .4659 for Resource Situation 1 and .5341 for Resource Situation 2.

The figures indicate imposing the quantity restriction of 1.5 acre feet per acre irrigated would reduce annual net farm income $388 and $1,314 for Resource Situations 1 and 2, respectively, when program payments are not considered. Weighting the two resource situations indicates imposing the quantity limitation would reduce net farm income $883 per unit. Imposing the quantity restriction would reduce water use per unit by much larger amounts on Resource Situation 2 than 1. Aggregating the two situations indicates imposing the quantity restriction would reduce average annual water withdrawal 12.76 percent for a given irrigated acreage.

The change in net farm income per acre of irrigated land resulting from the imposition of a method of water-use regulation provides a basis to estimate the corresponding change in primary economic benefits for the study area. The firm simulation estimates indicate primary economic benefits resulting from the imposition of a quantity limitation (of 1.5 acre feet per acre irrigated) would decline $1.23 per irrigated acre ($388 ÷ 315) in Resource Situation 1, $4.17 ($1,314 ÷ 315) per irrigated acre in Resource Situation 2 and $2.80 ($883 ÷ 315) per irrigated acre as an average over the study area. Primary economic benefits projected by Model II decline from $18.44 per acre in 1970 ($49.257 million ÷ 2,670,965 acres) to $15.16 per acre in 1990 ($51.004 million ÷ 3,363,921 acres). A reduction in net benefits or $2.80 per acre represents a decline of more than 15 percent (15.2 percent to 18.5 percent). If one accepts the irrigation strategies followed as being a reasonable approximation of the way farmers would operate under a quantity restriction, then it can also be concluded imposing such a restriction would result in a larger reduction in the primary economic benefits per acre irrigated (15 percent or more) than in the amount of water used (12.76 percent). Thus the analysis suggests imposing the quantity restriction would result in reduced primary economic benefits both per acre irrigated and per unit of water used.

The Impact of A Graduated Tax

A comparison of the net returns under the graduated tax alternative appears to hold more promise as a method of increasing primary irrigation benefits per year and also over time. This alternative imposes a tax of $6.00 per acre foot on all water pumped in excess of 1.5 acre feet per acre irrigated. The results indicate that the more rational water use would result in an annual increase of $1,020 per unit on Resource Situation 1, but restriction through taxation has little effect on net farm income for units in Resource Situation 2. These results suggest that imposition of a graduated tax would not adversely effect annual net farm income and the increased efficiency of water use it encourages may actually increase it.

The increase in net farm income per acre of irrigated land resulting from the imposition of the graduated tax to restrict water use is $3.24 per acre in Resource Situation 1 ($1,020 ÷ 315), nil in Resource Situation 2, and $1.50 per irrigated acre as an average over the study area. This represents an increase of more than 8 percent (8.1 percent to 9.9 percent) in benefits per acre irrigated when compared to the primary economic benefits of $18.44 to $15.16 per acre projected by Model II for the 1970 to 1990 period. The graduated tax would restrict water use per year by a much larger amount on Resource Situation 2 than 1. Aggregating the two situations suggests a reduction of 8.24 percent in water use per acre irrigated. Thus the figures indicate the use of a graduated tax will increase primary economic benefits per acre irrigated and per unit of water used. In addition, the analysis indicates average annual tax collections of $.46 ($146 ÷ 315) per irrigated acre would be available to defray administrative costs of the regulatory program and to provide other services of benefit to irrigators as well as other residents of the community.

The analysis clearly suggests that a graduated taxation plan may be an effective means of limiting water use per acre irrigated and increasing primary economic benefits per unit of water used without adversely affecting net farm income. However, additional analysis is needed with alternative sizes of farms on a wider range of resource situations to thoroughly analyze this tax alternative. In addition, similar analyses are needed using alternative taxing arrangements and levels before a specific taxing arrangement is recommended for use.

LISTING OF SIGNIFICANT CONCLUSIONS

1. The acreage of irrigated crop production in the semi-arid region overlying the Central Ogallala Formation has increased from approximately 70,000 in 1950 to 1.5 million in 1965. Annual water withdrawals for irrigation and other uses in the study area have been estimated to increase from .23 million acre feet to 2.7 million acre feet over the fifteen-year period. Natural recharge occurs primarily through percolation from the surface and is estimated to average .27 million acre feet per year. Estimated withdrawals have exceeded estimated recharge each year since 1954. The quantity of water in storage in 1965 in the study area was estimated at approximately 370 million acre feet. These figures suggest the aquifer is being mined at an increasingly rapid rate from year to year, but that a sufficient amount of water is currently in storage to permit such mining for sometime to come.

2. Estimates developed in this study indicate approximately 8.0 million of the 11.1 million acres of land in the study area are suitable for irrigation. Of the 8.0 million, approximately 6.3 million are suitable for surface irrigation while the remainder should only be irrigated with sprinkler systems.

3. Estimates of the land area for each of eight depth to water and six saturated-thickness intervals were developed in this study. The estimates indicate approximately 24 percent of the area has less than 100 feet of saturated thickness and an additional 23 percent has from 101-200 feet. An additional 26 percent has from 201-300 feet of saturated thickness with less than 28 percent having more than 400 feet of saturated thickness. While the portion of the study area having 200 feet or less of saturated thickness

comprises approximately 46 percent of the total land area and 44 percent of the total irrigable land, it has only 21 percent of the total water supply. The portion of the study area having more than 400 feet of saturated thickness comprises approximately 10 percent of the total land area and 11.7 percent of the irrigable acres, but overlies 22 percent of the stock water supply.

4. The availability of land and water resources suggest irrigation will expand in the area if the demand for irrigated crop production is sufficient to maintain the profitability of these alternatives. A related study completed at the Oklahoma Agricultural Experiment Station indicated the demand for food grains, feed grains and forages to feed cattle in the study area will provide the necessary market for increased irrigation production.

5. The first objective of this project was to project irrigation development, the rate of water use and aquifer depletion over time for the Central Basin of the Ogallala Formation. The projections were made using an aggregate linear programming model for the study area. One set of projections was completed assuming the study area is limited to its historic share of U.S. domestic production requirements as estimated by the U.S. Department of Agriculture. These projections, referred to as Model I, indicate the area could produce its historic share of U.S. needs with relatively minor increases in irrigated acreage. The projections indicated producing at this level would result in irrigated acreage increasing to 1.6 million acres by 1990 and a declining irrigated acreage in later years. The average cumulative water table decline over the study area would be 54 feet by the year 2000. A decline of this magnitude would eliminate irrigation on the part of the study area having less than 100 feet of saturated thickness (about 24 percent of the study area) and significantly reduce the profitability of irrigation in areas having 101 to 200 feet of saturated

thickness (an additional 22 percent of the study area). Estimated net returns from irrigation for the study area increased from 40.6 million in 1965 to a high of 48.2 million in 1990 and then declined to 42.3 million by the year 2000. Defining annual primary irrigation benefits as the increase in area net farm income resulting from irrigation, indicates primary economic benefits remain in the $18.4 to $21.9 million range through out the period. Significant declines would be anticipated after the year 2000 however.

6. A second set of projections completed under objective 1 used the recursive linear programming model, but assumed irrigation development is restricted only by the profitability of irrigation, the availability of land, labor, water, capital and managerial resources and the current institutional restrictions on development. The results of this analysis, referred to as Model II results, indicate the number of acres irrigated annually will increase to a peak of 3.4 million in 1990 and then decline to 2.8 million in the year 2000. Water withdrawn annually increases from 2.8 million acre feet annually in 1965 to 6.0 million acre feet in 1990 and decreases to 5.0 million by the year 2000. The amount of water remaining in storage declines from the initial level of 370 million acre feet to about 209 million acre feet--approximately 56.5 percent of the original level--by the year 2000. The water table decline will average approximately 100 feet by the turn of the century. The well capacity in the 0-100-foot saturated-thickness class declines rapidly and becomes inadequate to maintain irrigation systems by 1980. This is an area of 1.8 million acres or about 24 percent of the study area. Well yields in the second saturated-thickness class--an area of 1.7 million acres or 22 percent of the area--approach

uneconomic levels by 2000. Thus, development at the rate projected by Model II will result in about 44 percent of the land area not having an adequate water supply for profitable irrigation by the year 2000. Estimated net returns from irrigation increase from $49.7 million in 1965 to $79.3 million for the 2.7 million acres irrigated in 1970. Irrigated net returns remain relatively constant, reaching a peak of 81.0 million as irrigation increases to 3.4 million acres by 1990 and then declines to $94.1 million for the 2.8 million acres irrigated by the year 2000. The increase in net income attributable to irrigation (primary economic benefits) increases from 23.3 million in 1965 to 49.3 million in 1970. It remains relatively constant until 1990 and then declines rapidly to $27.5 million by the year 2000.

7. The results of Models I and II exhibit similar trends over time. In both cases growth of irrigation in the study area occurs from 1965 to 1990. The extent of irrigation in both models declines during the closing years of the twentieth century. In both models the direction of change in the level of crop production, water use, underground water storage and well capacities is the same. The results differ only in magnitude and timing which arise from differences in the basic assumptions of the two models. Model I's basic assumption, that the production of the study area will not surpass its historic share of the projected national supply of the eight crops, effectively restricts a rapid growth of irrigation. Consequently, the ground water supply is depleted at a slower rate than in Model II. Model II places no upper restriction on production of irrigated crops, except for sugar beets and cotton, but does impose restrictions limiting growth in irrigation to a somewhat slower rate than experienced in the

recent past. Model II increases the irrigated acreage of crops at a more accelerated rate than Model I which results in a faster depletion of the water resources. The assumptions of Model II are judged to more closely approximate the restrictions on irrigation development in the Central Ogallala. Thus the projections resulting from Model II are considered more indicative of the irrigation development that will result providing further institutional constraints on development are not imposed.

8. Because the solution to Model I includes irrigation on only 1.4 to 1.6 million acres, the water withdrawals, the net income from irrigation and primary economic benefits can be used to indicate the effect of limiting irrigation development in the area to approximately the 1965 level--1.5 million acres. A comparison of Model I and II results indicates the tradeoff between water saved for the future and economic benefits to be derived during the remainder of the twentieth century.

9. The second objective of this project was to test whether the projected rate of basin-wide withdrawals represents a potential misallocation of the water resource over time. The moot question is, "Can irrigation firms making a decision as a group increase the present value of net irrigation benefits by reducing the current rate of water use?" A multistage sequential decision model was used to select the rate of water use which would maximize the present value of the stream of future net farm income for the area. The model assumed no change in technology over the 100-year planning horizon and hence constant benefits per unit of water used over time. It was solved for discount rates of 4 and 8 percent to test the sensitivity of the solution to the level of the discount rate used.

The withdrawal rates projected by the recursive linear programming model (Model II) are substantially less during the periods 1970-79 and 1980-89 than those selected as optimal by the multi-stage sequential decision model for discount rates of either 4 or 8 percent. The multi-stage sequential decision model indicates an additional 21 million acre feet of water should be withdrawn annually during the 1970's and 17 million acre feet more should be withdrawn during the 1980's than is projected by the recursive linear program. Although the projected rate of ground water withdrawal (Model II) exceeds the rate selected as optimal after 1989, the projected levels of storage are greater than those resulting from optimal withdrawal rates for the remainder of the twentieth century. The projected storage level in the year 2000 is 204.9 million acre feet (with Model II). The year 2000 study area levels of storage resulting from withdrawal policies prescribed by the multi-stage sequential decision model are 176.0 and 174.0 million acre feet for 4 and 8 percent discount rates, respectively. The conclusion that can be drawn is that the projected rates of withdrawal do not exceed the rate required to maximize the present value of primary irrigation benefits over the next 20 to 30 years under the assumptions used in this study.

10. A comparison of the results of the recursive linear programming model and the multi-stage sequential decision model indicates the misallocation of ground water is not a direct corollary of its being a common property stock resource. Whether such a resouce will be intertemporally misallocated depends to a large degree on whether or not it is the most limiting factor of production at the margin. Factors such as (a) a high discount rate, (b) constraints on the quantity of crops produced due to

market or government program conditions, and (c) limited availability of capital and labor that complement the expansion of irrigated production may sufficiently constrain expansion by individual operators so that the mining of ground water from the closed exhaustible aquifer does not result in automatic intertemporal misallocation.

11. Although the analysis under the second objective indicates the projected rates of withdrawal for the next twenty years do not exceed the optimum withdrawal rates, many area residents and policy makers are interested in the effect of alternative methods of water-use regulation on the amount of water used over time and its effect on primary economic benefits to the stock water supply. Some individuals place a high value on the conservation of water and other natural resources. Others may question the procedures used in objective 2 to establish the "true value" of the stock water supply in future years.[16] To the extent future values are underestimated, the multi-stage sequantial decision model will tend to recommend using larger quantities in the near future and saving smaller quantities for the more distant future than are actually optimum. For these reasons three alternative methods of water-use regulation on representative farms in the study area were evaluated under the third objective of this study.

12. A simulation model which estimates the yield of crops as a function of soil moisture conditions throughout the growing season was

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[16] In general, any change in technology which either reduces the water and distribution costs or increases production per acre foot of water used (such as improved varieties or reduced evapotranspiration) would tend to increase the value of water in the future. Changes in demand and supply conditions resulting in higher prices for the products would also increase the value of water in the future. Given the current federal farm programs designed to reduce production of feed and food grains, it can also be argued that the current value of water used in production of irrigated crops in the study area is much lower--perhaps even zero--when considered from a national standpoint.

developed to evaluate the alternative methods of water-use regulation. This model is referred to as the Production Subset and determines final crop yields as a function of the length and severity of soil moisture and atmospheric stress in relation to critical stages of plant development for each dryland and irrigated crop included in the analysis. Components of the model include discrete probability distributions for rainfall, lognormally distributed pan evaporation distributions, a set of relationships between pan evaporation and evapotranspiration for each crop, a series of equations composing a soil moisture balance system and coefficients relating soil moisture and atmospheric stress to yield reductions for each crop. Daily values of rainfall and pan evaporation are generated probabilistically. Daily soil moisture values are maintained for each crop. Daily yield reductions are a function of severity of soil moisture and atmospheric stress for each crop. Daily yield reductions are summed across three critical stages of grain sorghum development, four critical stages of wheat development and five critical stages of corn development. Final yield for each crop is determined by subtracting yield reduction from a potential yield which may be reached under adequate soil moisture conditions throughout the growing season. The Production Subset was given an extensive evaluation by agronomists familiar with crop production in the study area and judged to be a "very good" predictor of yield levels for alternative moisture and atmospheric stress conditions prevailing in the study area. Specification and development of this model are certainly significant results of this study, but are not repeated in this section as they are covered in detail in the body of this report.

13. Three water-use alternatives were simulated in this study. The first alternative is continued development and pumping without restrictions. This alternative assumes irrigators base irrigation decisions on the level of available soil moisture and provides no incentive to conserve water use at the current time for future use. The second alternative requires irrigators to restrict pumping to 1.5 acre feet per acre of water rights. The third water-use alternative simulated assumes the irrigator can pump as much water as desired providing he pays a tax of $.50 per acre inch for each acre inch pumped above 1.5 acre feet per acre of water rights. These three water-use alternatives were simulated for a 640-acre farm with 315 acres of irrigated crops for each of two resource situations.

Resource Situation 1 represents the "poor water" situation within the study area. It was assumed the weighted average saturated thickness of 100 feet will support a well yield of approximately 780 g.p.m. Well yields decline rapidly over time causing the operator to add a second and third well, and then reduce the number of acres devoted to irrigated crop production as it becomes uneconomic to irrigate the entire acreage. The effects of unrestricted pumping, quantity limitation and graduated taxation on total acre inches pumped, net farm income and net worth for the irrigators in Resource Situation 1 were evaluated.

An analysis of the simulation results indicates that the total acre inches pumped over the 20-year period under the three institutional alternatives do not differ significantly. However, the distribution of water use over the 20-year planning horizon does differ to some extent. The unrestricted irrigator pumps more water during early years of the 20-year period, depleted his pumping capacity rapidly and pumps the smallest number of acre inches in years 12 through 20. The quantity limitation results in fewer acre inches pumped during early years, but leaves the irrigator the capacity to pump the greatest number of acre inches per year from years 12 through 20. Water use under the graduated tax alternative is between the two extremes. The three water-use alternatives, though differing somewhat in timing of applications, result in essentially the same saturated thickness and decline in the water table at the end of the 20-year period. The average feet of saturated thickness remaining after 20 years of operation are 35.8, 38.4 and 37.7 for the unrestricted, quantity restriction and graduated tax alternatives, respectively.

Mean net farm income under the graduated tax alternative is significantly above mean net farm income under unrestricted pumping and a quantity limitation. Also the mean under unrestricted pumping is significantly larger than the mean under a quantity restriction on water use. The somewhat surprising conclusion that mean net farm income under the taxing alternative is greater than under unrestricted pumping results from more rational use of irrigation water when a tax on additional use is imposed. The taxed irrigator achieves more timely irrigation in relation to plant needs and higher crop yields for the same amount of water. Since pumping costs rise more slowly, net returns per acre and net farm income are higher despite the tax payments. It is also significant to note that the variability of net farm income as measured by the coefficient of variation is greatest under the quantity restriction and least under the graduated tax alternative.

The mean net worth of the firm under graduated taxation also exceeds the mean net worth under unrestricted pumping or the quantity limitation. This is not surprising given the results concerning net farm income above.

Mean net worth with unrestricted pumping exceeds that under the quantity limitation.

14. Resource Situation 2 represents the "adequate" water situation within the study area. The weighted average saturated thickness of the Ogallala Formation for this resource situation is 325 feet--a sufficient saturated thickness to maintain a pumping capacity of 1,000 g.p.m. throughout the 20-year planning horizon. Irrigators in this situation experience some increase in pumping costs as the water table declines, but are neither required to add additional wells to maintain their pumping capacity nor to revert a portion of their acreage to dryland production over the 20-year period.

The effects of unrestricted pumping, quantity limitation and graduated tax alternatives on total acre inches pumped, net farm income and net worth for the representative firm were simulated over a 20-year period. Operation with the unrestricted alternative allows the irrigator to pump at the capacity of the system for the entire growing season and thus pump significantly more water than the firm operating under either the quantity limitation or the graduated tax. The amount of water pumped under the graduated taxation alternative is significantly greater than the amount pumped under the quantity limitation. Since capacity does not decline over time, the firm has the same ability to pump water during the latter years of the planning horizon as during the initial years. The amount of water pumped per year does vary depending on the weather conditions simulated for the year. Variability of acre inches pumped is greater under the unrestricted pumping alternative. The least relative variability is observed under a quantity limitation because the irrigator is prohibited from pumping more than the upper limit, even during very dry years.

The feet of decline in saturated thickness are 90.0, 73.2 and 79.4 for the three water-use alternatives, respectively. The three policies result in declines of 4.5, 3.7 and 4.0 feet per year, respectively. Projecting these rates of decline linearly, the irrigator pumping without restriction should have an additional 24 years (a total of 44) before encountering significant declines in pumping capacity. The graduated tax alternative should provide an additional 30 years of pumping (a total of 50 years) before significant reductions in well yields occur. The quantity limitation should provide an additional 35 years pumping (a total of 55) before significant reductions in well yields occur. The difference between the maximum (55) and minimum (44) number of years prior to encountering well yield reductions is sizable (11 years).

Mean net farm income under unrestricted pumping is significantly greater than mean net farm income under either the graduated tax of the quantity limitation for Resource Situation 2. Also, mean net farm income under the graduated tax is above that under the quantity limitation. Thus the unrestricted irrigator in Resource Situation 2 is able to maintain the highest level of net farm income while pumping the greatest quantity of water. Mean net farm income under graduated taxation, while significantly lower from a statistical standpoint, remains at a reasonable level. The relative variability of net farm income, as measured by the coefficient of variation is greatest under the quantity limitation and least under unrestricted pumping.

An analysis of the net worth generated under the three water-use alternatives indicates that the net worth under unrestricted pumping exceeds that of both the graduated tax and quantity limitation alternatives. Mean net worth under the graduated tax exceeds that under the quantity limitation. These results are expected based upon the difference in net farm income for each alternative.

15. Converting the streams of income over the 20-year period of the analysis to their present value does not change the general conclusions based on the undiscounted patterns of net farm income. Discount rates of 1 to 8 percent were used to test the sensitivity of the results to the discount rate. For Resource Situation 1, the present value of net farm income was greatest under graduated taxation, followed by unrestricted pumping and the quantity limitation regardless of the discount rate used. For Resource Situation 2, the present value of net farm income under unrestricted pumping exceeds present values under both graduated taxation and a quantity limitation. Thus the time pattern of net farm income under the three alternatives does not change the conclusions reached above.

16. The fourth objective of this study was to estimate the primary economic benefits from irrigation under projected use of the stock water supply with current restrictions and the effect of each method of water-use regulation on the pattern of primary benefits realized. Primary economic benefits were defined as the increase in net returns to land and management resulting from irrigated crop production that accrue to residents of the study area.

17. The primary economic benefits corresponding to the projected rates of irrigation development, production levels and water use under objective 1 were made. The first set of projections (Model I) indicated the study area's historic proportion of U.S. domestic needs can be met over the period with about 1.4 to 1.6 million acres of irrigated production. This is approximately the number of acres that were irrigated in 1965. Thus the Model I estimates of primary economic benefits approximate the level of primary benefits that would be projected if restrictions are

imposed limiting irrigation development to approximately the 1965 level (1.5 million acres). Estimates of primary economic benefits derived from Model I projections indicate annual benefits of 18.4 million to 21.9 million will result over the 35-year period if irrigation is limited to approximately 1.5 million acres per year. Production at this level will use approximately 23 percent of the stock water supply by the year 2000.

18. Removing the limitation on area production and restricting the development of irrigation only on the basis of resource availability in the study area resulted in a relatively rapid rate of irrigation development. The primary economic benefits estimated under these assumptions (Model II) resulted in a projected increase from $23.3 million for the acres irrigated in 1965 to $49.0 million for 2.7 million acres irrigated in 1970. The level of annual primary benefits remain relatively constant due to the increase of irrigation costs as the acreage increased to 3.4 million in 1990. Projections developed with Model II indicate 43.5 percent of the stock water supply will be exhausted by the year 2000, resulting in rapidly declining primary economic benefits.

19. The projections of primary economic benefits made using Models I and II assume current irrigation technology and hold it constant over the 1965-2000 period. They assume no real incentive is provided encouraging farmers to maximize efficiency of water use at the firm level. The firm level simulation results completed under objective 3 provide some information concerning the difference in efficiency of water use and primary economic benefits to be expected under two alternative methods of water-use regulation. The figures indicate imposing a quantity restriction of 1.5 acre feet per acre irrigated on farms having approximately 100 feet

saturated thickness would reduce net farm income approximately $1.23 per acre irrigated. Imposing the quantity restriction on firms having 325 feet of saturated thickness would reduce net farm income approximately $4.17 per acre. Applying weights to these two representative situations suggests that imposing the quantity restriction would reduce net farm income an average of $2.80 per acre irrigated in the study area. The reduction in annual primary economic benefits would be of approximately the same magnitued as the reduction in net farm income.

The reduction in net primary economic benefits resulting from imposing the quantity restriction would reduce primary economic benefits derived by more than 15 percent per acre irrigated, while reducing the amount of water used by 12.76 percent. Thus the analysis suggests imposing the quantity restriction would result in reduced primary economic benefits both per irrigated acre and per unit of water used.

20. An analysis of the primary economic benefits per acre irrigated under the graduated tax indicates an increase of more than 8 percent in primary economic benefits would result. The graduated tax would also restrict water use per acre irrigated by an estimated 8.24 percent. Thus the analysis indicates the use of a graduated tax would increase primary economic benefits per acre irrigated and per unit of water used. In addition, the analysis indicates annual average tax collections of approximately $.46 per irrigated acre would be available to defray administrative costs of a regulatory program and to provide other services of benefit to irrigators as well as other residents of the community.

21. The analysis suggests that a graduated taxation plan may be an effective means of limiting water use per acre irrigated and increasing

primary economic benefits per unit of water used without adversely affecting net farm income of individual farmers. However, additional analysis is needed with alternative arrangements and levels before a specific taxing arrangement is recommended for use in the Central Ogallala Formation.

PUBLICATIONS RESULTING FROM PROJECT

A. Ph. D. Dissertations

1. Bekure, Solomon E., "An Economic Analysis of the Intertemporal Allocation of Ground Water in the Central Ogallala Formation," (Unpublished Ph.D. thesis) Oklahoma State University, May, 1971.

2. Mapp, Harry P., Jr., "An Economic Analysis of Water-Use Regulation in the Central Ogallala Formation," (Unpublished Ph.D. Thesis) Oklahoma State University, May, 1972.

B. Publications

1. Mapp, Harry P., Jr., and Vernon R. Eidman, "Sequential Sampling and Simulation: An Optimizing Procedure," Western Agricultural Economics Association Proceedings, 1970, pp. 103-107.

2. Bekure, Solomon E., and Vernon R. Eidman, "Intertemporal Allocation of Ground Water in the Central Ogallala Formation: An Application of A Multistage Sequential Decision Model," Southern Journal of Agricultural Economics, Vol. III, December, 1971, pp. 155-160.

3. Eidman, Vernon R., "Some Projections for Irrigation in the Central Basin of the Ogallala Formation," A.E. Paper 7105 (mimeographed), Oklahoma Agricultural Experiment Station, Stillwater, February, 1971, 22 pages.

4. Clements, Alvin M., Jr., Harry P. Mapp, Jr., and Vernon R. Eidman, A Procedure for Correlating Events in Farm Firm Simulation Models, Oklahoma Agricultural Experiment Station Technical Bulletin T-131, August, 1971, 32 pages.

5. Eidman, Vernon, and Solomon Bekure, "Irrigation Development in the Central Basin of the Ogallala Formation--The Past and the Future," Current Farm Economics, Vol. 45, No. 1, March, 1972, pp. 3-13.

Several additional reports and articles are being prepared. The contribution of OWRR will be noted and copies of the publication furnished as they become available.

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