WATER RESOURCES PLANNING STUDIES
OKLAHOMA AND ARKANSAS, PHASE II (QUALITY)

PERIOD: SEPTEMBER 1, 1970 TO AUGUST 31, 1971

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REPORT PREPARED: AUGUST 1971

FINAL REPORT

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B-006-OKLA

Starting Date	September 1, 1970
Date of Report	August 31, 1971

INVESTIGATORS

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Enos L. Stover, Undergraduate Research Assistant

SUPPORTING AGENCIES

Office of Water Resources Research

Department of the Interior

Oklahoma State University

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WATER RESOURCES PLANNING STUDY
OKLAHOMA AND ARKANSAS, PHASE II (QUALITY)

Don F. Kincannon, Wen-hsiung Kao, and Enos L. Stover

1. Introduction

This is the final report for the Research Project Water Resources Planning Studies, Oklahoma and Arkansas, Phase II (Quality). The primary objective of the study was to develop a model that could be used for decision making in water quality management of the Arkansas River. The model developed centers around a zoned-optimization model which requires a uniform treatment level for groups of waste waters only within grouped zones and which minimizes the overall treatment.

This research project not only provided the opportunity to conduct this research but also it provided an opportunity for the training of two students. Mr. Kao is working toward the PhD degree and his thesis will be directed toward the work conducted under this research project. His thesis has not been completed at this time and he will continue in the present line of research. Mr. Stover has had the opportunity to work as an undergraduate research assistant which has provided some professional training during his undergraduate course work. He will be a graduate student in the Bioenvironmental Engineering program this coming school year.

II. Summary and Conclusions

A. Low Flow Studies

The distributions of low flows at Stations 7-1775, 7-1645, and 7-1945 in the Arkansas River Basin were investigated for the purpose of making design flow selections and for evaluating the goodness-of-fit of the distribution function applied. Two distribution functions were investigated. These being Johnson S_B distribution and Gumbel's limited distribution of the smallest value. The low flows used in the distribution studies were the annual minimum flows for various numbers of consecutive days. They were generated from computer analysis of daily streamflow records. Number of consecutive days taken were 1, 3, 7, 14, 30, 60, and 90 days.

Maximum deviation and sum-of-squares were the two criteria used for judging the distribution forms studied. It was found at Stations 1645 and 1945 (two stations without zero daily flow) Johnson S_B distribution had better goodness-of-fit than Gumbel's distribution did. It was also found that for stations having several zero flow days, such as Station 1775, only the minimum flows with significantly large number(s) of consecutive days, e.g., 14 days or more, could be fitted by either theoretical distribution function. This is a significant finding when one considers that the 7-day flow with one-tenth occurrence probability is the most widely used criterion for selection of design low flows.

Under a specified probability of occurrence the design flow determined from the distribution of minimum flow with small number of consecutive days, was found to be less than that determined from the distribution of minimum flows with large number of consecutive days. Therefore, when the minimum flow with large number of consecutive days is used, a small probability of occurrence, e.g., 0.05, should be selected in order to keep the design flow a moderate value.

B. Water Quality Management With the Application of Zone-Treatment Principle

In extending the application of the zone-treatment principle for regional water quality management, the feasibility of three zoning criteria were investigated. These three criteria are sub-basin, weight of influent BOD, and BOD-flow ratio. With the first criterion, a river basin is divided into sub-basins according to the distribution of its stream system. Treatment plants which discharge their wastes into the receiving waters in the same sub-basin are considered belonging to the same treatment zone. With the second criterion, treatment plants are grouped into treatment zones basing on their daily pounds of influent BOD. Within treatment zones, the greater the pounds of influent BOD, the higher the required percent BOD removal. Finally, with the third criterion, treatment plants are zoned according to their ratios of influent BOD and flow. The flow in this case is the summation of influent wastewater and design streamflow at the location of waste discharged. Similarly, between each zone, the larger the ratio, the higher the required percent BOD removal. For all three classifications, the percent BOD removal required should be uniform for all plants within each treatment zone; however, they can be different between zones.

In order to make a more complete comparison, two other methods of quality management were also considered. The first allows different percents BOD removal among treatment plants within a basin. The second requires a uniform percent BOD removal throughout the whole river basin.

A regional water quality management model involving the interrelationship of organic waste discharged, the stream assimilative capacity and the dissolved oxygen in the stream was formulated into a linear structure with an application of the Streeter-Phelps equation. The technique of linear programming was used to obtain a solution of the problem. In the selection of an objective function for optimizing the water quality system, a cost function of the activated sludge treatment plant was investigated. It was found that the relationship between treatment cost and treatment degree could be expressed by an exponential function. The expression is $Y = A + B \cdot R^X$, where Y is the cost in dollars, X is the percent BOD removal, and A, B, and R are coefficients. Because of much uncertainty and complexity involved in minimizing the summation of non-linear cost functions from all treatment plants in the basin, the cost function was not taken as the objective function. Instead, the total BOD removal within the basin in pounds per day, was selected as the optimization objective.

This study has shown that the "sub-basin" and "BOD-flow ratio" are two good zoning criteria. Both of them would be convenient to implement administratively, because they have shown the virtues of economy and equity in allocating the stream assimilative capacity.

III. Low Flow Studies

A. General

In the evaluation of the assimilative capacity of a stream, the characteristics of its low flow is one of the major considerations. Therefore, distributions of annual minimum flows with various numbers of consecutive days were investigated. Johnson S_B distribution and Gumbel's limited distribution of the smallest value were applied in this study for fitting the low flow data. Comparison of the goodness-of-fit among these two functions was made. The selection of a design low flow based upon distribution of minimum flow has also been developed.

B. Generation of Low Flow Data

In this study the minimum discharges, rather than the average minimum discharges, for various numbers of consecutive days within a year were used as low flow data. In order to cover the driest period as a whole, a year was defined as beginning on April 1 and ending on March 31. The average minimum flow is equal to the minimum flow divided by its corresponding number of consecutive days, e.g., average minimum 7-day flow = minimum 7-day flow divided by 7.

The determination of a design flow by using either the distribution of minimum flows or by using average minimum flows is essentially the same. However, for a small flow station, the use of minimum flows can provide more accurate results.

Three stations in the Oklahoma part of the Arkansas River basin were investigated. They are Stations 7-1775, 7-1645, and 7-1945. Station 1775, located at Bird Creek near Sperry, Oklahoma, has rather small flows. There are a significant number of days with zero flow at this station. Stations 1645 and 1945, located respectively on the Arkansas River near Tulsa and Muskogee have moderate to large magnitudes of flow. Daily flows recorded at these two stations are all greater than zero.

Annual minimum flows for various numbers of consecutive days at the three stations are shown in Appendix A. These flow data were generated from a computer analysis of daily flow records. Numbers of consecutive days taken into consideration include 1, 3, 7, 14, 30, 60, and 90 days. Flows in Appendix A are listed in a non-decreasing order of magnitude. Each flow has an assigned cumulative probability. The cumulative probability of the M^{th} order of flow is given by $M/(N+1)$, where N is the total years of records.

C. Johnson Distribution

The Johnson Distribution is an empirical distribution based on the transformation of a standard normal variate proposed by N. L. Johnson in 1949. Although it has been known since then, no application of it has been used in the field of water resources. Because of two considerations, the feasibility of applying this distribution in low flow studies was investigated. The first one that this distribution function has both lower and upper limits. Intuitively, the distribution of low flows should have limits at both ends. The second is that after transformation this distribution is in a form of standard normal distribution.

The general form of Johnson distribution is

$$z = \gamma + \eta T(x,\varepsilon,\lambda),\ \eta > o,\ -\infty < \gamma < \infty,\ \lambda > o,\ -\infty < \varepsilon < \infty, \qquad (1)$$

where z is a standard normal variate, x is the observation value, γ, η ε, and λ are parameters, and T is an arbitrary function. Three forms of function T were proposed by Johnson as follow.

$$T_1(x;\varepsilon,\lambda) = \ln\left(\frac{x - \varepsilon}{\lambda}\right),\ x \geq \varepsilon \qquad (2)$$

$$T_2(x;\varepsilon,\lambda) = \ln\left(\frac{x - \varepsilon}{\lambda + \varepsilon - x}\right),\ \varepsilon \leq x \leq \varepsilon + \lambda \qquad (3)$$

$$T_2(x;\varepsilon,\lambda) = \sinh^{-1}\left(\frac{x - \varepsilon}{\lambda}\right),\ -\infty < x < \infty \qquad (4)$$

The Johnson distribution with function 2 is known as Johnson S_L distribution which in fact is a log-normal distribution. Function 3 is Johnson S_B distribution. Function 4 is called Johnson S_U distribution.

To determine which of these three Johnson families should be applied for a given set of data, the first procedure is to estimate the relative measures of skewness and kurtosis of the data distribution, $\sqrt{\beta_1}$ and β_2, respectively. Then after entering these two estimated into a chart of β_1-β_2 relationship, an appropriate form of Johnson distribution could be decided. Figure 1 gives the chart for determining an appropriate distribution. This chart was plotted by the use of the following parametric equations. They are

$$\beta_1 = (w - 1)(w + 2)^2, \tag{5}$$

and

$$\beta_2 = w^4 + 2w^3 + 3w^2 - 3. \tag{6}$$

The range of the curves in Figure 1 can be extended as desired by the use of larger values of w.

The estimated of $\sqrt{\beta_1}$, and β_2 denoted as $\sqrt{b_1}$ and b, respectively, are

$$\sqrt{b_1} = \frac{m_3}{(m_2)^{3/2}} \tag{7}$$

and

$$b_2 = \frac{m_4}{(m_2)^2} \tag{8}$$

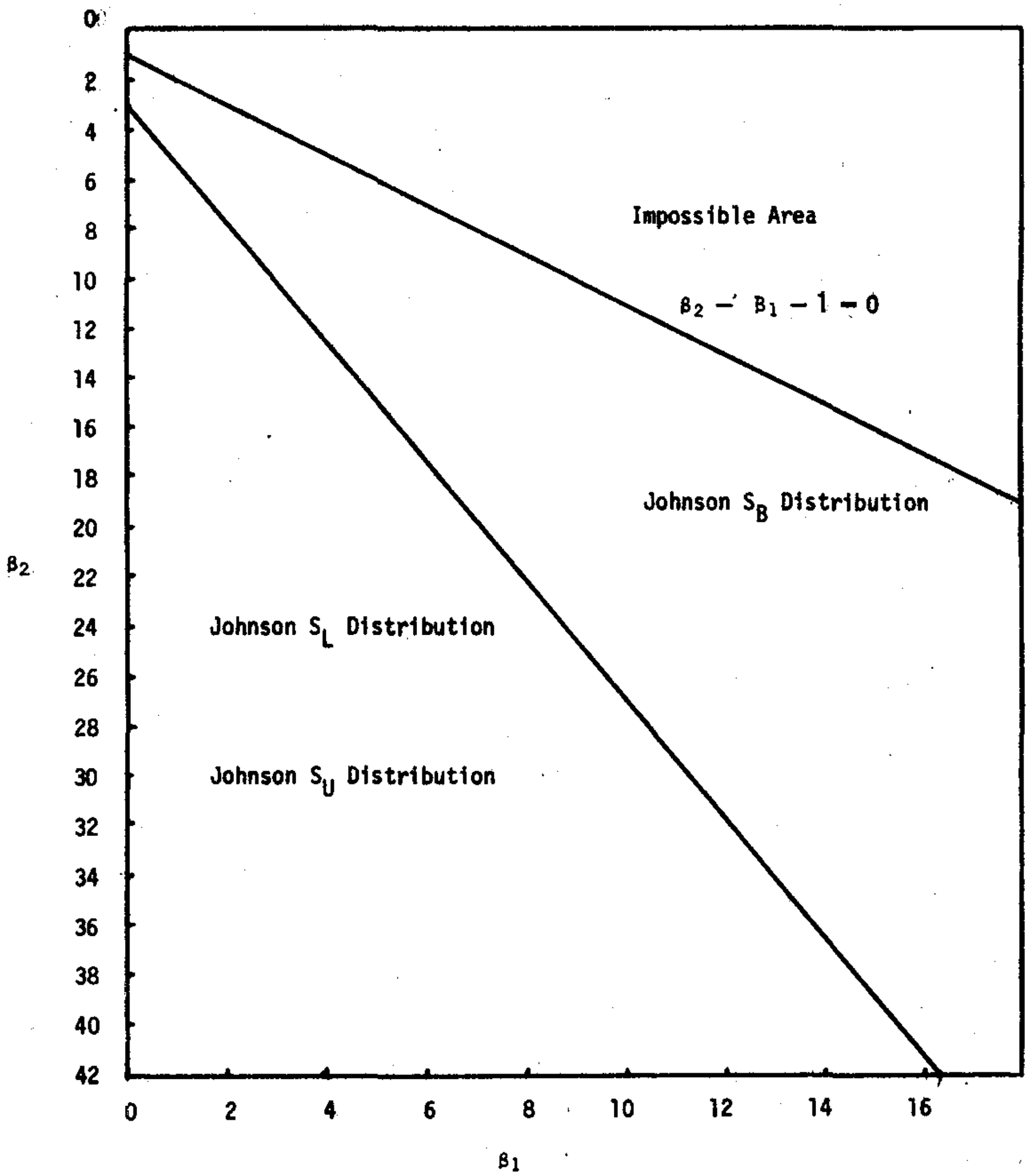

0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
40
42
β2
Impossible Area
β2 – β1 – 1 = 0
Johnson S_B Distribution
Johnson S_L Distribution
Johnson S_U Distribution
0
2
4
6
8
10
12
14
16
β1

FIGURE 1.

M_k is the estimated k^{th} central moment of distribution. It is given by

$$M_k = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^k}{n} \qquad (9)$$

where $\bar{x}$ is the data mean, and n is the sample size. In equation (7) and (8), k equals 3/2 and 2, respectively.

Values of b_1 and b_2 of low flow data for various numbers of consecutive days were calculated. Results for the three stations investigates are shown in Table 1. With no exception, these values indicate that among the three families considered Johnson S_B distribution is the most appropriate one to express the distribution of low flows.

From equations (1) and (3), the probability density functions of Johnson S_B distribution is given by

$$f(x) = \frac{\eta}{\sqrt{2\pi}} \frac{\lambda}{(x - \varepsilon)(\lambda - x + \varepsilon)} \exp\left\{ - \frac{1}{2}\left[\lambda + \eta \ln\left(\frac{x - \varepsilon}{\lambda - x + \varepsilon}\right)\right]^2 \right. ,$$

$$\varepsilon \leq x \leq \varepsilon + \lambda,\ \eta > o,\ -\infty < \gamma < \infty,\ \lambda > o,\ -\infty < \varepsilon < \infty. \qquad (10)$$

With the application of this equation, it is obvious that the distribution of the low flows is confined in a range between a lower limit, ε, and an upper limit $\varepsilon + \lambda$. Because the magnitude of streamflow can never be less than zero, the range of variation for the lower limit, ε, is specified as

$$o \leq \varepsilon < \infty$$

In order to use this distribution function to fit the flow data, the

TABLE I

APPROPRIATE JOHNSON DISTRIBUTIONS FOR MINIMUM FLOWS WITH VARIOUS NUMBERS OF CONSECUTIVE DAYS

	1-day	3-day	7-day	14-day	30-day	60-day	90-day
Station 1775							
	1-day	3-day	7-day	14-day	30-day	60-day	90-day
b_1	3.9628	3.7611	3.7347	4.5198	10.4529	4.6897	3.3262
b_2	6.6880	6.3422	6.1789	7.0526	13.0210	7.1918	6.3966
Distribution form	S_B	"	"	"	"	"	"
Station 1645							
	1-day	3-day	7-day	14-day	30-day	60-day	90-day
b_1	0.4663	0.7457	0.8154	0.7785	0.9324	1.2668	0.5415
b_2	2.8626	3.0624	3.1292	3.1976	3.4870	4.1825	2.8385
Distribution form	S_B	"	"	"	"	"	"
Station 1945							
	1-day	3-day	7-day	14-day	30-day	60-day	90-day
b_1	0.6703	0.8811	0.8498	1.6322	2.1417	2.3525	0.8444
b_2	3.2385	3.5142	3.2532	4.3787	5.0114	5.3478	2.9354
Distribution	S_B	"	"	"	"	"	"

magnitude of flow, x, was first transformed into a normal standard variate, z, by the equation

$$z = \gamma + \eta \ln \frac{x - \varepsilon}{\lambda + \varepsilon - x} \tag{11}$$

which is a summation form of equations (1) and (3). Then the cumulative probability from ε to x, i.e., $\int_{\varepsilon}^{x} F(x)dx$, was determined through the calculation of cumulative probability of normal distribution from $-\infty$ to z. This procedure of calculations is equivalent to the following expression

$$\int_{\varepsilon}^{x} f(x)dx = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{1}{2}z^2} \, dz, \tag{12}$$

where the right side is the cumulative probability function of the standard normal distribution.

An approximation method of determining cumulative probability for a known normal standard variate, z, was used in this study. This method was listed in IBM System/360 Scientific Subroutine Package, which is particularly convenient for the use of electronic computer. The procedures of this method are expressed in the following equations

(a) $T = 1.0/(1.0 + 0.2316419 \times |z|)$

(b) $D = 0.3989423 \exp\left(-\frac{1}{2}z^2\right)$

(c) $A = DT((((1.330274T - 1.821256)T + 1.781478)T - 0.3565638)T + 0.3193815)$

(d) if z is zero or positive,

$P = 1.0 - A$

if z is negative,

P = A

where P is the cumulative probability.

A computer program, basically written at Chevron Research Corporation, was used for the equation fitting. The listing of the program is given in Appendix B. This program accepts as input the data to be fitted and the initial estimates of the parameters in the equation used. From the use of the subroutines in the program, the parameter estimates are improved until the changes in all parameter values given by successive estimates are not greater than a percentage specified. The set giving the least sum-of-squares fit is then taken as a convergence. Printout includes original and calculated values of dependent variable, the percent deviation for each datum point, as well as the parameter values. Average and maximum deviations are also printed out.

Results from the application of the Johnson S_B distribution function in fitting low flow data are shown numerically in Appendix C. Also shown are the sum-of-squares, maximum deviations of cumulative probability, and estimates of parameters. The results are also expressed graphically in Figures 2 through 22. These figures also show the results from the application of Gumbel's limited distribution, a distribution function which will be discussed in the next section.

D. Gumbel's Limited Distribution of the Smallest Value

This is a probability distribution widely used in low flow studies. It was first proposed by E. J. Gumbel in 1954. It is essentially a modification of the Weibull distribution which is also called Type III asymptotic distribution for minimum values.

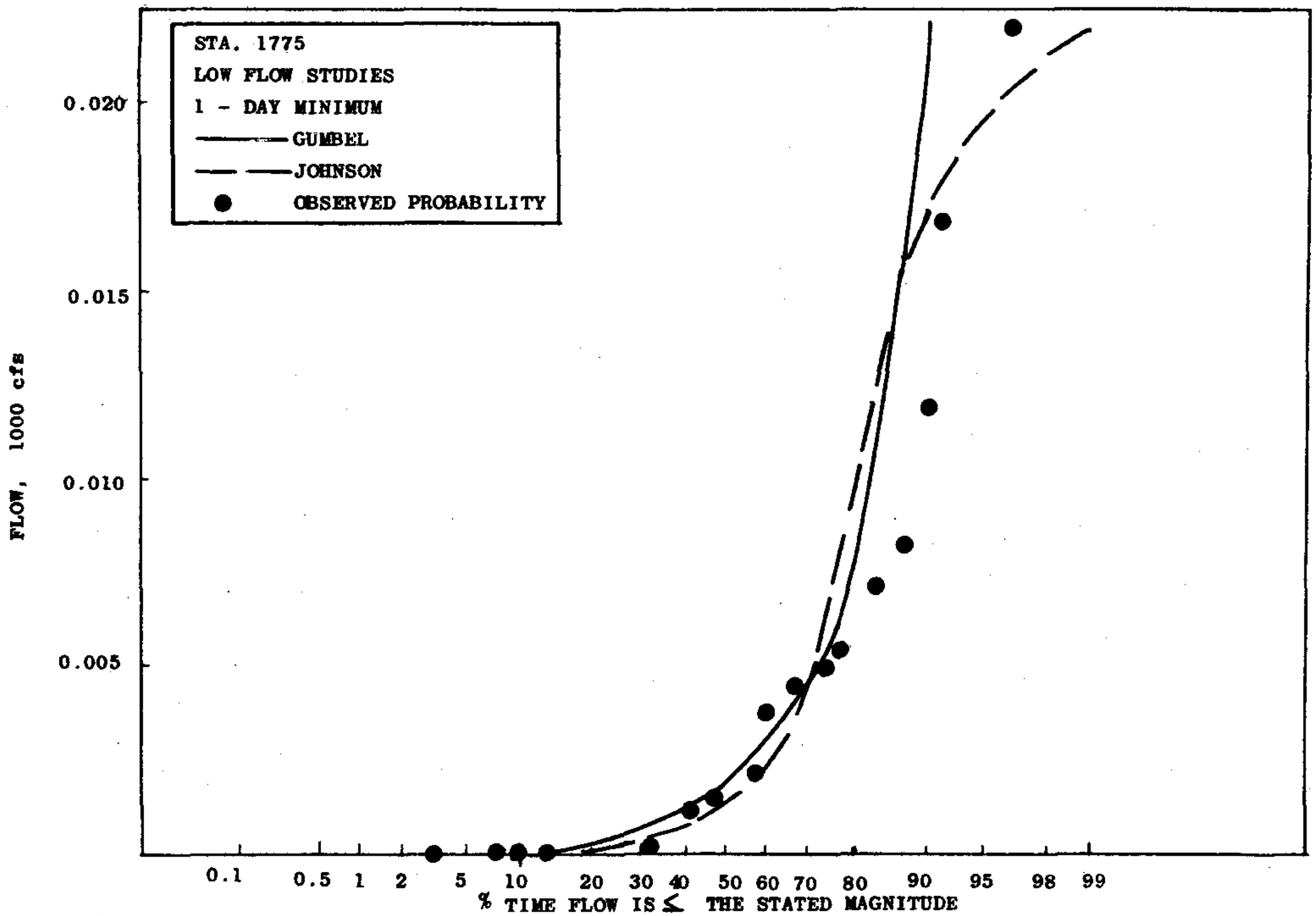
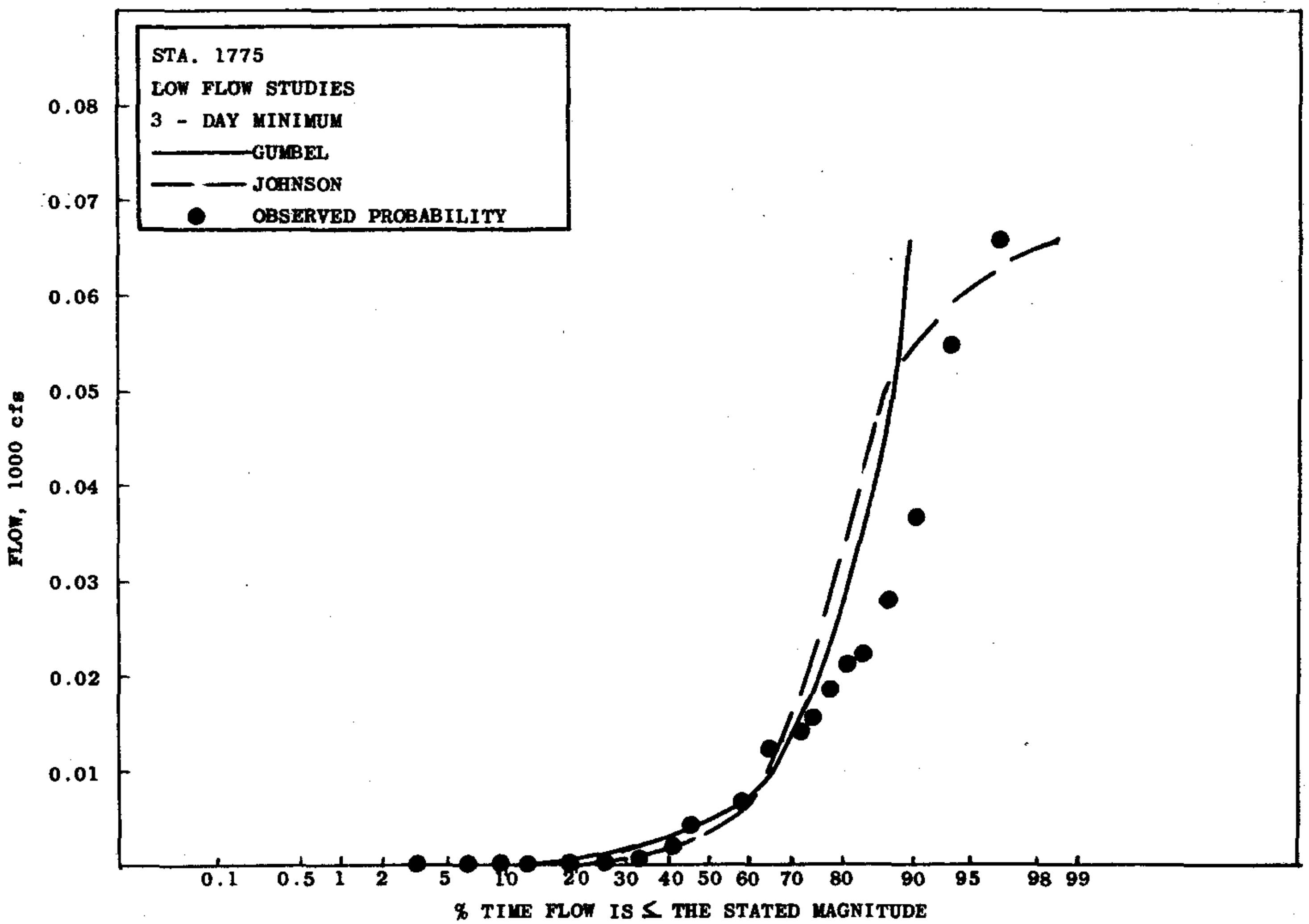

STA. 1775
LOW FLOW STUDIES
1 - DAY MINIMUM
GUMBEL
JOHNSON
OBSERVED PROBABILITY
0.020
0.015
0.010
0.005
FLOW, 1000 cfs
0.1
0.5
1
2
5
10
20
30
40
50
60
70
80
90
95
98
99
% TIME FLOW IS ≤ THE STATED MAGNITUDE

FIGURE 2.



STA. 1775
LOW FLOW STUDIES
3 - DAY MINIMUM
GUMBEL
JOHNSON
OBSERVED PROBABILITY
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
FLOW, 1000 cfs
0.1
0.5
1
2
5
10
20
30
40
50
60
70
80
90
95
98
99
% TIME FLOW IS ≤ THE STATED MAGNITUDE

FIGURE 3.

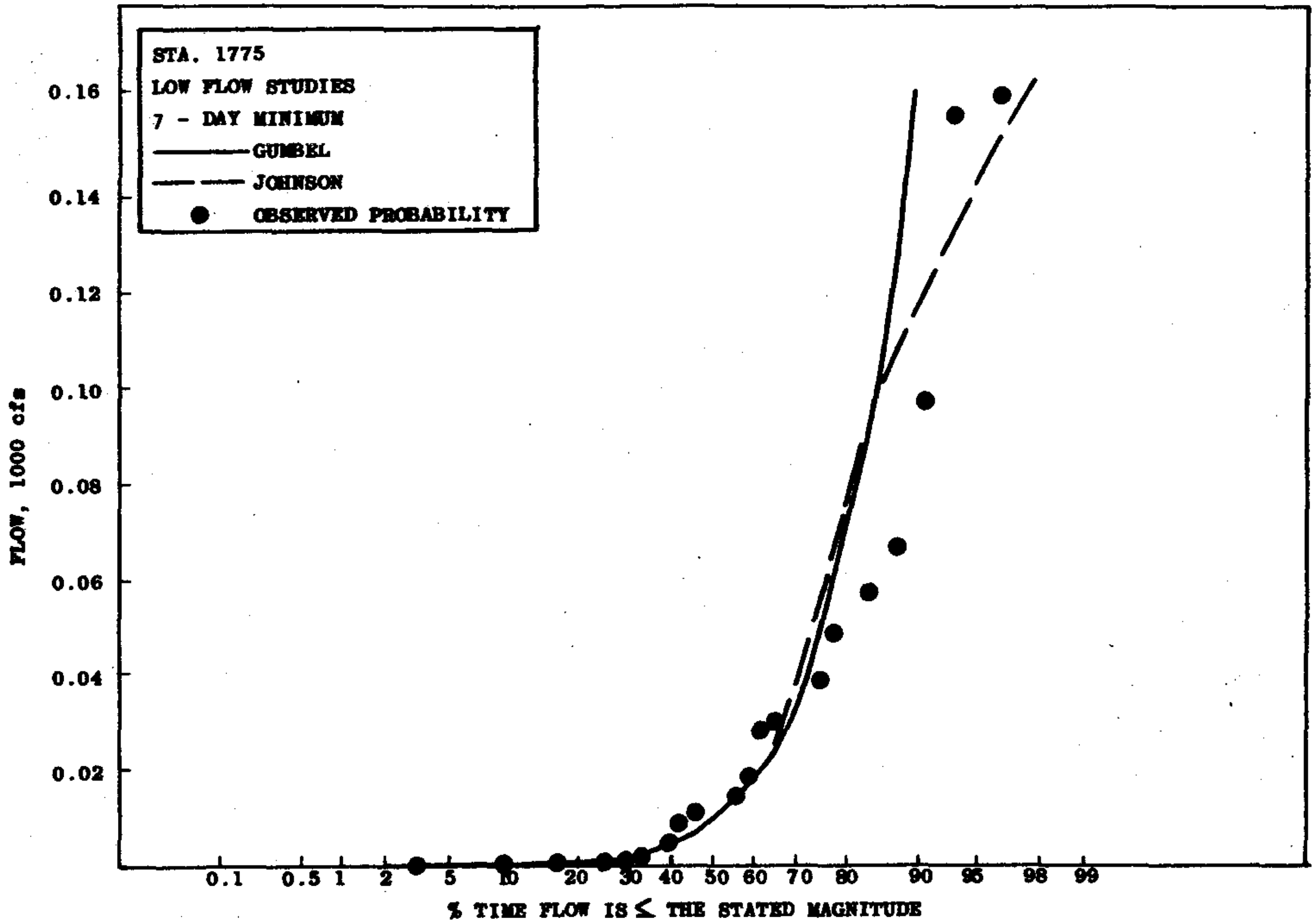

STA. 1775
LOW FLOW STUDIES
7 - DAY MINIMUM
GUMBEL
JOHNSON
OBSERVED PROBABILITY
FLOW, 1000 cfs
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.1
0.5
1
2
5
10
20
30
40
50
60
70
80
90
95
98
99
% TIME FLOW IS ≤ THE STATED MAGNITUDE

FIGURE 4.

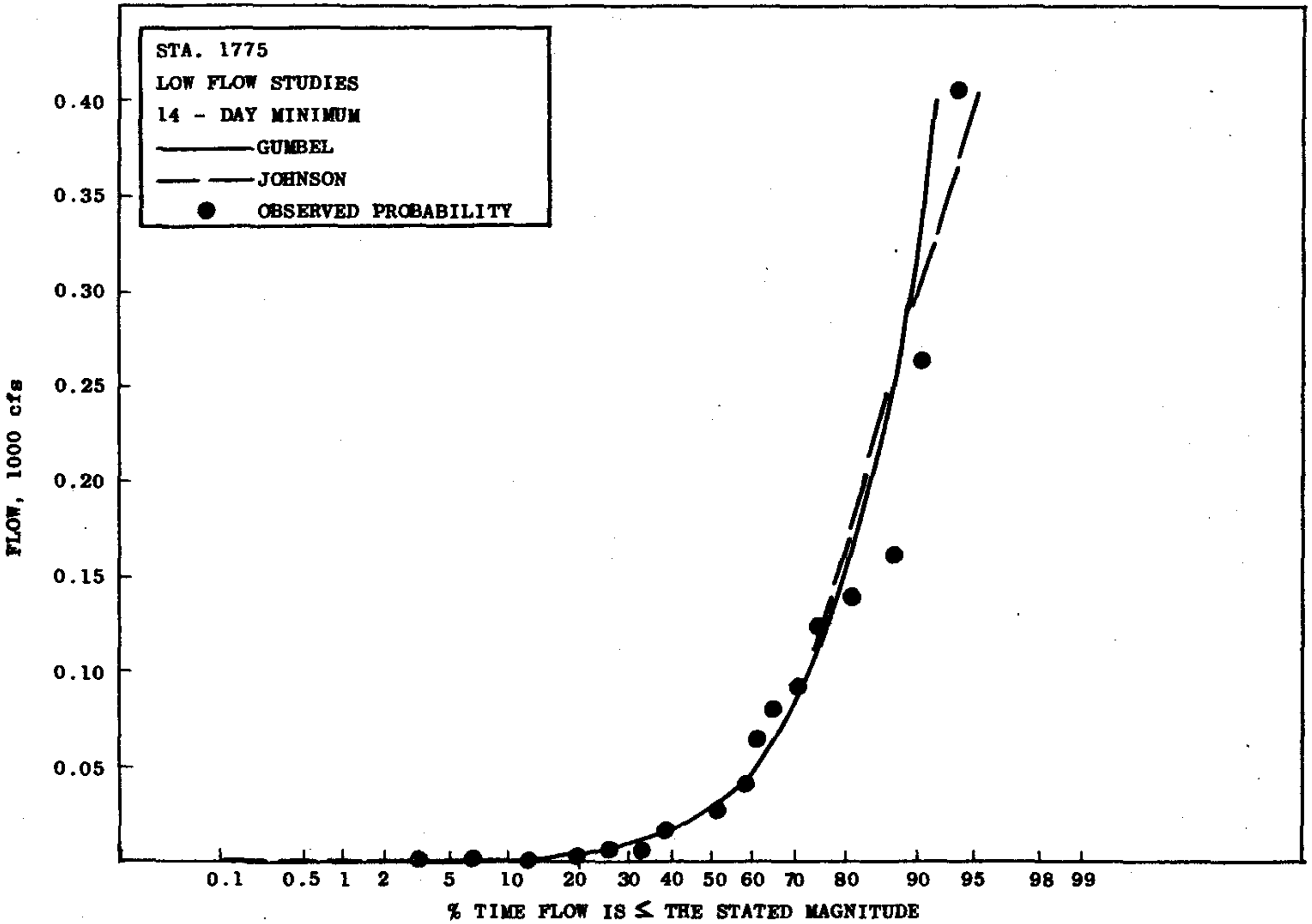

STA. 1775
LOW FLOW STUDIES
14 - DAY MINIMUM
GUMBEL
JOHNSON
OBSERVED PROBABILITY
FLOW, 1000 cfs
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.1
0.5
1
2
5
10
20
30
40
50
60
70
80
90
95
98
99
% TIME FLOW IS ≤ THE STATED MAGNITUDE

FIGURE 5.

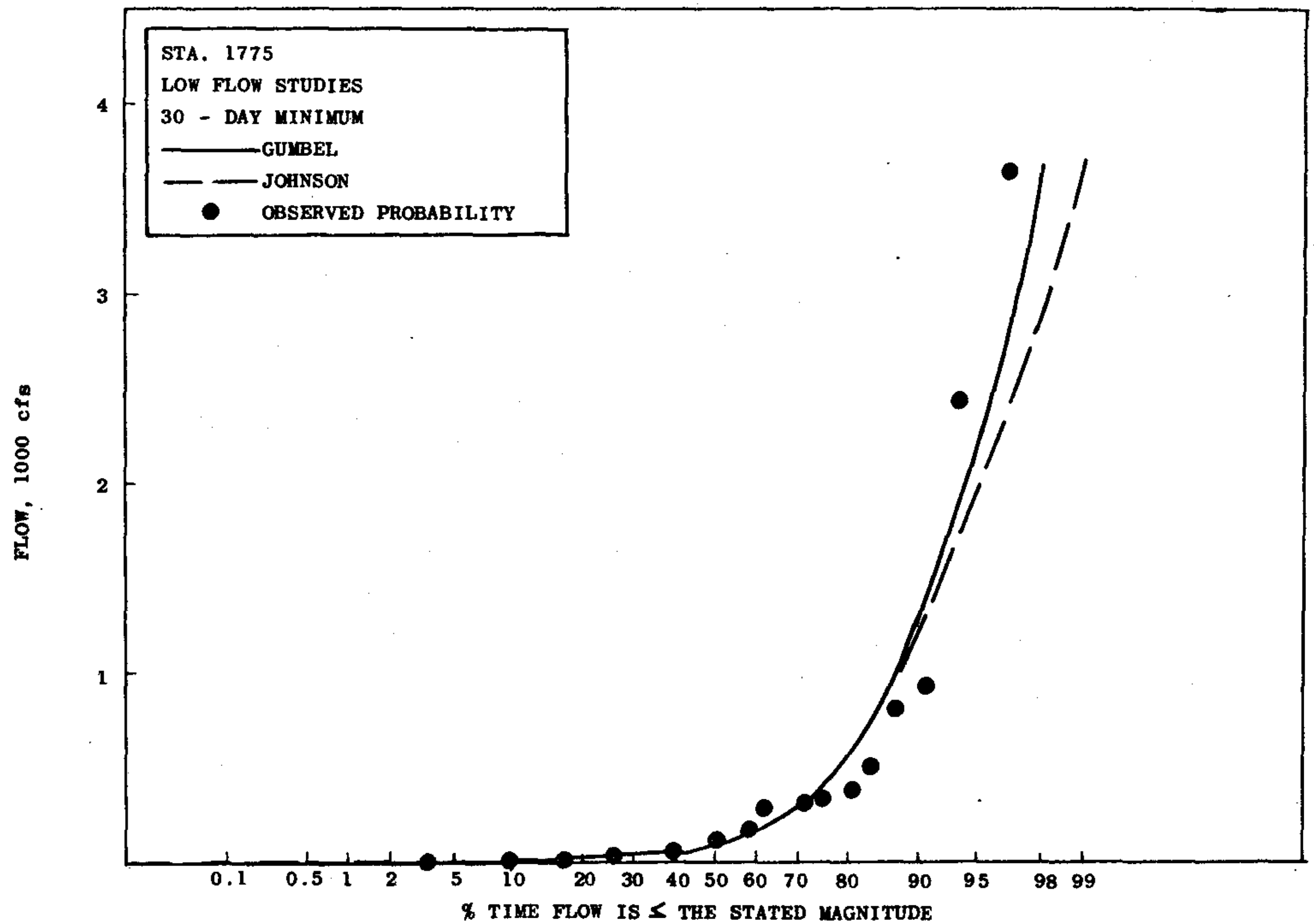
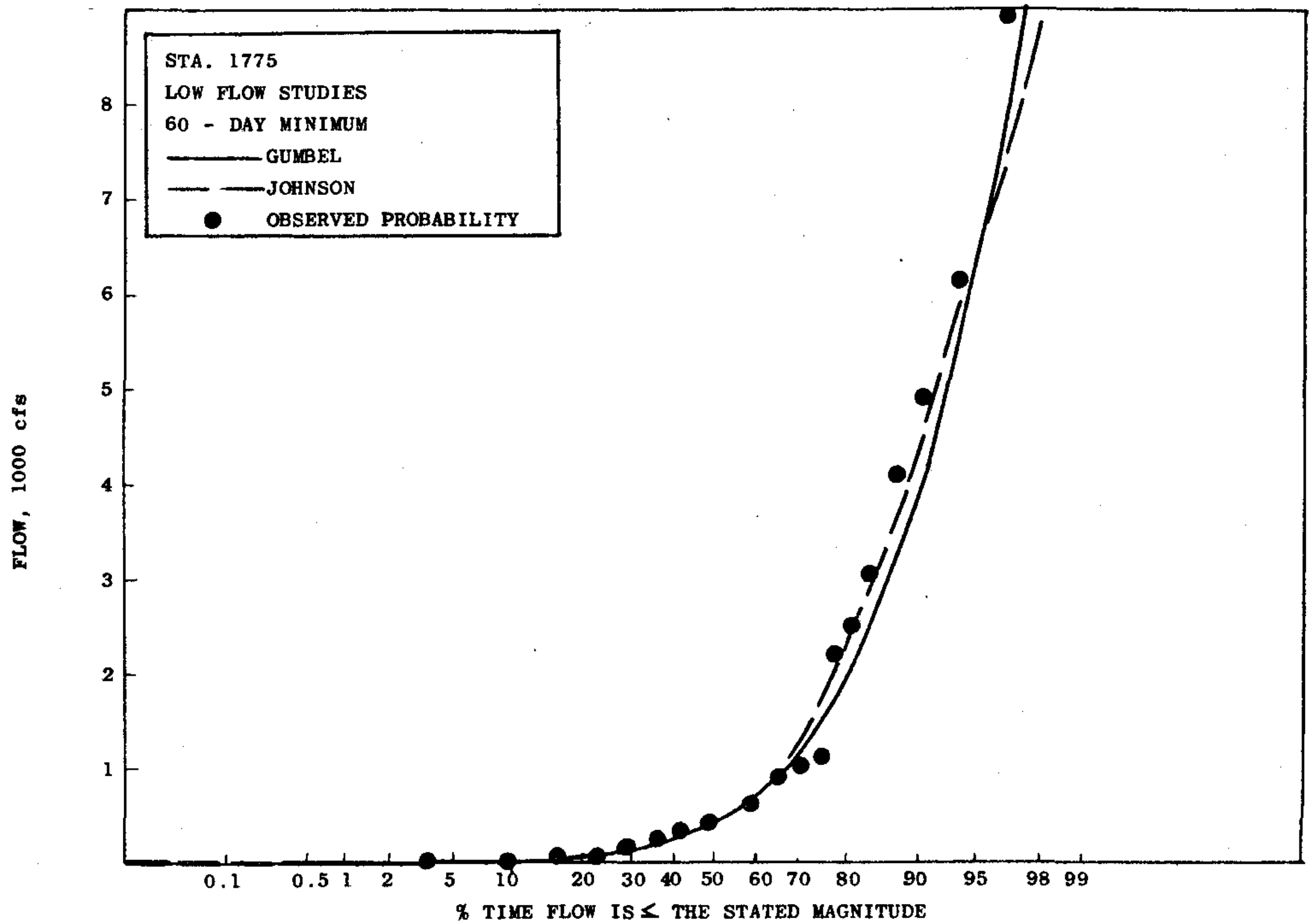

STA. 1775
LOW FLOW STUDIES
30 - DAY MINIMUM
GUMBEL
JOHNSON
OBSERVED PROBABILITY
FLOW, 1000 cfs
4
3
2
1
0.1
0.5
1
2
5
10
20
30
40
50
60
70
80
90
95
98
99
% TIME FLOW IS ≤ THE STATED MAGNITUDE

FIGURE 6.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LOW FLOW STUDIES
60 - DAY MINIMUM
GUMBEL
JOHNSON
OBSERVED PROBABILITY
FLOW, 1000 cfs
8
7
6
5
4
3
2
1
0.1
0.5 1
2
5
10
20
30 40
50
60 70
80
90
95
98 99
% TIME FLOW IS ≤ THE STATED MAGNITUDE

FIGURE 7.

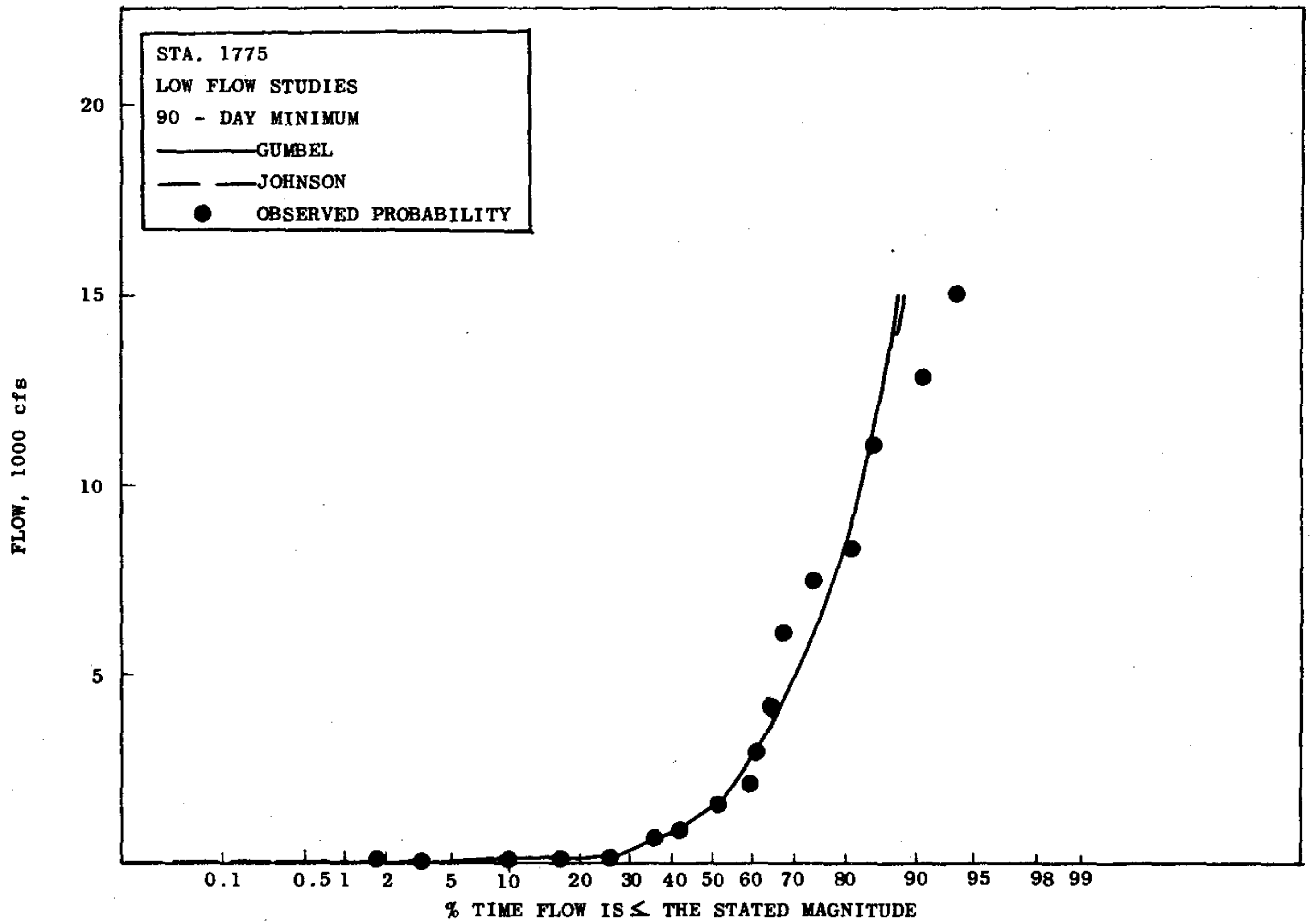

STA. 1775
LOW FLOW STUDIES
90 - DAY MINIMUM
GUMBEL
JOHNSON
OBSERVED PROBABILITY
20
15
10
5
FLOW, 1000 cfs
0.1
0.5 1
2
5
10
20
30
40
50
60
70
80
90
95
98 99
% TIME FLOW IS ≤ THE STATED MAGNITUDE

FIGURE 8.

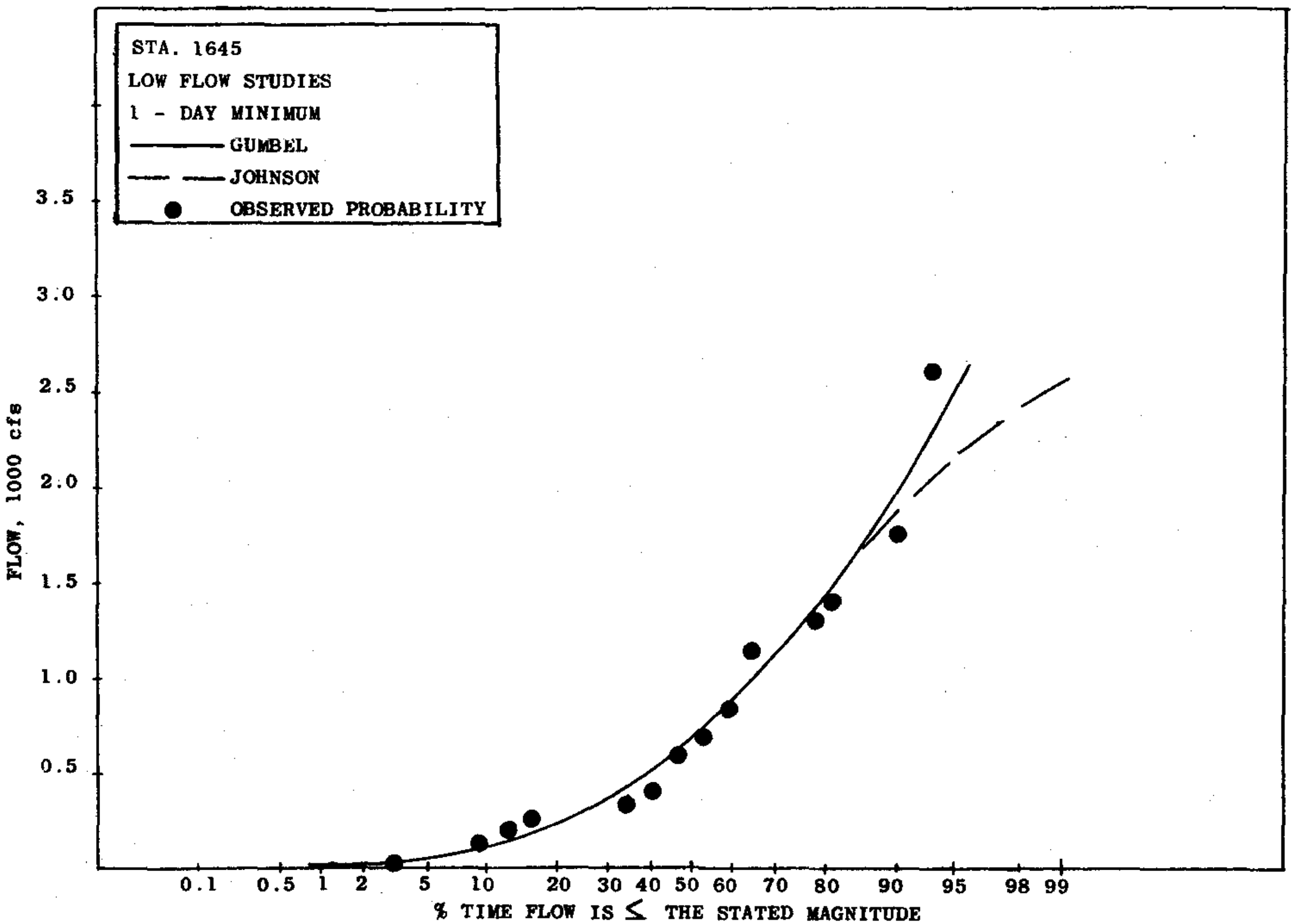

STA. 1645
LOW FLOW STUDIES
1 - DAY MINIMUM
GUMBEL
JOHNSON
OBSERVED PROBABILITY
FLOW, 1000 cfs
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.1
0.5
1
2
5
10
20
30
40
50
60
70
80
90
95
98
99
% TIME FLOW IS ≤ THE STATED MAGNITUDE

FIGURE 9.

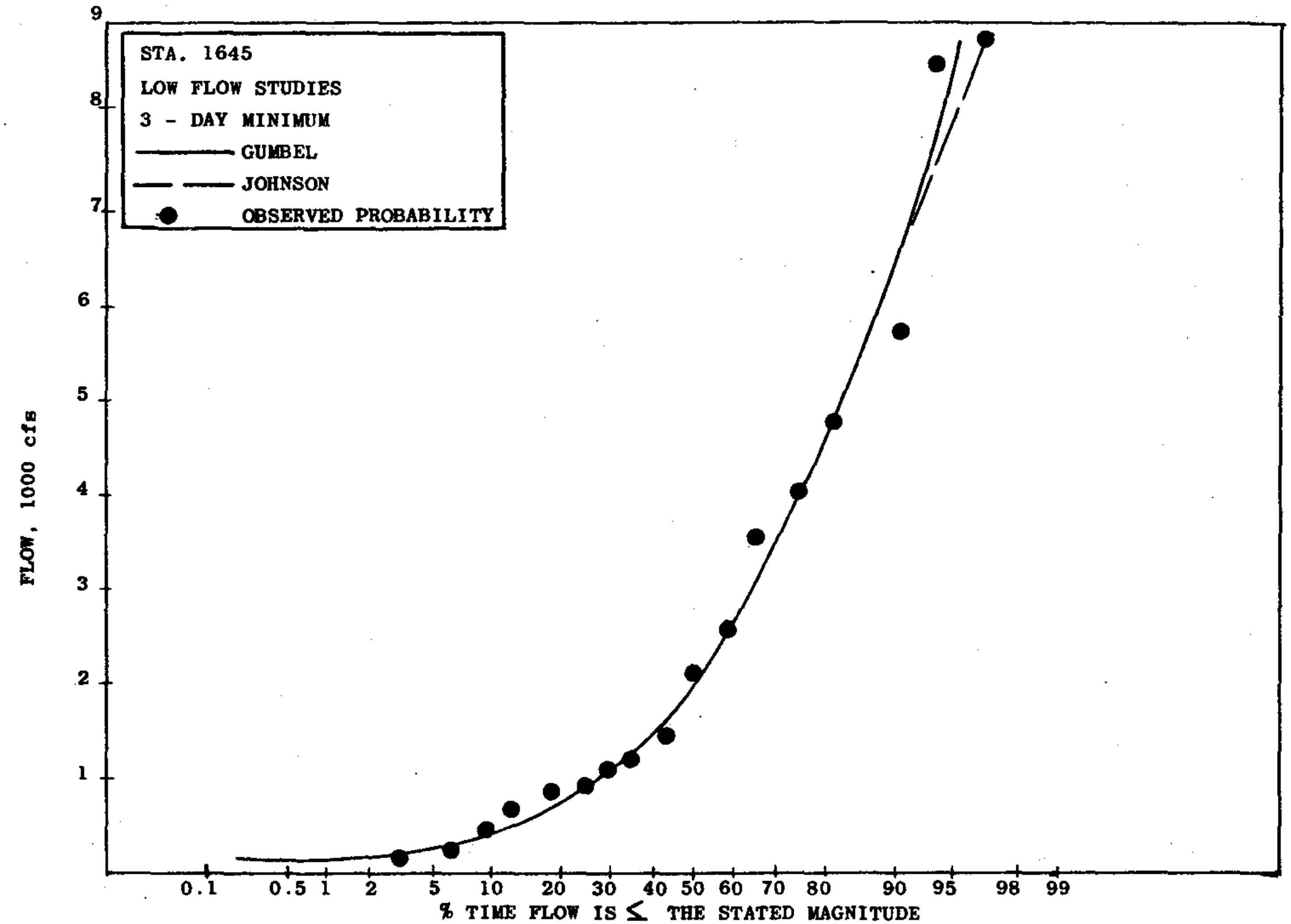

STA. 1645
LOW FLOW STUDIES
3 - DAY MINIMUM
GUMBEL
JOHNSON
OBSERVED PROBABILITY
FLOW, 1000 cfs
9
8
7
6
5
4
3
2
1
0.1
0.5
1
2
5
10
20
30
40
50
60
70
80
90
95
98
99
% TIME FLOW IS ≤ THE STATED MAGNITUDE

FIGURE 10.

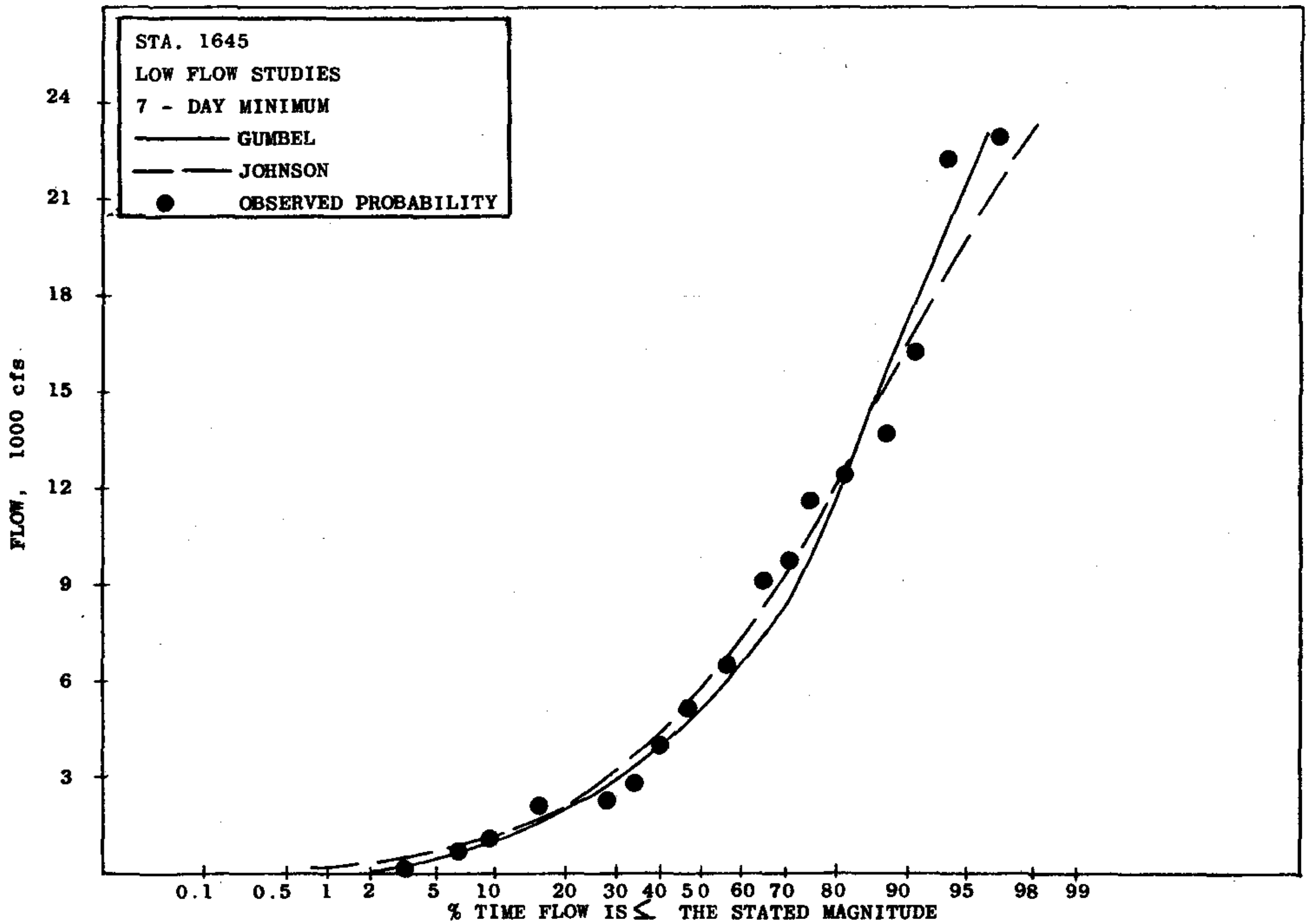

STA. 1645
LOW FLOW STUDIES
7 - DAY MINIMUM
GUMBEL
JOHNSON
OBSERVED PROBABILITY
FLOW, 1000 cfs
24
21
18
15
12
9
6
3
0.1
0.5
1
2
5
10
20
30
40
50
60
70
80
90
95
98
99
% TIME FLOW IS ≤ THE STATED MAGNITUDE

FIGURE 11.

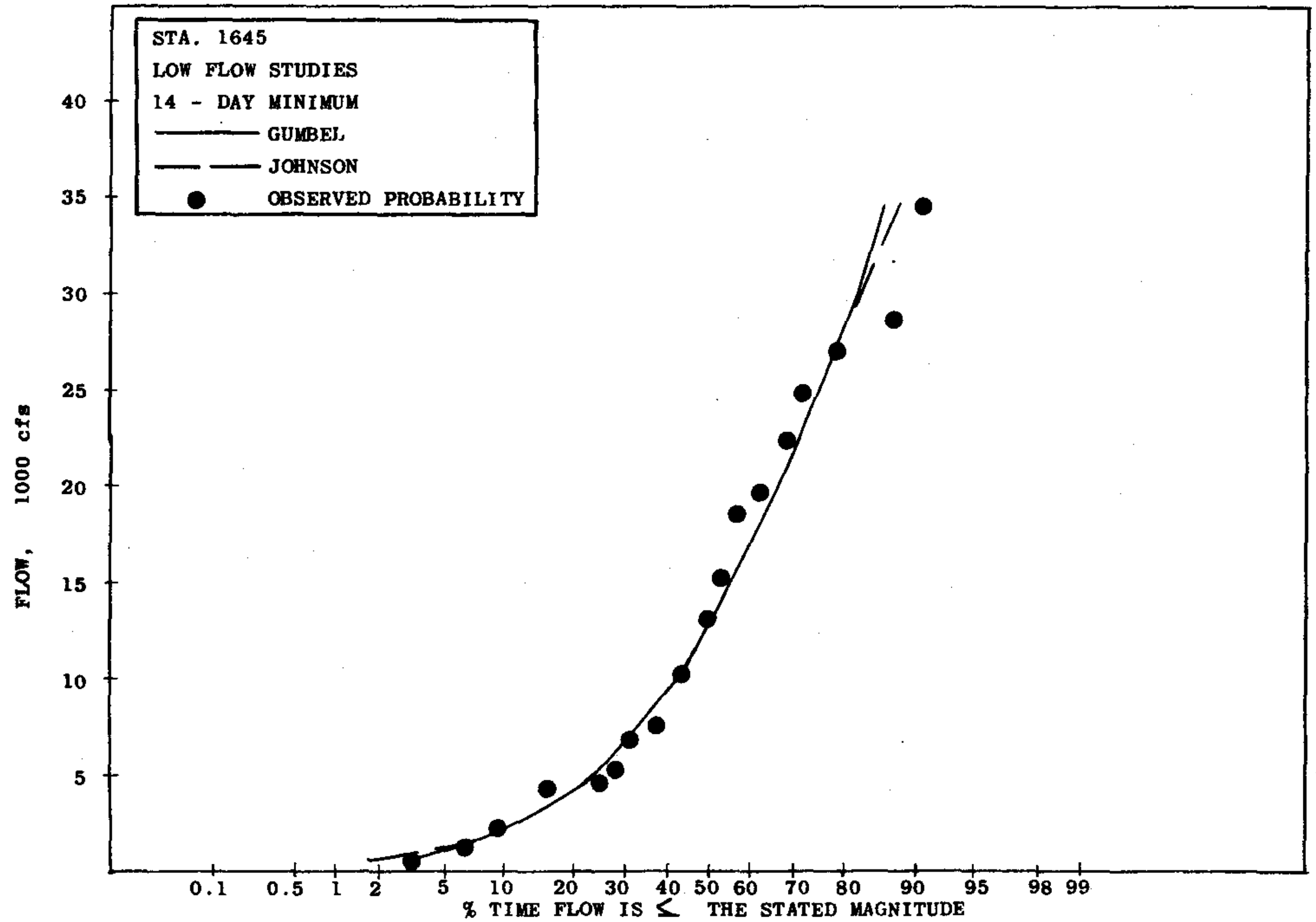
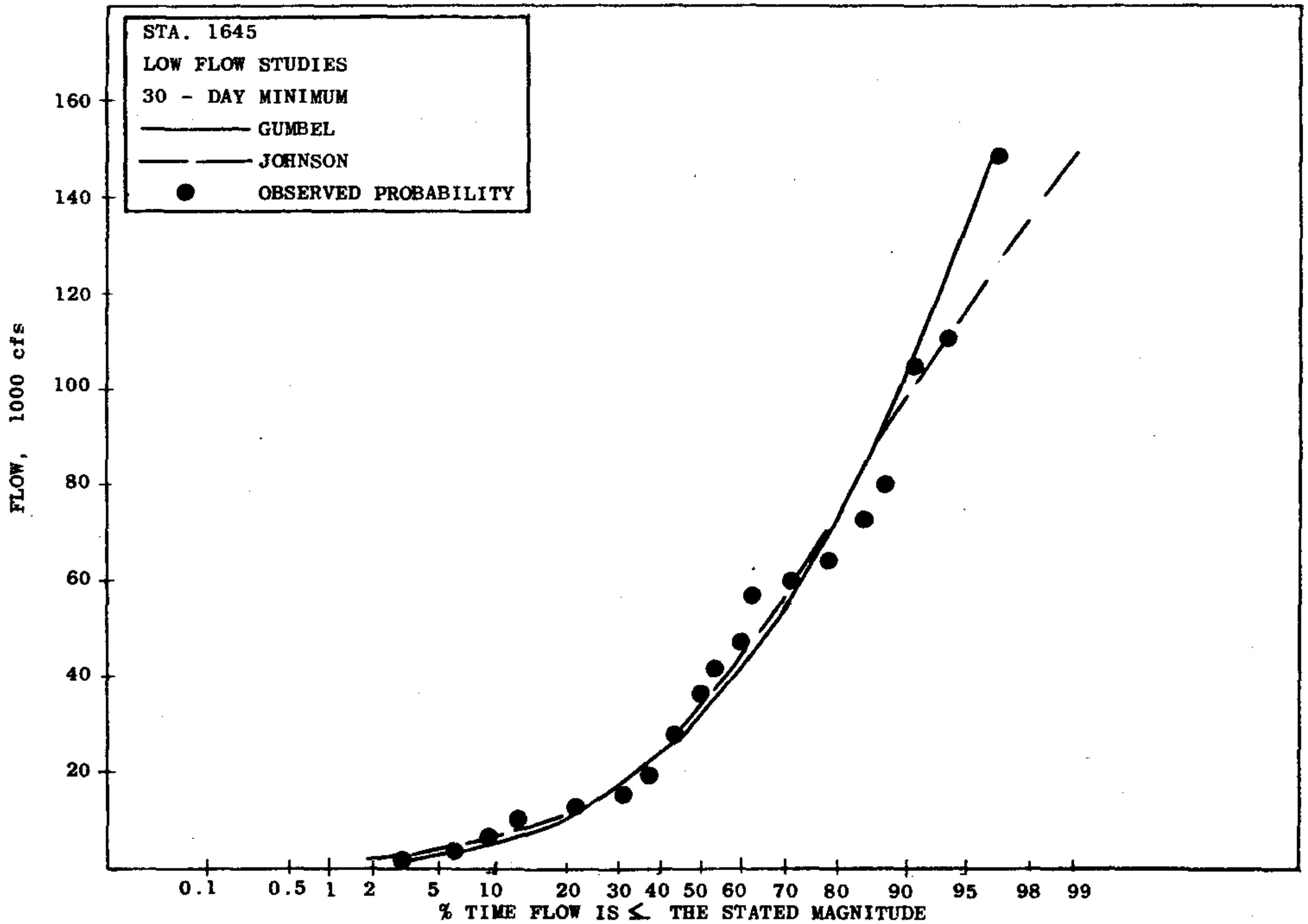

STA. 1645
LOW FLOW STUDIES
14 - DAY MINIMUM
GUMBEL
JOHNSON
OBSERVED PROBABILITY
FLOW, 1000 cfs
40
35
30
25
20
15
10
5
0.1 0.5 1 2 5 10 20 30 40 50 60 70 80 90 95 98 99
% TIME FLOW IS ≤ THE STATED MAGNITUDE

FIGURE 12.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LOW FLOW STUDIES
30 - DAY MINIMUM
GUMBEL
JOHNSON
OBSERVED PROBABILITY
FLOW, 1000 cfs
160
140
120
100
80
60
40
20
0.1
0.5
1
2
5
10
20
30
40
50
60
70
80
90
95
98
99
% TIME FLOW IS ≤ THE STATED MAGNITUDE

FIGURE 13.

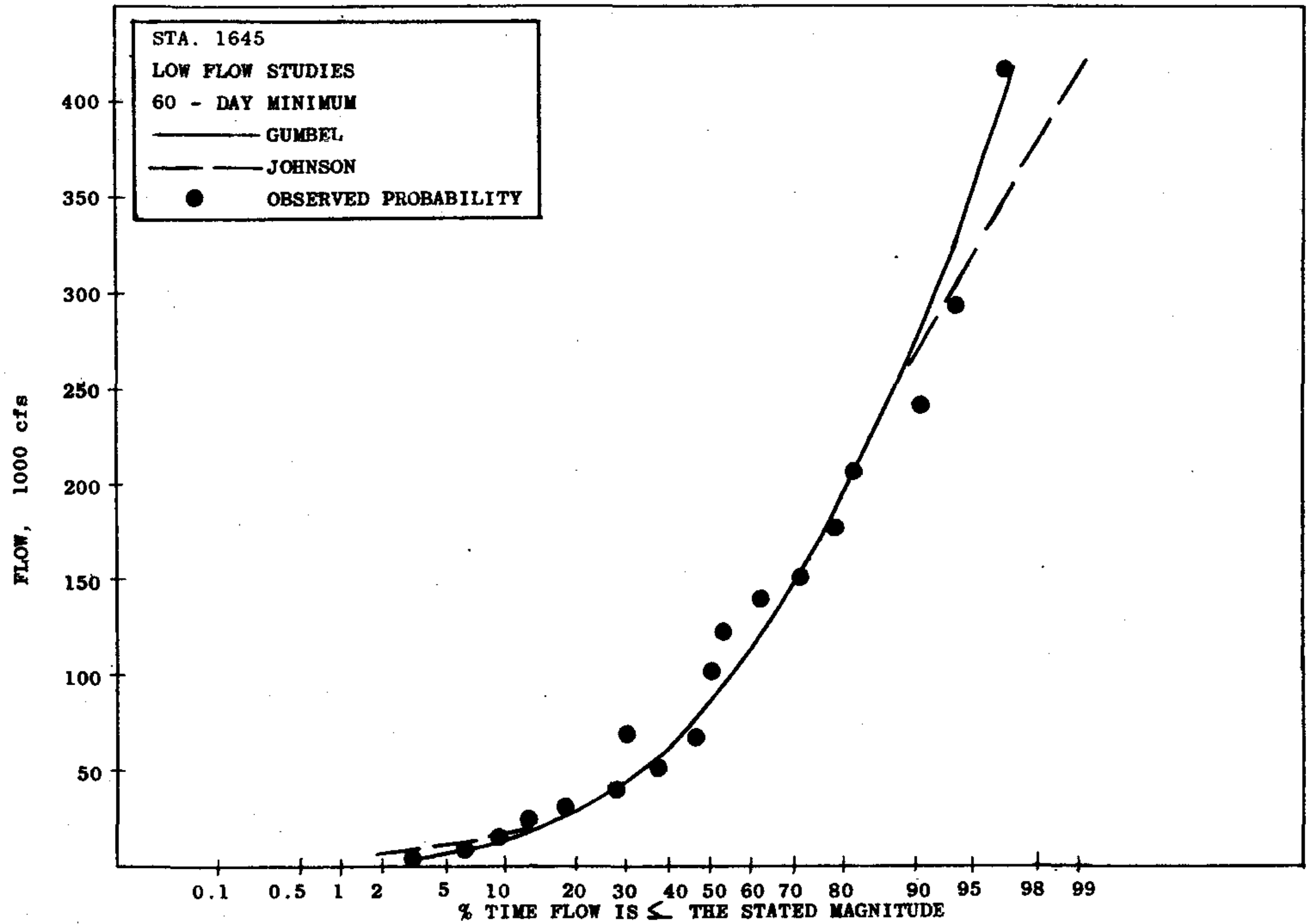

STA. 1645
LOW FLOW STUDIES
60 - DAY MINIMUM
GUMBEL
JOHNSON
OBSERVED PROBABILITY
FLOW, 1000 cfs
400
350
300
250
200
150
100
50
0.1 0.5 1 2 5 10 20 30 40 50 60 70 80 90 95 98 99
% TIME FLOW IS ≤ THE STATED MAGNITUDE

FIGURE 14.

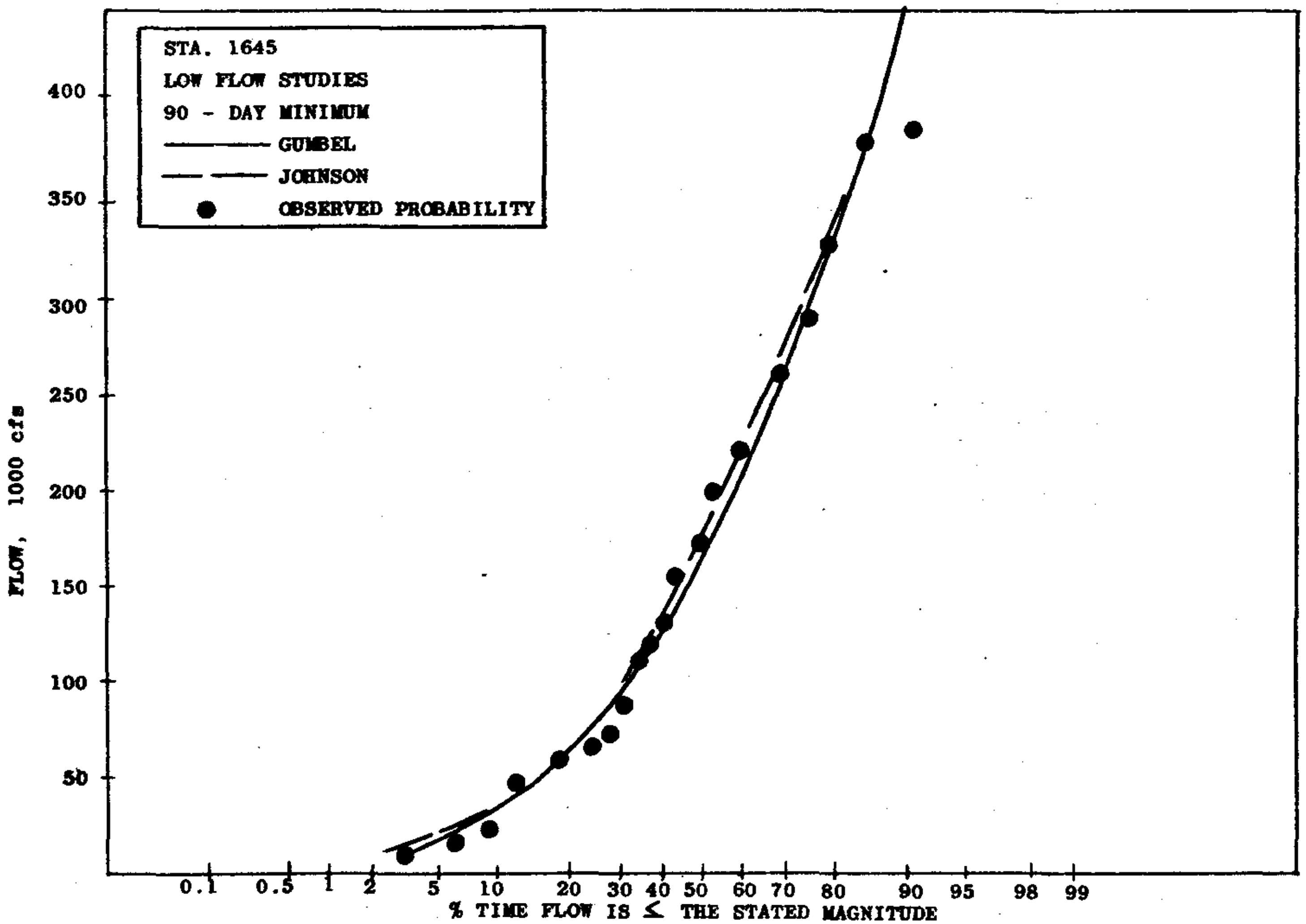

STA. 1645
LOW FLOW STUDIES
90 - DAY MINIMUM
GUMBEL
JOHNSON
OBSERVED PROBABILITY
FLOW, 1000 cfs
400
350
300
250
200
150
100
50
0.1
0.5
1
2
5
10
20
30
40
50
60
70
80
90
95
98
99
% TIME FLOW IS ≤ THE STATED MAGNITUDE

FIGURE 15.

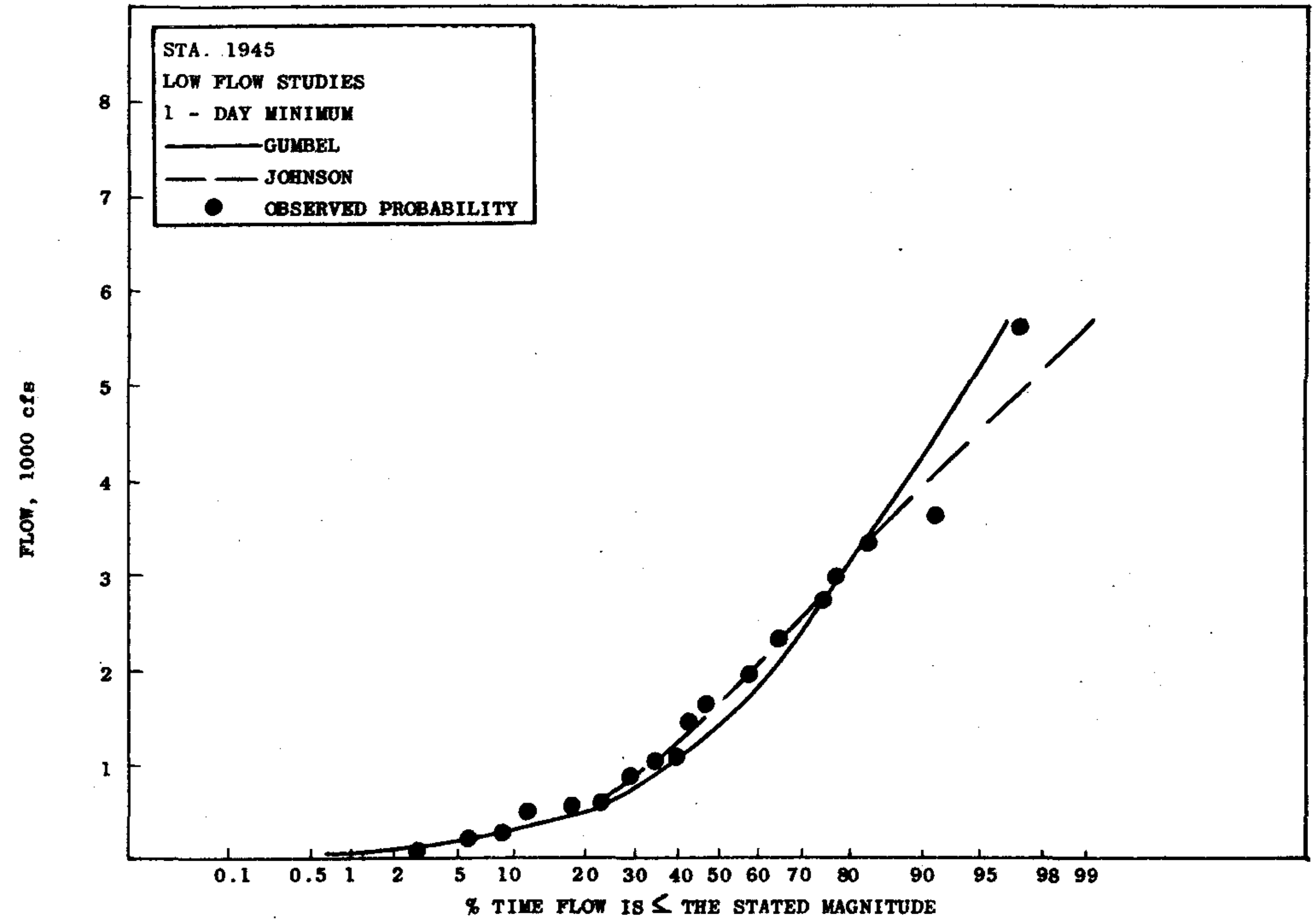
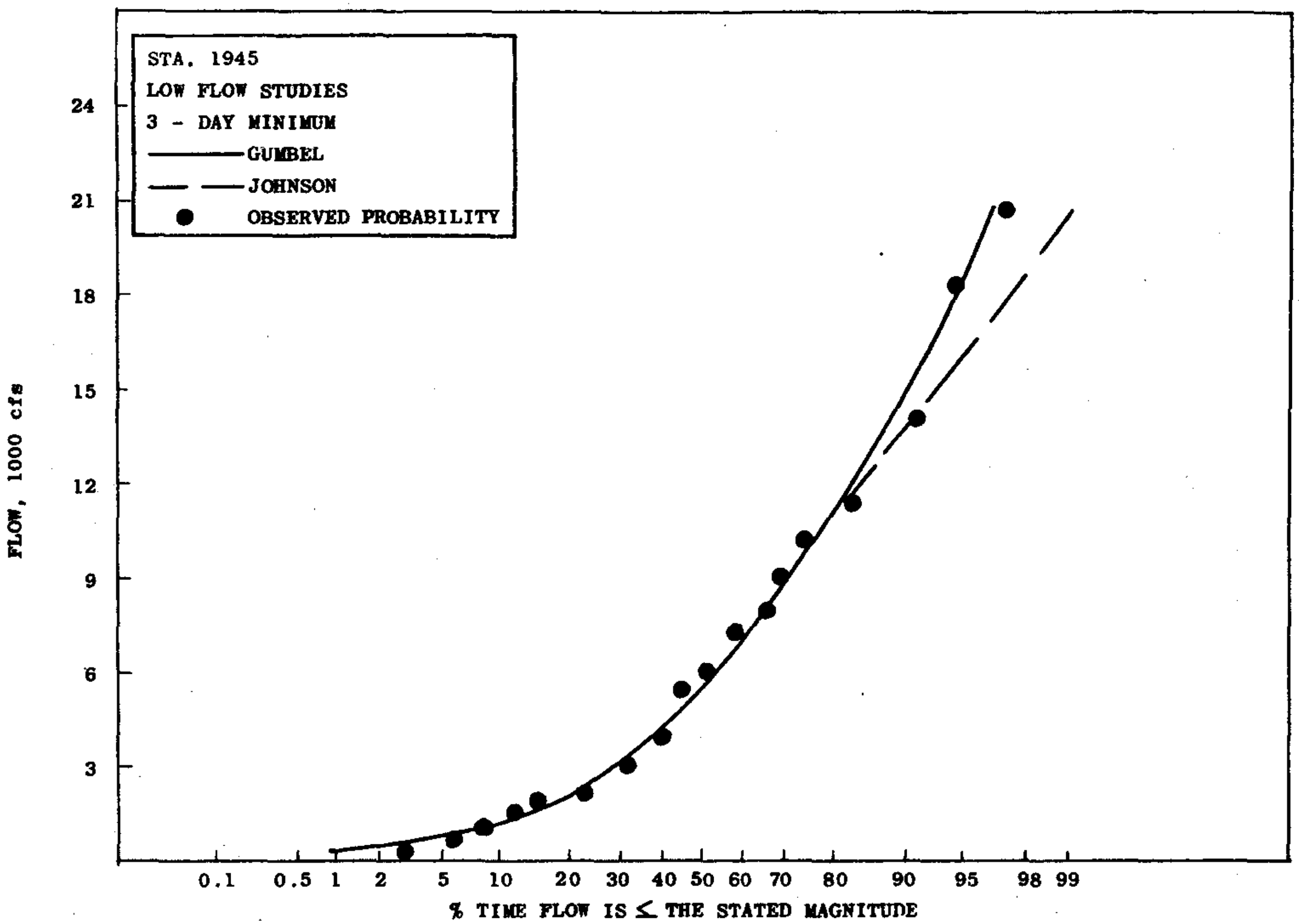

STA. 1945
LOW FLOW STUDIES
1 - DAY MINIMUM
GUMBEL
JOHNSON
OBSERVED PROBABILITY
FLOW, 1000 cfs
1
2
3
4
5
6
7
8
0.1
0.5
1
2
5
10
20
30
40
50
60
70
80
90
95
98
99
% TIME FLOW IS ≤ THE STATED MAGNITUDE

FIGURE 16.



STA. 1945
LOW FLOW STUDIES
3 - DAY MINIMUM
GUMBEL
JOHNSON
OBSERVED PROBABILITY
24
21
18
15
12
9
6
3
FLOW, 1000 cfs
0.1 0.5 1 2 5 10 20 30 40 50 60 70 80 90 95 98 99
% TIME FLOW IS ≤ THE STATED MAGNITUDE

FIGURE 17.

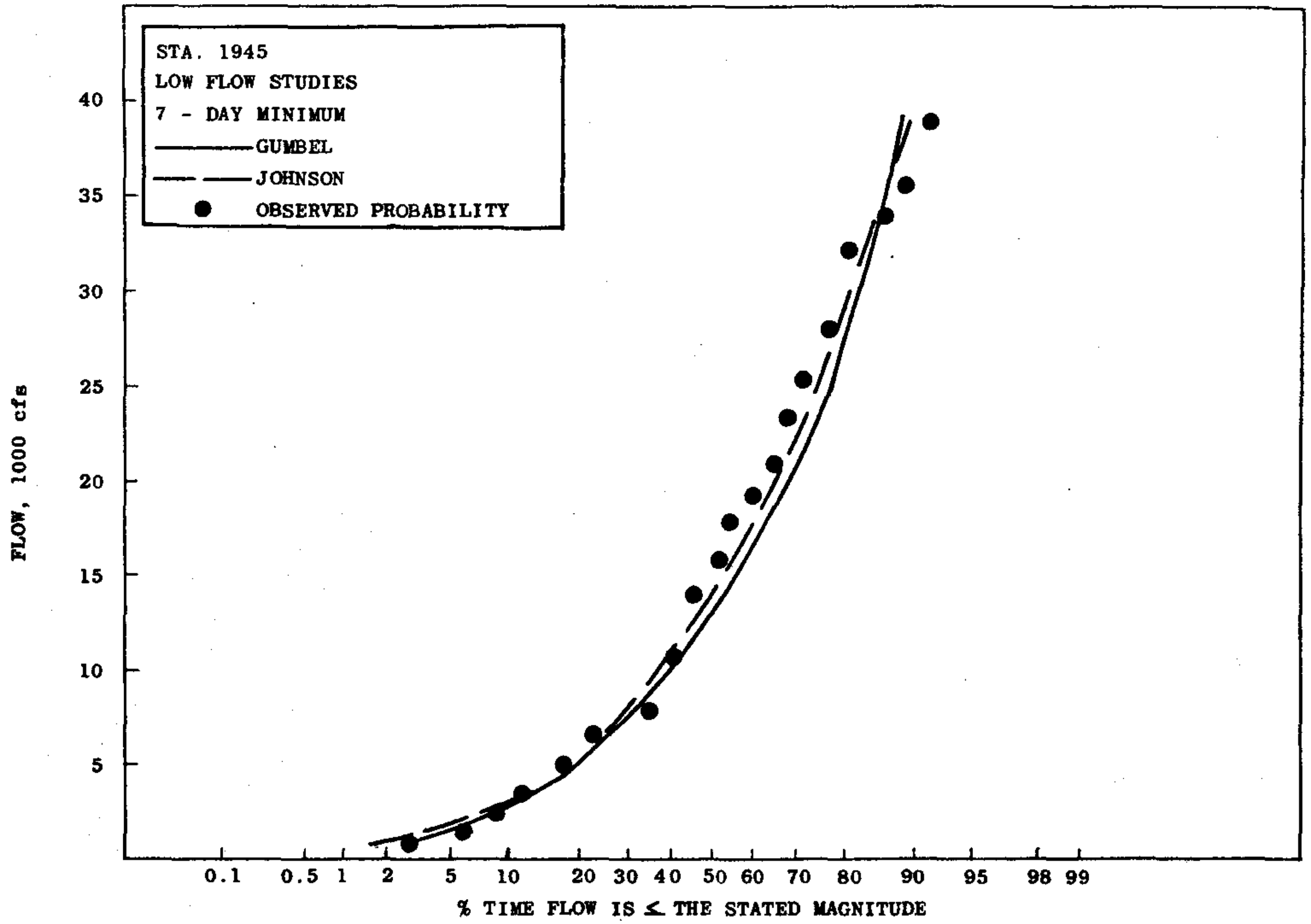

STA. 1945
LOW FLOW STUDIES
7 - DAY MINIMUM
GUMBEL
JOHNSON
OBSERVED PROBABILITY
FLOW, 1000 cfs
40
35
30
25
20
15
10
5
0.1
0.5
1
2
5
10
20
30
40
50
60
70
80
90
95
98
99
% TIME FLOW IS ≤ THE STATED MAGNITUDE

FIGURE 18.

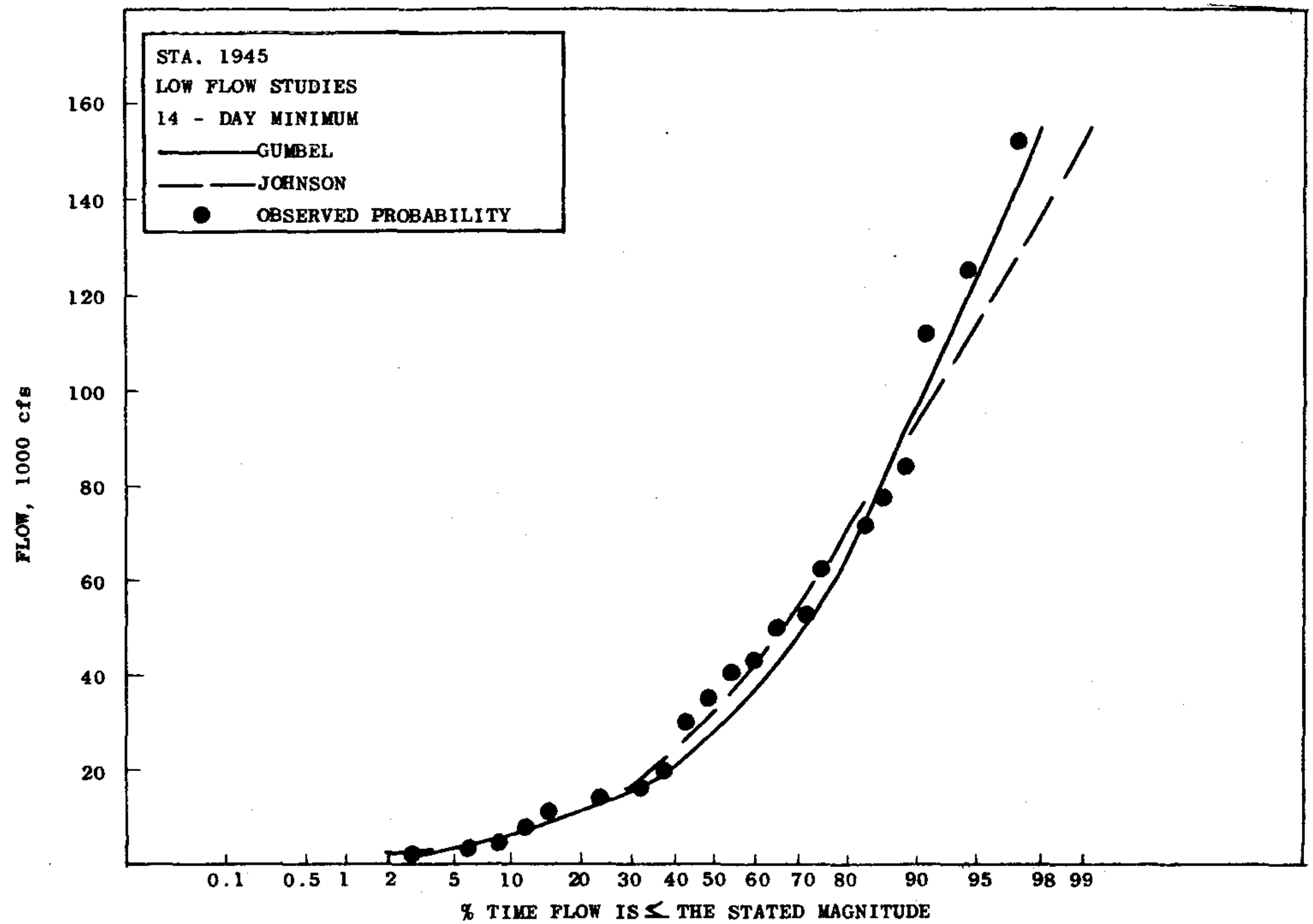

STA. 1945
LOW FLOW STUDIES
14 - DAY MINIMUM
GUMBEL
JOHNSON
OBSERVED PROBABILITY
FLOW, 1000 cfs
160
140
120
100
80
60
40
20
0.1
0.5
1
2
5
10
20
30
40
50
60
70
80
90
95
98
99
% TIME FLOW IS ≤ THE STATED MAGNITUDE

FIGURE 19.

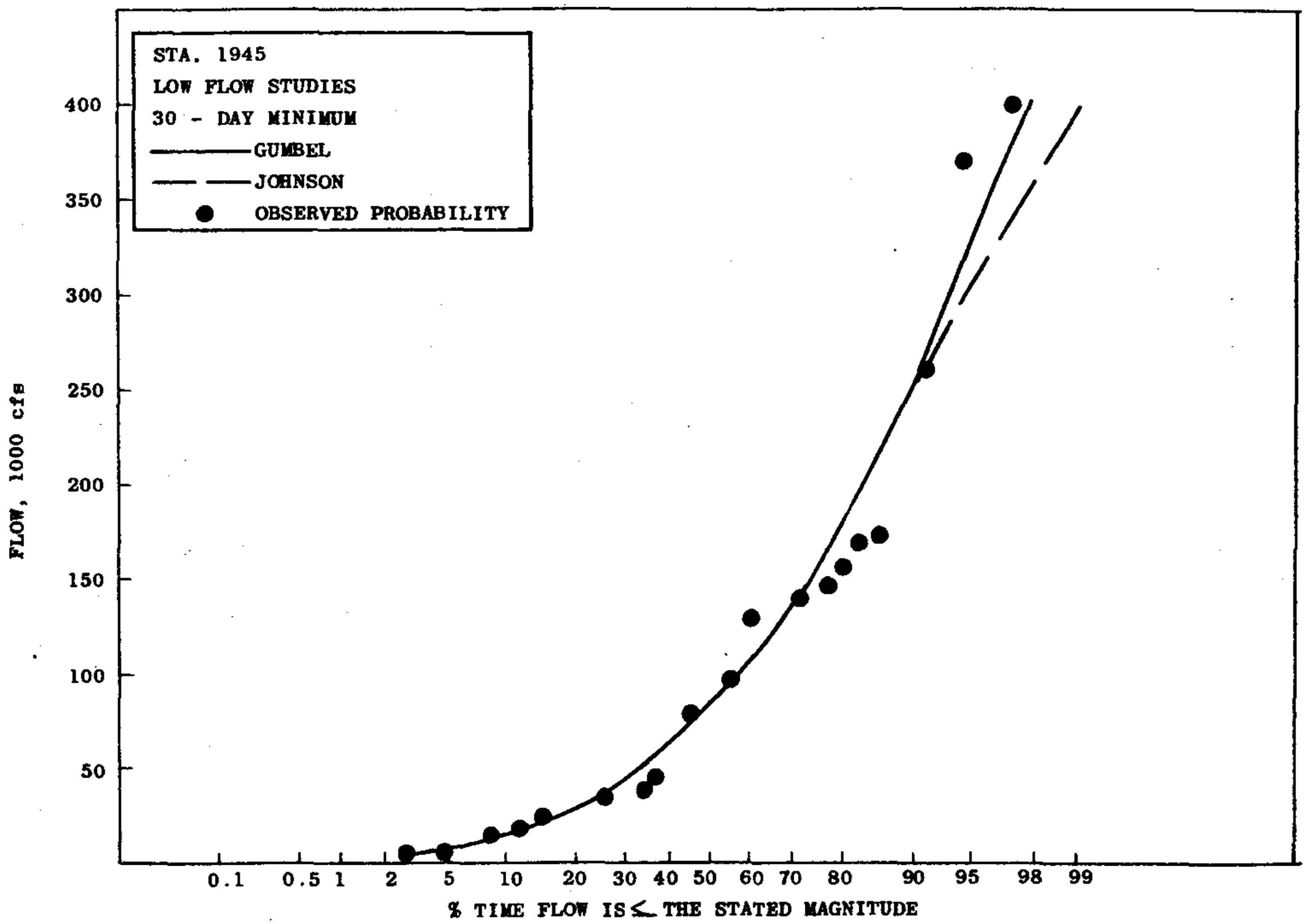

STA. 1945
LOW FLOW STUDIES
30 - DAY MINIMUM
GUMBEL
JOHNSON
OBSERVED PROBABILITY
FLOW, 1000 cfs
400
350
300
250
200
150
100
50
0.1
0.5
1
2
5
10
20
30
40
50
60
70
80
90
95
98
99
% TIME FLOW IS ≤ THE STATED MAGNITUDE

FIGURE 20.

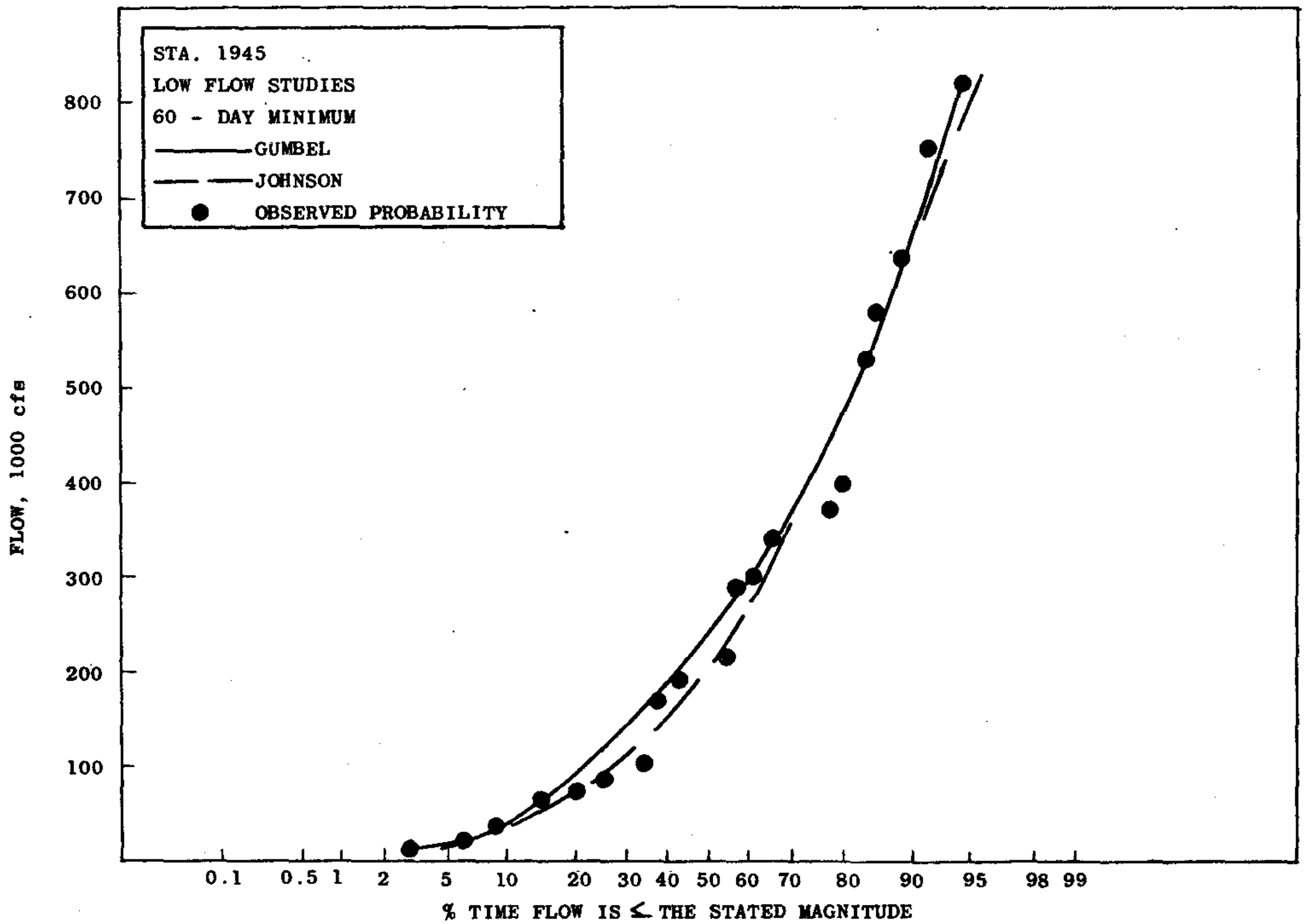
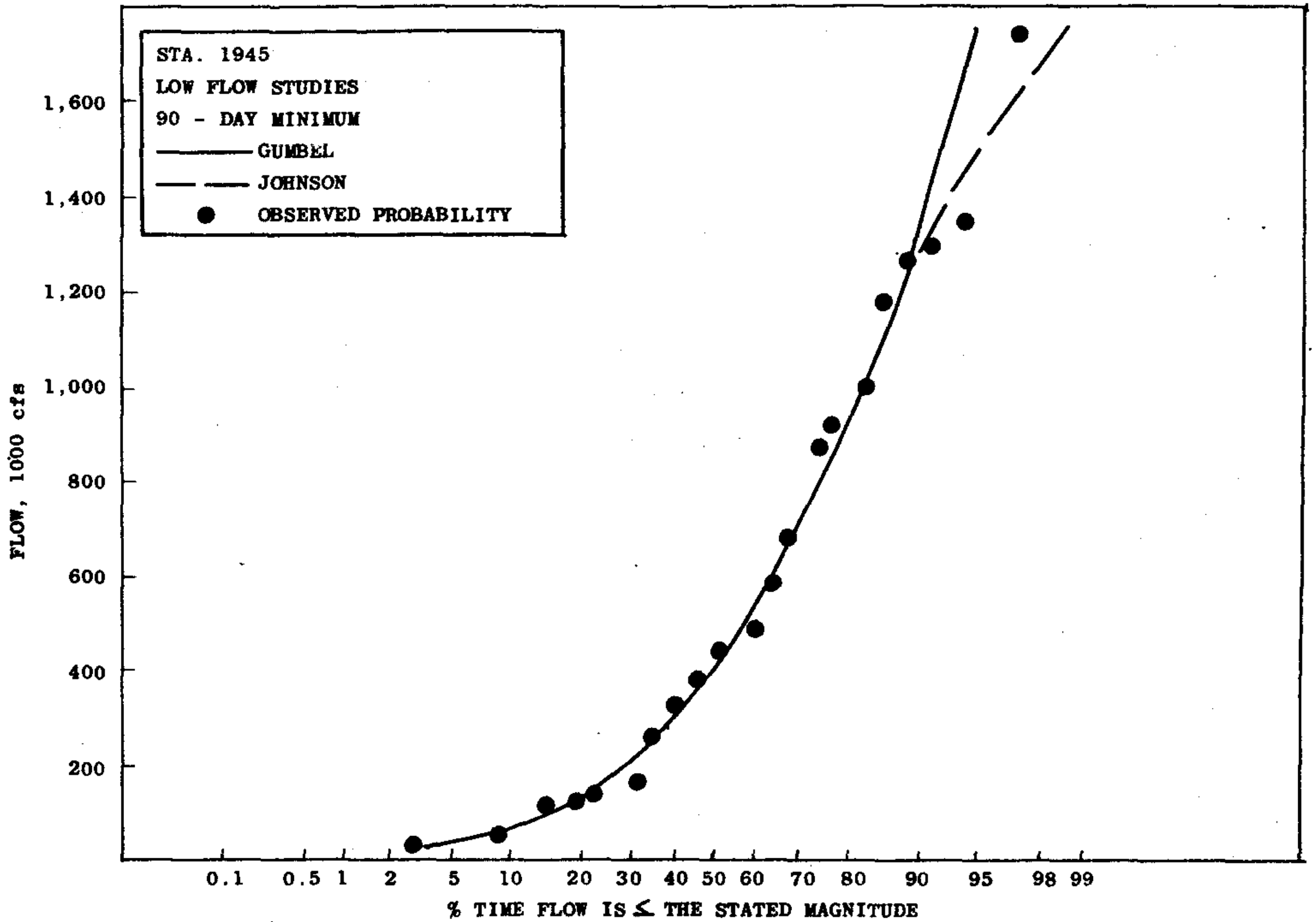

STA. 1945
LOW FLOW STUDIES
60 - DAY MINIMUM
GUMBEL
JOHNSON
OBSERVED PROBABILITY
FLOW, 1000 cfs
100
200
300
400
500
600
700
800
0.1
0.5
1
2
5
10
20
30
40
50
60
70
80
90
95
98
99
% TIME FLOW IS ≤ THE STATED MAGNITUDE

FIGURE 21.



STA. 1945
LOW FLOW STUDIES
90 - DAY MINIMUM
GUMBEL
JOHNSON
OBSERVED PROBABILITY
1,600
1,400
1,200
1,000
800
600
400
200
FLOW, 1000 cfs
0.1
0.5
1
2
5
10
20
30
40
50
60
70
80
90
95
98
99
% TIME FLOW IS ≤ THE STATED MAGNITUDE

FIGURE 22.

The probability density function of this distribution is

$$f(x) = \begin{cases} \frac{\eta}{\sigma - \varepsilon}\left(\frac{x - \varepsilon}{\sigma - \varepsilon}\right)^{\eta - 1} \exp\left[-\left(\frac{x - \varepsilon}{\sigma - \varepsilon}\right)^{\eta}\right], \; x \geq \varepsilon, \\ \varepsilon \geq 0, \; \sigma > 0, \; \eta > 0, \; 0 \text{ elsewhere} \end{cases} \tag{13}$$

where σ is the scale parameter, η is the shape parameter, and ε denotes the lower limit. When ε is equal to zero, equation (13) then becomes the probability density function of Weibull distribution.

The cumulative probability function of Gumbel's limited distribution is

$$F(x) = 1 - \exp\left[-\left(\frac{x - \varepsilon}{\sigma - \varepsilon}\right)^{\eta}\right], \; x \geq \varepsilon \tag{14}$$

which defines the cumulative probability of x, a variable, obtained by integrating equation 13 from ε to an arbitrary value of x.

Low flow data and their corresponding observed cumulative probabilities used previously for Johnson S_B distribution were also used for this distribution function. Equation (14) was applied to fit the distribution of low flows. Numerical results which include the magnitudes of flows and their corresponding cumulative probabilities, both calculated and observed, are given in Appendix D. Also shown are the values of sum-of-square, the maximum deviations of cumulative probability, and the estimates of parameters. These results are also shown in Figures 2 through 22.

E. Comparison of Goodness-of-Fit

Maximum deviation and sum-of-squares were the criteria used in this investigation to compare the goodness-of-fits of the two types of distribution functions applied. Average deviation and average absolute deviation were also calculated to serve the reference purpose. These values are shown in Appendixes C and D along with observed and calculated probabilities for various numbers of consecutive days at each station.

Maximum deviation is defined as the largest absolute deviation between the calculated cumulative probability and the observed probability. In the mathematical expression, it is given by

$$D = \text{Max.}\ |F_0(X) - S_N(X)|, \tag{15}$$

where $F_0(X)$ is the theoretical cumulative distribution function, and $S_N(X)$ is the observed cumulative probability distribution of a sample with N observations. In this low flow study, $S_N(X) = M/N+1)$, where X is any flow, M denotes number of flows equal to or less than X. The maximum deviation is used in Kolmogorov-Smirnov one-sample test, non-parametric statistical test of goodness-of-fit. It is a measure of the degree of agreement between the distri btion of observations and the theoretical distribution assumed. Therefore, it was adopted in this investigation to compare the goodness-of-fits between Gumbel's limited distribution and Johnson S_B distribution.

Sum-of-squares is the summation of squares of the differences between observed values and their corresponding calculated values from the fitted distribution. Although it is not highly sensitive, it is still a good indicator in comparing goodness-of-fits of different distribution functions applied.

Values of maximum deviation and sum-of-squares for the two distribution functions are listed in Table II.

TABLE II

VALUES OF MAXIMUM DEVIATION AND SUM-OF-SQUARES FROM THE USES OF GUMBEL'S LIMITED DISTRIBUTION AND JOHNSON S_B DISTRIBUTION

Station	Distribution	Min. 1-Day Flow		Min. 3-Day Flow		Min. 7-Day Flow		Min. 14-Day Flow		Min. 30-Day Flow		Min. 60-Day Flow		Min. 90-Day Flow	
		Max. D	S.S.	Max. D	S.S.	Max. D	S.S.	Max. D	S.S.	Max. D	S.S.	Max. D	S.S.	Max. D	S.S.
1775	Gumbel	0.1251	0.09709	0.1612	0.11676	0.1553	0.10349	0.1172	0.05381	0.1026	0.08654	0.0640	0.03570	0.0638	0.02910
	Johnson S_B	0.1351	0.10601	0.1690	0.14022	0.1730	0.12741	0.1457	0.07581	0.1262	0.06257	0.1001	0.04712	0.0651	0.03245
1645	Gumbel	0.0763	0.04693	0.0717	0.03786	0.0721	0.02623	0.0790	0.04374	0.0726	0.04432	0.0894	0.05320	0.0581	0.01880
	Johnson S_B	0.0757	0.04216	0.0695	0.03463	0.0699	0.02637	0.0774	0.04038	0.0655	0.04116	0.0865	0.04838	0.0632	0.01962
1945	Gumbel	0.0651	0.02973	0.0668	0.02779	0.0558	0.01772	0.0797	0.02777	0.0863	0.06064	0.0707	0.03498	0.0811	0.02686
	Johnson S_B	0.0550	0.02692	0.0636	0.02551	0.0487	0.01616	0.0725	0.03169	0.0857	0.06463	0.0661	0.03808	0.0740	0.02462

Table II shows that for Stations 1645 and 1945 (stations with large magnitude of low flows) Johnson S_B distribution fives better goodness-of-fit in the characterization of low flow distribution. However, for Station 1775 (station with many zero flows), Gumbel's limited distribution provides better fitting. With few exceptions both the maximum deviation and sum-of-squares show this. All maximum deviations for Station 1945 and six of seven maximum deviations for Station 1645 have smaller values associated with Johson S_B distribution than those associated with Gumbel's distribution. Also five of seven sum-of-squares for Station 1945 and four of seven sum-of-squares for Station 1645 give the same indication in favor of Johnson S_B distribution. On the contrary, for Station 1775, all maximum deviations and six of seven sum-of-squares show that the use of Gumbel's distribution is better. Intuitively, to assume that the distribution of low flows are bounded at both ends, rather than bounded only at one, is reasonable. The results of this investigation show that for most cases Johnson S_B distribution fitted low flow data better than Gumbel's distribution did.

The relationship between each of the criteria, maximum deviation and sum-of-squares, and the number of consecutive days is another area of interest. Table II indicates that there was no such relationship at Stations 1645 and 1945. However, it was found that at Station 1775, both maximum deviation and sum-of-squares decrease significantly when the number of consecutive days is 14 days or greater. The cause of such phenomena can be fairly well pointed out, if not completely, by a review of the Tables in Appendix A. All flows for Stations 1645 and 1945 are significantly greater than zero. But for Station 1775, each set of flow data has one or more of zero magnitude of flow. It is unlikely that a theoretical distribution function could provide a good fitting to a set of observations having too many zero flows. Obviously, at the small flow station such as Station 1775, only the low flows with significantly large number of consecutive days could possibly be fitted well by a theoretical distribution function.

F. The Selection of a Design Flow

The selection of a design flow is a critical aspect in the management of water quality. The evaluation of stream assimilative capacity depends heavily on the magnitude of the design flow used.

Whether there should be a single criterion or a flexible standard to determine design flows of all streams in a basin will not be the concern in this section. INstead, the selection of a design flow from the distribution of low flows will be considered. Tables III and IV show various design flows determined from the Johnson S_B distribution and from Gumbel's limited distribution respectively. Each value in these tables was obtained through two steps. The first was to obtain a flow associated with a specified probability from the minimum flow distribution. Seven types of minimum flow were considered. They are 1-Day, 3-Day, 7-Day, 14-Day, 30-Day, 60-Day, and 90-Day. The probability of occurrence included 0.01, 0.05, 0.1, 0.15, 0.2, and 0.25. The second step was to average the selected flow by its corresponding number of consecutive days.

Tables III and IV indicate that with a specified probability the magnitude of design flow varies significantly with the change in the number of consecutive days. The amount of increase is particularly pronounced when the number of consecutive days are 30 or greater. When

TABLE III

DESIGN FLOWS, CFS, FROM JOHNSON S_B DISTRIBUTION

STATION 1775

PROB.	1-DAY	3-DAY	7-DAY	14-DAY	30-DAY	60-DAY	90-DAY
0.01	0.00	0.00	0.00	0.00	0.03	0.05	0.04
0.05	0.00	0.00	0.01	0.03	0.11	0.21	0.24
0.10	0.01	0.01	0.02	0.07	0.24	0.46	0.62
0.15	0.03	0.02	0.04	0.13	0.40	0.79	1.18
0.20	0.05	0.05	0.08	0.22	0.61	1.21	1.96
0.25	0.09	0.09	0.14	0.34	0.87	1.74	3.03

STATION 1645

PROB.	1-DAY	3-DAY	7-DAY	14-DAY	30-DAY	60-DAY	90-DAY
0.01	31.02	37.66	39.91	35.00	43.88	72.25	86.34
0.05	79.76	91.57	100.88	98.31	120.06	173.50	230.89
0.10	130.76	146.13	164.06	168.69	203.46	283.32	385.29
0.15	181.69	199.78	226.90	241.42	289.04	395.49	540.52
0.20	234.35	254.83	291.79	318.35	379.27	513.48	701.17
0.25	290.85	313.71	361.40	402.23	477.59	641.87	873.03

STATION 1945

PROB.	1-DAY	3-DAY	7-DAY	14-DAY	30-DAY	60-DAY	90-DAY
0.01	93.91	97.76	96.94	111.53	121.80	158.80	228.09
0.05	233.62	247.01	253.80	287.09	318.79	409.18	504.72
0.10	374.90	400.28	419.55	471.08	528.22	672.98	817.69
0.15	512.20	550.98	585.65	655.02	739.74	938.35	1145.91
0.20	650.83	704.69	757.58	845.47	960.56	1214.93	1497.91
0.25	796.21	867.41	941.89	1050.05	1199.57	1514.20	1887.11

TABLE IV

DESIGN FLOWS, CFS, FROM GUMBEL DISTRIBUTION

STATION 1775

PROB.	1-DAY	3-DAY	7-DAY	14-DAY	30-DAY	60-DAY	90-DAY
0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.05	0.00	0.00	0.00	0.01	0.01	0.07	0.09
0.10	0.01	0.01	0.02	0.04	0.05	0.27	0.40
0.15	0.03	0.03	0.04	0.10	0.13	0.58	0.96
0.20	0.06	0.07	0.09	0.19	0.25	1.03	1.81
0.25	0.12	0.13	0.17	0.32	0.44	1.63	3.02

STATION 1645

PROB.	1-DAY	3-DAY	7-DAY	14-DAY	30-DAY	60-DAY	90-DAY
0.01	31.83	39.16	16.82	17.75	20.10	27.00	54.22
0.05	73.85	80.49	75.07	81.87	94.77	126.91	211.91
0.10	126.01	133.96	145.35	160.82	187.96	251.36	386.89
0.15	179.55	190.06	216.36	241.49	283.88	379.33	555.90
0.20	235.21	249.28	289.43	325.12	383.81	512.56	724.67
0.25	293.56	312.08	365.43	412.60	488.74	652.37	896.21

STATION 1945

PROB.	1-DAY	3-DAY	7-DAY	14-DAY	30-DAY	60-DAY	90-DAY
0.01	56.02	52.67	50.01	52.20	53.30	127.57	183.55
0.05	205.52	207.49	210.80	227.23	245.38	345.66	442.77
0.10	364.89	380.15	397.92	435.07	481.58	624.18	785.36
0.15	515.54	547.36	583.34	643.29	722.75	914.45	1149.15
0.20	663.83	714.62	771.69	856.33	972.65	1219.40	1536.14
0.25	812.91	884.87	965.64	1076.95	1233.96	1541.66	1949.06

the probability of occurrence is greater than or equal to 0.1, the magnitude of design flow increases consistently as the number of consecutive days increases. This shows that for a specified probability of occurrence, the design flow determined from the distribution of minimum flow with 30 or more of consecutive days is noticeably greater than that determined from the distribution of flow when the number of consecutive days is less than 30.

The use of the minimum consecutive 7-day flow with 0.1 of occurrence probability as the design flow is the most widely used criterion today. It was the criterion adopted in the water quality study for Delaware River Basin. However, this investigation does not show any advantage of this selection over others. From the review of Table II, it seems that the use of minimum flow with a large number of consecutive days, e.g., 30, 60, or 90 days, to determine the design flow is a better alternative. Table II indicates that for a large flow station, the assumed distribution function will fit either flows with large number of consecutive days or flows with small number of consecutive days. It also shows that for a small flow station, the distribution function is acceptable only for flows with large number of consecutive days. This phenomenon is probably because the larger number of consecutive days can remove the effect of fluctuation resulting from minor river regulation. With the use of minimum flows with large number of consecutive days, the probability of occurrence specified should be less than 0.1 if it is desired to keep the magnitude of design flow in a reasonable size.

IV. Water Quality Management with the Application of Zone-Treatment Principle

A. General

In the achievement of basinwide water quality objectives, economy and equity of allocating the stream assimilative capacity are two major concerns. The primary purpose of this study was to apply the principle of zone-treatment in various ways to provide a management model. The idea of zone-treatment was first used in the water quality study of the Delaware River Basin. In that study the treatment zones were sections of stream which were grouped according to their geographical locations. The treatment levels required for various sources of wastes discharged within each section were uniform. However, treatment levels could be different for different sections of the river.

This study has expanded the application of the zone-treatment principle. Various kinds of criteria were used to group the treatment zones. This portion of the study will be continued in greater depth.

Bio-degradable organic matter was the type of waste considered in this study. Therefore, the dissolved oxygen (D.O.) concentration was adopted as the quality measured parameter and the percent BOD removal was considered as the degree of waste treatment.

In this study it was assumed that each or a number of waste sources were collected and treated at treatment plants, either municipal treatment plants or industrial treatment plants, before being discharged to the receiving waters.

B. Methods for Grouping of Treatment Zones

In the selection of methods for grouping the treatment zones in a river basin, practicability and equity must be considered. Several criteria for gouping of treatment zones have been considered in this study. They are described as follow:

1. Classifications of sub-basins

Sub-basins were classified in a river basin according to their geographical locations. Each was then considered as a treatment zone. The required percent BOD removal at each treatment plant that discharges its wastes into the receiving waters of a sub-basin are uniform. The percent BOD removal is allowed to be different for treatment plants which discharge wastes into streams in different sub-basins. The grouping of treatment plants depends on where their wastes are discharges, not on where they are located. Water quality objectives in each sub-basin could be either different or identical.

2. Pounds of influent BOD

Treatment plants in the basin are classified into groups according to their pounds of influent BOD, e.g., pounds per day. Between groups the larger the pounds of BOD, the higher the percent BOD removal is required. When the number of treatment plants is small, e.g., ten or less, each of the treatment plants can be considered as one group. The pounds of influent BOD for each plant can be determined from the analysis of the daily influent BOD records over a period of time.

3. Ratio of influent BOD to flow

The flow in this classification is the sum of the waste flow and the design stream flow at the point of discharge. As the design stream flows are selected, the weight ratio at each treatment plant can be determined from the analysis of influent BOD records. The number of treatment zones from the application of this criterion depends upon the number of treatment plants in the basin and the distribution of the weight ratios of treatment plants. Similarly, between groups the larger the ratio is, the higher the required treatment efficiency.

For the purpose of comparison, two other cases were also investigated. One was to allow different degrees of BOD removal at each treatment plant. The other was to require a uniform degree of BOD removal at all treatment plants throughout the basin.

C. Cost Function of the Treatment Plant

The purpose of making a cost function study was to investigate the relationship between the treatment cost and the percent BOD removal. The activated sludge treatment was the only plant considered. The basic information of cost estimation used was taken from "Cost of Wastewater Processes," a report from Robert A. Taft Water Research Center. From the combined applications of available cost data and design criteria, annual cost functions of treatment plants, both municipally and industrially, were formulated. Furthermore, an asymptotic regression analysis was made to relate the treatment level and annual treatment cost. This relationship was express by an exponential function as follows:

$$Y = A + B \circ R^{x} \qquad (16)$$

where

Y = treatment cost in dollars

x = percent of BOD removal

A = intercept

$B \circ R$ = regression coefficients.

Details in the formulation of the cost functions and the analysis of the relationship between cost and degree of treatment are shown in Appendix E.

Although the assumption of a linear relationship between treatment cost and treatment level (up to 80%) has been used for many years in a number of water quality management studies, the results from this investigation has indicated a linear relationship between costs and treatment level does not exist.

D. Formulation and Optimization of Regional Water Quality System

The Streeter-Phelps equation was used to formulate the regional water quality system. It expresses that the D.O. level in the stream is the function of deoxygenation rate due to organic waste discharged, the reaeration rate, the flowing time of water from one location in the stream to another, and initial levels of BOD and D.O. It is given by the equation

$$D_t = \frac{k_1 L_i}{k_2 - k_1} (e^{-k_1 t} - e^{-k_2 t}) + D_i e^{-k_2 t} \qquad (17)$$

where

D_i, D_t = D.O. deficits at time = 0 and time - t, respectively, mg/l

L_i = BOD concentration in the stream at time = 0, mg/l

K_1 = deoxygenation rate, day^{-1}

k_2 = reaeration rate, day^{-1}

T = flowing time, day.

Associated with Equation (17) an equation expressing the change of BOD with time was also used. It is given by

$$L_t = L_i \, e^{-k_1 t} \tag{18}$$

where

L_t = BOD concentration at time - t, mg/l.

let

$$\alpha = e^{-k_1 t}$$

$$\beta = e^{-k_2 t}$$

and

$$\gamma = \frac{k_1}{(k_2 - k_1)} \left(e^{-k_1 t} - e^{-k_2 t}\right),$$

then Equations (17) and (18) become

$$D_t = \gamma L_i + \beta D_i, \tag{19}$$

and

$$L_t = \alpha L_i. \tag{20}$$

Furthermore, let

O_t = D.O. in the stream at time = t,

O_s = saturated D.O. in the stream,

O_i = initial D.O. at time = 0,

then equation (19) becomes

$$\beta O_i - O_t - \gamma L_i = O_s(\beta - 1). \quad (21)$$

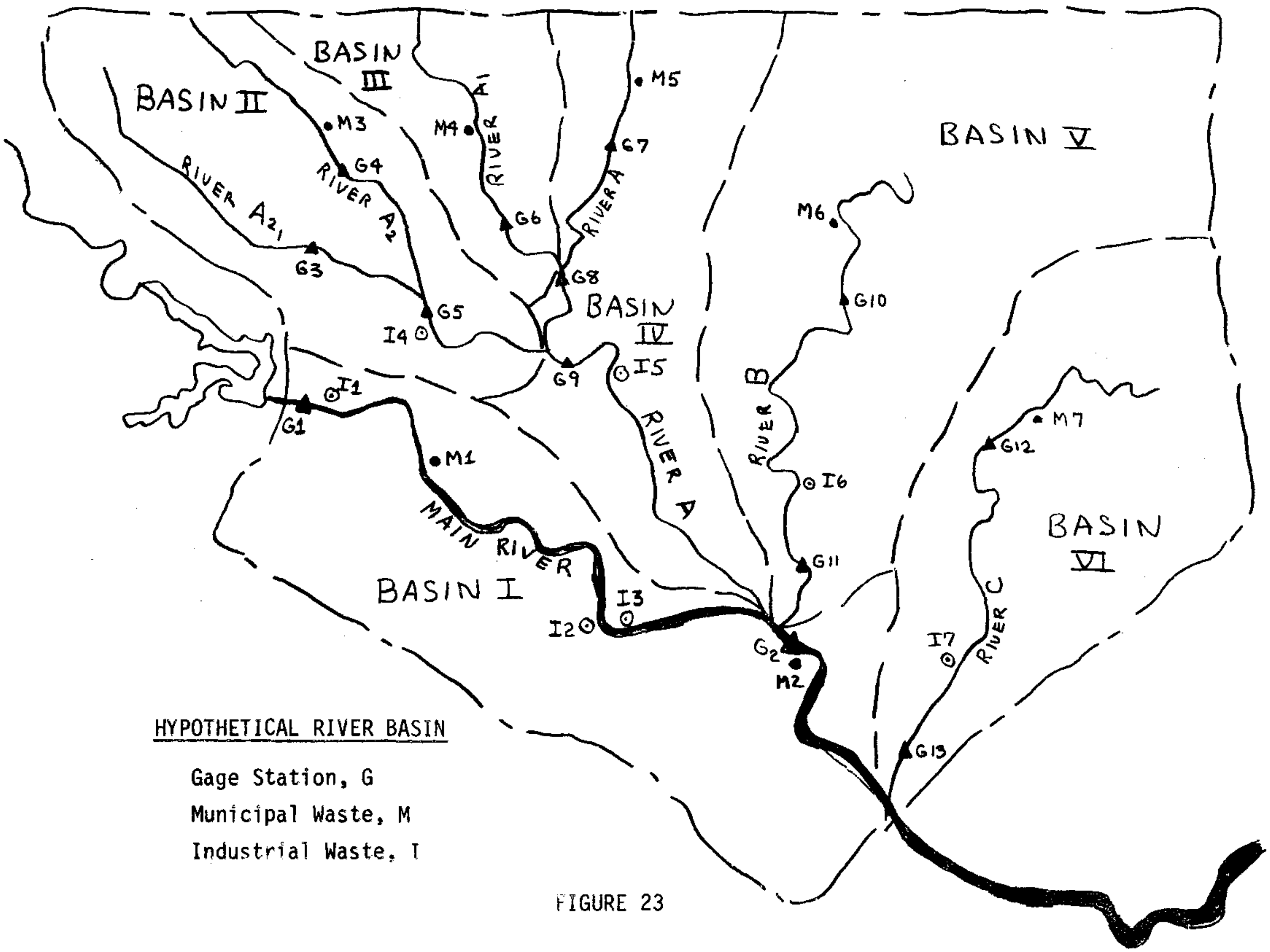
Therefore, when values of k_1's, k_2's and t's are known, with the use of Equations (20) and (21), a basin-wide water quality system can be formulated into a linear structure. Combining this linear system with the quality objectives of the receiving waters, the application of the zone-treatment principle, and with other constraints, the desired treatment levels of plants can be determined optimally through the use of linear programming techniques. The objective function should be linear in original form or after meaningful transformation.

Because of much uncertainty and complexity involved in minimizing the summation of non-linear cost functions from all treatment plants in a basin, the treatment cost was not selected as the objective criterion. Instead, total BOD removal, in pounds per day, of the complete basin was taken as the criterion for minimization.

E. A Hypothetical Study of Water Quality Management

In order to demonstrate the application of the zone-treatment principle in the management of water quality, a hypothetical study was made.

A river basin was hypothesized as shown Figure 23. The river system consisted of one main river and six major tributaries. Six sub-basins were defined in the basin according to their geographical locations. Locations of thirteen gage stations, seven municipal treatment plants,



HYPOTHETICAL RIVER BASIN

Gage Station, G
Municipal Waste, M
Industrial Waste, I

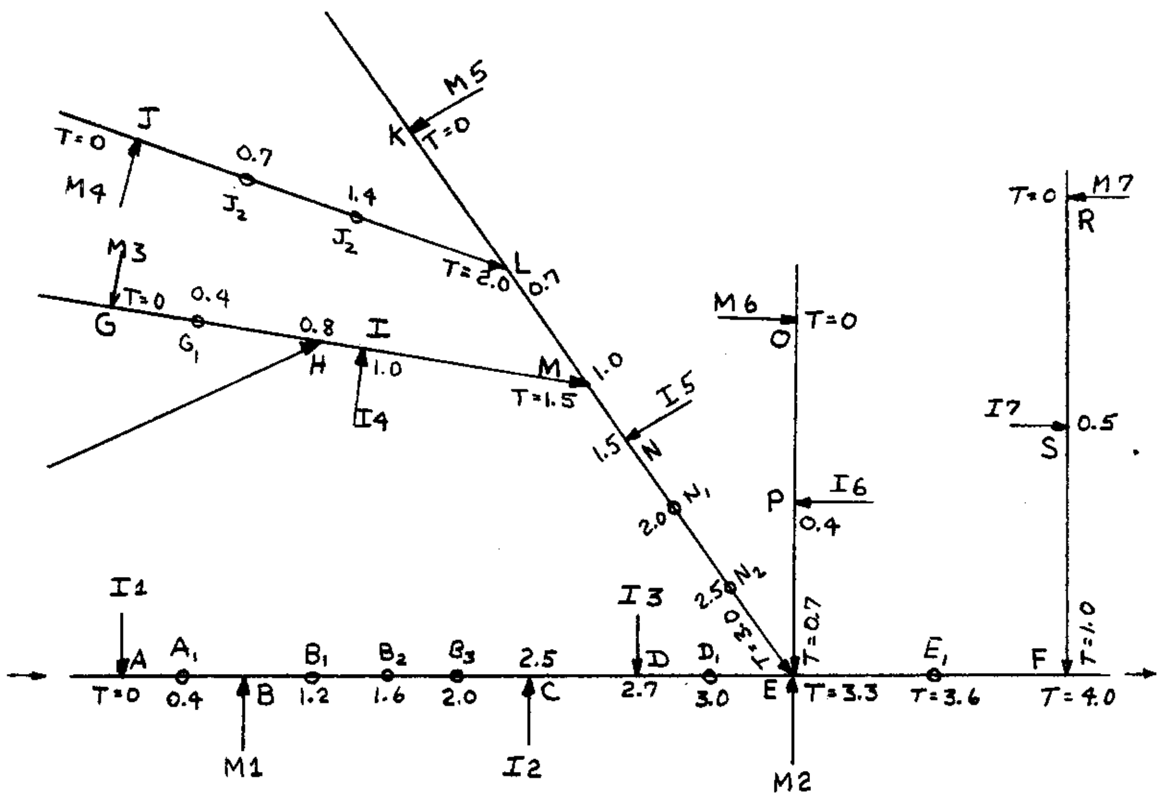
FIGURE 23

and seven industrial treatment plants were also assumed. For the convenience of formulating this basin-wide water quality system, a simplified layout of the river system was schemed as shown in Figure 24. In this scheme either discharge locations of treatment plants or stream intersections were considered as control points. For the purpose of reducing probability of D.O. violation in the stream, additional control points were added. The method of determining additional control points is shown in Appendix F. At each control point, the D.O. level should be equal or greater than the specified quality objective. Distance location of control points were expressed in terms of water flowing time in days. Each river section between two discharge locations of plants was counted as a reach. There are seventeen reaches in the stream system. Reach numbers are also shown in Figure 24. Design stream flows, discharges of treatment plants, influent BOD concentrations, D.O. and BOD in the upstream sections of rivers, deoxygenation rates, and reaeration rates are all listed in Table V. The D.O. level in the wastewater after treatment was assumed equal to 4.0 mg/l. D.O. saturation in the stream was assumed equal to 9.0 mg/l.

The quality system of the hypothetical river basin was formulated by using the data in Table V and the application of Equations (20) and (21). Details of the system formulation are shown in Appendix G. Appendix G also shows the determinations of treatment zones from the application of every criterion discussed in Section B.

A summary of results from the application of zoning criteria is given in Table VI. A uniform minimum quality objective with 4.0 mg/l of dissolved oxygen throughout the basin was used. Results with the uses of other quality objectives will be investigated later.

FIGURE 24: LAYOUT OF STREAM SYSTEM



Remarks:

(1) T = Distance in term of water flowing time, day.

(2) o = Additional control point

(3) Classification of reach number =

No. 1 = A-B¯, No. 2 = B-C¯, No. 3 = C-D¯, No. 4 = D-E¯,
No. 5 = E-F, No. 6 - G-H¯, No. 7 = H-I¯, No.8 = I-M¯,
No. 9 = J-L¯, No. 10 = K-L¯, No. 11 = L-M¯, No. 12 = M-N¯,
No. 13 = N-E¯, No. 14 = O-P¯, No. 15 = P-E¯, No. 16 = R-S¯
No. 17 = S-F¯

TABLE V

WATER QUALITY DATA OF HYPOTHETICAL RIVER BASINS

River	Location \ Items	Q. cfs: Design Flow	Q. cfs: Effluent	D.O., mg/l: Stream	D.O., mg/l: Effluent	BOD. mg/l: Stream	BOD. mg/l: Untreated Flow	k_1, day-1	k_2, day-2
Main River	A - + (I1)	250 "	5	7.4 *	4.0	1.6 *	500	0.23 0.30	0.45 "
	B - + (M1)	260 "	35	* *	4.0	* *	380	" 0.28	" "
	C - + (I2)	275 "	6	* *	4.0	* *	650	" 0.32	" "
	D - + (I3)	280 "	3	* *	4.0	* *	975	" 0.34	" "
	E - + (M2) + (River A) + (River B)	" 450 " "	12	* * * *	4.0	* * * *	240	" 0.31 " "	" 0.48 " "
	F - + (River C)	" 500		* *		* *		" 0.30	" "
River A2	G - + (M3)	10 "	8	7.5 *	4.0	1.0 *	220	0.24 0.26	0.35 "
	H - + (River A21)	" 20	5	* *	8.0	* *	1.0	" 0.23	" "
	I - + (I4)	" "	2	* *	4.0	* *	550	" 0.29	" "
	M -	"		*		*		"	"
River A1	J - + (M4)	25 "	7	7.6 *	4.0	0.9 *	270	0.20 0.25	0.36 "
	L -	30		*		*		"	"
River A	K - + (M5)	28 "	10	8.0 *	4.0	1.2 *	300	0.24 0.25	0.40 "
	L - + (River A1)	" 60		* *		* *		" "	" "
	M - + (River A2)	" 80		* *		* *		" 0.27	" 0.45
	N - + (I5)	" 84	4.2	* *	4.0	* *	1040	" 0.31	" "
	E -	"		*		*		"	"
River B	O - + (M6)	34 "	7.5	7.8 *	4.0	2.1 *	350	0.20 0.22	0.50 "
	P - + (I6)	" 50	3.6	* *	4.0	* *	700	" 0.27	" "
	E -	"		*		*		"	"
River C	R - + (M7)	32 "	7.5	7.7 *	4.0	1.5 *	275	0.21 0.23	0.46 "
	S - + (I7)	" 46	2.5	* *	4.0	* *	865	" 0.31	" "
	F -	"		*		*		"	"

- = Immediate upstream side
+ = Point of the inflow
* = Value needed to be determined

TABLE VI

OPTIMIZED RESULTS OF WATER QUALITY MANAGEMENT WITH USES OF VARIOUS ZONING CRITERIA

	Uniform Treatment	Minimum Treatment	Sub-basin	Influent BOD	BOD-flow Ratio
Total BOD Removal, lbs/day	224088	201566	209815	224088	211731
E_{I1}, %	88.38	80.22	83.25	88.38	80.64
E_{I2}, %	"	98.00	"	"	"
E_{I3}, %	"	30.00	"	"	"
E_{I4}, %	"	98.00	86.88	"	"
E_{I5}, %	"	98.00	92.13	"	"
E_{I6}, %	"	30.00	70.89	"	"
E_{I7}, %	"	44.08	77.72	"	"
E_{M1}, %	"	98.00	83.25	"	"
E_{M2}, %	"	30.00	"	"	"
E_{M3}, %	"	83.66	86.88	"	95.17
E_{M4}, %	"	93.12	76.72	"	"
E_{M5}, %	"	69.78	92.13	"	"
E_{M6}, %	"	88.74	70.89	"	"
E_{M7}, %	"	98.00	77.72	"	80.64

Remark:

E_{I1} and E_{M3}, for example, denote percents of BOD removal required at industrial treatment plant 1 and municipal treatment plant 3, respectively.

From reviewing the values of minimized total BOD removal in pounds per day and the required treatment degree at each plant, two points are obvious. First, the method which allows a different treatment degree at each plant is extremely inequitable, though it requires the least amount of total BOD removal. The most obvious disadvantage is that this solution is very likely to create large difference in treatment requirement among dischargers, even those located close to each other. The case of treatment plants 12 and 13 is an outstanding example. They are located at opposite banks of the Main River and could be expected to cause an almost equal degree of damages. But through the use of this minimum treatment method, the treatment requirements are 98% and 30% for plants 12 and 13, respectively. Second, these results indicate that "BOD flow ratio" and "Sub-basin" are two potential criteria which could provide good management of regional water quality. The values of total BOD removal from the uses of these two methods are almost equal to each other and are only slightly larger than that from the use of the minimum treatment method. Both methods would be convenient to implement administratively because they have shown the virtues of economy and equity.

APPENDIX A

ANNUAL MINIMUM FLOWS AT STATIONS 1645, 1775 AND 1945

ANNUAL MINIMUM FLOWS, CFS, FOR VARIOUS NUMBERS

OF CONSECUTIVE DAYS AT STATION 7-1645

M	M/(N+1)	1-DAY	3-DAY	7-DAY	14-DAY	30-DAY	60-DAY	90-DAY
1	0.0313	30.0	92.0	240.0	653.0	1834.0	5593.0	12427.0
2	0.0625	85.0	263.0	624.0	1350.0	3210.0	8986.0	16007.0
3	0.0938	147.0	447.0	1071.0	2440.0	6151.0	15685.0	24598.0
4	0.1250	202.0	692.0	1996.0	4323.0	10266.0	26962.0	49495.0
5	0.1563	270.0	852.0	2160.0	4400.0	11229.0	28140.0	51318.0
6	0.1875	280.0	880.0	2209.0	4501.0	11284.0	31910.0	61844.0
7	0.2188	290.0	880.0	2209.0	4700.0	12595.0	37301.0	63046.0
8	0.2500	300.0	920.0	2210.0	4730.0	12865.0	39817.0	67259.0
9	0.2813	326.0	989.0	2361.0	5474.0	13576.0	42319.0	74883.0
10	0.3125	327.0	1116.0	2698.0	6899.0	15873.0	45610.0	88535.0
11	0.3438	342.0	1193.0	3070.0	7058.0	17236.0	48002.0	112252.0
12	0.3750	372.0	1218.0	3731.0	7683.0	19896.0	51414.0	120559.0
13	0.4063	394.0	1307.0	4250.0	9982.0	25484.0	57286.0	133623.0
14	0.4375	436.0	1494.0	5047.0	10500.0	28440.0	68012.0	155117.0
15	0.4688	615.0	2116.0	5306.0	11933.0	29939.0	69021.0	160678.0
16	0.500	688.0	2134.0	5973.0	13217.0	36667.0	103918.0	174297.0
17	0.5313	694.0	2170.0	6377.0	15498.0	42070.0	124057.0	201910.0
18	0.5625	790.0	2403.0	6458.0	18800.0	47100.0	134120.0	208360.0
19	0.5938	850.0	2638.0	7354.0	19850.0	47848.0	136730.0	220550.0
20	0.6250	944.0	3540.0	8850.0	19954.0	56880.0	141910.0	228580.0
21	0.6563	1150.0	3550.0	9180.0	22210.0	57980.0	142100.0	233200.0
22	0.6875	1200.0	3640.0	9240.0	22530.0	59550.0	149754.0	261520.0
23	0.7188	1220.0	3880.0	9840.0	25000.0	60710.0	151420.0	288520.0
24	0.7500	1260.0	4070.0	11750.0	25860.0	62450.0	154920.0	288780.0
25	0.7813	1310.0	4770.0	11890.0	27060.0	64360.0	177120.0	328920.0
26	0.8125	1400.0	4820.0	12430.0	27190.0	66290.0	207900.0	377600.0
27	0.8438	1500.0	5320.0	13220.0	27960.0	72870.0	207910.0	380890.0
28	0.8750	1540.0	5400.0	13830.0	28710.0	80220.0	218970.0	386440.0
29	0.9063	1750.0	5790.0	16360.0	34820.0	105390.0	241390.0	388510.0
30	0.9375	2100.0	8600.0	22310.0	48770.0	111790.0	294520.0	508610.0
31	0.9688	2620.0	8880.0	23000.0	51700.0	149190.0	418200.0	606430.0

Remarks:

1. Period of records: April 1, 1938 - March 31, 1969
2. M: Ranking number of flows in a non-decreasing order
3. N: Total number of years
4. M/(N+1) = Observed cumulative probability

ANNUAL MINIMUM FLOWS, CFS, FOR VARIOUS NUMBERS
OF CONSECUTIVE DAYS AT STATION 7-1775

M	M/(N+1)	1-DAY	3-DAY	7-DAY	14-DAY	30-DAY	60-DAY	90-DAY
1	0.0323	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0645	0.0	0.0	0.0	0.0	0.0	0.1	0.1
3	0.0968	0.0	0.0	0.0	0.0	0.0	6.8	24.8
4	0.1290	0.0	0.0	0.0	0.0	0.3	7.2	29.8
5	0.1613	0.0	0.0	0.0	0.1	11.4	40.1	112.1
6	0.1935	0.0	0.0	0.0	1.8	15.3	52.6	116.5
7	0.2258	0.0	0.0	0.6	3.2	17.2	83.3	237.7
8	0.2581	0.1	0.3	0.7	5.2	19.8	126.6	382.8
9	0.2903	0.1	0.5	1.8	5.5	43.9	194.2	429.4
10	0.3226	0.2	0.8	2.2	6.9	46.1	249.0	466.7
11	0.3548	0.3	0.9	2.4	12.6	46.7	263.2	725.6
12	0.3871	0.3	1.7	4.5	15.3	50.4	318.2	925.4
13	0.4194	1.2	3.6	9.0	20.7	84.7	349.3	934.1
14	0.4516	1.3	4.3	11.5	27.8	85.0	352.1	1203.7
15	0.4839	1.5	5.4	13.5	29.4	107.0	407.5	1217.4
16	0.5161	1.7	5.5	14.7	29.6	135.4	424.8	1507.8
17	0.5484	2.1	6.3	14.7	35.7	176.6	592.7	1858.2
18	0.5806	2.2	6.6	18.4	43.6	177.8	666.4	2043.2
19	0.6129	3.8	11.4	28.2	66.0	254.0	685.8	2935.3
20	0.6452	3.8	12.2	30.8	66.8	307.8	901.2	4130.6
21	0.6774	4.5	13.5	36.0	80.8	308.8	925.0	6080.9
22	0.7097	4.8	14.4	39.7	89.7	315.2	1052.3	7258.7
23	0.7419	5.0	15.8	39.7	124.3	345.3	1158.3	7482.3
24	0.7742	5.6	18.4	48.8	134.4	369.1	2274.0	7935.1
25	0.8065	7.2	21.6	54.8	137.3	376.4	2503.2	8369.0
26	0.8387	7.2	22.3	57.9	159.5	521.3	3069.0	11036.5
27	0.8710	8.6	28.0	67.9	161.4	832.4	4163.1	11316.0
28	0.9032	12.0	37.0	98.0	264.0	923.0	4939.8	12768.6
29	0.9355	17.0	55.0	157.0	405.0	2437.0	6194.0	15034.0
30	0.9677	22.0	66.0	161.0	482.0	3665.0	8973.0	25535.6

Remarks:

1. Period of records: April 1, 1939 - March 31, 1969
2. M: Ranking number of flows in a non-decreasing order
3. N: Total number of years
4. M/(N+1) = Observed cumulative probability

ANNUAL MINIMUM FLOWS, CFS, FOR VARIOUS NUMBERS

OF CONSECUTIVE DAYS AT STATION 7-1945

M	M/(N+1)	1-DAY	3-DAY	7-DAY	14-DAY	30-DAY	60-DAY	90-DAY
1	0.0286	76.0	276.0	843.0	2060.0	6969.0	17379.0	36425.0
2	0.0571	200.0	605.0	1445.0	3284.0	8276.0	24204.0	54024.0
3	0.0857	255.0	1020.0	2460.0	5115.0	14015.0	37733.0	59913.0
4	0.1143	500.0	1500.0	3500.0	7460.0	17917.0	38725.0	111559.0
5	0.1429	566.0	1956.0	4823.0	10837.0	25831.0	66089.0	120878.0
6	0.1714	585.0	2045.0	4958.0	10857.0	29514.0	75640.0	128605.0
7	0.2000	638.0	2054.0	5213.0	10953.0	30373.0	75868.0	132510.0
8	0.2286	650.0	2177.0	6272.0	14552.0	32780.0	83351.0	141192.0
9	0.2571	678.0	2366.0	7190.0	14990.0	35331.0	85342.0	149230.0
10	0.2857	910.0	2980.0	7400.0	15810.0	35842.0	90820.0	157350.0
11	0.3143	983.0	3049.0	7506.0	15909.0	37030.0	91110.0	162670.0
12	0.3429	1040.0	3120.0	7799.0	15910.0	38629.0	101440.0	265740.0
13	0.3714	1040.0	3430.0	8830.0	19940.0	47380.0	170140.0	299780.0
14	0.4000	1110.0	4040.0	10800.0	29360.0	76890.0	172620.0	329880.0
15	0.4286	1470.0	4630.0	12870.0	29430.0	77180.0	194541.0	371390.0
16	0.4571	1680.0	5400.0	13970.0	31720.0	79080.0	202240.0	380670.0
17	0.4857	1810.0	5790.0	14120.0	35120.0	87150.0	212460.0	395390.0
18	0.5143	1850.0	6050.0	15910.0	37340.0	93840.0	217430.0	451421.0
19	0.5429	1890.0	6750.0	17930.0	40880.0	96580.0	217550.0	464560.0
20	0.5714	1960.0	7250.0	19350.0	42890.0	126930.0	291020.0	482670.0
21	0.6000	2190.0	7650.0	19500.0	42970.0	129070.0	303380.0	496740.0
22	0.6286	2240.0	7780.0	19740.0	47670.0	131580.0	308910.0	595450.0
23	0.6571	2340.0	8050.0	20940.0	50050.0	137430.0	341810.0	615680.0
24	0.6857	2530.0	9060.0	23410.0	52430.0	139700.0	343930.0	680130.0
25	0.7143	2640.0	9200.0	25220.0	53000.0	139830.0	360260.0	684220.0
26	0.7429	2790.0	10300.0	27660.0	62720.0	143380.0	366000.0	877090.0
27	0.7714	3020.0	10530.0	28170.0	63290.0	148200.0	374240.0	936940.0
28	0.8000	3090.0	11340.0	32240.0	65580.0	157710.0	402530.0	969290.0
29	0.8286	3380.0	11390.0	33520.0	70950.0	169580.0	535280.0	1010460.0
30	0.8571	3490.0	11440.0	34030.0	76930.0	174510.0	581470.0	1181510.0
31	0.8857	3510.0	11880.0	35740.0	83610.0	261040.0	633030.0	1275460.0
32	0.9143	3650.0	14170.0	39130.0	111870.0	262280.0	754560.0	1309740.0
33	0.9429	5100.0	18450.0	50120.0	125310.0	367930.0	822730.0	1352030.0
34	0.9714	5650.0	20900.0	56750.0	152230.0	415410.0	1169300.0	1741100.0

Remarks:

1. Period of records: April 1, 1935 - March 31, 1969
2. M: Ranking number of flows in a non-decreasing order
3. N: Total number of years
4. M/(N+1) = Observed cumulative probability

APPENDIX B

COMPUTER PROGRAM

Subroutine YCOMP

This routine accepts the equation to which the data are to be fitted, and calculates the value of the dependent variable for each data point using successive estimates of the parameters as determined in the main routine.

Subroutine DERIV

In this section, the values of partial derivaties of the correlating equation are estimated by a differencing method. Thus, we avoid the problem of determining and programming partial derivatives which may be very complex. Comparisons of the results obtained by this method with those obtained by use of actual partial derivatives show no appreciable advantage in convergence from mathematically accurate derivatives.

This routine obtains the current parameter values from the main program, and changes each parameter successively by a small percentage, both plus and minus. Subroutine YCOMP is then used to find the dependent variable values for these slightly differing parameter values. The result gives estimates of the partial derivatives, which are used to establish the parameter values for the succeeding iteration.

Subroutine SOLV

The value obtained from the YCOMP and DERIV subroutines are used by the main program to formulate a matrix which is fed to the SOLV subroutine for solution. The results are returned to the main program, which uses them to determine the parameter values for the next iteration.

This routine uses the largest available element of each row as a pivotal element, rather than solving on the diagonal. The major advantage of this is that it eliminates the possibility of using a zero element as a pivot unless there is an entire zero row or column in the matrix. This eventually is also provided for. If it occurs, the program terminates with a printout informing the programmer of the reason.

Parobolic Fit

After each third iteration, the next estimate is made by applying a parabolic fit to the three previous sum-of-squares values. In general, this will materially increase the convergence rate.

Programming of Correlating Equation

The program statement of the equation must have the digit 1 (one) in column 5. This controls a DO loop which cycles through the data points. The dependent variable must be specified as CY(N), where the subscript, N, is the index for the DO loop.

The independent variables are designated as Z (i, N), where the i identifies the particular variable. The parameters are identified as B, subscripted.

For example, the equation

$$W = (A+BX^2)\ e^{(CX-DY)}$$

where the parameter values to be optimized are A, B, C, and D, and the independent variables are X and Y, can be programmed as

```
1 CY(N) = (B(1)+B(2)*Z(1,N)**2)* EXP(B(3)*Z(1,N)-B(4)*Z(2,N))
```

This is inserted in subroutine YCOMP after the statement

DO 1 N = 1,NUMBER

Programming of Data

The first data card contains specifications in 1216 format which are listed in the program as subscripted MM. They represent the following:

MM(1) = number of data points

MM(2) = index of dependent variable, or one digit higher than the highest indexed independent variable

MM(3) = number of parameters

MM(4) = limit on number of iterations. A value of 30 to 60 is usually sufficient for convergence.

MM(5), if negative, skips reading of Z(i, N) values from input.

MM(6) = -1 gives results of calculations for each iteration.

MM(6) = 0 gives final results only.

MM(6) = 1 gives results for only the first and last iterations.

MM(7) = 1 gives straight fit

MM(7) = 0 gives parabolic fit. Recommended.

MM(8) = 1 for input. This value is changed internally during the program run.

MM(9) = 1 prints input data as part of output.

MM(9) = 0 bypasses this recording.

MM(10) = -1 records each matrix.

MM(10) = 0 bypasses these.

MM(10) = 1 records first matrix only.

MM(11) = number of problems in the run. Allows multiple problems on one computer run.

MM(12) = if negative, nullifies printing of all output except final solution.

The second through fifth data cards are the initial estimates of parameter values in 6F12.12 format. The maximum number of parameters is twenty. The twenty-first through twenty-fourth fields are used as follows:

B(21) = tolerance on convergence. Recommend 0.000/.

B(22) = is used internally in DERIV routine. Leave this field blank.

B(23) = limit on magnitude of iteration changes. Recommend 1.0.

B(24) = is not normally used.

The program reads 24 values regardless of the number of parameters actually used. Therefore, there must be four data cards in this section, even though one or more may be blank.

The sixth card specifies the format of the remaining cards. All variable values for the first data point are listed first, followed by those of the second data point, etc. The dependent variable is the last value listed for each point. There are no unused fields. The first value of one data point must be in the first following that of the last value of the previous data point. The maximum number of data are three hundred points. And the maximum number of variable are twelve.

Program Lists

The following program lists are shown on succeeding pages:

List I = The general program excluding YCOMP subroutine.

List II = YCOMP subroutine for Johnson S_B distribution.

List III = YCOMP subroutine for Gumbel's limited distribution.

LIST I

```
C      EMBEDDING PROGRAM FOR GAUSS
       DIMENSION B(24),Z(12,300),MM(12)
       DIMENSION FTM (18)
       COMMON NUM,B,Z
       COMMON /COMA/ MM
       COMMON /COMB/ JJ
     1 READ (5,2) (MM(J),J=1,12)
     2 FORMAT (12I6)
       NUM=MM(1)
       NSET=MM(2)
       IF(NUM) 4,4,10
     4 WRITE (6,5)
       CALL EXIT
     5 FORMAT (40H0   GAUSS INPUT ZERO, PROGRAM STOP       /1H1)
    10 READ (5,11) (B(J),J=1,24)
    11 FORMAT (6F12.4)
       READ (5,100) (FMT(I),I=1,18)
   100 FORMAT (18A4)
       IF (MM(5)) 15,14,14
    13 FORMAT (4F12.4)
    14 READ (5,FMT) ((Z(J,N),J=1,NSET),N=1,NUM)
    15 CALL GAUSS
       IF (MM(8)-2) 30,20,30
    20 WRITE (6,21)
    21 FORMAT (40H0  GAUSS CONVERGENCE                    //)
       MM(8)=1
    30 MM(11)=MM(11)-1
       IF (MM(11)) 1,1,14
       END

       SUBROUTINE GAUSS
       DIMENSION A(20,21),B(24),BMIN(20),BSTART(20),C(20,1),X(20,1),
      *Z(12,300),DEL(20),E(20),MM(12),RECORD(100),CY(300),FP(20,300)
       COMMON NUMBER,B,Z
       COMMON /COMA/ MM
       COMMON /COMB/ JJ
       COMMON /COMC/ CY
       COMMON /COMD/ FP
       COMMON /COME/ A,C,M
       EQUIVALENCE  (A,X)
       NUMBER = MM(1)
       NSET =  MM(2)
       JJ =    MM(3)
       LIMIT = MM(4)
       NULL = MM(12)
       MM(12) = MM(12) + 1
       IDNTFC = MM(12)
       IZERO = 1.0
```

LIST I (CONTINUED)

```
      SCALE1 = 0.2
      SCALE2 = 1.5
      SCALE3 = 1.0
      TOL1 = B(21)
      X NORM = 0.0
      MARK P = 0
      KKPATH = -1
      NDOWN = 0
      NN = 0
      NNPARA = 0
      MPATH = 1
      NTZERO = -1
      SUMSQ = 0.0
      T = 0.0
      X3 = 3.0
      X2 = 2.0
      Y2 = 2.0
      Y3 = 3.0
      IF (LIMIT - 100) 2,47,47
    2 IF (TOL1) 420,420,1
    1 DO 4 J = 1,JJ
      BMIN(J) = B(J)
      BSTART(J) = B(J)
      XNORM = X NORM + B(J)**2
      DEL(J) = 0.05*ABS (B(J))
      IF (DEL(J)) 4,3,4
    3 DEL(J) = 0.05
    4 CONTINUE
      WRITE (6,5)
    5 FORMAT (51H1    GAUSSIAN PARAMETER SUBROUTINE        Z(12,300)
      WRITE (6,412)               (MM(L), L=1,12)
      WRITE (6,108)               (B(J), J = 1,24)
      IF (MM(9)) 400,6,400
    6 IF (MM(8) - 1) 7,80,7
    7 IF (B(23)) 8,8,430
    8 JPARA = -1
      MPATH = -1
      T = 0.0
      MM(8) = 2
      WRITE (6,59)
      DO 9  J = 1,JJ
    9 BSTART(J) = B(J)
   10 SQLAST = SUMSQ
      SUMSQ = 0.0
      NTZERO = NTZERO + 1
      NN = NN+1
      IF (NN - LIMIT) 12,12,11
   11 MM(8) = -2
```

LIST I (CONTINUED)

```
      GO TO 80
   12 CALL YCOMP
      DO 17 N = 1,NUMBER
      YC = CY(N)
      DELY = YC - Z(NSET,N)
      SUMSQ = SUMSQ + DELY**2
      IF (NULL) 17,13,13
   13 IF (MM(6)) 14,17,14
   14 IF (N-1)  16,15,16
   15 WRITE (6,410)
   16 WRITE (6,18)                N,YC,Z(NSET,N),DELY
      MARK P = 1
   17 CONTINUE
      RECORD(NN) = SUMSQ
   18 FORMAT (I6,4E18.7)
      GO TO 440
   19 IF (NN - 1) 20,22,30
   20 IF (SUMSQ-SQMIN) 21,21,27
   21 NDOWN = 1
   22 SQMIN = SUMSQ
      DO 24 J=1,JJ
   24 BMIN(J) = B(J)
   25 IF (MPATH) 301,200,38
   27 IF (NDOWN) 28,28,29
   28 NDOWN = -1
   29 IF (MPATH) 301,200,36
   30 IF (MM(6)) 32,32,31
   31 MM(6) = 0
   32 IF (MM(10)) 20,20,33
   33 NTZERO = 1
   38 DO 39 J = 1,JJ
      B(J) = BMIN(J)
   39 BSTART(J) = BMIN(J)
      Y1 = SQMIN
      X1 = 0.0
      JPARA = -1
      MPATH = -1
      GO TO 301
   40 SUM2 = SUM1
      SUM1 = SUMSQ
      NNPARA = 0
      IF (SUM1 - SUM2) 19,45,19
   45 TZERO = SCALE1*TZERO
      NDOWN = 0
      T = 0.0
      GO TO 8
   47 LIMIT = 99
      GO TO 2
```

LIST I (CONTINUED)

```
49 T = -0.5*((X1*X1-X2*X2)*(Y1-Y3)-(X1*X1-X3*X3)*(Y1-Y2))/
  *      ((X1-X3)*(Y1-Y2)-(X1-X2)*(Y1-Y3))
   MPATH = 1
   JPARA = -1
   NNPARA = 1
   NDOWN = 0
   GO TO 366
53 WRITE (6,54)
54 FORMAT (24H0 OVER-UNDERFLOW          //)
   MM(8) = -1
   MM(10) = -1
   GO TO 301
56 WRITE (6,57)
57 FORMAT (24H0 MATRIX IS SINGULAR      //)
   MM(8) = -1
   MM(10) = -1
   GO TO 301
59 FORMAT (114H0CYCLE   . SUM OF SQUARES      ***********************
  *************** PARAMETERS *************************************//)
58 FORMAT (I6, F18.5, 5E18.6/(E42.6,4E18.6))
60 DO 66  J=1,JJ
   BTEST = B(J)-BSTART(J)-DEL(J)
   IF (BTEST)  63,63,62
62 B(J) = BSTART(J) + DEL(J)
63 CONTINUE
   BTEST = B(J) -BSTART(J) + DEL(J)
   IF (BTEST) 65,65,66
65 B(J) = BSTART(J)-DEL(J)
66 CONTINUE
   MPATH = -1
67 DO 69 J=1,JJ
69 BSTART(J) = B(J)
   GO TO 10
80 IF (NULL) 1000,82,82
82 AV = 0.0
   AV1 = 0.0
   AV2 = 0.0
   YMAX = 0.0
   ZMAX = 0.0
   ZZMAX = 0.0
   DO 81  J=1,JJ
81 B(J) = BMIN(J)
   N = 1
   DO 90  J=1,JJ
90 WRITE (6,91) J,B(J)
91 FORMAT (4H   B I2, E14.5)
   WRITE (6,100)
92 WRITE (6,93)
```

LIST I (CONTINUED)

```
 93 FORMAT (82HONUMBER             Y OBSERVED      Y CALCULATED
   * DELTA Y       PCT DEVIATION ///)
 94 CALL YCOMP
 98 YC = CY(N)
    DELY = YC - Z(NSET,N)
    RATIO = 100.0 * (DELY / Z(NSET,N))
    ABSRAT = ABS (RATIO)
    AV = AV + DELY
    AV1 = AV1 + RATIO
    AV2 = AV2 + ABSRAT
    WRITE (6,95) N,Z(NSET,N),YC,DELY,RATIO
 95 FORMAT (I5,E23.5,E17.5,2E19.5)
    ABSVAL = ABS (DELY)
    IF (YMAX - ABSVAL) 96,96,97
 96 YMAX = ABSVAL
    YYMAX = DELY
    MARK = N
 97 IF (ZMAX-ABSRAT) 971,971,972
971 ZMAX = ABSRAT
    ZZMAX = RATIO
    MARK1 = N
972 N = N+1
    IF (N - NUMBER) 98,98,99
 99 D = NUMBER
    AV = AV/D
    AV1 = AV1/D
    AV2 = AV2/D
    RTMNSQ = SQRT (SUMSQ/D)
    WRITE (6,100)
100 FORMAT(118H0*******************************************************
   ****************************************************************/,
   */)
    WRITE (6,101)  AV,AV1,AV2
101 FORMAT (30H0  AVERAGE DEVIATION          E14.5,
   *        20H     AVERAGE PCT DEV          E14.5,
   *        20H     AVE ABS PCT DEV          E14.5)
    WRITE (6,103)  YYMAX,MARK
103 FORMAT (30H0  MAXIMUN DEVIATION          E14.5,I6)
    WRITE (6,104)  ZMAX,MARK1
104 FORMAT (30H0  MAXIMUN   PCT DEV          E14.5,I6)
    WRITE (6,105)  RTMNSQ
105 FORMAT (30H0  ROOT MEAN SQUARE DEVIATION E14.5)
107 FORMAT ( 21H0       AT ITERATION I3,  24H, THE SUM OF SQUARES IS
   *    E16.7/ 22H0 FOR PARAMETER VALUES /1H0//(6E20.7))
108 FORMAT (5F20.5)
109 FORMAT (///)
110 FORMAT (I20, F20.8)
```

LIST I (CONTINUED)

```
      WRITE (6,5)
      IF (MM(8) + 2) 114,111,114
  111 WRITE (6,112)
  112 FORMAT (30H0 EXCEEDED ITERATION LIMIT          ///)
      GO TO 999
  114 IF (MM(8) - 1) 999,8,999
  200 IF (NDOWN) 201,201,202
  201 T = T*SCALE1
      GO TO 203
  202 T = T*SCALE2
  203 MPATH = 0
      JPARA = JPARA + 1
      GO TO 366
  301 MPATH = 0
  302 DO 5 N = 1,JJ
  305 A(M,N) = 0.0
      CALL DERIV
      CALL YCOMP
      DO 313 N = 1,NUMBER
      DO 313 K = 1,JJ
      C(K,1) = C(K,1) + FP(K,N) * (Z(NSET,N) - CY(N))
      DO 313 J = K,JJ
  313 A(K,J) = A(K,J) + FP(K,N) * FP(J,N)
      IF (NTZERO) 318,318,317
  317 TZERO = 1.0
  318 T = TZERO
      DO 316 I = 2,JJ
      II = I-1
      DO 316 J=1,II
  316 A(I,J) = A(J,I)
      IF (MM(10)) 319,331,319
  319 WRITE (6,320)  NN
  320 FORMAT (19H0 MATRIX, ITERATION I3)
      MMPATH = 0
  322 DO 323 I=1,JJ
  323 WRITE (6,324)  (A(I,J), J=1,JJ)
  324 FORMAT (9E13.5)
      DO 328 I=1,JJ
  328 WRITE (6,324)  C(I,1)
      IF (MMPATH) 350,331,350
  331 DO 340 I=1,JJ
      DENOM = ABS (A(I,1))
      DO 336 J=2,JJ
      IF (DENOM-ABS (A(I,J))) 335,336,336
  335 DENOM = ABS (A(I,J))
  336 CONTINUE
```

LIST I (CONTINUED)

```
      DO 338  K=1,JJ
  338 A(I,K) = A(I,K)/DENOM*SCALE3
  340 C(I,1) = C(I,1)/DENOM*SCALE3
      MMPATH = 1
      IF (MM(10)) 322,350,322
  350 DD = 1.0
      IF (MM(8)) 999,354,354
  354 CALL SOLV
      GO TO (351,53,56),M
  351 IF (MM(6)) 352,363,352
  352 WRITE (6,353)  (X(J,1), J=1,JJ)
  353 FORMAT (13H0 DELTA B(J)/(9E13.5))
  363 Y NORM = 0.0
      DO 364 J=1,JJ
  364 Y NORM = Y NORM + X(J,1)**2
      IF (Y NORM - X NORM) 366,366,365
  365 T = 0.5*SQRT (X NORM)/SQRT (Y NORM)
      X1 = T
  366 DO 367 J=1,JJ
  367 B(J) = BSTART(J) + T*X(J,1)
  371 DO 376 J=1,JJ
      IF (B(J)) 372,374,372
  372 XX = ABS ((B(J) - BSTART(J))/B(J))
      GO TO 375
  374 XX = ABS (B(J) - BSTART(J))
  375 IF (XX-TOL1)  376,376,378
  376 CONTINUE
      MM(8) = 2
      GO TO 80
  378 IF (MM(7)) 60,379,60
  379 IF (NDOWN) 10,10,380
  380 IF (JPARA) 10,10,49
  400 IF (NULL)  6,401,401
  401 WRITE (6,100)
      IF (MM(5)) 406,403,403
  403 WRITE (6,402)
  402 FORMAT (15H   OBSERVATIONS//)
      DO 404 N=1,NUMBER
  404 WRITE (6,405)  N, (Z(J,N), J=1,12)
  405 FORMAT (I4,8E14.5/(E18.5,7E14.5))
  406 WRITE (6,5)  IDNTFC
      GO TO 6
  410 FORMAT ( 60H0 DATA        Y COMP                   Y OBS                DIFFER
     *ENCE                          )
  411 FORMAT (I6,F20.7)
  412 FORMAT (12I6)
  420 TOL1 = 0.0001
```

LIST I (CONTINUED)

```
      GO TO 1
  430 IF (B(23) - 1.0) 431,8,8
  431 TZERO = B(23)
      WRITE (6,433) TZERO
  433 FORMAT ( 30HO VECTOR SCALE FACTOR = B(23),    E12.4//)
      GO TO 8
  440 IF (NULL) 446,441,441
  441 N SPIN = N SPIN + 1
  442 IF (MARK P) 444,443,444
  443 IF (N SPIN - 15) 445,444,444
  444 N SPIN = 0
      WRITE (6,59)
  445 WRITE (6,58)  NN,SUMSQ,(B(J), J=1,JJ)
  446 X3 = X2
      X2 = X1
      X1 = T
      Y3 = Y2
      Y2 = Y1
      Y1 = SUMSQ
      IF (NNPARA) 40,19,40
  999 WRITE (6,991)
      DO 990  J=1,NN
  999 WRITE (6,405)   J,RECORD(J)
  991 FORMAT ( 28HO  RECORD OF SUM OF SQUARES       // )
  993 FORMAT (24HO MINIMIZING PARAMETERS    //)
      WRITE (6,993)
      WRITE (6,108)  (B(MIN(J),J=1,JJ)
 1000 RETURN
      END

      SUBROUTINE SOLV
      DIMENSION A(20,21), C(20,1),LOC(20), CK(20)
      COMMON /COMB/ JJ
      COMMON /COME/ A,C,M
      M = 1
      NP = JJ + 1
      DO 11 I = 1,JJ
      CK(I) = 0.0
   11 A(I,NP) = C(I,1)
      DO 50 I = 1,JJ
      IP = I + 1
C*****FIND MAX ELEMENT IN I'TH COLUMN.
      AMAX = 0.
      DO 2 K = 1,JJ
      IF (AMAX - ABS(A(K,I))) 3,2,2
C*****IS NEW MAX IN ROW PREVIOUSLY USED AS PIVOT
    3 IF (CK(K)) 4,4,2
```

LIST I (CONTINUED)

```
    4 LOC(I) = K
      AMAX = ABS(A(K,I))
    2 CONTINUE
      CK(L) = 1.0
C****PERFORM ELIMINATION. L IS PIVOT ROW, A(L,I) IS PIVOT ELEMENT.
      DO 50 J = 1,JJ
      IF (L-J) 6,50,6
    6 F = -A(J,I) / A(L,I)
      DO 40 K = IP,NP
   40 A(J,K) = A(J,K) + F * A(L,K)
   50 CONTINUE
      DO 200 I = 1,JJ
      L = LOC(I)
  200 A(I,1) = A(L,NP) / A(L,I)
      RETURN
   99 M = 3
      RETURN
      END

      SUBROUTINE DERIV
      DIMENSION B(24),Z(12,300),CY(300),FP(20,300),H(20),Y(300)
      COMMON NUMBER,B,Z
      COMMON /COMB/ JJ
      COMMON /COMC/ CY
      COMMON /COMD/ FP
      IF (B(22)) 20,1,20
    1 B(22) = 1.0
      DO 7 J = 1,JJ
      TEST = ABS(B(J))
      IF (TEST - 0.001) 5,6,6
    5 H(J) = 0.001
      GO TO 7
    6 H(J) = 0.0001 * TEST
    7 CONTINUE
   20 DO 22 J = 1,JJ
      TEMP = B(J)
      B(J) = TEMP + H(J)
      CALL YCOMP
      DO 21 N = 1,NUMBER
   21 Y(N) = CY(N)
      B(J) = TEMP - H(J)
      CALL YCOMP
      B(J) = TEMP
      DO 22 N = 1,NUMBER
   22 FP(J,N) = (Y(N) - CY(N))/(2. * H(J))
      RETURN
      END
```

LIST II

```
      SUBROUTINE YCOMP
      DIMENSION B(24),Z(12,300),CY(300)
      COMMON NUM,B,Z
      COMMON /COMC/ CY
      M = 31
''''''M = SAMPLE SIZE
      IF (B(3)-Z((1,1)) 20,20,30
   30 B(3) = Z(1,1)-1.0
      GO TO 80
   20 IF (B(3)) 60,80,80
   60 B(3) = 0.0
   80 BB = B(3) + B(4)
      IF (BB - Z(1,M)) 2,2,4
    2 B(4) = Z(1,M) - B(3) + 1.0
    4 DO 1 N = 1,NUM
      X = B(1) + B(2)*ALOG((Z(1,N)-B(3))/(B(3)+B(4)-Z(1,N)))
      AX = ABS(X)
      T = 1.0/(1.0+0.2316419*AX)
      D = 0.3989423*EXP(-X*X/2.0)
      P = 1. - D*T*((((1.330274*T-1.821256)*T+1.781478)*T-0.3565638)*T
     * 0.3193815)
      IF (X) 10,1,1
   10 P = 1.0-P
    1 CY(N) = P
      RETURN
      END
```

LIST III

```
      SUBROUTINE YCOMP
      DIMENSION B(24),Z(12,300),CY(300)
      COMOMN NUM,B,Z
      COMMON /COMC/ CY
      IF (B(3)-Z(1,1)) 20,20,30
   30 B(3) = Z(1,1)
      GO TO 3
   20 IF (B(3)) 2,3,3
    2 B(3) = 0.0
    3 DO 1 N = 1,NUM
      EX = ((Z(1,N)-B(3))/(B(1)-B(3)))**B(2)
      EY = EXP(EX)
    1 CY(N) = 1.0 - (1.0/EY)
      RETURN
      END
```

APPENDIX C

JOHNSON S_B DISTRIBUTION

JOHNSON SB DISTRIBUTION FOR MIN. 1-DAY FLOWS AT STATION 7-1775

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	0.01	0.0943	0.0323	0.0620
2	0.01	0.0943	0.0645	0.0297
3	0.01	0.0943	0.0968	-0.0025
4	0.01	0.0943	0.1290	-0.0348
5	0.01	0.0943	0.1613	-0.0670
6	0.01	0.0943	0.1935	-0.0993
7	0.01	0.0943	0.2258	-0.1315
8	0.10	0.2565	0.2581	-0.0016
9	0.10	0.2565	0.2903	-0.0338
10	0.20	0.3248	0.3226	0.0022
11	0.30	0.3681	0.3548	0.0133
12	0.30	0.3681	0.3871	-0.0190
13	1.20	0.5288	0.4193	0.1094
14	1.30	0.5384	0.4516	0.0868
15	1.50	0.5558	0.4839	0.0719
16	1.70	0.5710	0.5161	0.0549
17	2.10	0.5968	0.5484	0.0484
18	2.20	0.6025	0.5806	0.0219
19	3.80	0.6701	0.6129	0.0572
20	3.80	0.6701	0.6452	0.0250
21	4.50	0.6914	0.6774	0.0140
22	4.80	0.6996	0.7097	-0.0101
23	5.00	0.7048	0.7419	-0.0371
24	5.60	0.7194	0.7742	-0.0548
25	7.20	0.7524	0.8064	-0.0541
26	7.20	0.7524	0.8387	-0.0863
27	8.60	0.7767	0.8710	-0.0943
28	12.00	0.8262	0.9032	-0.0771
29	17.00	0.8912	0.9355	-0.0443
30	22.00	0.9892	0.9677	0.0214

SUM OF SQUARES	0.10601	** GAMMA =	0.89156
MAX. DEVIATION	0.1315	** ETA =	0.28644
AVE. DEVIATION	-0.0076	** EPSILON =	0.0
AVE. ABS. DEVIATION	0.0489	** LAMBDA =	22.16277

JOHNSON SB DISTRIBUTION FOR MIN. 3-DAY FLOWS AT STATION 7-1775

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	0.01	0.0568	0.0323	0.0245
2	0.01	0.0568	0.0645	-0.0077
3	0.01	0.0568	0.0968	-0.0400
4	0.01	0.0568	0.1290	-0.0722
5	0.01	0.0568	0.1613	-0.1045
6	0.01	0.0568	0.1935	-0.1367
7	0.01	0.0568	0.2258	-0.1690
8	0.30	0.2593	0.2581	0.0012
9	0.50	0.3071	0.2903	0.0168
10	0.80	0.3543	0.3226	0.0318
11	0.90	0.3666	0.3548	0.0118
12	1.70	0.4355	0.3871	0.0484
13	3.60	0.5208	0.4193	0.1014
14	4.30	0.5414	0.4516	0.0898
15	5.40	0.5681	0.4839	0.0842
16	5.50	0.5703	0.5161	0.0541
17	6.30	0.5863	0.5484	0.0379
18	6.60	0.5918	0.5806	0.0112
19	11.40	0.6574	0.6129	0.0445
20	12.20	0.6657	0.6452	0.0206
21	13.50	0.6782	0.6774	0.0008
22	14.40	0.6862	0.7097	-0.0235
23	15.80	0.6978	0.7419	-0.0441
24	18.40	0.7172	0.7742	-0.0570
25	21.60	0.7381	0.8064	-0.0683
26	22.30	0.7424	0.8387	-0.0964
27	28.00	0.7737	0.8710	-0.0973
28	37.00	0.8162	0.9032	-0.0870
29	55.00	0.8977	0.9355	-0.0378
30	66.00	0.9846	0.9677	0.0168

SUM OF SQUARES	0.14022	** GAMMA =	0.83905
MAX. DEVIATION	0.1690	** ETA =	0.27505
AVE. DEVIATION	-0.0149	** EPSILON =	0.0
AVE. ABS. DEVIATION	0.0546	** LAMBDA =	66.54346

JOHNSON SB DISTRIBUTION FOR MIN. 7-DAY FLOWS AT STATION 7-1775

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	0.01	0.0205	0.0323	-0.0117
2	0.01	0.0205	0.0645	-0.0440
3	0.01	0.0205	0.0968	-0.0762
4	0.01	0.0205	0.1290	-0.1085
5	0.01	0.0205	0.1613	-0.1407
6	0.01	0.0205	0.1935	-0.1730
7	0.60	0.2069	0.2258	-0.0189
8	0.70	0.2204	0.2581	-0.0376
9	1.80	0.3134	0.2903	0.0230
10	2.20	0.3352	0.3226	0.0126
11	2.40	0.3449	0.3548	-0.0100
12	4.50	0.4179	0.3871	0.0308
13	9.00	0.5034	0.4193	0.0840
14	11.50	0.5345	0.4516	0.0829
15	13.50	0.5551	0.4839	0.0712
16	14.70	0.5661	0.5161	0.0499
17	14.70	0.5661	0.5484	0.0177
18	18.40	0.5952	0.5806	0.0146
19	28.20	0.6515	0.6129	0.0386
20	30.80	0.6633	0.6452	0.0182
21	36.00	0.6844	0.6774	0.0070
22	39.70	0.6978	0.7097	-0.0119
23	39.70	0.6978	0.7419	-0.0442
24	48.80	0.7265	0.7742	-0.0477
25	54.80	0.7431	0.8064	-0.0634
26	57.90	0.7511	0.8387	-0.0876
27	67.90	0.7748	0.8710	-0.0961
28	98.00	0.8355	0.9032	-0.0678
29	157.00	0.9608	0.9355	0.0254
30	161.00	0.9764	0.9677	0.0087

SUM OF SQUARES	0.12741	** GAMMA =	0.86115
MAX. DEVIATION	0.1730	** ETA =	0.29906
AVE. DEVIATION	-0.0185	** EPSILON =	0.0
AVE. ABS. DEVIATION	0.0508	** LAMBDA =	164.76128

JOHNSON SB DISTRIBUTION FOR MIN. 14-DAY FLOWS AT STATION 7-1775

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	0.01	0.0012	0.0323	-0.0311
2	0.01	0.0012	0.0645	-0.0633
3	0.01	0.0012	0.0968	-0.0956
4	0.01	0.0012	0.1290	-0.1278
5	0.10	0.0156	0.1613	-0.1457
6	1.80	0.1473	0.1935	-0.0462
7	3.20	0.2042	0.2258	-0.0216
8	5.20	0.2613	0.2581	0.0033
9	5.50	0.2684	0.2903	-0.0219
10	6.90	0.2982	0.3226	-0.0244
11	12.60	0.3841	0.3548	0.0292
12	15.30	0.4136	0.3871	0.0265
13	20.70	0.4608	0.4193	0.0415
14	27.80	0.5081	0.4516	0.0565
15	29.40	0.5171	0.4839	0.0333
16	29.60	0.5182	0.5161	0.0021
17	35.70	0.5487	0.5484	0.0003
18	43.60	0.5814	0.5806	0.0008
19	66.00	0.6493	0.6129	0.0364
20	66.80	0.6513	0.6452	0.0061
21	80.80	0.6823	0.6774	0.0049
22	89.70	0.6993	0.7097	-0.0104
23	124.30	0.7521	0.7419	0.0101
24	134.40	0.7646	0.7742	-0.0095
25	137.30	0.7681	0.8064	-0.0384
26	159.50	0.7922	0.8387	-0.0465
27	161.40	0.7941	0.8710	-0.0769
28	264.00	0.8744	0.9032	-0.0288
29	405.00	0.9515	0.9355	0.0161
30	482.00	0.9910	0.9677	0.0233

SUM OF SQUARES	0.07581	** GAMMA =	1.10408
MAX. DEVIATION	0.1457	** ETA =	0.38274
AVE. DEVIATION	-0.0166	** EPSILON =	0.0
AVE. ABS. DEVIATION	0.0359	** LAMBDA =	499.75635

JOHNSON SB DISTRIBUTION FOR MIN. 30-DAY FLOWS AT STATION 7-1775

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	0.01	0.0000	0.0323	-0.0323
2	0.01	0.0000	0.0645	-0.0645
3	0.01	0.0000	0.0968	-0.0968
4	0.30	0.0028	0.1290	-0.1262
5	11.40	0.1437	0.1613	-0.0175
6	15.30	0.1774	0.1935	-0.0161
7	17.20	0.1921	0.2258	-0.0337
8	19.80	0.2108	0.2581	-0.0473
9	43.90	0.3345	0.2903	0.0442
10	46.10	0.3430	0.3226	0.0204
11	46.70	0.3453	0.3548	-0.0096
12	50.40	0.3587	0.3871	-0.0284
13	84.70	0.4544	0.4193	0.0350
14	85.00	0.4550	0.4516	0.0034
15	107.00	0.4990	0.4839	0.0151
16	135.40	0.5442	0.5161	0.0281
17	176.60	0.5949	0.5484	0.0465
18	177.80	0.5962	0.5806	0.0156
19	254.00	0.6625	0.6129	0.0496
20	307.80	0.6970	0.6452	0.0518
21	308.80	0.6975	0.6774	0.0201
22	315.20	0.7011	0.7097	-0.0086
23	345.30	0.7170	0.7419	-0.0250
24	369.10	0.7284	0.7742	-0.0458
25	376.40	0.7317	0.8064	-0.0748
26	521.30	0.7846	0.8387	-0.0541
27	832.40	0.8523	0.8710	-0.0187
28	923.00	0.8658	0.9032	-0.0374
29	2437.00	0.9661	0.9355	0.0307
30	3665.00	0.9931	0.9677	0.0254

SUM OF SQUARES	0.06257	** GAMMA =	1.73426
MAX. DEVIATION	0.1262	** ETA =	0.46931
AVE. DEVIATION	-0.0117	** EPSILON =	0.0
AVE. ABS. DEVIATION	0.0374	** LAMBDA =	4437.83594

JOHNSON SB DISTRIBUTION FOR MIN. 60-DAY FLOWS AT STATION 7-1775

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	0.01	0.0000	0.0323	-0.0323
2	0.10	0.0001	0.0645	-0.0645
3	6.80	0.0273	0.0968	-0.0695
4	7.20	0.0290	0.1290	-0.1001
5	40.10	0.1328	0.1613	-0.0285
6	52.60	0.1613	0.1935	-0.0322
7	83.30	0.2181	0.2258	-0.0077
8	126.60	0.2788	0.2581	0.0208
9	194.20	0.3487	0.2903	0.0584
10	249.00	0.3922	0.3226	0.0697
11	263.20	0.4022	0.3548	0.0474
12	318.20	0.4368	0.3871	0.0497
13	349.30	0.4541	0.4193	0.0347
14	352.10	0.4556	0.4516	0.0040
15	407.50	0.4829	0.4839	-0.0009
16	424.80	0.4908	0.5161	-0.0254
17	592.70	0.5539	0.5484	0.0055
18	666.40	0.5761	0.5806	-0.0046
19	685.80	0.5815	0.6129	-0.0314
20	901.20	0.6328	0.6452	-0.0124
21	925.00	0.6376	0.6774	-0.0398
22	1052.30	0.6614	0.7097	-0.0483
23	1158.30	0.6789	0.7419	-0.0631
24	2274.00	0.7949	0.7742	0.0207
25	2503.20	0.8102	0.8064	0.0038
26	3069.00	0.8416	0.8387	0.0028
27	4163.10	0.8855	0.8710	0.0145
28	4939.80	0.9085	0.9032	0.0052
29	6194.00	0.9370	0.9355	0.0016
30	8973.00	0.9790	0.9677	0.0113

SUM OF SQUARES	0.04712	** GAMMA =	1.46252
MAX. DEVIATION	0.1001	** ETA =	0.45524
AVE. DEVIATION	-0.0070	** EPSILON =	0.0
AVE. ABS. DEVIATION	0.0303	** LAMBDA =	11529.59766

JOHNSON SB DISTRIBUTION FOR MIN. 90-DAY FLOWS AT STATION 7-1775

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	0.01	0.0000	0.0323	-0.0323
2	0.10	0.0001	0.0645	-0.0644
3	24.80	0.0556	0.0968	-0.0411
4	29.80	0.0640	0.1290	-0.0651
5	112.10	0.1550	0.1613	-0.0063
6	116.50	0.1585	0.1935	-0.0350
7	237.70	0.2337	0.2258	0.0079
8	382.80	0.2937	0.2581	0.0357
9	429.40	0.3092	0.2903	0.0189
10	466.70	0.3207	0.3226	-0.0018
11	725.60	0.3847	0.3548	0.0298
12	925.40	0.4218	0.3871	0.0347
13	934.10	0.4232	0.4193	0.0039
14	1203.70	0.4631	0.4516	0.0115
15	1217.40	0.4649	0.4839	-0.0190
16	1507.80	0.4993	0.5161	-0.0169
17	1858.20	0.5333	0.5484	-0.0151
18	2043.20	0.5488	0.5806	-0.0318
19	2935.30	0.6086	0.6129	-0.0043
20	4130.60	0.6653	0.6452	0.0201
21	6080.90	0.7294	0.6774	0.0520
22	7258.70	0.7588	0.7097	0.0491
23	7482.30	0.7638	0.7419	0.0219
24	7935.10	0.7736	0.7742	-0.0006
25	8369.00	0.7825	0.8064	-0.0240
26	11036.50	0.8291	0.8387	-0.0096
27	11316.00	0.8334	0.8710	-0.0376
28	12768.60	0.8542	0.9032	-0.0490
29	15034.00	0.8833	0.9355	-0.0522
30	25535.60	1.0000	0.9677	0.0323

SUM OF SQUARES	0.03245	** GAMMA =	1.05479
MAX. DEVIATION	0.0651	** ETA =	0.38166
AVE. DEVIATION	-0.0063	** EPSILON =	0.0
AVE. ABS. DEVIATION	0.0275	** LAMBDA =	25536.59766

JOHNSON SB DISTRIBUTION FOR MIN. 1-DAY FLOWS AT STATION 7-1649

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	30.00	0.0094	0.0313	-0.0219
2	85.00	0.0550	0.0625	-0.0075
3	147.00	0.1161	0.0938	0.0223
4	202.00	0.1696	0.1250	0.0446
5	270.00	0.2320	0.1563	0.0757
6	280.00	0.2408	0.1875	0.0533
7	290.00	0.2494	0.2188	0.0306
8	300.00	0.2580	0.2500	0.0080
9	326.00	0.2797	0.2813	-0.0016
10	327.00	0.2805	0.3125	-0.0320
11	342.00	0.2927	0.3438	-0.0511
12	372.00	0.3165	0.3750	-0.0585
13	394.00	0.3333	0.4063	-0.0730
14	436.00	0.3642	0.4375	-0.0733
15	615.00	0.4798	0.4688	0.0110
16	688.00	0.5205	0.5000	0.0205
17	694.00	0.5237	0.5313	-0.0076
18	790.00	0.5723	0.5625	0.0098
19	850.00	0.6004	0.5938	0.0066
20	944.00	0.6411	0.6250	0.0161
21	1150.00	0.7186	0.6563	0.0623
22	1200.00	0.7353	0.6875	0.0478
23	1220.00	0.7418	0.7188	0.0230
24	1260.00	0.7544	0.7500	0.0044
25	1310.00	0.7696	0.7813	-0.0117
26	1400.00	0.7954	0.8125	-0.0171
27	1500.00	0.8217	0.8438	-0.0221
28	1540.00	0.8316	0.8750	-0.0434
29	1750.00	0.8784	0.9063	-0.0279
30	2100.00	0.9389	0.9375	0.0014
31	2620.00	0.9918	0.9688	0.0230

SUM OF SQUARES	0.04216	** GAMMA =	0.88846
MAX. DEVIATION	0.0757	** ETA =	0.70837
AVE. DEVIATION	0.0004	** EPSILON =	0.0
AVE. ABS. DEVIATION	0.0293	** LAMBDA =	2930.84131

JOHNSON SB DISTRIBUTION FOR MIN. 3-DAY FLOWS AT STATION 7-1645

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	92.00	0.0065	0.0313	-0.0248
2	263.00	0.0466	0.0625	-0.0159
3	447.00	0.1026	0.0938	0.0088
4	692.00	0.1783	0.1250	0.0533
5	852.00	0.2252	0.1563	0.0689
6	880.00	0.2331	0.1875	0.0456
7	880.00	0.2331	0.2188	0.0143
8	920.00	0.2443	0.2500	-0.0057
9	989.00	0.2632	0.2813	-0.0181
10	1116.00	0.2966	0.3125	-0.0159
11	1193.00	0.3160	0.3438	-0.0278
12	1218.00	0.3222	0.3750	-0.0528
13	1307.00	0.3436	0.4063	-0.0627
14	1494.00	0.3861	0.4375	-0.0514
15	2116.00	0.5060	0.4688	0.0372
16	2134.00	0.5091	0.5000	0.0091
17	2170.00	0.5151	0.5313	-0.0162
18	2403.00	0.5522	0.5625	-0.0103
19	2638.00	0.5865	0.5938	-0.0073
20	3540.00	0.6945	0.6250	0.0695
21	3550.00	0.6956	0.6563	0.0393
22	3640.00	0.7046	0.6875	0.0171
23	3880.00	0.7275	0.7188	0.0087
24	4070.00	0.7445	0.7500	-0.0055
25	4770.00	0.7990	0.7813	0.0177
26	4820.00	0.8025	0.8125	-0.0100
27	5320.00	0.8345	0.8438	-0.0093
28	5400.00	0.8392	0.8750	-0.0358
29	5790.00	0.8606	0.9063	-0.0457
30	8600.00	0.9597	0.9375	0.0222
31	8880.00	0.9656	0.9688	-0.0032

SUM OF SQUARES	0.03463	** GAMMA =	1.26156
MAX. DEVIATION	0.0695	** ETA =	0.75587
AVE. DEVIATION	-0.0002	** EPSILON =	0.0
AVE. ABS. DEVIATION	0.0268	** LAMBDA =	13123.80469

JOHNSON SB DISTRIBUTION FOR MIN. 7-DAY FLOWS AT STATION 7-1645

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	240.00	0.0074	0.0313	-0.0239
2	624.00	0.0412	0.0625	-0.0213
3	1071.00	0.0910	0.0938	-0.0028
4	1996.00	0.1949	0.1250	0.0699
5	2160.00	0.2123	0.1563	0.0560
6	2209.00	0.2175	0.1875	0.0300
7	2209.00	0.2175	0.2188	-0.0013
8	2210.00	0.2176	0.2500	-0.0324
9	2361.00	0.2331	0.2813	-0.0482
10	2698.00	0.2667	0.3125	-0.0458
11	3070.00	0.3019	0.3438	-0.0419
12	3731.00	0.3598	0.3750	-0.0152
13	4250.00	0.4014	0.4063	-0.0049
14	5047.00	0.4596	0.4375	0.0221
15	5306.00	0.4772	0.4688	0.0084
16	5973.00	0.5197	0.5000	0.0197
17	6377.00	0.5437	0.5313	0.0124
18	6458.00	0.5483	0.5625	-0.0142
19	7354.00	0.5968	0.5938	0.0030
20	8850.00	0.6671	0.6250	0.0421
21	9180.00	0.6810	0.6563	0.0247
22	9240.00	0.6835	0.6875	-0.0040
23	9840.00	0.7074	0.7188	-0.0114
24	11750.00	0.7739	0.7500	0.0239
25	11890.00	0.7782	0.7813	-0.0031
26	12430.00	0.7945	0.8125	-0.0180
27	13220.00	0.8167	0.8438	-0.0271
28	13830.00	0.8327	0.8750	-0.0423
29	16360.00	0.8892	0.9063	-0.0171
30	22310.00	0.9732	0.9375	0.0357
31	23000.00	0.9792	0.9688	0.0104

SUM OF SQUARES	0.02637	** GAMMA =	1.01037
MAX. DEVIATION	0.0699	** ETA =	0.72258
AVE. DEVIATION	-0.0005	** EPSILON =	0.0
AVE. ABS. DEVIATION	0.0236	** LAMBDA =	28557.96875

JOHNSON SB DISTRIBUTION FOR MIN. 14-DAY FLOWS AT STATION 7-1645

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	653.00	0.0163	0.0313	-0.0150
2	1350.00	0.0487	0.0625	-0.0138
3	2440.00	0.1039	0.0938	0.0101
4	4323.00	0.1939	0.1250	0.0689
5	4400.00	0.1974	0.1563	0.0411
6	4501.00	0.2019	0.1875	0.0144
7	4700.00	0.2106	0.2188	-0.0082
8	4730.00	0.2119	0.2500	-0.0381
9	5474.00	0.2437	0.2813	-0.0376
10	6899.00	0.3001	0.3125	-0.0124
11	7058.00	0.3060	0.3438	-0.0378
12	7683.00	0.3289	0.3750	-0.0461
13	9982.00	0.4058	0.4063	-0.0005
14	10500.00	0.4217	0.4375	-0.0158
15	11933.00	0.4635	0.4688	-0.0053
16	13217.00	0.4984	0.5000	-0.0016
17	15498.00	0.5552	0.5313	0.0239
18	18800.00	0.6278	0.5625	0.0653
19	19850.00	0.6489	0.5938	0.0551
20	19954.00	0.6510	0.6250	0.0260
21	22210.00	0.6933	0.6563	0.0370
22	22530.00	0.6990	0.6875	0.0115
23	25000.00	0.7410	0.7188	0.0222
24	25860.00	0.7548	0.7500	0.0048
25	27060.00	0.7733	0.7813	-0.0080
26	27190.00	0.7753	0.8125	-0.0372
27	27960.00	0.7867	0.8438	-0.0571
28	28710.00	0.7976	0.8750	-0.0774
29	34820.00	0.8767	0.9063	-0.0296
30	48770.00	0.9940	0.9375	0.0565
31	51700.00	1.0000	0.9688	0.0312

SUM OF SQUARES	0.04038	** GAMMA =	0.68905
MAX. DEVIATION	0.0774	** ETA =	0.64851
AVE. DEVIATION	0.0008	** EPSILON =	0.0
AVE. ABS. DEVIATION	0.0293	** LAMBDA =	51701.00000

JOHNSON SB DISTRIBUTION FOR MIN. 30-DAY FLOWS AT STATION 7-1645

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	1834.00	0.0177	0.0313	-0.0136
2	3210.00	0.0424	0.0625	-0.0201
3	6151.00	0.1009	0.0938	0.0071
4	10266.00	0.1799	0.1250	0.0549
5	11229.00	0.1972	0.1563	0.0409
6	11284.00	0.1982	0.1875	0.0107
7	12595.00	0.2211	0.2188	0.0023
8	12865.00	0.2257	0.2500	-0.0243
9	13576.00	0.2377	0.2813	-0.0436
10	15873.00	0.2749	0.3125	-0.0376
11	17236.00	0.2959	0.3438	-0.0479
12	19896.00	0.3346	0.3750	-0.0404
13	25484.00	0.4080	0.4063	0.0017
14	28440.00	0.4429	0.4375	0.0054
15	29939.00	0.4598	0.4688	-0.0090
16	36667.00	0.5290	0.5000	0.0290
17	42070.00	0.5782	0.5313	0.0469
18	47100.00	0.6196	0.5625	0.0571
19	47848.00	0.6255	0.5938	0.0317
20	56880.00	0.6905	0.6250	0.0655
21	57980.00	0.6978	0.6563	0.0415
22	59550.00	0.7080	0.6875	0.0205
23	60710.00	0.7153	0.7188	-0.0035
24	62450.00	0.7261	0.7500	-0.0239
25	64360.00	0.7376	0.7813	-0.0437
26	66290.00	0.7489	0.8125	-0.0636
27	72870.00	0.7850	0.8438	-0.0588
28	80220.00	0.8214	0.8750	-0.0536
29	105390.00	0.9200	0.9063	0.0137
30	111790.00	0.9394	0.9375	0.0019
31	149190.00	1.0000	0.9688	0.0312

SUM OF SQUARES	0.04116	** GAMMA =	0.81994
MAX. DEVIATION	0.0655	** ETA =	0.66632
AVE. DEVIATION	-0.0007	** EPSILON =	0.0
AVE. ABS. DEVIATION	0.0305	** LAMBDA =	149191.00000

JOHNSON SB DISTRIBUTION FOR MIN. 60-DAY FLOWS AT STATION 7-1645

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	5593.00	0.0170	0.0313	-0.0143
2	8986.00	0.0396	0.0625	-0.0229
3	15685.00	0.0899	0.0938	-0.0039
4	26962.00	0.1733	0.1250	0.0483
5	28140.00	0.1815	0.1563	0.0252
6	31910.00	0.2073	0.1875	0.0198
7	37301.00	0.2425	0.2188	0.0237
8	39817.00	0.2583	0.2500	0.0083
9	42319.00	0.2735	0.2813	-0.0078
10	45610.00	0.2930	0.3125	-0.0195
11	48002.00	0.3067	0.3438	-0.0371
12	51414.00	0.3257	0.3750	-0.0493
13	57286.00	0.3570	0.4063	-0.0493
14	68012.00	0.4096	0.4375	-0.0279
15	69021.00	0.4143	0.4688	-0.0545
16	103918.00	0.5533	0.5000	0.0533
17	124057.00	0.6178	0.5313	0.0865
18	134120.00	0.6466	0.5625	0.0841
19	136730.00	0.6538	0.5938	0.0600
20	141910.00	0.6677	0.6250	0.0427
21	142100.00	0.6682	0.6563	0.0119
22	149754.00	0.6878	0.6875	0.0003
23	151420.00	0.6919	0.7188	-0.0269
24	154920.00	0.7005	0.7500	-0.0495
25	177120.00	0.7506	0.7813	-0.0307
26	207900.00	0.8100	0.8125	-0.0025
27	207910.00	0.8100	0.8438	-0.0338
28	218970.00	0.8288	0.8750	-0.0462
29	241390.00	0.8636	0.9063	-0.0427
30	294520.00	0.9297	0.9375	-0.0078
31	418200.00	1.0000	0.9688	0.0312

SUM OF SQUARES	0.04838	** GAMMA =	0.88796
MAX. DEVIATION	0.0865	** ETA =	0.67673
AVE. DEVIATION	-0.0010	** EPSILON =	751.85596
AVE. ABS. DEVIATION	0.0330	** LAMBDA =	417449.12500

JOHNSON SB DISTRIBUTION FOR MIN. 90-DAY FLOWS AT STATION 7-1645

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	12427.00	0.0226	0.0313	-0.0087
2	16007.00	0.0338	0.0625	-0.0287
3	24598.00	0.0635	0.0938	-0.0303
4	49495.00	0.1531	0.1250	0.0281
5	51318.00	0.1595	0.1563	0.0032
6	61844.00	0.1957	0.1875	0.0082
7	63046.00	0.1997	0.2188	-0.0191
8	67259.00	0.2137	0.2500	-0.0363
9	74883.00	0.2385	0.2813	-0.0428
10	88535.00	0.2809	0.3125	-0.0316
11	112252.00	0.3491	0.3438	0.0053
12	120559.00	0.3715	0.3750	-0.0035
13	133623.00	0.4052	0.4063	-0.0011
14	155117.00	0.4573	0.4375	0.0198
15	160678.00	0.4701	0.4688	0.0013
16	174297.00	0.5004	0.5000	0.0004
17	201910.00	0.5578	0.5313	0.0265
18	208360.00	0.5704	0.5625	0.0079
19	220550.00	0.5937	0.5938	-0.0001
20	228580.00	0.6085	0.6250	-0.0165
21	233200.00	0.6169	0.6563	-0.0394
22	261520.00	0.6658	0.6875	-0.0217
23	288520.00	0.7089	0.7188	-0.0099
24	288780.00	0.7093	0.7500	-0.0407
25	328920.00	0.7676	0.7813	-0.0137
26	377600.00	0.8302	0.8125	0.0177
27	380890.00	0.8342	0.8438	-0.0096
28	386440.00	0.8407	0.8750	-0.0343
29	388510.00	0.8431	0.9063	-0.0632
30	508610.00	0.9584	0.9375	0.0209
31	606430.00	1.0000	0.9688	0.0312

SUM OF SQUARES	0.01962	** GAMMA =	0.61588
MAX. DEVIATION	0.0632	** ETA =	0.67717
AVE. DEVIATION	-0.0091	** EPSILON =	0.0
AVE. ABS. DEVIATION	0.0201	** LAMBDA =	606431.00000

JOHNSON SB DISTRIBUTION FOR MIN. 1-DAY FLOWS AT STATION 7-1945

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	76.00	0.0065	0.0286	-0.0220
2	200.00	0.0390	0.0571	-0.0181
3	255.00	0.0572	0.0857	-0.0285
4	500.00	0.1457	0.1143	0.0314
5	566.00	0.1696	0.1429	0.0267
6	585.00	0.1764	0.1714	0.0050
7	638.00	0.1954	0.2000	-0.0046
8	650.00	0.1996	0.2286	-0.0290
9	678.00	0.2095	0.2571	-0.0477
10	910.00	0.2879	0.2857	0.0021
11	983.00	0.3113	0.3143	-0.0030
12	1040.00	0.3291	0.3429	-0.0138
13	1040.00	0.3291	0.3714	-0.0423
14	1110.00	0.3505	0.4000	-0.0495
15	1470.00	0.4524	0.4286	0.0238
16	1680.00	0.5059	0.4571	0.0488
17	1810.00	0.5371	0.4857	0.0514
18	1850.00	0.5465	0.5143	0.0322
19	1890.00	0.5557	0.5429	0.0128
20	1960.00	0.5715	0.5714	0.0000
21	2190.00	0.6208	0.6000	0.0208
22	2240.00	0.6310	0.6286	0.0024
23	2340.00	0.6509	0.6571	-0.0062
24	2530.00	0.6871	0.6857	0.0014
25	2640.00	0.7070	0.7143	-0.0073
26	2790.00	0.7330	0.7429	-0.0098
27	3020.00	0.7705	0.7714	-0.0009
28	3090.00	0.7813	0.8000	-0.0187
29	3380.00	0.8236	0.8286	-0.0049
30	3490.00	0.8386	0.8571	-0.0185
31	3510.00	0.8412	0.8857	-0.0445
32	3650.00	0.8593	0.9143	-0.0550
33	5100.00	0.9880	0.9429	0.0452
34	5650.00	1.0000	0.9714	0.0286

SUM OF SQUARES	0.02692	** GAMMA =	0.64003
MAX. DEVIATION	0.0550	** ETA =	0.72690
AVE. DEVIATION	-0.0027	** EPSILON =	0.0
AVE. ABS. DEVIATION	0.0223	** LAMBDA =	5650.29297

JOHNSON SB DISTRIBUTION FOR MIN. 3-DAY FLOWS AT STATION 7-1945

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	276.00	0.0089	0.0286	-0.0197
2	605.00	0.0363	0.0571	-0.0208
3	1020.00	0.0799	0.0857	-0.0058
4	1500.00	0.1332	0.1143	0.0189
5	1956.00	0.1831	0.1429	0.0402
6	2045.00	0.1926	0.1714	0.0211
7	2054.00	0.1935	0.2000	-0.0065
8	2177.00	0.2065	0.2286	-0.0220
9	2366.00	0.2262	0.2571	-0.0310
10	2980.00	0.2871	0.2857	0.0014
11	3049.00	0.2937	0.3143	-0.0206
12	3120.00	0.3004	0.3429	-0.0424
13	3430.00	0.3290	0.3714	-0.0424
14	4040.00	0.3823	0.4000	-0.0177
15	4630.00	0.4301	0.4286	0.0015
16	5400.00	0.4876	0.4571	0.0305
17	5790.00	0.5149	0.4857	0.0292
18	6050.00	0.5324	0.5143	0.0181
19	6750.00	0.5771	0.5429	0.0343
20	7250.00	0.6071	0.5714	0.0356
21	7650.00	0.6299	0.6000	0.0299
22	7780.00	0.6371	0.6286	0.0085
23	8050.00	0.6518	0.6571	-0.0054
24	9060.00	0.7032	0.6857	0.0175
25	9200.00	0.7100	0.7143	-0.0043
26	10300.00	0.7597	0.7429	0.0168
27	10530.00	0.7694	0.7714	-0.0021
28	11340.00	0.8019	0.8000	0.0019
29	11390.00	0.8038	0.8286	-0.0248
30	11440.00	0.8057	0.8571	-0.0514
31	11880.00	0.8221	0.8857	-0.0636
32	14170.00	0.8962	0.9143	-0.0181
33	18450.00	0.9852	0.9429	0.0423
34	20900.00	1.0000	0.9714	0.0286

SUM OF SQUARES	0.02551	** GAMMA =	0.72585
MAX. DEVIATION	0.0636	** ETA =	0.71770
AVE. DEVIATION	-0.0007	** EPSILON =	0.0
AVE. ABS. DEVIATION	0.0228	** LAMBDA =	20901.00000

JOHNSON SB DISTRIBUTION FOR MIN. 7-DAY FLOWS AT STATION 7-1945

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	843.00	0.0149	0.0286	-0.0137
2	1445.00	0.0365	0.0571	-0.0206
3	2460.00	0.0791	0.0857	-0.0066
4	3500.00	0.1244	0.1143	0.0101
5	4823.00	0.1804	0.1429	0.0375
6	4958.00	0.1859	0.1714	0.0145
7	5213.00	0.1963	0.2000	-0.0037
8	6272.00	0.2380	0.2286	0.0094
9	7190.00	0.2723	0.2571	0.0151
10	7400.00	0.2799	0.2857	-0.0058
11	7506.00	0.2837	0.3143	-0.0306
12	7799.00	0.2942	0.3429	-0.0487
13	8830.00	0.3296	0.3714	-0.0418
14	10800.00	0.3923	0.4000	-0.0077
15	12870.00	0.4517	0.4286	0.0232
16	13970.00	0.4810	0.4571	0.0239
17	14120.00	0.4849	0.4857	-0.0008
18	15910.00	0.5292	0.5143	0.0149
19	17930.00	0.5752	0.5429	0.0323
20	19350.00	0.6052	0.5714	0.0338
21	19500.00	0.6083	0.6000	0.0083
22	19740.00	0.6132	0.6286	-0.0154
23	20940.00	0.6369	0.6571	-0.0203
24	23410.00	0.6823	0.6857	-0.0034
25	25220.00	0.7130	0.7143	-0.0013
26	27660.00	0.7512	0.7429	0.0084
27	28170.00	0.7588	0.7714	-0.0126
28	32240.00	0.8145	0.8000	0.0145
29	33520.00	0.8304	0.8286	0.0018
30	34030.00	0.8365	0.8571	-0.0206
31	35740.00	0.8562	0.8857	-0.0295
32	39130.00	0.8918	0.9143	-0.0225
33	50120.00	0.9756	0.9429	0.0327
34	56750.00	0.9988	0.9714	0.0273

SUM OF SQUARES	0.01616	** GAMMA =	0.76350
MAX. DEVIATION	0.0487	** ETA =	0.69388
AVE. DEVIATION	0.0001	** EPSILON =	0.0
AVE. ABS. DEVIATION	0.0180	** LAMBDA =	58931.39063

JOHNSON SB DISTRIBUTION FOR MIN. 14-DAY FLOWS AT STATION 7-1945

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	2060.00	0.0167	0.0286	-0.0119
2	3284.00	0.0366	0.0571	-0.0205
3	5115.00	0.0709	0.0857	-0.0148
4	7460.00	0.1169	0.1143	0.0026
5	10837.00	0.1815	0.1429	0.0387
6	10857.00	0.1819	0.1714	0.0105
7	10953.00	0.1837	0.2000	-0.0163
8	14552.00	0.2476	0.2286	0.0191
9	14990.00	0.2550	0.2571	-0.0021
10	15810.00	0.2687	0.2857	-0.0170
11	15909.00	0.2703	0.3143	-0.0440
12	15910.00	0.2703	0.3429	-0.0725
13	19940.00	0.3333	0.3714	-0.0381
14	29360.00	0.4584	0.4000	0.0584
15	29430.00	0.4592	0.4286	0.0306
16	31720.00	0.4856	0.4571	0.0285
17	35120.00	0.5225	0.4857	0.0367
18	37340.00	0.5451	0.5143	0.0308
19	40880.00	0.5791	0.5429	0.0362
20	42890.00	0.5973	0.5714	0.0259
21	42970.00	0.5980	0.6000	-0.0020
22	47670.00	0.6379	0.6286	0.0093
23	50050.00	0.6568	0.6571	-0.0004
24	52430.00	0.6748	0.6857	-0.0109
25	53000.00	0.6790	0.7143	-0.0353
26	62720.00	0.7444	0.7429	0.0016
27	63290.00	0.7479	0.7714	-0.0235
28	65580.00	0.7617	0.8000	-0.0383
29	70950.00	0.7918	0.8286	-0.0368
30	76930.00	0.8223	0.8571	-0.0349
31	83610.00	0.8529	0.8857	-0.0328
32	111870.00	0.9485	0.9143	0.0342
33	125310.00	0.9771	0.9429	0.0343
34	152230.00	1.0000	0.9714	0.0286

SUM OF SQUARES	0.03169	** GAMMA =	0.90892
MAX. DEVIATION	0.0725	** ETA =	0.70794
AVE. DEVIATION	-0.0008	** EPSILON =	0.0
AVE. ABS. DEVIATION	0.0258	** LAMBDA =	152231.00000

JOHNSON SB DISTRIBUTION FOR MIN. 30-DAY FLOWS AT STATION 7-1945

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	6969.00	0.0307	0.0286	0.0022
2	8276.00	0.0402	0.0571	-0.0169
3	14015.00	0.0852	0.0857	-0.0005
4	17917.00	0.1165	0.1143	0.0022
5	25831.00	0.1778	0.1429	0.0350
6	29514.00	0.2050	0.1714	0.0335
7	30373.00	0.2111	0.2000	0.0111
8	32780.00	0.2281	0.2286	-0.0004
9	35331.00	0.2457	0.2571	-0.0114
10	35842.00	0.2492	0.2857	-0.0365
11	37030.00	0.2571	0.3143	-0.0571
12	38629.00	0.2677	0.3429	-0.0751
13	47380.00	0.3224	0.3714	-0.0490
14	76890.00	0.4741	0.4000	0.0741
15	77180.00	0.4753	0.4286	0.0468
16	79080.00	0.4837	0.4571	0.0265
17	87150.00	0.5174	0.4857	0.0317
18	93840.00	0.5437	0.5143	0.0294
19	96580.00	0.5540	0.5429	0.0111
20	126930.00	0.6540	0.5714	0.0825
21	129070.00	0.6602	0.6000	0.0602
22	131580.00	0.6673	0.6286	0.0387
23	137430.00	0.6834	0.6571	0.0263
24	139700.00	0.6895	0.6857	0.0038
25	139830.00	0.6898	0.7143	-0.0244
26	143380.00	0.6991	0.7429	-0.0437
27	148200.00	0.7114	0.7714	-0.0600
28	157710.00	0.7344	0.8000	-0.0656
29	169580.00	0.7610	0.8286	-0.0676
30	174510.00	0.7715	0.8571	-0.0857
31	261040.00	0.9090	0.8857	0.0233
32	262280.00	0.9105	0.9143	-0.0038
33	367930.00	0.9917	0.9429	0.0489
34	415410.00	1.0000	0.9714	0.0286

SUM OF SQUARES	0.06463	** GAMMA =	0.96851
MAX. DEVIATION	0.0857	** ETA =	0.69731
AVE. DEVIATION	0.0005	** EPSILON =	0.0
AVE. ABS. DEVIATION	0.0357	** LAMBDA =	415411.00000

JOHNSON SB DISTRIBUTION FOR MIN. 60-DAY FLOWS AT STATION 7-1945

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	17379.00	0.0291	0.0286	0.0005
2	24204.00	0.0489	0.0571	-0.0082
3	37733.00	0.0915	0.0857	0.0058
4	38725.00	0.0946	0.1143	-0.0197
5	66089.00	0.1798	0.1429	0.0370
6	75640.00	0.2078	0.1714	0.0364
7	75868.00	0.2085	0.2000	0.0085
8	83351.00	0.2296	0.2286	0.0011
9	85342.00	0.2351	0.2571	-0.0220
10	90820.00	0.2501	0.2857	-0.0356
11	91110.00	0.2508	0.3143	-0.0634
12	101440.00	0.2780	0.3429	-0.0649
13	170140.00	0.4307	0.3714	0.0593
14	172620.00	0.4355	0.4000	0.0355
15	194541.00	0.4753	0.4286	0.0468
16	202240.00	0.4886	0.4571	0.0314
17	212460.00	0.5055	0.4857	0.0198
18	217430.00	0.5135	0.5143	-0.0008
19	217550.00	0.5137	0.5429	-0.0291
20	291020.00	0.6171	0.5714	0.0457
21	303380.00	0.6321	0.6000	0.0321
22	308910.00	0.6387	0.6286	0.0101
23	341810.00	0.6752	0.6571	0.0181
24	343930.00	0.6775	0.6857	-0.0083
25	360260.00	0.6942	0.7143	-0.0201
26	366000.00	0.6999	0.7429	-0.0430
27	374240.00	0.7078	0.7714	-0.0636
28	402530.00	0.7339	0.8000	-0.0661
29	535280.00	0.8319	0.8286	0.0033
30	581470.00	0.8586	0.8571	0.0015
31	633030.00	0.8849	0.8857	-0.0009
32	754560.00	0.9340	0.9143	0.0198
33	822730.00	0.9550	0.9429	0.0121
34	1169300.00	1.0000	0.9714	0.0286

SUM OF SQUARES	0.03808	** GAMMA =	1.08188
MAX. DEVIATION	0.0661	** ETA =	0.70972
AVE. DEVIATION	0.0002	** EPSILON =	0.0
AVE. ABS. DEVIATION	0.0264	** LAMBDA =	1169301.00000

JOHNSON SB DISTRIBUTION FOR MIN. 90-DAY FLOWS AT STATION 7-1945

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	36425.00	0.0344	0.0286	0.0058
2	54024.00	0.0652	0.0571	0.0081
3	59913.00	0.0757	0.0857	-0.0100
4	111559.00	0.1638	0.1143	0.0495
5	120878.00	0.1785	0.1429	0.0357
6	128605.00	0.1905	0.1714	0.0190
7	132510.00	0.1964	0.2000	-0.0036
8	141192.00	0.2094	0.2286	-0.0192
9	149230.00	0.2212	0.2571	-0.0360
10	157350.00	0.2328	0.2857	-0.0529
11	162670.00	0.2403	0.3143	-0.0740
12	265740.00	0.3665	0.3429	0.0236
13	299780.00	0.4018	0.3714	0.0303
14	329800.00	0.4308	0.4000	0.0308
15	371390.00	0.4682	0.4286	0.0396
16	380670.00	0.4761	0.4571	0.0189
17	395390.00	0.4884	0.4857	0.0027
18	451421.00	0.5322	0.5143	0.0180
19	464560.00	0.5419	0.5429	-0.0009
20	482670.00	0.5549	0.5714	-0.0165
21	496740.00	0.5647	0.6000	-0.0353
22	595450.00	0.6278	0.6286	-0.0008
23	615680.00	0.6396	0.6571	-0.0176
24	680130.00	0.6750	0.6857	-0.0108
25	684220.00	0.6771	0.7143	-0.0372
26	877090.00	0.7663	0.7429	0.0234
27	936940.00	0.7899	0.7714	0.0185
28	969290.00	0.8020	0.8000	0.0020
29	1010460.00	0.8168	0.8286	-0.0117
30	1181510.00	0.8715	0.8571	0.0144
31	1275460.00	0.8974	0.8857	0.0117
32	1309740.00	0.9061	0.9143	-0.0081
33	1352030.00	0.9165	0.9429	-0.0264
34	1741100.00	0.9869	0.9714	0.0154

SUM OF SQUARES	0.02462	** GAMMA =	0.86016
MAX. DEVIATION	0.0740	** ETA =	0.64153
AVE. DEVIATION	0.0002	** EPSILON =	7095.14063
AVE. ABS. DEVIATION	0.0214	** LAMBDA =	1941364.00000

APPENDIX D

GUMBEL'S DISTRIBUTION

GUMBEL DISTRIBUTION FOR MIN. 1-DAY FLOWS AT STATION 7-1775

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	0.01	0.1007	0.0323	0.0685
2	0.01	0.1007	0.0645	0.0362
3	0.01	0.1007	0.0968	0.0040
4	0.01	0.1007	0.1290	-0.0283
5	0.01	0.1007	0.1613	-0.0606
6	0.01	0.1007	0.1935	-0.0928
7	0.01	0.1007	0.2258	-0.1251
8	0.10	0.2347	0.2581	-0.0233
9	0.10	0.2347	0.2903	-0.0556
10	0.20	0.2976	0.3226	-0.0249
11	0.30	0.3401	0.3548	-0.0147
12	0.30	0.3401	0.3871	-0.0470
13	1.20	0.5158	0.4193	0.0964
14	1.30	0.5271	0.4516	0.0755
15	1.50	0.5476	0.4839	0.0637
16	1.70	0.5657	0.5161	0.0495
17	2.10	0.5966	0.5484	0.0482
18	2.20	0.6034	0.5806	0.0228
19	3.80	0.6839	0.6129	0.0710
20	3.80	0.6839	0.6452	0.0387
21	4.50	0.7084	0.6774	0.0310
22	4.80	0.7177	0.7097	0.0080
23	5.00	0.7236	0.7419	-0.0184
24	5.60	0.7396	0.7742	-0.0346
25	7.20	0.7743	0.8064	-0.0322
26	7.20	0.7743	0.8387	-0.0645
27	8.60	0.7978	0.8710	-0.0732
28	12.00	0.8391	0.9032	-0.0641
29	17.00	0.8777	0.9355	-0.0578
30	22.00	0.9027	0.9677	-0.0650

SUM OF SQUARES	0.09709	** SIGMA =	2.67279
MAX. DEVIATION	0.1251	** ETA =	0.40131
AVE. DEVIATION	-0.0090	** EPSILON =	0.0
AVE. ABS. DEVIATION	0.0498		

GUMBEL DISTRIBUTION FOR MIN. 3-DAY FLOWS AT STATION 7-1775

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	0.01	0.0646	0.0323	0.0323
2	0.01	0.0646	0.0645	0.0000
3	0.01	0.0646	0.0968	-0.0322
4	0.01	0.0646	0.1290	-0.0645
5	0.01	0.0646	0.1613	-0.0967
6	0.01	0.0646	0.1935	-0.1290
7	0.01	0.0646	0.2258	-0.1612
8	0.30	0.2296	0.2581	-0.0284
9	0.50	0.2740	0.2903	-0.0163
10	0.80	0.3206	0.3226	-0.0020
11	0.90	0.3332	0.3548	-0.0216
12	1.70	0.4072	0.3871	0.0201
13	3.60	0.5066	0.4193	0.0872
14	4.30	0.5316	0.4516	0.0800
15	5.40	0.5644	0.4839	0.0805
16	5.50	0.5671	0.5161	0.0509
17	6.30	0.5869	0.5484	0.0385
18	6.60	0.5937	0.5806	0.0130
19	11.40	0.6741	0.6129	0.0612
20	12.20	0.6840	0.6452	0.0389
21	13.50	0.6988	0.6774	0.0213
22	14.40	0.7081	0.7097	-0.0016
23	15.80	0.7214	0.7419	-0.0205
24	18.40	0.7429	0.7742	-0.0313
25	21.60	0.7651	0.8064	-0.0414
26	22.30	0.7694	0.8387	-0.0693
27	28.00	0.7996	0.8710	-0.0714
28	37.00	0.8343	0.9032	-0.0690
29	55.00	0.8784	0.9355	-0.0571
30	66.00	0.8963	0.9677	-0.0714

SUM OF SQUARES	0.11676	** SIGMA =	8.56882
MAX. DEVIATION	0.1612	** ETA =	0.40082
AVE. DEVIATION	-0.0154	** EPSILON =	0.0
AVE. ABS. DEVIATION	0.0503		

GUMBEL DISTRIBUTION FOR MIN. 7-DAY FLOWS AT STATION 7-1775

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	0.01	0.0383	0.0323	0.0060
2	0.01	0.0383	0.0645	-0.0263
3	0.01	0.0383	0.0968	-0.0585
4	0.01	0.0383	0.1290	-0.0908
5	0.01	0.0383	0.1613	-0.1230
6	0.01	0.0383	0.1935	-0.1553
7	0.60	0.1959	0.2258	-0.0299
8	0.70	0.2075	0.2581	-0.0505
9	1.80	0.2924	0.2903	0.0021
10	2.20	0.3137	0.3226	-0.0089
11	2.40	0.3232	0.3548	-0.0316
12	4.50	0.3986	0.3871	0.0115
13	9.00	0.4936	0.4193	0.0742
14	11.50	0.5296	0.4516	0.0780
15	13.50	0.5537	0.4839	0.0698
16	14.70	0.5666	0.5161	0.0505
17	14.70	0.5666	0.5484	0.0182
18	18.40	0.6010	0.5806	0.0204
19	28.20	0.6670	0.6129	0.0541
20	30.80	0.6805	0.6452	0.0353
21	36.00	0.7043	0.6774	0.0269
22	39.70	0.7190	0.7097	0.0093
23	39.70	0.7190	0.7419	-0.0229
24	48.80	0.7495	0.7742	-0.0246
25	54.80	0.7663	0.8064	-0.0402
26	57.90	0.7741	0.8387	-0.0646
27	67.90	0.7962	0.8710	-0.0748
28	98.00	0.8437	0.9032	-0.0596
29	157.00	0.8959	0.9355	-0.0396
30	161.00	0.8984	0.9677	-0.0694

SUM OF SQUARES	0.10349	** SIGMA =	22.50313
MAX. DEVIATION	0.1553	** ETA =	0.42026
AVE. DEVIATION	-0.0171	** EPSILON =	0.0
AVE. ABS. DEVIATION	0.0476		

GUMBEL DISTRIBUTION FOR MIN. 14-DAY FLOWS AT STATION 7-1775

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	0.01	0.0146	0.0323	-0.0177
2	0.01	0.0146	0.0645	-0.0499
3	0.01	0.0146	0.0968	-0.0822
4	0.01	0.0146	0.1290	-0.1144
5	0.10	0.0441	0.1613	-0.1172
6	1.80	0.1680	0.1935	-0.0255
7	3.20	0.2160	0.2258	-0.0098
8	5.20	0.2652	0.2581	0.0072
9	5.50	0.2715	0.2903	-0.0189
10	6.90	0.2979	0.3226	-0.0247
11	12.60	0.3775	0.3548	0.0226
12	15.30	0.4060	0.3871	0.0189
13	20.70	0.4531	0.4193	0.0337
14	27.80	0.5017	0.4516	0.0501
15	29.40	0.5112	0.4839	0.0273
16	29.60	0.5123	0.5161	-0.0038
17	35.70	0.5446	0.5484	-0.0038
18	43.60	0.5798	0.5806	-0.0009
19	66.00	0.6538	0.6129	0.0409
20	66.80	0.6559	0.6452	0.0108
21	80.80	0.6897	0.6774	0.0123
22	89.70	0.7081	0.7097	-0.0016
23	124.30	0.7638	0.7419	0.0219
24	134.40	0.7766	0.7742	0.0025
25	137.30	0.7801	0.8064	-0.0263
26	159.50	0.8039	0.8387	-0.0348
27	161.40	0.8058	0.8710	-0.0652
28	264.00	0.8753	0.9032	-0.0279
29	405.00	0.9230	0.9355	-0.0125
30	482.00	0.9386	0.9677	-0.0292

SUM OF SQUARES	0.05381	** SIGMA =	58.47423
MAX. DEVIATION	0.1172	** ETA =	0.48641
AVE. DEVIATION	-0.0139	** EPSILON =	0.0
AVE. ABS. DEVIATION	0.0305		

GUMBEL DISTRIBUTION FOR MIN. 30-DAY FLOWS AT STATION 7-1775

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	0.01	0.0104	0.0323	-0.0218
2	0.01	0.0104	0.0645	-0.0541
3	0.01	0.0104	0.0968	-0.0863
4	0.30	0.0490	0.1290	-0.0800
5	11.40	0.2353	0.1613	0.0740
6	15.30	0.2645	0.1935	0.0710
7	17.20	0.2769	0.2258	0.0511
8	19.80	0.2925	0.2581	0.0344
9	43.90	0.3930	0.2903	0.1026
10	46.10	0.3998	0.3226	0.0772
11	46.70	0.4017	0.3548	0.0468
12	50.40	0.4125	0.3871	0.0254
13	84.70	0.4911	0.4193	0.0718
14	85.00	0.4917	0.4516	0.0401
15	107.00	0.5287	0.4839	0.0449
16	135.40	0.5676	0.5161	0.0515
17	176.60	0.6123	0.5484	0.0639
18	177.80	0.6135	0.5806	0.0328
19	254.00	0.6738	0.6129	0.0609
20	307.80	0.7059	0.6452	0.0607
21	308.80	0.7064	0.6774	0.0290
22	315.20	0.7098	0.7097	0.0002
23	345.30	0.7248	0.7419	-0.0171
24	369.10	0.7357	0.7742	-0.0385
25	376.40	0.7388	0.8064	-0.0676
26	521.30	0.7898	0.8387	-0.0489
27	832.40	0.8556	0.8710	-0.0154
28	923.00	0.8686	0.9032	-0.0347
29	2437.00	0.9581	0.9355	0.0227
30	3665.00	0.9783	0.9677	0.0105

SUM OF SQUARES	0.08654	** SIGMA =	198.50133
MAX. DEVIATION	0.1026	** ETA =	0.46048
AVE. DEVIATION	0.0169	** EPSILON =	0.0
AVE. ABS. DEVIATION	0.0479		

GUMBEL DISTRIBUTION FOR MIN. 60-DAY FLOWS AT STATION 7-1775

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	0.01	0.0017	0.0323	-0.0305
2	0.10	0.0062	0.0645	-0.0583
3	6.80	0.0631	0.0968	-0.0337
4	7.20	0.0650	0.1290	-0.0640
5	40.10	0.1607	0.1613	-0.0006
6	52.60	0.1843	0.1935	-0.0092
7	83.30	0.2315	0.2258	0.0057
8	126.60	0.2828	0.2581	0.0248
9	194.20	0.3443	0.2903	0.0539
10	249.00	0.3841	0.3226	0.0615
11	263.20	0.3934	0.3548	0.0386
12	318.20	0.4263	0.3871	0.0392
13	349.30	0.4431	0.4193	0.0237
14	352.10	0.4445	0.4516	-0.0071
15	407.50	0.4716	0.4839	-0.0123
16	424.80	0.4794	0.5161	-0.0367
17	592.70	0.5443	0.5484	-0.0041
18	666.40	0.5679	0.5806	-0.0128
19	685.80	0.5737	0.6129	-0.0392
20	901.20	0.6295	0.6452	-0.0157
21	925.00	0.6348	0.6774	-0.0426
22	1052.30	0.6612	0.7097	-0.0485
23	1158.30	0.6808	0.7419	-0.0612
24	2274.00	0.8105	0.7742	0.0363
25	2503.20	0.8270	0.8064	0.0206
26	3069.00	0.8599	0.8387	0.0212
27	4163.10	0.9027	0.8710	0.0317
28	4939.80	0.9229	0.9032	0.0197
29	6194.00	0.9454	0.9355	0.0099
30	8973.00	0.9720	0.9677	0.0042

SUM OF SQUARES	0.03570	** SIGMA =	912.96802
MAX. DEVIATION	0.0640	** ETA =	0.55741
AVE. DEVIATION	-0.0028	** EPSILON =	0.0
AVE. ABS. DEVIATION	0.0289		

GUMBEL DISTRIBUTION FOR MIN. 90-DAY FLOWS AT STATION 7-1775

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	0.01	0.0018	0.0323	-0.0305
2	0.10	0.0056	0.0645	-0.0589
3	24.80	0.0837	0.0968	-0.0131
4	29.80	0.0913	0.1290	-0.0377
5	112.10	0.1691	0.1613	0.0078
6	116.50	0.1721	0.1935	-0.0215
7	237.70	0.2361	0.2258	0.0103
8	382.80	0.2892	0.2581	0.0311
9	429.40	0.3033	0.2903	0.0130
10	466.70	0.3139	0.3226	-0.0087
11	725.60	0.3745	0.3548	0.0197
12	925.40	0.4112	0.3871	0.0241
13	934.10	0.4126	0.4193	-0.0067
14	1203.70	0.4532	0.4516	0.0016
15	1217.40	0.4551	0.4839	-0.0288
16	1507.80	0.4910	0.5161	-0.0251
17	1858.20	0.5273	0.5484	-0.0211
18	2043.20	0.5441	0.5806	-0.0365
19	2935.30	0.6097	0.6129	-0.0032
20	4130.60	0.6721	0.6452	0.0270
21	6080.90	0.7413	0.6774	0.0638
22	7258.70	0.7715	0.7097	0.0619
23	7482.30	0.7766	0.7419	0.0347
24	7935.10	0.7863	0.7742	0.0122
25	8369.00	0.7950	0.8064	-0.0114
26	11036.50	0.8378	0.8387	-0.0009
27	11316.00	0.8414	0.8710	-0.0295
28	12768.60	0.8585	0.9032	-0.0447
29	15034.00	0.8801	0.9355	-0.0554
30	25535.60	0.9368	0.9677	-0.0309

SUM OF SQUARES	0.02910	** SIGMA =	3318.01807
MAX. DEVIATION	0.0638	** ETA =	0.49772
AVE. DEVIATION	-0.0053	** EPSILON =	0.0
AVE. ABS. DEVIATION	0.0257		

GUMBEL DISTRIBUTION FOR MIN. 1-DAY FLOWS AT STATION 7-1645

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	30.00	0.0083	0.0313	-0.0230
2	85.00	0.0607	0.0625	-0.0018
3	147.00	0.1198	0.0938	0.0260
4	202.00	0.1704	0.1250	0.0454
5	270.00	0.2301	0.1563	0.0738
6	280.00	0.2386	0.1875	0.0511
7	290.00	0.2470	0.2188	0.0282
8	300.00	0.2554	0.2500	0.0054
9	326.00	0.2767	0.2813	-0.0046
10	327.00	0.2775	0.3125	-0.0350
11	342.00	0.2896	0.3438	-0.0542
12	372.00	0.3132	0.3750	-0.0618
13	394.00	0.3300	0.4063	-0.0763
14	436.00	0.3613	0.4375	-0.0762
15	615.00	0.4804	0.4688	0.0116
16	688.00	0.5228	0.5000	0.0228
17	694.00	0.5262	0.5313	-0.0051
18	790.00	0.5768	0.5625	0.0143
19	850.00	0.6058	0.5938	0.0120
20	944.00	0.6475	0.6250	0.0225
21	1150.00	0.7247	0.6563	0.0684
22	1200.00	0.7409	0.6875	0.0534
23	1220.00	0.7471	0.7188	0.0283
24	1260.00	0.7590	0.7500	0.0090
25	1310.00	0.7732	0.7813	-0.0081
26	1400.00	0.7968	0.8125	-0.0157
27	1500.00	0.8202	0.8438	-0.0236
28	1540.00	0.8288	0.8750	-0.0462
29	1750.00	0.8678	0.9063	-0.0385
30	2100.00	0.9145	0.9375	-0.0230
31	2620.00	0.9555	0.9688	-0.0133

SUM OF SQUARES	0.04693	** SIGMA =	908.18384
MAX. DEVIATION	0.0763	** ETA =	1.05705
AVE. DEVIATION	-0.0011	** EPSILON =	20.39815
AVE. ABS. DEVIATION	0.0316		

GUMBEL DISTRIBUTION FOR MIN. 3-DAY FLOWS AT STATION 7-1645

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	92.00	0.0018	0.0313	-0.0295
2	263.00	0.0568	0.0625	-0.0057
3	447.00	0.1137	0.0938	0.0199
4	692.00	0.1846	0.1250	0.0596
5	852.00	0.2280	0.1563	0.0717
6	880.00	0.2354	0.1875	0.0479
7	880.00	0.2354	0.2188	0.0166
8	920.00	0.2458	0.2500	-0.0042
9	989.00	0.2635	0.2813	-0.0178
10	1116.00	0.2949	0.3125	-0.0176
11	1193.00	0.3134	0.3438	-0.0304
12	1218.00	0.3192	0.3750	-0.0558
13	1307.00	0.3398	0.4063	-0.0665
14	1494.00	0.3811	0.4375	-0.0564
15	2116.00	0.5010	0.4688	0.0322
16	2134.00	0.5041	0.5000	0.0041
17	2170.00	0.5103	0.5313	-0.0210
18	2403.00	0.5484	0.5625	-0.0141
19	2638.00	0.5839	0.5938	-0.0099
20	3540.00	0.6962	0.6250	0.0712
21	3550.00	0.6973	0.6563	0.0410
22	3640.00	0.7067	0.6875	0.0192
23	3880.00	0.7303	0.7188	0.0115
24	4070.00	0.7477	0.7500	-0.0023
25	4770.00	0.8026	0.7813	0.0213
26	4820.00	0.8060	0.8125	-0.0065
27	5320.00	0.8373	0.8438	-0.0065
28	5400.00	0.8418	0.8750	-0.0332
29	5790.00	0.8621	0.9063	-0.0442
30	8600.00	0.9489	0.9375	0.0114
31	8880.00	0.9537	0.9688	-0.0151

SUM OF SQUARES	0.03786	** SIGMA =	2991.70093
MAX. DEVIATION	0.0717	** ETA =	1.01353
AVE. DEVIATION	-0.0003	** EPSILON =	86.43230
AVE. ABS. DEVIATION	0.0279		

GUMBEL DISTRIBUTION FOR MIN. 7-DAY FLOWS AT STATION 7-1645

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	240.00	0.0216	0.0313	-0.0097
2	624.00	0.0600	0.0625	-0.0025
3	1071.00	0.1054	0.0938	0.0116
4	1996.00	0.1971	0.1250	0.0721
5	2160.00	0.2128	0.1563	0.0565
6	2209.00	0.2174	0.1875	0.0299
7	2209.00	0.2174	0.2188	-0.0014
8	2210.00	0.2175	0.2500	-0.0325
9	2361.00	0.2317	0.2813	-0.0496
10	2698.00	0.2628	0.3125	-0.0497
11	3070.00	0.2960	0.3438	-0.0478
12	3731.00	0.3521	0.3750	-0.0229
13	4250.00	0.3936	0.4063	-0.0127
14	5047.00	0.4530	0.4375	0.0155
15	5306.00	0.4711	0.4688	0.0023
16	5973.00	0.5156	0.5000	0.0156
17	6377.00	0.5408	0.5313	0.0095
18	6458.00	0.5457	0.5625	-0.0168
19	7354.00	0.5971	0.5938	0.0033
20	8850.00	0.6712	0.6250	0.0462
21	9180.00	0.6858	0.6563	0.0295
22	9240.00	0.6883	0.6875	0.0008
23	9840.00	0.7131	0.7188	-0.0057
24	11750.00	0.7801	0.7500	0.0301
25	11890.00	0.7844	0.7813	0.0031
26	12430.00	0.8002	0.8125	-0.0123
27	13220.00	0.8214	0.8438	-0.0224
28	13830.00	0.8362	0.8750	-0.0388
29	16360.00	0.8861	0.9063	-0.0202
30	22310.00	0.9525	0.9375	0.0150
31	23000.00	0.9571	0.9688	-0.0117

SUM OF SQUARES	0.02623	** SIGMA =	8026.34375
MAX. DEVIATION	0.0721	** ETA =	1.08954
AVE. DEVIATION	-0.0005	** EPSILON =	0.0
AVE. ABS. DEVIATION	0.0225		

GUMBEL DISTRIBUTION FOR MIN. 14-DAY FLOWS AT STATION 7-1645

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	653.00	0.0278	0.0313	-0.0035
2	1350.00	0.0592	0.0625	-0.0033
3	2440.00	0.1084	0.0938	0.0146
4	4323.00	0.1904	0.1250	0.0654
5	4400.00	0.1936	0.1563	0.0373
6	4501.00	0.1979	0.1875	0.0104
7	4700.00	0.2062	0.2188	-0.0126
8	4730.00	0.2074	0.2500	-0.0426
9	5474.00	0.2379	0.2813	-0.0434
10	6899.00	0.2936	0.3125	-0.0189
11	7058.00	0.2997	0.3438	-0.0441
12	7683.00	0.3229	0.3750	-0.0521
13	9982.00	0.4028	0.4063	-0.0035
14	10500.00	0.4196	0.4375	-0.0179
15	11933.00	0.4639	0.4688	-0.0049
16	13217.00	0.5011	0.5000	0.0011
17	15498.00	0.5613	0.5313	0.0300
18	18800.00	0.6366	0.5625	0.0741
19	19850.00	0.6579	0.5938	0.0641
20	19954.00	0.6599	0.6250	0.0349
21	22210.00	0.7015	0.6563	0.0452
22	22530.00	0.7070	0.6875	0.0195
23	25000.00	0.7463	0.7188	0.0275
24	25860.00	0.7588	0.7500	0.0088
25	27060.00	0.7752	0.7813	-0.0061
26	27190.00	0.7769	0.8125	-0.0356
27	27960.00	0.7868	0.8438	-0.0570
28	28710.00	0.7960	0.8750	-0.0790
29	34820.00	0.8581	0.9063	-0.0482
30	48770.00	0.9390	0.9375	0.0015
31	51700.00	0.9490	0.9688	-0.0198

SUM OF SQUARES	0.04374	** SIGMA =	18586.69922
MAX. DEVIATION	0.0790	** ETA =	1.06610
AVE. DEVIATION	-0.0019	** EPSILON =	0.0
AVE. ABS. DEVIATION	0.0299		

GUMBEL DISTRIBUTION FOR MIN. 30-DAY FLOWS AT STATION 7-1645

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	1834.00	0.0318	0.0313	0.0005
2	3210.00	0.0566	0.0625	-0.0059
3	6151.00	0.1090	0.0938	0.0152
4	10266.00	0.1795	0.1250	0.0545
5	11229.00	0.1953	0.1563	0.0390
6	11284.00	0.1962	0.1875	0.0087
7	12595.00	0.2175	0.2188	-0.0013
8	12865.00	0.2218	0.2500	-0.0282
9	13576.00	0.2330	0.2813	-0.0483
10	15873.00	0.2685	0.3125	-0.0440
11	17236.00	0.2889	0.3438	-0.0549
12	19896.00	0.3273	0.3750	-0.0477
13	25484.00	0.4021	0.4063	-0.0042
14	28440.00	0.4386	0.4375	0.0011
15	29939.00	0.4563	0.4688	-0.0125
16	36667.00	0.5295	0.5000	0.0295
17	42070.00	0.5815	0.5313	0.0502
18	47100.00	0.6251	0.5625	0.0626
19	47848.00	0.6311	0.5938	0.0373
20	56880.00	0.6976	0.6250	0.0726
21	57980.00	0.7049	0.6563	0.0486
22	59550.00	0.7150	0.6875	0.0275
23	60710.00	0.7222	0.7188	0.0034
24	62450.00	0.7327	0.7500	-0.0173
25	64360.00	0.7439	0.7813	-0.0374
26	66290.00	0.7546	0.8125	-0.0579
27	72870.00	0.7882	0.8438	-0.0556
28	80220.00	0.8204	0.8750	-0.0546
29	105390.00	0.8985	0.9063	-0.0078
30	111790.00	0.9123	0.9375	-0.0252
31	149190.00	0.9630	0.9688	-0.0058

SUM OF SQUARES	0.04432	** SIGMA =	47968.93750
MAX. DEVIATION	0.0726	** ETA =	1.05114
AVE. DEVIATION	-0.0019	** EPSILON =	0.0
AVE. ABS. DEVIATION	0.0310		

GUMBEL DISTRIBUTION FOR MIN. 60-DAY FLOWS AT STATION 7-1645

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	5593.00	0.0364	0.0313	0.0051
2	8986.00	0.0592	0.0625	-0.0033
3	15685.00	0.1040	0.0938	0.0102
4	26962.00	0.1766	0.1250	0.0516
5	28140.00	0.1839	0.1563	0.0276
6	31910.00	0.2070	0.1875	0.0195
7	37301.00	0.2392	0.2188	0.0204
8	39817.00	0.2539	0.2500	0.0039
9	42319.00	0.2683	0.2813	-0.0130
10	45610.00	0.2868	0.3125	-0.0257
11	48002.00	0.3000	0.3438	-0.0438
12	51414.00	0.3185	0.3750	-0.0565
13	57286.00	0.3493	0.4063	-0.0570
14	68012.00	0.4024	0.4375	-0.0351
15	69021.00	0.4072	0.4688	-0.0616
16	103918.00	0.5527	0.5000	0.0527
17	124057.00	0.6207	0.5313	0.0894
18	134120.00	0.6509	0.5625	0.0884
19	136730.00	0.6584	0.5938	0.0646
20	141910.00	0.6727	0.6250	0.0477
21	142100.00	0.6733	0.6563	0.0170
22	149754.00	0.6934	0.6875	0.0059
23	151420.00	0.6976	0.7188	-0.0212
24	154920.00	0.7063	0.7500	-0.0437
25	177120.00	0.7560	0.7813	-0.0253
26	207900.00	0.8118	0.8125	-0.0007
27	207910.00	0.8118	0.8438	-0.0320
28	218970.00	0.8286	0.8750	-0.0464
29	241390.00	0.8584	0.9063	-0.0479
30	294520.00	0.9102	0.9375	-0.0273
31	418200.00	0.9694	0.9688	0.0006

SUM OF SQUARES	0.05320	** SIGMA =	127759.31250
MAX. DEVIATION	0.0894	** ETA =	1.05322
AVE. DEVIATION	-0.0012	** EPSILON =	0.0
AVE. ABS. DEVIATION	0.0337		

GUMBEL DISTRIBUTION FOR MIN. 90-DAY FLOWS AT STATION 7-1645

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	12427.00	0.0303	0.0313	-0.0010
2	16007.00	0.0407	0.0625	-0.0218
3	24598.00	0.0672	0.0938	-0.0266
4	49495.00	0.1482	0.1250	0.0232
5	51318.00	0.1542	0.1563	-0.0021
6	61844.00	0.1889	0.1875	0.0014
7	63046.00	0.1929	0.2188	-0.0259
8	67259.00	0.2067	0.2500	-0.0433
9	74883.00	0.2314	0.2813	-0.0499
10	88535.00	0.2750	0.3125	-0.0375
11	112252.00	0.3476	0.3438	0.0038
12	120559.00	0.3720	0.3750	-0.0030
13	133623.00	0.4091	0.4063	0.0028
14	155117.00	0.4668	0.4375	0.0293
15	160678.00	0.4810	0.4688	0.0122
16	174297.00	0.5146	0.5000	0.0146
17	201910.00	0.5776	0.5313	0.0463
18	208360.00	0.5913	0.5625	0.0288
19	220550.00	0.6163	0.5938	0.0225
20	228580.00	0.6320	0.6250	0.0070
21	233200.00	0.6408	0.6563	-0.0155
22	261520.00	0.6910	0.6875	0.0035
23	288520.00	0.7330	0.7188	0.0142
24	288780.00	0.7334	0.7500	-0.0166
25	328920.00	0.7866	0.7813	0.0053
26	377600.00	0.8383	0.8125	0.0258
27	380890.00	0.8413	0.8438	-0.0025
28	386440.00	0.8463	0.8750	-0.0287
29	388510.00	0.8482	0.9063	-0.0581
30	508610.00	0.9258	0.9375	-0.0117
31	606430.00	0.9597	0.9688	-0.0091

SUM OF SQUARES	0.01880	** SIGMA =	228641.93750
MAX. DEVIATION	0.0581	** ETA =	1.19576
AVE. DEVIATION	-0.0036	** EPSILON =	0.0
AVE. ABS. DEVIATION	0.0192		

GUMBEL DISTRIBUTION FOR MIN. 1-DAY FLOWS AT STATION 7-1945

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	76.00	0.0146	0.0286	-0.0139
2	200.00	0.0484	0.0571	-0.0088
3	255.00	0.0650	0.0857	-0.0207
4	500.00	0.1448	0.1143	0.0305
5	566.00	0.1670	0.1429	0.0241
6	585.00	0.1734	0.1714	0.0020
7	638.00	0.1913	0.2000	-0.0087
8	650.00	0.1953	0.2286	-0.0332
9	678.00	0.2048	0.2571	-0.0524
10	910.00	0.2821	0.2857	-0.0036
11	983.00	0.3059	0.3143	-0.0084
12	1040.00	0.3242	0.3429	-0.0187
13	1040.00	0.3242	0.3714	-0.0473
14	1110.00	0.3463	0.4000	-0.0537
15	1470.00	0.4538	0.4286	0.0252
16	1680.00	0.5108	0.4571	0.0536
17	1810.00	0.5439	0.4857	0.0581
18	1850.00	0.5537	0.5143	0.0394
19	1890.00	0.5634	0.5429	0.0205
20	1960.00	0.5799	0.5714	0.0085
21	2190.00	0.6310	0.6000	0.0310
22	2240.00	0.6414	0.6286	0.0128
23	2340.00	0.6615	0.6571	0.0043
24	2530.00	0.6972	0.6857	0.0115
25	2640.00	0.7164	0.7143	0.0021
26	2790.00	0.7409	0.7429	-0.0020
27	3020.00	0.7750	0.7714	0.0035
28	3090.00	0.7845	0.8000	-0.0155
29	3380.00	0.8205	0.8286	-0.0080
30	3490.00	0.8327	0.8571	-0.0244
31	3510.00	0.8349	0.8857	-0.0508
32	3650.00	0.8492	0.9143	-0.0651
33	5100.00	0.9437	0.9429	0.0009
34	5650.00	0.9621	0.9714	-0.0094

SUM OF SQUARES	0.02973	** SIGMA =	2195.52930
MAX. DEVIATION	0.0651	** ETA =	1.25398
AVE. DEVIATION	-0.0034	** EPSILON =	0.0
AVE. ABS. DEVIATION	0.0227		

GUMBEL DISTRIBUTION FOR MIN. 3-DAY FLOWS AT STATION 7-1945

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	276.00	0.0193	0.0286	-0.0093
2	605.00	0.0484	0.0571	-0.0088
3	1020.00	0.0881	0.0857	0.0024
4	1500.00	0.1358	0.1143	0.0215
5	1956.00	0.1813	0.1429	0.0385
6	2045.00	0.1902	0.1714	0.0188
7	2054.00	0.1911	0.2000	-0.0089
8	2177.00	0.2033	0.2286	-0.0253
9	2366.00	0.2219	0.2571	-0.0352
10	2980.00	0.2811	0.2857	-0.0046
11	3049.00	0.2876	0.3143	-0.0266
12	3120.00	0.2943	0.3429	-0.0485
13	3430.00	0.3230	0.3714	-0.0484
14	4040.00	0.3775	0.4000	-0.0225
15	4630.00	0.4273	0.4286	-0.0013
16	5400.00	0.4879	0.4571	0.0307
17	5790.00	0.5167	0.4857	0.0310
18	6050.00	0.5351	0.5143	0.0209
19	6750.00	0.5821	0.5429	0.0392
20	7250.00	0.6132	0.5714	0.0418
21	7650.00	0.6367	0.6000	0.0367
22	7780.00	0.6441	0.6286	0.0155
23	8050.00	0.6590	0.6571	0.0018
24	9060.00	0.7101	0.6857	0.0244
25	9200.00	0.7166	0.7143	0.0023
26	10300.00	0.7636	0.7429	0.0207
27	10530.00	0.7725	0.7714	0.0010
28	11340.00	0.8015	0.8000	0.0015
29	11390.00	0.8031	0.8286	-0.0254
30	11440.00	0.8048	0.8571	-0.0523
31	11880.00	0.8189	0.8857	-0.0668
32	14170.00	0.8784	0.9143	-0.0359
33	18450.00	0.9441	0.9429	0.0012
34	20900.00	0.9647	0.9714	-0.0067

SUM OF SQUARES	0.02779	** SIGMA =	7570.32813
MAX. DEVIATION	0.0668	** ETA =	1.18890
AVE. DEVIATION	-0.0023	** EPSILON =	0.0
AVE. ABS. DEVIATION	0.0228		

GUMBEL DISTRIBUTION FOR MIN. 7-DAY FLOWS AT STATION 7-1945

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	843.00	0.0268	0.0286	-0.0017
2	1445.00	0.0489	0.0571	-0.0083
3	2460.00	0.0875	0.0857	0.0018
4	3500.00	0.1276	0.1143	0.0133
5	4823.00	0.1782	0.1429	0.0353
6	4958.00	0.1833	0.1714	0.0119
7	5213.00	0.1929	0.2000	-0.0071
8	6272.00	0.2322	0.2286	0.0037
9	7190.00	0.2655	0.2571	0.0083
10	7400.00	0.2729	0.2857	-0.0128
11	7506.00	0.2767	0.3143	-0.0376
12	7799.00	0.2870	0.3429	-0.0558
13	8830.00	0.3225	0.3714	-0.0489
14	10800.00	0.3869	0.4000	-0.0131
15	12870.00	0.4494	0.4286	0.0208
16	13970.00	0.4805	0.4571	0.0233
17	14120.00	0.4846	0.4857	-0.0011
18	15910.00	0.5318	0.5143	0.0175
19	17930.00	0.5806	0.5429	0.0377
20	19350.00	0.6122	0.5714	0.0407
21	19500.00	0.6154	0.6000	0.0154
22	19740.00	0.6205	0.6286	-0.0081
23	20940.00	0.6451	0.6571	-0.0121
24	23410.00	0.6913	0.6857	0.0056
25	25220.00	0.7216	0.7143	0.0073
26	27660.00	0.7582	0.7429	0.0154
27	28170.00	0.7653	0.7714	-0.0061
28	32240.00	0.8153	0.8000	0.0153
29	33520.00	0.8288	0.8286	0.0003
30	34030.00	0.8340	0.8571	-0.0232
31	35740.00	0.8502	0.8857	-0.0355
32	39130.00	0.8780	0.9143	-0.0363
33	50120.00	0.9382	0.9429	-0.0046
34	56750.00	0.9595	0.9714	-0.0120

SUM OF SQUARES	0.01772	** SIGMA =	20299.34766
MAX. DEVIATION	0.0558	** ETA =	1.13301
AVE. DEVIATION	-0.0015	** EPSILON =	0.0
AVE. ABS. DEVIATION	0.0176		

GUMBEL DISTRIBUTION FOR MIN. 14-DAY FLOWS AT STATION 7-1945

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	2060.00	0.0312	0.0286	0.0026
2	3284.00	0.0517	0.0571	-0.0054
3	5115.00	0.0832	0.0857	-0.0026
4	7460.00	0.1236	0.1143	0.0093
5	10837.00	0.1809	0.1429	0.0380
6	10857.00	0.1812	0.1714	0.0098
7	10953.00	0.1828	0.2000	-0.0172
8	14552.00	0.2416	0.2286	0.0131
9	14990.00	0.2486	0.2571	-0.0085
10	15810.00	0.2616	0.2857	-0.0241
11	15909.00	0.2631	0.3143	-0.0512
12	15910.00	0.2631	0.3429	-0.0797
13	19940.00	0.3244	0.3714	-0.0470
14	29360.00	0.4523	0.4000	0.0523
15	29430.00	0.4532	0.4286	0.0246
16	31720.00	0.4811	0.4571	0.0239
17	35120.00	0.5202	0.4857	0.0345
18	37340.00	0.5443	0.5143	0.0300
19	40880.00	0.5806	0.5429	0.0377
20	42890.00	0.6000	0.5714	0.0286
21	42970.00	0.6008	0.6000	0.0008
22	47670.00	0.6431	0.6286	0.0145
23	50050.00	0.6629	0.6571	0.0058
24	52430.00	0.6817	0.6857	-0.0040
25	53000.00	0.6861	0.7143	-0.0282
26	62720.00	0.7525	0.7429	0.0096
27	63290.00	0.7560	0.7714	-0.0155
28	65580.00	0.7694	0.8000	-0.0306
29	70950.00	0.7983	0.8286	-0.0303
30	76930.00	0.8264	0.8571	-0.0307
31	83610.00	0.8534	0.8857	-0.0323
32	111870.00	0.9295	0.9143	0.0152
33	125310.00	0.9506	0.9429	0.0077
34	152230.00	0.9760	0.9714	0.0046

SUM OF SQUARES	0.02777	** SIGMA =	46405.31641
MAX. DEVIATION	0.0797	** ETA =	1.10823
AVE. DEVIATION	-0.0013	** EPSILON =	0.0
AVE. ABS. DEVIATION	0.0226		

GUMBEL DISTRIBUTION FOR MIN. 30-DAY FLOWS AT STATION 7-1945

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	6969.00	0.0472	0.0286	0.0187
2	8276.00	0.0565	0.0571	-0.0007
3	14015.00	0.0970	0.0857	0.0113
4	17917.00	0.1242	0.1143	0.0099
5	25831.00	0.1779	0.1429	0.0351
6	29514.00	0.2022	0.1714	0.0308
7	30373.00	0.2078	0.2000	0.0078
8	32780.00	0.2233	0.2286	-0.0053
9	35331.00	0.2394	0.2571	-0.0177
10	35842.00	0.2426	0.2857	-0.0431
11	37030.00	0.2501	0.3143	-0.0642
12	38629.00	0.2600	0.3429	-0.0829
13	47380.00	0.3123	0.3714	-0.0591
14	76890.00	0.4662	0.4000	0.0662
15	77180.00	0.4676	0.4286	0.0390
16	79080.00	0.4763	0.4571	0.0192
17	87150.00	0.5121	0.4857	0.0264
18	93840.00	0.5400	0.5143	0.0257
19	96580.00	0.5510	0.5429	0.0082
20	126930.00	0.6577	0.5714	0.0863
21	129070.00	0.6642	0.6000	0.0642
22	131580.00	0.6718	0.6286	0.0432
23	137430.00	0.6887	0.6571	0.0315
24	139700.00	0.6950	0.6857	0.0093
25	139830.00	0.6954	0.7143	-0.0189
26	143380.00	0.7051	0.7429	-0.0378
27	148200.00	0.7177	0.7714	-0.0537
28	157710.00	0.7412	0.8000	-0.0588
29	169580.00	0.7679	0.8286	-0.0607
30	174510.00	0.7782	0.8571	-0.0790
31	261040.00	0.9012	0.8857	0.0155
32	262280.00	0.9024	0.9143	-0.0119
33	367930.00	0.9645	0.9429	0.0217
34	415410.00	0.9777	0.9714	0.0062

SUM OF SQUARES	0.06064	** SIGMA =	118923.50000
MAX. DEVIATION	0.0863	** ETA =	1.06756
AVE. DEVIATION	-0.0005	** EPSILON =	0.0
AVE. ABS. DEVIATION	0.0344		

GUMBEL DISTRIBUTION FOR MIN. 60-DAY FLOWS AT STATION 7-1945

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	17379.00	0.0398	0.0286	0.0112
2	24204.00	0.0605	0.0571	0.0034
3	37733.00	0.1008	0.0857	0.0151
4	38725.00	0.1037	0.1143	-0.0106
5	66089.00	0.1810	0.1429	0.0381
6	75640.00	0.2066	0.1714	0.0351
7	75868.00	0.2072	0.2000	0.0072
8	83351.00	0.2267	0.2286	-0.0019
9	85342.00	0.2318	0.2571	-0.0253
10	90820.00	0.2458	0.2857	-0.0399
11	91110.00	0.2465	0.3143	-0.0678
12	101440.00	0.2721	0.3429	-0.0707
13	170140.00	0.4231	0.3714	0.0517
14	172620.00	0.4279	0.4000	0.0279
15	194541.00	0.4692	0.4286	0.0406
16	202240.00	0.4830	0.4571	0.0258
17	212460.00	0.5007	0.4857	0.0150
18	217430.00	0.5092	0.5143	-0.0051
19	217550.00	0.5094	0.5429	-0.0335
20	291020.00	0.6190	0.5714	0.0476
21	303380.00	0.6349	0.6000	0.0349
22	308910.00	0.6419	0.6286	0.0133
23	341810.00	0.6805	0.6571	0.0233
24	343930.00	0.6828	0.6857	-0.0029
25	360260.00	0.7003	0.7143	-0.0140
26	366000.00	0.7062	0.7429	-0.0367
27	374240.00	0.7145	0.7714	-0.0569
28	402530.00	0.7413	0.8000	-0.0587
29	535280.00	0.8374	0.8286	0.0089
30	581470.00	0.8618	0.8571	0.0047
31	633030.00	0.8848	0.8857	-0.0009
32	754560.00	0.9250	0.9143	0.0107
33	822730.00	0.9411	0.9429	-0.0017
34	1169300.00	0.9829	0.9714	0.0115

SUM OF SQUARES	0.03498	** SIGMA =	301149.31250
MAX. DEVIATION	0.0707	** ETA =	1.02665
AVE. DEVIATION	-0.0000	** EPSILON =	4292.26563
AVE. ABS. DEVIATION	0.0251		

GUMBEL DISTRIBUTION FOR MIN. 90-DAY FLOWS AT STATION 7-1945

NUMBER	MIN. FLOW	CAL. PROB.	OBS. PROB.	DELTA PROB.
1	36425.00	0.0443	0.0286	0.0157
2	54024.00	0.0734	0.0571	0.0162
3	59913.00	0.0829	0.0857	-0.0028
4	111559.00	0.1620	0.1143	0.0477
5	120878.00	0.1755	0.1429	0.0326
6	128605.00	0.1865	0.1714	0.0150
7	132510.00	0.1920	0.2000	-0.0080
8	141192.00	0.2041	0.2286	-0.0245
9	149230.00	0.2151	0.2571	-0.0420
10	157350.00	0.2261	0.2857	-0.0596
11	162670.00	0.2332	0.3143	-0.0811
12	265740.00	0.3586	0.3429	0.0157
13	299780.00	0.3952	0.3714	0.0238
14	329800.00	0.4258	0.4000	0.0258
15	371390.00	0.4655	0.4286	0.0370
16	380670.00	0.4740	0.4571	0.0169
17	395390.00	0.4872	0.4857	0.0015
18	451421.00	0.5344	0.5143	0.0201
19	464560.00	0.5448	0.5429	0.0019
20	482670.00	0.5587	0.5714	-0.0127
21	496740.00	0.5693	0.6000	-0.0307
22	595450.00	0.6365	0.6286	0.0079
23	615680.00	0.6489	0.6571	-0.0082
24	680130.00	0.6857	0.6857	-0.0000
25	684220.00	0.6879	0.7143	-0.0264
26	877090.00	0.7757	0.7429	0.0329
27	936940.00	0.7976	0.7714	0.0261
28	969290.00	0.8085	0.8000	0.0085
29	1010460.00	0.8215	0.8286	-0.0071
30	1181510.00	0.8667	0.8571	0.0096
31	1275460.00	0.8865	0.8857	0.0008
32	1309740.00	0.8929	0.9143	-0.0214
33	1352030.00	0.9004	0.9429	-0.0425
34	1741100.00	0.9487	0.9714	-0.0228

SUM OF SQUARES	0.02686	** SIGMA =	588474.56250
MAX. DEVIATION	0.0811	** ETA =	0.99200
AVE. DEVIATION	-0.0010	** EPSILON =	10926.56641
AVE. ABS. DEVIATION	0.0219		

APPENDIX E

TREATMENT PLANT COST FUNCTION

TREATMENT PLANT COST FUNCTION

I. Municipal Waste Treatment Plant

1. Assumptions

(1) The treatment system consists of a sedimentation unit, aeration tank, final clarifier, and activated carbon unit.

(2) Upper limit of primary treatment = 30% of BOD removal

(3) Upper limit of secondary treatment = 95% of BOD removal

(4) Upper limit of tertiary treatment = 98% of BOD removal

(5) Service life of treatment facility = 20 years, interest rate = 4%. Therefore, capital recover factor = 0.7358

(6) Data of unit cost were taken from "Cost of Wastewater Treatment Processes," a report of Robert A. Taft Water Research Center.

2. Annual Cost of Primary Treatment

Assume treatment capacity per unit area of sedimentation basin = 800 gal/day/ft^2 (equivalent to 0.0012377 cfs/ft^2), req'd area = Q/0.0012377 = 807.95Q, in ft^2.

Unit cost per sq. ft. of sedimentation basin = 10^s, where s = 1.0/[0.233 log (807.95Q/1000) + 0.758].

Capital recovery = (capital recovery factor)(req'd area)(unit cost) = 59.4490 x 10^s.

Annual maintenance cost = 3% of the total capital investment = 0.03(req'd area)(unit cost)

Therefore, total annual cost of primary treatment = capital recovery + annual maintenance cost

or

total annual cost of primary treatment = 83.688Q x 10^S (1)

3. Annual Cost of Activated Sludge System

(1) Determination of aeration time

Let

X = effluent BOD, mg/l

Y = influent BQD before entering primary sedimentation unit, mg/l

R = % BOD removal due to primary treatment

E = % BOD removal due to primary and secondary treatment

E'= % BOD removal of the remaining BOD after primary treatment

then

$$E' = [(Y - YR/100 - X)/(Y - YR/100)] \times 100, \quad (2)$$

and

$$E = [(Y - X)/Y] \times 100. \quad (3)$$

From Equations 2 and 3,

$$E' = 100(E - R)/(100 - R). \quad (4)$$

Combining Equation 4 with the NRC formula

$$E' = 100/\left[1 + M\left(\frac{P}{WT}\right)^N\right], \quad (5)$$

where

P = pounds of BOD entering activated sludge system,

or

$P = Y - \frac{R}{100} Qc$, c is a conversion factor

W = 1000 pounds of sludge returned to aeration tank,

or

$$W = \frac{\text{Suspended solids conc. x \% Q returned x c}}{1000}$$

T = hours of contact or aeration,

M, N = coefficients, $M = 0.03$, $N = 0.42$,

then

$$T = \frac{P}{W}\left(\frac{E - R}{100 - E} M\right)^{1/N}. \tag{6}$$

Let

suspended solids conc. = 10,000 mg/l,

% Q returned = 0.25Q.

and

$$T = 0.000067\ Y\left(\frac{E - 30}{1000 - E}\right)^{2.38}. \tag{7}$$

(2) Annual Cost of Primary Treatment

Volume of aeration tank req'd

$$V = 3600\ QT. \tag{8}$$

Using the capital cost function

log (capital cost/1000) = 0.806 (V/1000) + 0.306

and the service life and interest rate previously assumed,

capital recovery = $0.07358 \times 10^{[0.806 \log QT + 3.754]}$

Assuming operation hours per day = 24 hrs., then

annual operation cost = 365 x 24 x 1.42(V.100.000),

or

annual operation cost = 447.811 QT. (10)

The annual maintenance cost = 3% of capital cost.

(3) Annual cost of final clarifier

Assume treatment capacity = 600 gal/day/ft^2 (equivalent to 9.0009283 cfs.ft^2)

Area required A = Q/0.0009283 = 1077.238Q, in sq. ft.

Capital cost in dollars per sq. ft.

$$= 10^{\left[\frac{1}{0.2 \log (A/100) + 0.57}\right]}$$

$$\text{Total capital cost} = 1077.238Q \times 10^{\left[\frac{1}{0.2 \log (A/100) + 0.57}\right]} \quad (11)$$

Therefore, with life of service = 20 years, and interest rate = 4%

$$\text{the capital recovery} = 79.263Q \times 10^{\left[\frac{1}{0.2 \log (10.77238Q) + 0.57}\right]} \quad (12)$$

Annual maintenance cost = 3% of total capital cost.

4. Tertiary treatment (by powered activated carbon)

$$\text{Treatment cost (c/1000 gal)} = 10^{\left[\frac{1}{0.396 \log (0.646317Q) + 0.83}\right]} \quad (13)$$

Assume

Operation hours = 24 hrs/day, therefore total amount of influent wastewater treated annually - 0.646317Q x 365 = 235906Q, in thousand gallons.

$$\text{Annual cost} = 2369.06Q \times 10^{\left[\frac{1}{0.396 \log (0.646417Q) + 0.83}\right]}$$

5. Summary of Annual Costs, in February 1968 dollars, at different treatment levels

(1) Primary treatment (BOD removal = 30%)

$$C_p = 83.688Q \times 10^{\left[\frac{1}{0.233 \log (0.807950) + 0.758}\right]} \quad (14)$$

(2) Primary treatment + Activated sludge process (30% < % BOD removal ≤ 95%)

$$C_A = C_p + 111.58Q \times 10^{\left[\frac{1}{0.2 \log (10.77238Q) + 0.57}\right]}$$

$$+ 10358 \times 10^{[0.806 \log (QT) + 3.754]} + 447.811\,QT \qquad (15)$$

$$T = \left[0.00067Y \left(\frac{E - 30}{100 - E}\right)^{2.38},\ 30\% < E \leq 95\%\right]$$

(3) Primary treatment + Activated sludge process + Tertiary treatment

Assume the upper limit of BOD removal = 98%

$$C_T = \text{Max. } C_A + 2359.06Q \times 10^{\left[\frac{1}{0.396 \log (0.646317Q) + 0.83}\right]} \qquad (16)$$

(Max C_A = the value of C_A when E = 95%)

Notation:

Q = Influent flow, cfs

E = Percent of BOD removal

T = Detention time in aeration tank, hr.

Y = Initial BOD conc., mg/l.

II. Industrial Waste Treatment Plant

1. Assumptions

(1) Completely soluble organic waste

(2) Except without primary treatment, other assumptions are the the same as those for municipal waste treatment plant.

2. Annual Cost of Activated Sludge System

Applying the same principles as those in Section II the aeration time is

$$T = 0.000096Y[E/(100 - E)]^{2.38} \qquad (17)$$

Others are the same as before.

3. Tertiary Treatment

   Same as Section II

4. Summary of Annual costs, in February 1968 dollars, at different treatment levels

   (1) Activated sludge process (0< % BOD removal $\leq$ 95)

$$C_A = 111.58Q \times 10^{\left[\frac{1}{0.2 \log (10.77238Q) + 0.57}\right]}$$

$$+ 0.10358 \times 10^{[0.806 \log (QT) + 3.754]} + 447.811QT \qquad (18)$$

$$T = 0.000096Y\left(\frac{E}{1000 - E}\right)^{2.38}, \; 0\% < E \leq 95\%]$$

   (2) Activated sludge process + tertiary treatment

   Assume the upper limit of BOD removal = 98%.

$$C_T = \text{Max. } C_A + 2359.06Q \times 10^{\left[\frac{1}{0.396 \log (0.646317Q + 0.83}\right]} \qquad (19)$$

   (Max. C_A = the value of C_A when E = 95%)

   Notation:

   Q = Influent flow, cfs

   E = % BOD removal

   T = Detention time in aeration tank, hr.

   Y = Initial BOD conc., mg/l

III. Relationship of Treatment Cost and Treatment Level

A treatment cost can be determined when assuming a certain level of treatment by using the equations developed in Sections I and II. In investigating the general form of the relationship between percent BOD removal and treatment cost, it was found that this relationship generally followed the exponential function

$$Y = A + B \cdot R^{X},$$

where

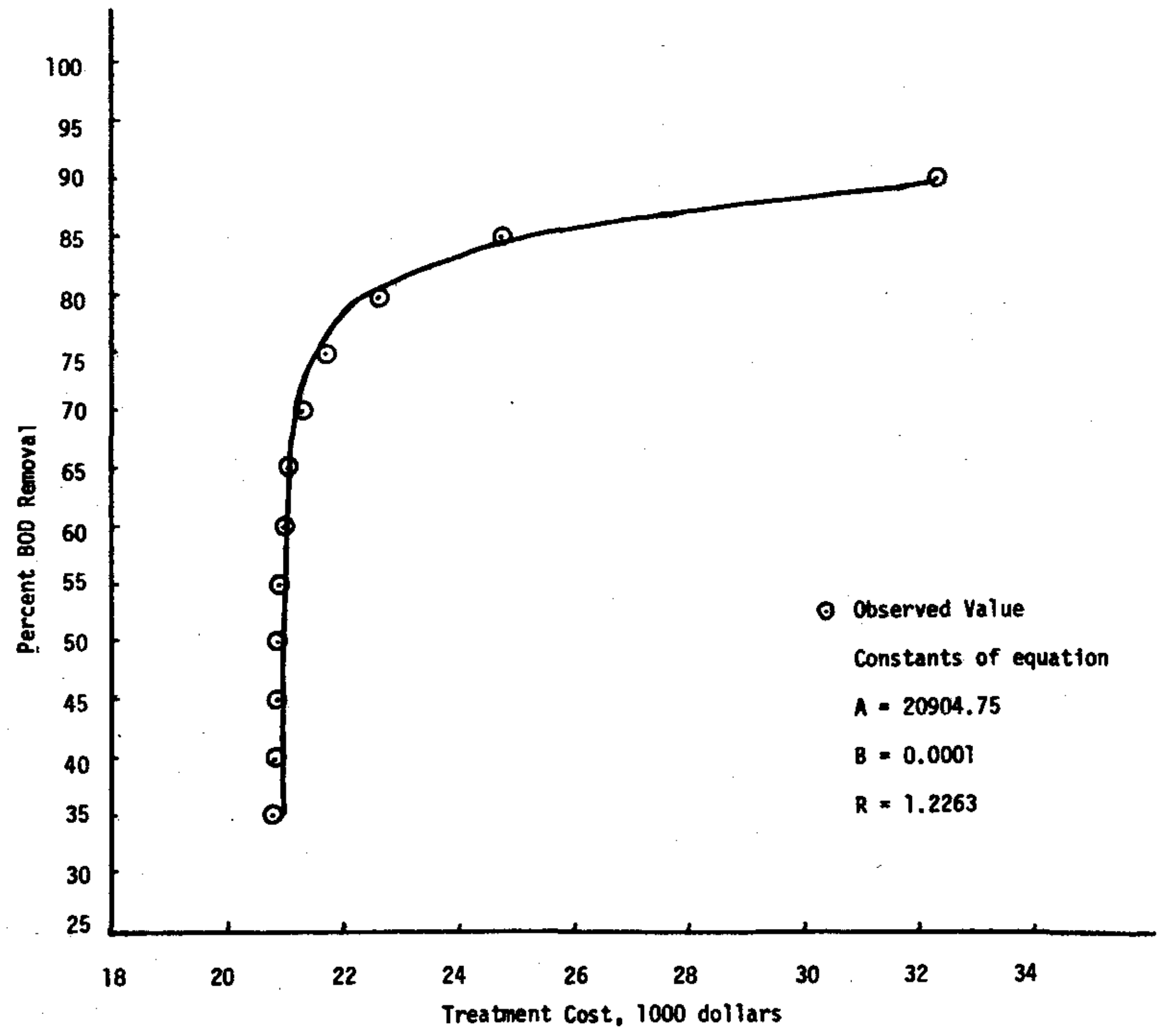
Y = treatment cost in dollars

X = percent of BOD removal

A = intercept of equation

B,R = coefficients

In performing the regression analysis of the above equation, a computer program of asymptotic regression analysis in BMD (Biomedical Computer Programs) was used. As an example, the analytical results of treatment plant M5 in the hypothetical basin is shown graphically in the following figure.

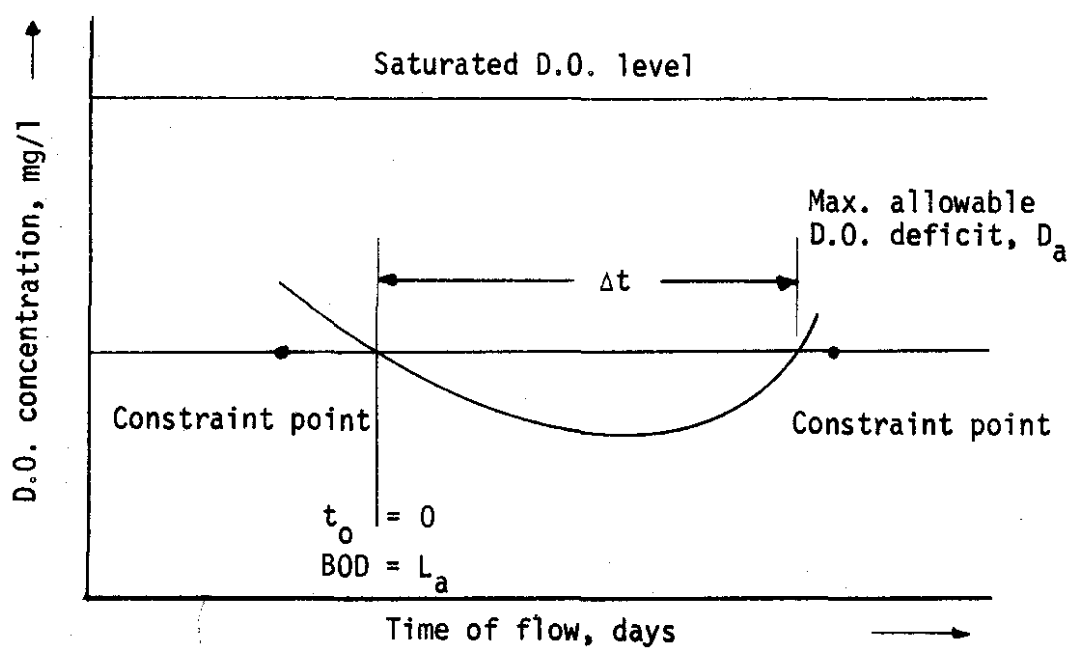

Percent BOD Removal
100
95
90
85
80
75
70
65
60
55
50
45
40
35
30
25
18
20
22
24
26
28
30
32
34
Treatment Cost, 1000 dollars
Observed Value
Constants of equation
A = 20904.75
B = 0.0001
R = 1.2263

APPENDIX F

SPACING OF D. O. CONSTANT POINTS

SPACING OF D.O. CONSTRAINT POINTS

The following sketch shows the lower part of an oxygen sag curve which shows a portion of the curve minimum D.O. level at a constraint point is equal or greater than the minimum required D.O. level, it is still possible to have a D.O. violation between two adjacent points. In



managing the water quality, the smaller the spacing of D.O. constraint points that is taken, the less the chance of a D.O. violation. However, if too many constraint points are adopted, the calculation work becomes unbearable. Therefore, it is necessary to determine an appropriate spacing at which the possible D.O. violation is insignificant.

The maximum possible D.O. violation occurs at the point where the critical D.O. deficit happens. An equation expressing the maximum violation in terms of the spacing between two constraint points can be derived as follows:

Let

Δt = interval of flow time between crossings of the maximum allowable D.O. deficit level, days,

and

t = any time of flow in the reach considered, days.

Thus, from the figure and the Streeter-Phelps equation, the value of the D.O. deficit with time is given by

$$D = \frac{k_1 L_a}{k_2 - k_1}\left(e^{-k_1 t} - e^{-k_2 t}\right) + D_a e^{-k_2 t} \tag{1}$$

but, at time $t_o + \Delta t$, the deficit again equals D_a. It follows that

$$D_a = \frac{k_1 L_a}{k_2 - k_1}\left(e^{-k_1 \Delta t} - e^{-k_2 \Delta t}\right) + D_a e^{-k_2 \Delta t} \tag{2}$$

The only unknown in Equation 2 is L_a.

$$L_a = \frac{D_a\left(1 - e^{-k_2 \Delta t}\right)\left(k_2 - k_1\right)}{k_1\left(e^{-k_1 \Delta t} - e^{-k_2 \Delta t}\right)} \tag{3}$$

The formula for the critial D.O. deficit, D_c, is

$$D_c = \frac{k_1 L_a}{k_2} \exp\left\{\frac{-k_1}{k_2 - k_1} \ln\left[\frac{k_2}{k_1}\left(1 - \frac{(k_2 - k_1)D_a}{k_1 L_a}\right)\right]\right\} \tag{4}$$

Substituting for L_a in Equation 4,

$$D_c = \frac{(k_2 - k_1)D_a\ (1 - e^{-k_2\Delta t})}{k_2(e^{-k_1\Delta t} - e^{-k_2\Delta t})} \exp\left\{\frac{-k_1}{k_2 - k_1} \ln\left[\frac{k_2}{k_1}\left(1 - \frac{(e^{-k_1\Delta t} - e^{-k_2\Delta t})}{(1 - e^{-k_2\ t})}\right)\right]\right\} \qquad (5)$$

The maximum violation, V, is the difference between D_c and D_a, i.e.,

$$V = D_c - D_a \qquad (6)$$

where D_c is given by Equation 5.

With the application of Equations 5 and 6, the appropriate spacing of each reach of the hypothetical basin was selected. Values of spacing and their corresponding values of D.O. violation are shown as follow. The marked values are the selected spacings whose corresponding D.O. violation are less or equal to 0.025 mg/l.

REACH NO. 1 (A - B)

SPACING, DAY	D.O. VIOLATED, MG/L
0.10	0.0008
0.20	0.0034
0.30	0.0076
** 0.40	0.0135
0.40	0.0211
0.60	0.0305
0.70	0.0415

REACH NO. 2 (B - C)

SPACING, DAY	D.O. VIOLATED, MG/L
0.10	0.0008
0.20	0.0032
0.30	0.0071
0.40	0.0126
** 0.50	0.0197
0.60	0.0284
0.70	0.0387
0.80	0.0506
0.90	0.0641
1.00	0.0792
1.10	0.0960
1.20	0.1143
1.30	0.1344
1.40	0.1561
1.60	0.1795
1.60	0.2046
1.70	0.2314
1.80	0.2599

REACH NO. 3 (C - D)

SPACING, DAY	D.O. VIOLATED, MG/L
0.10	0.0009
0.20	0.0036

REACH NO. 4 (D - E)

SPACING, DAY	D.O. VIOLATED, MG/L
0.10	0.0010
0.20	0.0038
** 0.30	0.0086
0.40	0.0153
0.50	0.0240
0.60	0.0345

REACH NO. 5 (E - F)

SPACING, DAY	D.O. VIOLATED, MG/L
0.10	0.0009
0.20	0.0037
0.30	0.0084
** 0.40	0.0149
0.50	0.0233
0.60	0.0336
0.70	0.0457

REACH NO. 6 (G - H)

SPACING, DAY	D.O. VIOLATED, MG/L
0.10	0.0006
0.20	0.0021
0.30	0.0046
** 0.40	0.0082
0.50	0.0128
0.60	0.0185
0.70	0.0251
0.80	0.0329

REACH NO. 7 (H - I)

SPACING, DAY	D.O. VIOLATED, MG/L
0.10	0.0005
0.20	0.0018

REACH NO. 8 (I - M)

SPACING, DAY	D.O. VIOLATED, MG/L
0.10	0.0006
0.20	0.0023
0.30	0.0052
0.40	0.0092
0.50	0.0143

REACH NO. 9 (J - L)

SPACING, DAY	D.O. VIOLATED, MG/L
0.10	0.0006
0.20	0.0020
0.30	0.0046
0.40	0.0081
0.50	0.0127
0.60	0.0183
0.70	0.0249
0.80	0.0325
** 0.90	0.0411
1.00	0.0508
1.10	0.0616
1.20	0.0733
1.30	0.0862
1.40	0.1001
1.50	0.1150
1.60	0.1310
1.70	0.1481
1.80	0.1663
1.90	0.1856
2.00	0.2059

REACH NO. 10 (K - L)

SPACING, DAY	D.O. VIOLATED, MG/L
0.10	0.0004
0.20	0.0018
0.30	0.0039
0.40	0.0070
0.50	0.0110
0.60	0.0158
0.70	0.0215

REACH NO. 11 (L - M)

SPACING, DAY	D.O. VIOLATED, MG/L
0.10	0.0004
0.20	0.0018
0.30	0.0039

REACH NO. 12 (M - N)

SPACING, DAY	D.O. VIOLATED, MG/L
0.10	0.0005
0.20	0.0021
0.30	0.0048
0.40	0.0085
0.50	0.0133

REACH NO. 13 (N - E)

SPACING, DAY	D.O. VIOLATED, MG/L
0.10	0.0006
0.20	0.0024
0.30	0.0055
0.40	0.0098
** 0.50	0.0153
0.60	0.0220
0.70	0.0300
0.80	0.0392
0.90	0.0497
1.00	0.0614
1.10	0.0744
1.20	0.0887
1.30	0.1043
1.40	0.1212
1.50	0.1393

REACH NO. 14 (O - P)

SPACING, DAY	D.O. VIOLATED, MG/L
0.10	0.0006
0.20	0.0022
0.30	0.0050
0.40	0.0088

REACH NO. 15 (P - E)

SPACING, DAY	D.O. VIOLATED, MG/L
0.10	0.0007
0.20	0.0027
0.30	0.0061

REACH NO. 16 (R - S)

SPACING, DAY	D.O. VIOLATED, MG/L
0.100.10	0.0006
0.200.20	0.0024
0.300.30	0.0054
0.400.40	0.0095
0.500.50	0.0149

REACH NO. 17 (S - F)

SPACING, DAY	D.O. VIOLATED, MG/L
0.10	0.0008
0.20	0.0032
0.30	0.0072
0.40	0.0128
0.50	0.0201

APPENDIX G

NUMERICAL ILLUSTRATION OF A HYPOTHETICAL STUDY

NUMERICAL ILLUSTRATION OF A HYPOTHETICAL STUDY

I. Notation

E_{ir} = percent BOD removal of the γ^{th} industrial treatment plant, $\gamma = 1, 2, \ldots, 7$.

E_{ms} = percent BOD removal of the s^{th} municipal treatment plant, $s = 1, 2, \ldots, 7$.

DO_{jk} = dissolved oxygen concentration at location j of reach k.

BOD_{jk} = BOD concentration at location j of reach k.

$$\alpha = e^{-k_1 t}$$

$$\beta = e^{-k_2 t}$$

$$\gamma = \frac{k_1}{k_2 - k_1} (e^{-k_1 t} - e^{-k_2 t})$$

II. Values of α, β, and γ

From the data of the hypothetical basin, values of α, β, and γ in each spacing of quality constraint points were calculated as shown in the following table.

REACH NO. 1 $(A - B^-)$

DISTANCE,	DAY	α	β	γ
$A-A_1$	0.40	0.88692	0.83527	0.10330
A_1-B^-	0.70	0.81058	0.72979	0.16159

REACH NO. 2 $(B - C^-)$

DISTANCE,	DAY	α	β	γ
$B-B_1$	0.50	0.86936	0.79852	0.11668
B_1-B_2	0.90	0.77724	0.66698	0.18162
B_2-B_3	1.30	0.69489	0.55711	0.22694
B_3-C^-	1.80	0.60411	0.44486	0.26230

REACH NO. 3 $(C - D^-)$

DISTANCE,	DAY	α	β	γ
$C-D^-$	0.20	0.93801	0.91393	0.05926

REACH NO. 4 $(D - E^-)$

DISTANCE,	DAY	α	β	γ
$D-D_1$	0.30	0.90303	0.87372	0.09061
D_1-E^-	0.60	0.81546	0.76338	0.16098

REACH NO. 5 $(E - F^-)$

DISTANCE,	DAY	α	β	γ
$E-E_1$	0.30	0.91119	0.86589	0.08262
E_1-F^-	0.70	0.80493	0.71462	0.16468

REACH NO. 6 $(G - H^-)$

DISTANCE,	DAY	α	β	γ
$G-G_1$	0.40	0.90123	0.86936	0.09206
G_1-H^-	0.80	0.81221	0.75578	0.17300

REACH NO. 7 $(H - I^-)$

DISTANCE,	DAY	α	β	γ
$H-I^-$	0.20	0.95504	0.93239	0.04341

REACH NO. 8 $(I - M^-)$

DISTANCE,	DAY	α	β	γ
$I-M^-$	0.50	0.86502	0.83946	0.12357

REACH NO. 9 $(J - L^-)$

DISTANCE,	DAY	α	β	γ
$J-J_1$	0.70	0.83946	0.77724	0.14139
J_1-J_2	1.40	0.70469	0.60411	0.22859
J_2-L^-	2.00	0.60653	0.48675	0.27222

REACH NO. 10 $(K - L^-)$

DISTANCE,	DAY	α	β	γ
$K-L^-$	0.70	0.83946	0.75578	0.13946

REACH NO. 11 $(L - M^-)$

DISTANCE,	DAY	α	β	γ
$L-M^-$	0.30	0.92774	0.88692	0.06804

REACH NO. 12 $(M - N^-)$

DISTANCE,	DAY	α	β	γ
$M-N^-$	0.50	0.87372	0.79852	0.11280

REACH NO. 13 $(N - E^-)$

DISTANCE,	DAY	α	β	γ
$N-N_1$	0.50	0.85642	0.79852	0.12820
N_1-N_2	1.00	0.73345	0.63763	0.21217
N_2-E^-	1.50	0.62814	0.50916	0.26345

REACH NO. 14 $(O - P^-)$

DISTANCE,	DAY	α	β	γ
$O-P^-$	0.40	0.91576	0.81873	0.07624

REACH NO. 15 $(P - E^-)$

DISTANCE,	DAY	α	β	γ
$P-E^-$	0.30	0.92219	0.86071	0.07218

REACH NO. 16 $(R - S^-)$

DISTANCE,	DAY	α	β	γ
$R-S^-$	0.50	0.89137	0.79453	0.09683

REACH NO. 17 $(S - F^-)$

DISTANCE,	DAY	α	β	γ
$S-F^-$	0.50	0.85642	0.79453	0.12789

III. Inventory Constraints of BOD and D.O.

Using Equations 20 and 21 in Section E, Part IV of this report and of values in Section II of this appendix, constraints of BOD and D.O. fo the regional quality system was developed as follows. For this case, the minimum D.O. requirement is 4.0 mg/l, and the saturated D.O. concentration in the streat is assumed equal to 9.0 mg/l.

Reach 1

At A

$DO_{1a} = (250 \times 7.4 + 5 \times 4)/(250 + 5)$

$DO_{1a} = 7.33$ (1)

$BOD_{1a} = [250 \times 1.6 + 5 \times 500 \times (100 - E_{i1})/100]/(250 + 5)$

$255\ BOD_{1a} + 25\ E_{i1} = 2900$ (2)

At A_i

$0.83527\ DO_{1a} - DO_{1a1} - 0.1033\ BOD_{1a} = -1.48257$ (3)

$BOD_{1a1} - 0.88692\ BOD_{1a} = 0$ (4)

At B^-

$0.72979\ DO_{1a} - DO_{1b} - 0.16159\ BOD_{1a} = -2.43189$ (5)

$BOD_{1b} - 0.81058\ BOD_{1a} = 0$ (6)

Reach 2

At B

$DO_{2b} = (260\ DO_{1b} + 35 \times 4)/(260 + 35)$

$295\ DO_{2b} - 260\ DO_{1b} = 140$ (7)

$BOD_{2b} = [260\ BOD_{1b} + 35 \times 380 \times (100 - E_{m1})/100]/260 + 35)$

$295\ BOD_{2b} - 260\ BOD_{1b} + 133\ E_{m1} = 13300$ (8)

At B_1

$$0.79852\ DO_{2b} - DO_{2b1} - 0.11668\ BOD_{2b} = -1.81332 \quad (9)$$

$$BOD_{2b1} - 0.86936\ BOD_{2b} = 0 \quad (10)$$

At B_2

$$0.66698\ DO_{2b} - DO_{2b2} - 0.18162\ BOD_{2b} = -2.99718 \quad (11)$$

$$BOD_{2b2} - 0.77724\ BOD_{2b} = 0 \quad (12)$$

At B_3

$$0.55711\ DO_{2b} - DO_{2b3} - 0.22694\ BOD_{2b} = -3.98601 \quad (13)$$

$$BOD_{2b3} - 0.69489\ BOD_{2b} = 0 \quad (14)$$

At C^-

$$0.44486\ DO_{2b} - DO_{2c} - 0.2623\ BOD_{2b} = -4.99626 \quad (15)$$

$$BOD_{2c} - 0.60411\ BOD_{2b} = 0 \quad (16)$$

Reach 3

At C

$$DO_{3c} = (275\ DO_{2c} + 6 \times 4)/(275 + 6)$$

$$281\ DO_{3c} - 275\ DO_{2c} = 24 \quad (17)$$

$$BOD_{3c} = [275\ BOD_{2c} + 6 \times 650 \times (100 - E_{i2})/100]/(275 + 6)$$

$$281\ BOD_{3c} - 275\ BOD_{2c} + 39.5\ E_{i2} = 3950 \quad (18)$$

At D^-

$$0.91393\ DO_{3c} - DO_{3d} - 0.05926\ BOD_{3c} = -0.77463 \quad (19)$$

$$BOD_{3d} - 0.93801\ BOD_{3c} = 0 \quad (20)$$

Reach 4

At D

$$DO_{4d} = (280\ DO_{3d} + 3 \times 4)/(280 + 3)$$

$$283\ DO_{4d} - 280\ DO_{3d} = 12 \tag{21}$$

$$BOD_{4d} = [280\ BOD_{3d} + 3 \times 975 \times (100 - E_{i3})/100]/(280 + 3)$$

$$283\ BOD_{4d} - 280\ BOD_{3d} + 20.25\ E_{i3} = 2925 \tag{22}$$

At D_1

$$0.87372\ DO_{4d} - DO_{4d1} - 0.09061\ BOD_{4d} = -1.13652 \tag{23}$$

$$BOD_{4d1} - 0.90303\ BOD_{4d} = 0 \tag{24}$$

At E^-

$$0.76338\ DO_{4d} - DO_{4e} - 0.16098\ BOD_{4d} = -2.12958 \tag{25}$$

$$BOD_{4e} - 0.81546\ BOD_{4d} = 0 \tag{26}$$

Reach 6

At G

$$DO_{6g} = (10 \times 7.5 + 8 \times 4)/(10 + 8)$$

$$DO_{6g} = 5.95 \tag{27}$$

~~$BOD_{6g} = [10 \times 1 + 8 \times 220 \times (100 - E_{m3})/100]/(10 + 8)$~~

$$18\ BOD_{6g} + 17.6\ E_{m3} = 1770 \tag{28}$$

At G_1

$$0.86936\ DO_{6g} - DO_{6g1} - 0.09206\ BOD_{6g} = -1.17576 \tag{29}$$

$$BOD_{6g1} - 0.09123\ BOD_{6g} = 0 \tag{30}$$

At H^-

$$0.75578\ DO_{6g} - DO_{6h} - 0.163\ BOD_{6g} = -2.19798 \quad (31)$$

$$BOD_{6h} - 0.81221\ BOD_{6g} = 0 \quad (32)$$

Reach 7

At H

$$DO_{7h} = (15\ DO_{6h} + 5 \times 8)/20$$

$$20\ DO_{7h} - 15\ DO_{6h} = 40 \quad (33)$$

$$BOD_{7h} = (15\ BOD_{6h} + 5 \times 1)/20$$

$$20\ BOD_{7h} - 15\ BOD_{6h} = 5 \quad (34)$$

At I^-

$$0.93239\ DO_{7h} - DO_{7i} - 0.04341\ BOD_{7h} = -0.60849 \quad (35)$$

$$BOD_{7i} - 0.95504\ BOD_{7h} = 0 \quad (36)$$

Reach 8

At I

$$DO_{8i} = (20\ DO_{7i} + 2 \times 4)/(20 + 2)$$

$$22\ DO_{8i} - 20\ DO_{7i} = 8 \quad (37)$$

$$BOD_{8i} = [20\ BOD_{7i} + 2 \times 550 \times (100 - E_{i4})/100]/(20 + 2)$$

$$22\ BOD_{8i} - 20\ BOD_{7i} + 11\ E_{i4} = 1100 \quad (38)$$

At M^-

$$0.83946\ DO_{8i} - DO_{8m} - 0.12357\ BOD_{8i} = -1.44486 \quad (39)$$

$$BOD_{8m} - 0.86502\ BOD_{8i} = 0 \quad (40)$$

Reach 9

At J

$$DO_{9j} = (25 \times 7.6 + 7 \times 4)/(25 + 7)$$

$$DO_{9j} = 6.81 \qquad (41)$$

$$BOD_{9j} = [25 \times 0.9 + 7 \times 270 \times (100 - E_{m4})/100]/(25 + 7)$$

$$32\ BOD_{0j} + 18.9\ E_{m4} = 1912.5 \qquad (42)$$

At J_1

$$0.77724\ DO_{9j} - DO_{9j1} - 0.14139\ BOD_{9j} = -2.00484 \qquad (43)$$

$$BOD_{9j1} - 0.83926\ BOD_{9j} = 0 \qquad (44)$$

At J_2

$$0.60411\ DO_{9j} - DO_{9j2} - 0.22859\ BOD_{9j} = -3.56301 \qquad (45)$$

$$BOD_{9j2} - 0.70469\ BOD_{9j} = 0 \qquad (46)$$

At L^-

$$0.48675\ DO_{9j} - DO_{91} - 0.27222\ BOD_{9j} = -4.61925 \qquad (47)$$

$$BOD_{91} - 0.60653\ BOD_{9j} = 0 \qquad (48)$$

Reach 10

At K

$$DO_{10k} = (28 \times 8 + 10 \times 4)/(28 + 10)$$

$$DO_{10k} = 6.95 \qquad (49)$$

$$BOD_{10k} = [28 \times 1.2 + 10 \times 300 \times (100 - E_{m5})]/(28 + 10)$$

$$38\ BOD_{10k} + 30\ E_{m5} = 3033.6 \qquad (50)$$

At L^-

$$0.75578\ DO_{10k} - DO_{10L} - 0.13946\ BOD_{10k} = -2.19798 \quad (51)$$

$$BOD_{10L} - 0.83946\ BOD_{10k} = 0 \quad (52)$$

Reach 11

At L

$$DO_{11L} = (30\ DO_{9L} + 30\ DO_{10L})/60$$

$$60\ DO_{11L} - 30\ DO_{9L} - 30\ DO_{10L} = 0 \quad (53)$$

$$BOD_{11L} = (30\ BOD_{9L} + 30\ BOD_{10L})/60$$

$$60\ BOD_{11L} - 30\ BOD_{9L} - 30\ BOD_{10L} = 0 \quad (54)$$

At M^-

$$0.88692\ DO_{11L} - DO_{11m} - 0.06804\ BOD_{11L} = -1.01772 \quad (55)$$

$$BOD_{11m} - 0.92772\ BOD_{11L} = 0 \quad (56)$$

Reach 12

At M

$$DO_{12m} = (60\ DO_{11M} + 20\ DO_{8m})/80$$

$$80\ DO_{12m} - 60\ DO_{11m} - 20\ DO_{8m} = 0 \quad (57)$$

$$BOD_{12m} = (60\ BOD_{11m} + 20\ BOD_{8m})/80$$

$$80\ BOD_{12m} - 60\ BOD_{11m} - 20\ BOD_{8m} = 0 \quad (58)$$

At N^-

$$0.79852\ DO_{12m} - DO_{12n} - 0.1128\ BOD_{12m} = -1.81332 \quad (59)$$

$$BOD_{12n} - 0.87372\ BOD_{12m} = 0 \quad (60)$$

Reach 13

At N

$DO_{13n} = (84\ DO_{12n} + 4.2 \times 4)/(84 + 4.2)$

$88.2\ DO_{13n} - 84\ DO_{12n} = 16.8$ (61)

$BOD_{13n} = [84\ BOD_{12n} + 4.2 \times 1040 \times (100 - E_{i5})/100]/(84 + 4.2)$

$88.2\ BOD_{13n} - 84\ BOD_{12n} + 43.68\ E_{i5} = 4368$ (62)

At N_1

$0.79852\ DO_{13n} - DO_{13n1} - 0.1282\ BOD_{13n} = -1.81332$ (63)

$BOD_{13n1} - 0.85642\ BOD_{13n} = 0$ (64)

At N_2

$0.63763\ DO_{13n} - DO_{13n2} - 0.21217\ BOD_{13n} = -3.26133$ (65)

$BOD_{13n2} - 0.73345\ BOD_{13n} = 0$ (66)

At E^-

$0.50916\ DO_{13n} - DO_{13e} - 0.26345\ BOD_{13n} = -4.41756$ (67)

$BOD_{13e} - 0.62814\ BOD_{13n} = 0$ (68)

Reach 14

At O

$DO_{14o} = (34 \times 7.8 + 7.5 \times 4)/(34 + 7.5)$

$DO_{14o} = 7.1$ (69)

$BOD_{14o} = [34 \times 2.1 + 7.5 \times 350 \times (100 - E_{m6})/100]/(34 + 7.5)$

$41.5\ BOD_{14o} + 26.25\ E_{m6} = 2696.4$ (70)

At P^-

$$0.81873\ DO_{14o} - DO_{14p} - 0.07624\ BOD_{14o} = -1.63143 \quad (71)$$

$$BOD_{14p} - 0.91576\ BOD_{14o} = 0 \quad (72)$$

Reach 15

At P

$$DO_{15p} = (50\ DO_{14p} + 3.6 \times 4)/(50 + 3.6)$$

$$53.6\ DO_{15p} - 50\ DO_{14p} = 14.4 \quad (73)$$

$$BOD_{15p} = [50\ BOD_{14p} + 3.6 \times 700 \times (100 - E_{i6})/100](50 + 3.6)$$

$$53.6\ BOD_{15p} - 50\ BOD_{14p} + 25.2\ E_{i6} = 2520 \quad (74)$$

At E^-

$$0.86071\ DO_{15p} - DO_{15e} - 0.07218\ BOD_{15p} = -1.25361 \quad (75)$$

$$BOD_{15e} - 0.92219\ BOD_{15p} = 0 \quad (76)$$

Reach 5

At E

$$DO_{5e} = [(450 - 84 - 50)\ DO_{4e} + 84\ DO_{13e} + 50\ DO_{15e} + 12 \times 4]/(450 + 12)$$

$$462\ DO_{5e} - 316\ DO_{4e} - 84\ DO_{13e} - 50\ DO_{5e} = 48 \quad (77)$$

$$BOD_{5e} = [(450 - 84 - 50)\ BOD_{4e} + 84\ BOD_{13e} + 50\ BOD_{15e} + 12 \times 240 \times (100 - E_{m2})/100]/(450 + 12)$$

$$462\ BOD_{5e} - 316\ BOD_{4e} - 84\ BOD_{13e} - 50\ BOD_{15e} + 28.8\ E_{m2} = 2880 \quad (78)$$

At E_1

$$0.86589\ DO_{5e} - DO_{5e1} - 0.08262\ BOD_{5e} = -1.20699 \quad (79)$$

$$BOD_{5e1} - 0.91119\ BOD_{5e} = 0 \qquad (80)$$

At F^-

$$0.71462\ DO_{5e} - DO_{5f} - 0.16468\ BOD_{5e} = -2.56842 \qquad (81)$$

$$BOD_{5f} - 0.80493\ BOD_{5e} = 0 \qquad (82)$$

Reach 16

At R

$$DO_{16r} = (32 \times 7.7 + 7.5 \times 4)/(32 + 7.5)$$

$$DO_{16r} = 7.0 \qquad (83)$$

$$BOD_{16r} = [32 \times 1.5 + 7.5 \times 275 \times (100 - E_{m7})/100]/(32 + 7.5$$

$$39.5\ BOD_{16r} + 20.625\ E_{m7} = 2110.5 \qquad (84)$$

At S^-

$$0.79453\ DO_{16r} - DO_{16s} - 0.09683\ BOD_{16r} = -.184923 \qquad (85)$$

$$BOD_{16s} - 0.89137\ BOD_{16r} = 0 \qquad (86)$$

Reach 17

At S

$$DO_{17s} = (46\ DO_{16s} + 2.5 \times 4)/(46 + 2.5)$$

$$48.5\ DO_{17s} - 46\ DO_{16s} = 10 \qquad (87)$$

$$BOD_{17s} = [46\ BOD_{16s} + 2.5 \times 865 \times (100 - E_{i7})/100]/(46 + 2.5)$$

$$48.5\ BOD_{17s} - 46\ BOD_{16s} + 21.625\ E_{i7} = 2162.5 \qquad (88)$$

At F^-

$$0.79453\ DO_{17s} - DO_{17f} - 0.12789\ BOD_{17s} = -1.84923 \qquad (89)$$

$$BOD_{17f} - 0.85642\ BOD_{17s} = 0 \qquad (90)$$

At F

$DO_{17ff} = [(500 - 46)\ DO_{5f} + 46\ DO_{17f}]/500$

$500\ DO_{17ff} - 454\ DO_{5f} - 46\ DO_{17f} = 0$ (91)

$BOD_{17ff} = [(500 - 46)\ BOD_{5f} + 46\ BOD_{17f}]/500$

$500\ BOD_{17ff} - 454\ BOD_{5f} - 46\ BOD_{17f} = 0$ (92)

IV. Grouping of Treatment Zones

1. Treatment zones from sub-basin criterion

Zone No.	Sub-basin	Treatment Plant
1	1	I1, I2, I3, M1, M2
2	2	I4, M3
3	3	M4
4	4	I5, M5
5	5	I6, M6
6	6	I7, M7

2. Treatment zones from weight of influent BOD

Zone No.	BOD, lbs/day	Treatment Plant
1	5,933	I4
"	9.493	M3
2	10,193	M4
"	11,124	M7
"	11,663	I7
"	13,484	I1
"	13,592	I7
"	14,158	M6
"	15,533	M2
"	15,776	I3
"	16,181	M5
3	21,035	I2
"	23,559	I5
4	71,734	M1

3. Treatment zones from BOD-flow ratio

Zone No.	BOD-flow Ratio, 10^{-6}	Treatment Plant
1	6.24	M2
"	9.81	I1
"	10.34	I3
"	13.88	I2
2	44.60	I7
"	45.10	M1
"	47.03	I6
"	49.54	I5
"	50.02	I4
"	52.24	M7
3	59.09	M4
"	63.28	M6
4	78.98	M5
5	97.82	M3

V. Objective Function of System Optimization

The total weight of BOD removal, in pounds per day, from all treatment plants in the basin is the objective function. It is given by

$$\text{pounds} = \sum (\text{Inflow} \times \text{Influent BOD concentration} \times \%\ \text{BOD removal})$$
$$= (5 \times 500\ E_{i1} + 6 \times 650\ E_{i2} + 3 \times 975\ E_{i3} + 2 \times 550\ E_{i4} + 4.2 \times 1040\ E_{i5} + 3.6 \times 700\ E_{i6} + 2.5 \times 865\ E_{i7} + 35 \times 380\ E_{m1} + 12 \times 240\ E_{m2} + 8 \times 220\ E_{m3} + 2 \times 270\ E_{m4} + 10 \times 300\ E_{m5} + 7.5 \times 350\ E_{m6} + 7.5 \times 275\ E_{m7})c/100.$$

where

c = conversion factor

1 cfs = 0.646 million gals/day

1 mg/l = 8.345 lbs/million gals.

$c = 0.0646 \times 8.345 = 5.39,$

therefore

$$\text{pounds} = 134.75\ E_{i1} + 212.905\ E_{i2} + 157.657\ E_{i3} + 59.29\ E_{i4} + 235.435\ E_{i5} + 135.828\ E_{i6} + 116.558\ E_{i7} + 716.87\ E_{m1} + 155.232\ E_{m2} + 94.864\ E_{m3} + 101.871\ E_{m4} + 161.7\ E_{m5} + 141.487\ E_{m6} + 111.168\ E_{m7}.$$

VI. Linear Programming Formation of Water Quality System

1. Minimum treatment management - different percent BOD removal at each treatment plant is allowed.

   Minimize

   the objective function in Section V

   subject to

   constraints 1 to 92 in Section III.

2. Uniform treatment management.

   Minimize

   the objective function in Section V

   subject to

   constraints 1 to 92 in Section III and

   constraints

   $E_{i1} = E_{i2} = - - - = E_{m7}.$

3. Zone treatment from sub-basin criterion

   Minimize

   the objective function in Section V

   subject to

constraints 1 to 92 in Section III, and

$E_{i1} = E_{i2} = E_{i3} = E_{m1} = E_{m2}$,

$E_{i4} = E_{m3}$,

$E_{i5} = E_{m5}$,

$E_{i6} = E_{m6}$, and

$E_{i7} = E_{m7}$.

4. Zone-treatment from the criterion of influent BOD weight.

Minimize

the objective function in Section V

subject to

constraints 1 to 92 in Section III and

$E_{i4} = E_{m3}$,

$E_{m4} = E_{m7} = E_{i7} = E_{i1} = E_{i6} = E_{m6} = E_{m2} = E_{i3} = E_{m5}$,

$E_{i2} = E_{i5}$,

$E_{i4} \leq E_{m4}$,

$E_{m4} \leq E_{i2}$, and

$E_{i2} \leq E_{m1}$.

5. Zone-treatment from the criterion of BOD-flow ratio.

Minimize

the objective function in Section V

subject to

constraints 1 to 92 in Section III, and

$E_{m2} = E_{i1} = E_{i3} = E_{i2}$,

$E_{i7} = E_{m1} = E_{i6} = E_{i5} = E_{i4} = E_{m7}$,

$E_{m4} = E_{m6}$,

$E_{m2} \leq E_{i7}$,

$E_{i7} \leq E_{m4}$,

$E_{m4} \leq E_{m5}$, and

$E_{m5} \leq E_{m3}$.