TECHNICAL COMPLETION REPORT

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DISTRIBUTION AND MIXING OF INFLOW INTO STRATIFIED LAKES: A HYDRAULIC MODEL STUDY

(Phase 1)

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ABSTRACT

The similitude problems in modeling flow in stratified lakes was investigated both theoretically and experimentally. Models with transparent sides were built with representative inlets. Using salt solutions of varying density and coloration, the establishment of stratified conditions, the control and modeling of inlet flows, and the use of visualization methods were developed. Special attention was given to the problem of vertical distortion; in the modeling of lakes, it is necessary to use a different depth scale than the horizontal length scale. Too little vertical depth in the model allows boundary layer effects to dominate; too much depth and distortion may invalidate the similitude of the model to the lake. Two models with different amounts of vertical distortion were built and compared. Quantitative methods using photographic sequences and computer analysis of time lines were used to show the basic flow patterns of inlet flows into stratified lakes and to show the application and limitations of vertical scale distortion.

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NOMENCLATURE

D	width of diffusion layer
F	Froude number, $u/[gH]^{\frac{1}{2}}$
g	gravitational constant
H	depth of lake model
L_1	characteristic length
L_2	characteristic length
Re	Reynolds number, $\frac{U_c L}{\nu}$
Ri	Richardson number, $\frac{g(\partial\rho/\partial y)}{\rho(\partial U_c/\partial y)^2}$
t	characteristic time
U_c	characteristic velocity
V^*	nondimensional velocity, U/U_c
y	vertical dimension
Y^*	nondimensional vertical dimension y/H

Greek Letters

Δ	difference
μ	dynamic viscosity of the fluid
ν	kinematic viscosity of the fluid
ρ	density of the fluid
Γ	shearing stress per unit area

Subscripts

A	deep model--12 inch
B	shallow model--6 inch
i	inlet
in	interface
t	total

CHAPTER I

INTRODUCTION

Problem Statement

The purpose of this experimental investigation was to determine the effect of vertical scale exaggeration in free surface hydraulic models. Specifically, this was to determine whether a hydraulic model could be scaled in a manner where the scale factor of the vertical direction would not be as small as for the horizontal direction.

The word model is used in the restricted sense of an attempt at a smaller scale simulation of a large and nearly always unique, natural or artificial hydraulic system. For this investigation the hydraulic system is taken to be an idealized stratified lake with a river channel inflow configuration.

In hydraulic systems such as lakes with relatively large horizontal lengths, the depths are usually comparatively small. When these systems are modeled by scaling these dimensions down with the same scaling parameter, the result in a models that are very shallow, so that frictional forces dominate, resulting in an inaccurate model of the prototype. Therefore, exaggeration of the vertical scale to give an acceptable depth in the model has been utilized in various hydraulic studies, but this technique is not universally supported as being accurate.

The lack of universal acceptance of the concept of vertical scale exaggeration is the motivation for this experimental study.

In this investigation, flow patterns were established in a prototype model of a stratified lake and the flow patterns were simulated in another stratified lake model which had an exaggeration of the vertical scale. It was then attempted to relate the observed flow pattern and velocity profiles at selected locations in the prototype model to those at the same corresponding locations in the exaggerated model.

Background

The lack of universal acceptance of the vertical scale exaggeration concept stems from the fact that any modeling technique employed is a compromise at best. The relative importance placed on the interaction of the various pairs of forces in a fluid flow situation is the basis for the dispute.

The power of the method of models in solving problems in free boundary hydraulics has been demonstrated in thousands of successful studies. If these modeling techniques are examined in detail, it is found that where gravitational forces predominate and a fixed containing boundary is used, there is consistent agreement between the model and the prototype, so long as various well established precautions are taken in the design and use of the model. Where frictional forces have the same order of influence as the gravitational forces, success has also been met with, but to a lesser extent. In many cases involving large but relatively shallow prototypes, success has been dependent on the method of exaggeration of the vertical scale in comparison with the horizontal scale of the model.

At present, the school of thought that backs the utilization of vertical scale exaggeration are mainly researchers in England and Australia. In open literature there are many studies contributed by the researchers in these countries that have proven to be quite successful. Most notable of these studies are those by Barr [3, 4, 5, 7 and 8], and the work by Price and Kendrick [26]. Most of these model studies were made to simulate the effects of thermal discharges into rivers and tidal estuaries and the mechanism of exchange flow in estuaries. They have reported that due to the large horizontal dimensions involved, vertical scale exaggeration could be utilized very successfully with careful consideration of the effects of exaggeration.

The researchers that dispute the validity of exaggeration are mainly from France. Most of their investigations deal with model studies of large hydraulic systems, such as large estuaries and rivers, with regard to the overall redesign of these systems. Since they dispute the problem concept, their models are scaled on the basis of the natural model scaling parameter, i.e., all dimensions are scaled proportionally with regard to the actual hydraulic systems dimensions. This has resulted in models that are quite large, because of the need to have the vertical dimension large enough that frictional effects will not be inaccurate.

In the United States there have been reports by researchers who follow both lines of reasoning. Fischer and Holly [13] reported that the vertical scale exaggeration concept is not feasible, but they approached this argument with a one-dimensional analytical study. Miner, Hinley, and Cayot [22] have reported success with the use of the exaggeration concept in a comparison study between a model and a prototype which investigated thermal discharges. No reports appeared in the literature on the utilization of vertical scale exaggeration with stratified lake models as pursued in this report. The only studies related to this investigation dealt with the consideration of a homogeneous body of water with an inflow of a different density.

Scope of the Present Study

To validate the concept of vertical scale exaggeration in stratified lake models, two idealized lake models were constructed for testing purposes. Specifically, the lake models were designed to allow the visualization of the distribution and mixing patterns resulting from dye traced inflows into various types of lake stratification.

The two lake models were constructed to have geometric similarity in all aspects except for the vertical scale. One lake was considered as the prototype model with a natural scaling parameter and the other model had a vertical scale exaggeration of twice the natural scale.

The investigation is limited because only one type of vertical scale exaggeration was utilized, but with a large range of variation in flow patterns available. Along with the various types of stratification that could be set up in the lake models, it was felt that this would lead to a determination of the critical parameters and their effect on the proper simulation between models.

To determine the validity of the scale exaggeration concept it was important to determine the inflow patterns for selected locations corresponding between the two models for the same basic flow situations. This was accomplished by testing one model with a specified flow configuration and recording the visual traces of the dye patterns. The same flow configuration was tested on the other model with the flow parameters adjusted suitably and again the flow patterns were recorded. The data was then corrected with regard to proper scaling and the velocity profiles for the specified locations compared.

To properly model a specified flow configuration in the exaggerated model it was necessary to develop a modeling technique that would adjust the various control parameters to yield a flow situation as developed in the prototype model. The development of the modeling theory and technique is presented in Chapter II of this report.

A detailed explanation of the experimental testing facilities that were constructed to determine the validity of the problem considered, is presented in Chapter III. Also presented are sections concerned with the testing procedures, the data collection systems, and the data reduction techniques.

Chapter IV gives the results of the experimental testing along with the types of tests utilized. A detailed discussion of the results and the significance of the variance of the similitude parameters is presented in Chapter V. A summation of the work of this investigation and the major conclusions developed during this report is presented in Chapter VI.

Relevance

This experimental investigation has application to the development of modeling techniques for stratified lake models. Validation of the vertical scale exaggeration concept for these types of models would enable researchers to design models that would be of laboratory size, whereas natural models would tend to be quite large and thereby would require a greater expenditure in terms of funds and space required for the same type of investigation. This would aid in the development of accurate models of lakes and reservoirs, in particular, the study of the fluid mechanics within these models. The use of stratified lake models is a relatively recent research development. This concept in lake modeling is a more realistic method in terms of developing an accurate model of a real lake situation, since most lakes are stratified to some extent. Most of the previous model studies in this area dealt primarily with a lake model of one density, i.e., a homogeneous lake, and simulating various types of inflows by introduction of a fluid of another density.

From these model studies the many aspects of the lake's dynamic flow system can be determined with regard to various types of inflows. Proposed lake modification designs could be tested by the simulation of the dispersion and mixing patterns of different inflows. The results of this type of simulation could then be extrapolated to a real lake situation, thus enabling the best design to be selected without actual testing in the real lake.

The success of a lake or reservoir design could be enhanced by the use of lake models utilizing vertical scale exaggeration. These types of models could also be used in the development of lake or reservoir preservation programs and for water quality maintenance.

CHAPTER II

MODELING THEORY

This chapter presents the development of the modeling theory utilized in this investigation. Consideration is given to the various forces acting on the fluids in this problem and the development of similitude numbers which are representative of these forces.

Previous Research

Exchange flows in stratified systems have aroused spasmodic interest during the past forty years or so; O'Brien and Cherno [23], Yih [28], and Keulegan [17] have described experimental studies, while Keulegan [17] and Schijf and Schonfeld [27] have given analytical approaches to this type of flow. It is one facet of the group of phenomena variously known as density currents, stratified flows, sub-surface flows or internal flows. The existence of parallels between sub-surface and free surface hydraulic occurrences has been stressed by various writers, notably Keulegan [17] and more recently Harleman [15]. It is, however, important to remember that in practical circumstances small density difference phenomena are normally observed as between miscible fluids; fresh and salt water or warmer and colder water, warmer and colder air, or between air and other gases. In the laboratory miscible liquids may be used, as in the studies mentioned above.

The utilization of scale exaggeration was reported in Keulegan's [17] studies and also by Barr [7] as being a workable concept so long as the consideration of the effects of exaggeration were taken into account. Fischer and Holly [13] disagreed with the concept in their paper on dispersion studies.

Theoretical Development

Figure 1 is a sketch of the specific physical situation we have chosen to model. The lake has an initial stratification which consists of two distinct volumes of water of approximately 3% different densities. The density difference can be caused by either a temperature difference or a difference in saline concintration. Between the heavy and light layers there are layers of intermediate density fluid.

For the first group of experiments, reported here, the inflow situation consists of an inflow channel much like a stream or river, containing fluid whose density is some where between the two extremes of density present in the lake. It is well known that this inflow flows into the lake in a lens pattern between the two existing density layers. However the objective of our experimental program is to establish a technique which will model the inflow distribution quantitatively as well as qualitalively. To obtain quantitative data, a primary tool is dye marked visualization. Our models are quasi-two dimensional, and the lake model is trunkated on the sides with plexiglass walls. The position of these walls is shown in Figure 1.

There are three non-dimensional parameters that are important in the modeling of free-surface stratified hydraulic facilities. They are:

1) Reynolds number

$$Re = \frac{\rho U_c L}{\mu}$$

2) Richardson Number

$$Ri = \frac{g \;\; \partial\rho/\partial y}{\rho\left(\frac{\partial U}{\partial y}\right)^2}$$

3) Froude number

$$Fr = \frac{U}{[gH]^{\frac{1}{2}}}$$

In lake flows in many cases the surface waves do not play a major role in the dynamics of the flow situation. In the present work we are not attempting to model a situation where the surface phenomenon is important. In this case the Froude number is an unimportant parameter.

For modeling purposes the Richardson number is usually reduced to an overall form by assuming $\frac{\partial\rho}{\partial z} = \frac{-\Delta\rho}{L_1}$ and $\frac{\partial U}{\partial z} = \frac{U_c}{L_2}$.

This normally reduces the Richardson number to $\overline{Ri} = \frac{g\Delta\rho L_2^2}{\rho U_c^2 L_1}$ where the bar indicates an overall average, and if $L_1 = L_2$ this reduces further to $\overline{Ri} = \frac{g\Delta\rho L_1}{\rho U_c^2}$.

In lake models where scale exaggeration is employed the situation is greatly complicated because a choice must be made between a horizontal or a vertical length scale. The choice yields different results since the horizontal scale factor differs from the vertical scale factor.

With regard to the density gradient $\frac{\partial \rho}{\partial y}$ there are two characteristic lengths which can be used in forming the overall Richardson number. The first, and most commonly used is the characteristic depth of the lake H. The second is the width of the diffusion layer (D) between the heavy fluid on the bottom and the light fluid on top. In the case of the vertical gradient of the velocity profile $\frac{\partial U}{\partial y}$ there are three length scales which can be used in reducing this gradient to parametric form: 1) the depth of the lake H 2) the characteristic length of the lake $L\ell$ 3) the viscous length $\frac{\nu}{U}$. Since there are two possible density gradient parameters and three possible velocity gradient parameters, then there are 6 possible permutations of overall Richardson number which can be used.

In this first years work we have assumed the limited task of building the apparatus, proving the experimental procedure and testing the applicability of one of the overall Richardson numbers. Evaluation of the Reynolds number effect has also been undertaken. It is well known that in turbulent flow situations the fluid dynamics is not critically sensitive to changes in Reynolds number even as large as an order of magnitude. If this proves to be true in the stratified lake situation as well, then the viscous length

$\frac{\nu}{U_c}$ can be removed from the list of possible velocity gradient parameters reducing the number of possible Richardson numbers to 4.

CHAPTER III

EXPERIMENTAL TECHNIQUES

This chapter presents the details of the laboratory facilities, the lake models, the flow visualization technique and the data collection and reduction methods. The experimental procedure and testing methods are also described.

Lake Models and Inflow System

Two lake models were constructed for experimentally determining the validity of the vertical scale exaggeration concept. Both models were constructed from $\frac{1}{2}$-inch plexiglas, to allow observation of the bulk of the lake model's volume. Figure 2 shows a sketch of the experimental set-up, with the lake model shown in the center.

Both lake models are eight feet in length and have a horizontal dimension of eighteen inches. The vertical dimension is twelve inches for the exaggerated scale lake model and six inches for the natural scale lake model. This is a vertical exaggeration of twice the natural scale of model B. The lakes are designed to have a water depth of six and three inches respectively.

Both lake models have their own support stands which are identical in size and construction. Each lake model has an inlet contour molded into the upstream portion of the model volume. The contours are of arbitrary shape, to represent an idealized lake bottom contour. Both contours are geometrically similar with only the vertical dimension exaggerated in the large model to coincide with the proper simulation of the total lake's similarity.

Figure 3 shows the schematic of the inflow system and the dye injection system. Three 45 gallon plastic storage tanks are set on the upper deck of the laboratory to yield the required head necessary for the flow system to operate by gravity feed. The three storage tanks feed directly to the control panel, which is a central location of all valves and flow meters utilized in the project.

The flow lines for the entire system are made from either Polyvinyl Chloride (P. V. C.) pipe, $\frac{1}{2}$-inch in diameter or $\frac{1}{2}$-inch rubber garden hose to minimize the corrosion problem inherent with saline solutions. The valves utilized for the storage tank lines, the fresh water line and the drain are all made of stainless steel and brass for the same reason.

The valves from the different water sources are connected to a manifold constructed of $\frac{1}{2}$-inch P. V. C. pipe and the outflow from this manifold is connected to a plastic $\frac{1}{2}$-inch gate valve for flow control, then through a Fisher-Porter precision bore rotameter with a flow range from 0.2 GPM to 2.0 GPM. From the flowmeter the flow is introduced to the inlet channel entrance pipe.

The inflowing fluid enters the inlet channel which is shown in Figure 4. The inlet channel width can be varied from a maximum of two inches to a minimum of one inch width through the use of $\frac{1}{2}$-inch plexiglas inserts.

The flow then leaves the inlet channel and flows into the lake model upstream section. This is the section of the lake model that contains the contour of the lake bottom. The contours are constructed from $\frac{1}{4}$-inch fiberboard laminated to form the desired shape, then contoured to achieve geometric similarity. The interior portion of the lake models as well as the contours are sealed with a clear silicone sealing compound to ensure that the models are watertight.

The outflow end of the lake model has the previously mentioned weir arrangement. The weir attachment is designed to accommodate either a flat edge (or straight edge) weir or a V-notch weir. The arrangement allows variation in the vertical direction to insure proper lake depths. The weir system has a calibration system embossed on its surface to allow readings of the outflow rate.

Flow Visualization

To visualize the flow patterns of the different inflows into the stratified lake models, a dye injection system was constructed. A dye container was positioned by the control panel on a movable stand, to allow a variation in the gravity head for the dye system. The dye container was fitted with a small ball valve on the bottom side to allow regulation in the dye flow rate. The dye was then transmitted through 1/8 inch plastic tubing to a large bore hypodermic needle. The needle was inserted into the desired storage tank flow line for dye injection into the flow.

This arrangement was designed to allow complete mixing of the dye with the inlet fluid before it reached the lake inlet, and also to allow the inflow fluid in the storage tank to be kept clear. This was necessary because for each experimental test, the selected inflow fluid was introduced into the lake model in the un-dyed form until the flow became developed, thus allowing the dye traced fluid to be introduced that would accurately visualize the flow pattern.

Data Collection System

To record the dye-traced flow patterns of the various inflows a photographic system was devised to record all pertinent information of the flow field. Since the lake models were constructed of transparent plexiglass, visual observation of the flow patterns could be made from both a top and a side view. Utilizing a 35mm Ashia Pentax single lens reflex camera all pertinent data was recorded on a single frame of film for a certain period of time during the experimental test.

This was accomplished by means of mirror systems shown in Figure 5. Two mirror systems were utilized, one for the top view and one for the side view of the test section. The top mirror system consisted of a 24 x 36 inch double plated mirror inclined at a 45° angle. The side mirror system consisted of four 8 x 24 inch mirrors so arranged as to yield the same focal length for both the side view and the top view.

The 35mm camera was positioned approximately twelve feet from the lake model on a tripod. A 150mm lens was used to eliminate the depth of field problem inherent in this type of optical arrangement. The film used was Kodak TX-135 and PX-135, with ASA numbers of 400 and 125 respectively.

The lighting arrangement for the photographic set-up consisted of a four bulb florescent light as a back light for the side view of the lake model. Three 300 watt photoflood lights were positioned on the top and the bottom of the model to provide adequate illumination.

Both lake models had a grid pattern attached to the side surfaces with 1/16 inch black circuit tape to yield a grid pattern of one inch squares to aid in the determination of the location of the dye patterns. A similar grid system was utilized for the bottom of the lake models. Placed on the top of the side mirror system was a data information board and a digital timing clock with a readout in seconds.

A typical photograph recorded both a top view and a side view of the lake flow pattern at one time. By taking a series of photographs at approximately one to two second intervals a complete record of the flow pattern was recorded for each test.

Data Reduction Technique

Each test sequence was approximately 20 photographs in length. The film was developed in the laboratory darkroom, and checked to determine if all test results were recorded properly and if the test conditions were met.

If the developed film was suitable for data purposes, enlargements were made of all the frames of each test film roll. The enlargements were usually four by five inch prints, but on occasion when the flow pattern was unusual or unique, the prints were made at a larger size, usually eight by ten inch prints.

The prints of each run sequence were inspected to verify that all test data was shown and in proper focus. Then each print was analyzed and the dye front trace was determined from the dye front pattern against the reference grid system and the location of points was recorded in terms of vertical and horizontal positions. This method was utilized for both the top and the side view.

After the dye front profiles for each photograph (at successive periods of time in the test) were recorded, instantaneous velocity profiles were calculated from this data. This was accomplished through the use of a velocity computation program that is outlined in Appendix B. The program is rather simple and is used with the Hewlett-Packard series 9820 computer.

The velocity at a given location is determined by the standard time of flight technique. The test photographs record the dye front traces at a number of successive positions in the flowfield. The velocity at the point of interest is then computed from the data obtained from the two dye fronts which are in sequence ahead and behind the selected location.
A collection of the velocities at specified points was obtained through the use of this computing method. The data was then plotted as non-dimensional velocity versus non-dimensional distance for specified locations down the length of the lake model. Figure 5 is a typical example of a velocity profile plot obtained in the deeper lake model.

Since the flow is turbulent an instantaneous profile can take any shape, and useful interpretation is next to impossible. Hence, the standard experimental procedure is to determine an average velocity profile. However, in establishing velocity profiles from time-of-flight of dye fronts, an average profile can only be obtained by ensemble averaging many individual instantaneous profiles. The experimental effort involved in such a technique is prohibitive so that velocity profiles at only one X station are presented here.

In a sense, the shapes of dye fronts themselves represent integrated velocity profiles, and hence average velocity profiles averaged not over time at one station, but for a particular control volume moving with the fluid.

These velocity profile plots were obtained for both models for the same test configuration and combined to yield a comparative plot of both the individual velocity profiles at the same specified location. This was one method employed to determine whether the two lake models were similar to each other for the test configuration chosen.

Experimental Procedure

The experimental procedure consists of the methods used to determine how the various control parameters are determined and how they are introduced into the lake models. Also the testing sequence is outlined with discussion of various points that are considered significant.

The testing program was developed from the basic theory presented in Chapter II. Considering the shallow lake model, a density stratification configuration was determined with regard to the number of layers of fluids needed along with the density value of each layer. In the small lake it was essential to develop a stratification scheme that utilized the largest spread in density difference available. Since sodium chloride, i.e., salt, was used as the density altering agent, the maximum density range was from 62.4 pounds per cubic foot to 63.6 pounds per cubic foot. The large range was necessary in light of trying to develop the same overall Richardson number and the same Froude-Reynolds number for both models.

Recalling from the development of the modeling theory that the Richardson number,

$$\overline{Ri} = \frac{g \Delta\rho H}{\rho U_c^2}$$

and the Reynolds number in this study is taken to be,

$$R = \frac{U_c H}{\nu}$$

Therefore, for the models to have the same Richardson and Reynolds numbers, the inlet velocity Uc and the density difference $\Delta\rho$ are adjusted to account for the change in H from the deep model to the shallow model. It is obvious that for the small model to have the same similitude parameters as the large model, the density range of the small model must be as large as possible to allow for the small density ranges needed for the large model. Due to the limitation in accurately measuring the density of the fluids by means of a hydrometer, a minimum density range that is practical is from 62.4 pounds per cubic foot to 62.53 pounds per cubic foot.

Due to these limits in the density ranges only two basic configurations of lake model stratification could be imposed on either model. The other flow parameter that was controlled was the inflow velocity. This was controlled by means of the flowmeter mounted on the control panel. By setting a desired inlet width and lake depth, the flow rate could be adjusted to yield the desired inflow velocity.

The testing sequence consisted of determining the desired lake densities and loading the separate storage tanks accordingly. This was done by filling the storage tank with fresh water and adding the proper weight of salt to the water. The solution was then mixed and a hydrometer reading was taken to determine the specific gravity of the solution. When the desired density was reached the solution was allowed to set for one hour and measurements were taken at 15 minute intervals to insure that the correct density was obtained. All required density solutions were made up in this manner.

The lake model was filled with fresh tap water initially and allowed to set for a period of three to eight hours. This was to let all initial turbulence in the model die out. A temperature measurement and a specific gravity measurement was made of the lake model. If the density of the lake model was different than the desired density, the solutions made up in the storage tanks were changed to yield the proper density difference required for the test. When the lake had settled and the densities were correct, the heavier density fluid was then introduced into the lake model through the inlet hydraulic system. This was done at the flow rate desired for the test to check the inlet depth and lake level for the correct setup of the test conditions. If any discrepancies were noted they were corrected at this time. After the desired depth of the heaviest inflow solution was reached the flow was shut off and the lake was allowed to settle for approximately one hour. If another density layer was required, it was then introduced into the model at this time. The introduction of the heavier solutions into the fresh water at relatively low flow rates resulted in very little mixing between the density layers and yielded a stratified lake model in a relatively short period of time.

The test was initiated by turning the lighting system on. The proper flowmeter setting was then made to start the flow of the desired solution into the inlet section of the lake model. The inflow was allowed to run for a period of 20 seconds, this was determined by previous test that determined when the inflow became developed and the initial transient condition passed.

When the flow was considered developed the dye injection system was turned on, introducing a dye, usually red or green food coloring, into the inflow fluid. When the dyed fluid progressed to the inlet exit, the timing clock was initiated. Photographs of the flow pattern were then taken as soon as the dye-traced fluid entered the test section. The test section was at a position of 28 inches to 46 inches down the length of the lake model from the inlet. The photographs were taken at one to two second intervals.

This was continued until the flow had passed out of the test section.

When the test was in progress measurements of the inflow rate, the inlet depth and the depth of the lake were continuously monitored. Also readings were taken of the outflow rate at the weir exit.

At the conclusion of the test, the dye injection system and the inflow were discontinued. The lake model was then drained and flushed with fresh water to remove any saline solution that was left. The storage tank system was also drained and flushed. The equipment was then ready for the next test.

When the other lake model was to be used in the next test, the model in place was removed and the other model moved into it's place, then leveled and connected to the hydraulic system. The lake model was then filled with fresh water and the flow parameters for the next test were then set. This was done by the adjustment of the outlet weir to obtain the desired lake model depth. Also the inlet channel was aligned and tested to check for misalignment which would result in improper inflow. Then the testing procedure was again repeated.

CHAPTER IV

EXPERIMENTAL DATA

This chapter presents the experimental data obtained from various density inflows into the two stratified lake models. Briefly, the experiments can be classified in the following way. Velocity profiles were obtained by plotting the instantaneous velocities at specified distances down the lake models for a number of experimental tests and taking simple averages.

Experiments were performed on the two lake models at the same Reynolds number and the overall Richardson number selected to try in Chapter II. Since the only parameter that was different was the extent of vertical scale exaggeration this provided a critical test concept of vertical scale exaggeration. Table I is a summary of the test conditions and the data acquired during this study. It should be noted that each test was conducted repeatedly to yield a number of instantaneous velocity points at each selected location. The number of individual runs in each test is indicated in the table.

Mean Velocity Profiles

Velocity profiles were determined from composite plots of the velocity points obtained from the dye front photographs. Figures 6 through 12 indicate the mean velocity profiles determined for the selected locations down the length of the lake models. The velocity plots were nondimensional so as to yield a comparative profile between the models. As shown in Table I the test series B refers to the same test repeated on the shallow,

TABLE I

TESTING PROGRAM DATA

Test Number	Lake Model	Total Depth (H_t) (in)	Inlet Depth (H_i) (in)	Density Difference $\frac{\Delta\rho}{\rho}$	Flow Rate (Q) (GPM)	Inlet Velocity V_i (ft/sec)	Overall Richardson Number $\overline{Ri}$	Reynolds Number Re
A	Deep	6	$\frac{1}{2}$	0.00118	0.578	0.3714	2.695	18,569
B	Shallow	3	$\frac{1}{4}$	0.00943	0.58	0.744	2.695	18,600

6 inch depth model three times. This test was used as the base data in this study. Test A was the experiment on the deep lake model, which was tested at the same Richardson and Reynolds number as the shallow lake model.

The test section interval starts at a position 34 inches from the inlet and continues down the lake model for 24 inches. The locations selected for determination of velocity profiles start at the 34 inch location and continue in two inch increments to 42 inches.

These velocity profiles can then be used to indicate the degree of similitude obtained between the lake models for the various test conditions, as described in Chapter III. A discussion of these profiles and their significance is presented in Chapter V of this report.

Measurement of the velocity at the specified locations was difficult due to the design of the data acquisition system which severely reduced the accuracy of the measurements. The end result of this system was that the flow situation was recorded on film which when enlarged to the largest size (this was determined by the resolution), locations of the dye front could only be made within one tenth of an inch. Since the dye fronts were never more than two inches in height only twenty-four positions could be taken, which results in basically a rough outline of the velocity pattern. Since the inflow fluid was in the turbulent region the data points collected for each test showed a wide range of velocities. The only method to determine a mean velocity profile from this type of data was to repeat the same test conditions a number of times to yield a composite plot of velocity data, and from this a mean velocity profile could be obtained. It should be noted that this method is very rough and the amount of time needed to complete a velocity profile is excessive.

Dye Front Profiles

From the photographic data obtained during the test program, a record of the dye front profiles was obtained for each test. It is interesting to develop a comparative plot of these dye fronts at selected locations and times, to yield insight into the mechanism of the inflow fluid. Figures 13 through 17 are comparative plots of the nondimensionalized dye front profiles between tests A and B. It was observed that it was necessary to dye the inflowing fluid in order to be able to distinguish between the different density fluids. To the eye and to the camera, the dye colored water is dominant over the undyed water existing in the lake model.

CHAPTER V

DISCUSSION

Considerable effort has been devoted to determining if the flow in the two lake models is similar when the vertical scale of one model is exaggerated. By using the nondimensional parameters of Reynolds number and overall Richardson number (based on the depth of the lake models) as the basis of the modeling criteria, experimental tests were made to determine the validity of this concept. In this chapter some of the experimental evidence is discussed including the various aspects of the inflow patterns as related to inflow rates, slope distortion, mixing rates of the fluid, and density variation.

Discussion of Visual Observations

The primary information in this investigation is the time sequenced photographs of dye-traced inflow fluids. As seen in Figures 8 through 12, which are comparative plots of the dye front profiles for test A and B for selected locations, there is a difference in the flowfields of the two tests. If vertical scale exaggeration had no effect on the flowfield (suitably nondimensionalized) there would be no difference in the data from tests A and B since they are run at the same Richardson and Reynolds numbers. The gross features of the flowfields are similar, for example in both cases the inflow tends to "lens" out over the denser layer. However there are some noticeable differences indicating the similarity is not exact. In the test B, the shallow model with the larger density difference, interfacial waves are observed at the lower interface. But in the upper interface there is the predominance of the force of gravity which increases the slope of the inflow fluid and this tends to dampen the interfacial wave growth.

Browand and Winant [10] have described this mechanism in their paper on laboratory observations of instabilities of shear layers in stratified fluids. As observed in the lake models, there are initially instabilities introduced into the inflow fluid when it reaches the density interface in the lake model. The inflow fluid acting under a gravitational force increases its inertial energy as it flows down the contour slope. When the fluid reaches the density interface the buoyancy forces tend to counteract the inertial forces, which introduces instabilities into the inflow fluid. This area in the lake model is referred to as the mixing zone. In this stratified shear layer the instability is redistributed along the interface. This redistribution generates interfacial waves, which can be noted in the dye front profiles. These interfacial waves and the residual turbulence decay and the shear flows approach a laminar state. This can also be seen in the dye front profiles at distances farther down the lake model. This instability is related to the Richardson number of the inflow, with a higher Richardson number being more stable than a smaller Richardson number.

Also observed in this model study was the development of "rollers" in the inflow fluid. These "rollers" were the rolling up of the inflow fluid about a horizontal axis oriented towards the side of the lake model.

This observation was not evident from the photographs of the lake models but could be seen visually. These "rollers" were quite large in both tests and did not appear to similar. Various attempts to reproduce these "rollers" were not successful. This leads to the reasoning that these are not reproducible and therefore the flow will not be similar in this respect.

Due to the method of coloring the inflow fluid only tip velocities can be obtained for anyone location and internal velocities are prevented from being obtained from this coloring scheme. As regards mixing actions, two types were noted. At the tip, especially the underflow tip, there was a rolling up process similar to that shown by Prandtl's 1952 illustrations [25]. This was observed when the interface was otherwise completely smooth, which suggest laminar conditions of flow at the interface at least. In some cases the "roller" was seen quite distinctly as in Prandtl's idealized figure. As the depth or the density difference was decreased, more turbulent conditions prevailed. The rolling layers no longer appeared to have distinct existence, but the general pattern of movement seemed to be the same.

The other type of mixing action which was clearly observed, occurred behind the tip of the inflow profile, when the depth and density difference were such to give values of the Richardson number of the order of 2.0 or less. Interfacial waves, on the point of breaking, were observed in the region behind the underflow front in the inflows of the smaller lake model. When the Richardson Number was slightly increased, breaking of the waves could be seen and on further increase the individual waves could no longer be distinguished. The general impression gained by the author after watching many experiments, was that the turbulent mixing between the density layers grew more intense with decreasing values of Richardson number.

This type of interfacial wave formation was also noted by Ellison and Turner [12] as occurring behind the nose of an underflow layer progressing down a slight slope. It seemed reasonable to assume that similar waves could be obtained behind an inflow front, though such were not actually observed.

Another interesting observation was that due to the exaggeration of the vertical scale the contour in the deeper model is distorted. This distortion appears to be a critical factor in the design of the exaggerated scale model. This is because when the slope of the contour is greater than seven degrees the inflow tends to become unattached from the contour surface. This results in a flow pattern in the deeper model which does not model the shallow model's flow pattern.

Similarity of Flowfields

Utilization of the Richardson and the Reynolds numbers as the basis for the development of the model parameters between the lake models is one of the main aspects of this report. From the experimental data it can be noted that there is some similarity between the mean velocity profiles and the dye front traces for the tests in the deep and shallow models which were run at the same Richardson and Reynolds number.

It is believed that due to the inaccuracy of the data collection system, that there can be no definite conclusions drawn, outside of the fact that the flow patterns tend to be similar but not exactly so. In the figures noted for comparison, the velocity profiles can be seen to differ between tests in both a vertical displacement as well as a horizontal displacement. This is probably due to the slight variations in the test conditions, but for a generalized view this is not considered critical. The basic information to be incurred from the comparative plots of both the velocity profiles and the dye front traces is that when vertical scale exaggeration is used, the flow patterns are close but not exactly similar.

The intersurface phenomena occurring is thought to be not as dependent on total depth of the lake as originally stated in the development of the Richardson number. Perhaps it would be best to base the Richardson number on a characteristic length of a horizontal dimension. No definite statement can be made on this point with the results obtained in this study. Therefore it is suggested that further investigation of this area be considered and it is possible that comparison of the two types of investigations would lead to additional insight into this phenomena. This work might be directed in such a way as to vary the Richardson number over a wide range and utilize both a horizontal and vertical characteristic length as discussed in Chapter II.

Due to the limited amount of data and also the limited variations attempted, no positive statements as to the validity of the vertical scale exaggeration can be made. But it is worthwhile to describe the observation noted during the testing and this leads to a better understanding of the problems involved in the use of an exaggerated scale model.

As the data tentatively indicated, there is a degree of similarity achieved between the two lake models when the test conditions are set in accordance with the Richardson and Reynolds number criteria. Since both geometric and dynamical similarity were imposed on the models, then this indication of similarity between velocity profiles also tends to indicate a degree of similarity between the kinematic aspects of the flow in both models.

CHAPTER VI

SUMMARY AND CONCLUSIONS

Summary

The objectives of this study were to experimentally determine the validity of the concept of vertical scale exaggeration with stratified lake models. This was developed by establishing a modeling technique and visualizing the flow of an incoming fluid of an intermediate density to the two densities of the lake model stratification layers.

Numerous tests were carried out with both photographic recording of the flow and visual observation of the tests. The photographic data showed in some detail the interfacial waves, the rolling motion of the inflow front, and the mixing and dispersion patterns of the various tests.

Comparisons of the mean velocity profiles for the various tests were made to determine whether the lake models indicated kinematic similarity. Also dye front profiles were recorded to outline the major flow characteristics of the inflows. Visual observations of the many experiments were made to yield an insight into the flow mechanism that could not be properly recorded by the photographic system utilized in this report.

The problem inherent to vertical scale exaggeration, slope distortion, is discussed and the effects of this distortion are critical to an exact modeling of the lake models.

Conclusions

The conclusions of this study may be stated as follows:

1. Modeling a stratified lake according to the Richardson and Reynolds number concept, a degree of similarity can be obtained for a model utilizing vertical scale exaggeration.
2. To properly model a stratified lake when vertical scale exaggeration is used requires cautious design in regard to the distortion inherent in the exaggeration. In this study the distortion factor was introduced in the contours and leads to a non-similarity of flow patterns. The critical factor may be that when the contour slope exceeds about seven degrees the flow becomes detached.
3. There appears to be a rolling motion in the inflow which is quite large in structure and is non-reproducible between the models. This is considered to verify that there exist a dissimilarity between the lake models tests.
4. Measurements in one model at constant Richardson number and various Reynolds numbers showed good agreement. This indicates a week dependence on Reynolds numbers as is common in turbulent flow situations.
5. Measurements in one model at constant Reynolds number and varying Richardson number indicated a very strong dependence on the latter

parameter.

6. Measurements in the two models of different scale distortions did not agree exactly indicating that the overall Richardson number based on the depth of the lake is not the similarity parameter that should be used in modeling.

Future work should include experiments on the two models using alternate choices of characteristic lengths for the density and velocity gradients in the overall Richardson number as discussed in Chapter II.

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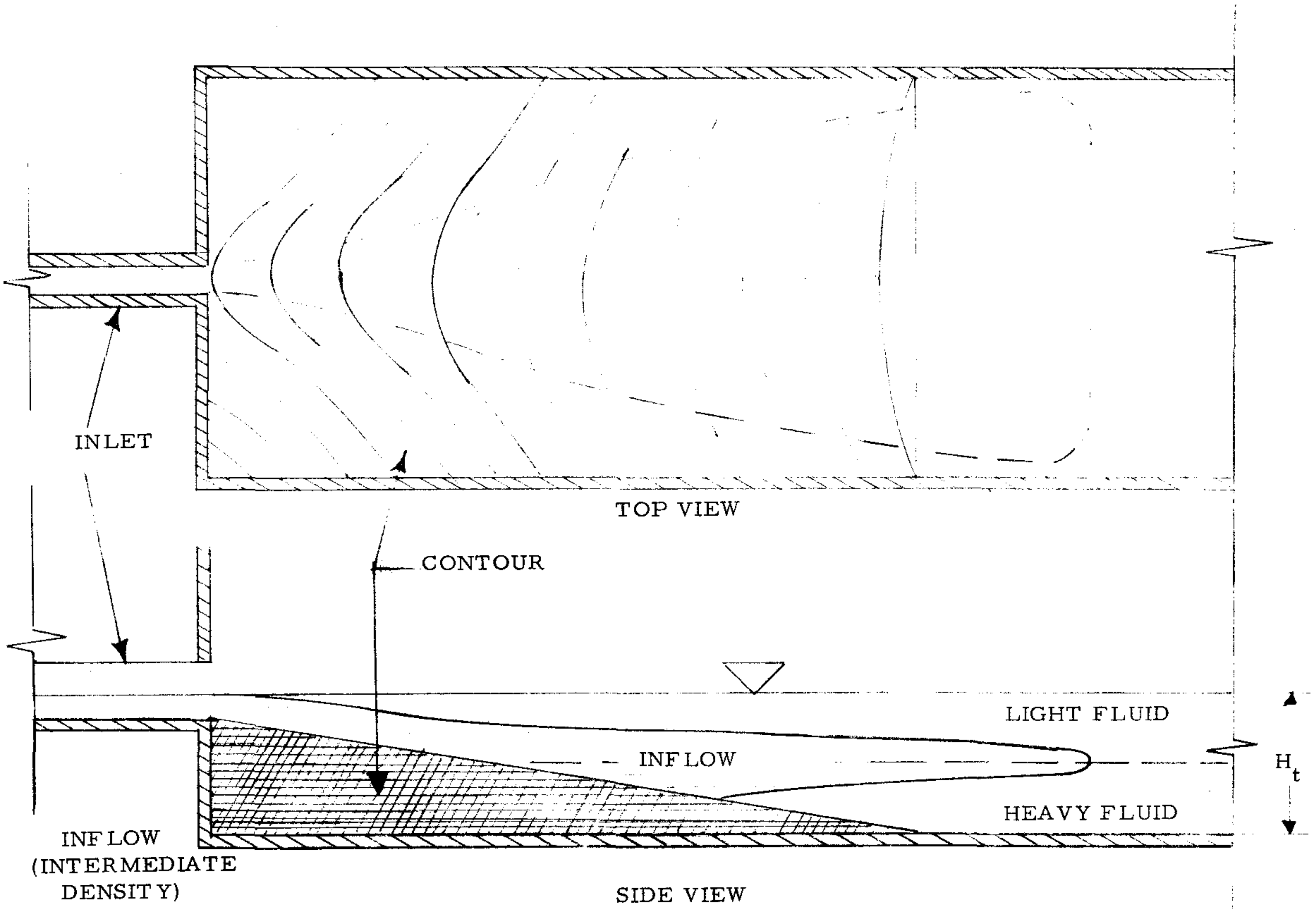

Figure 1. Sketch of physical system being modelled.

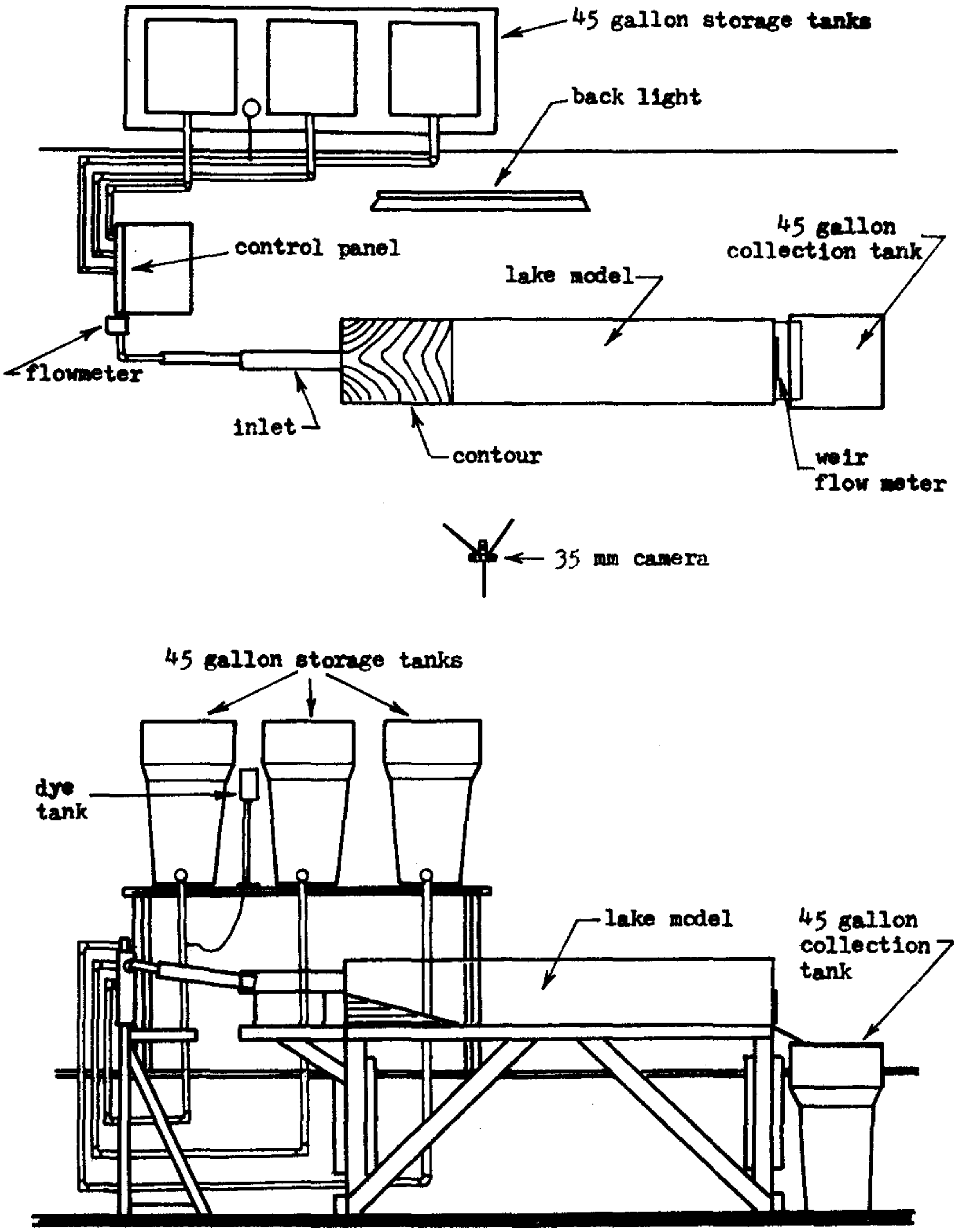

Figure 2. Lake Model and Inflow System

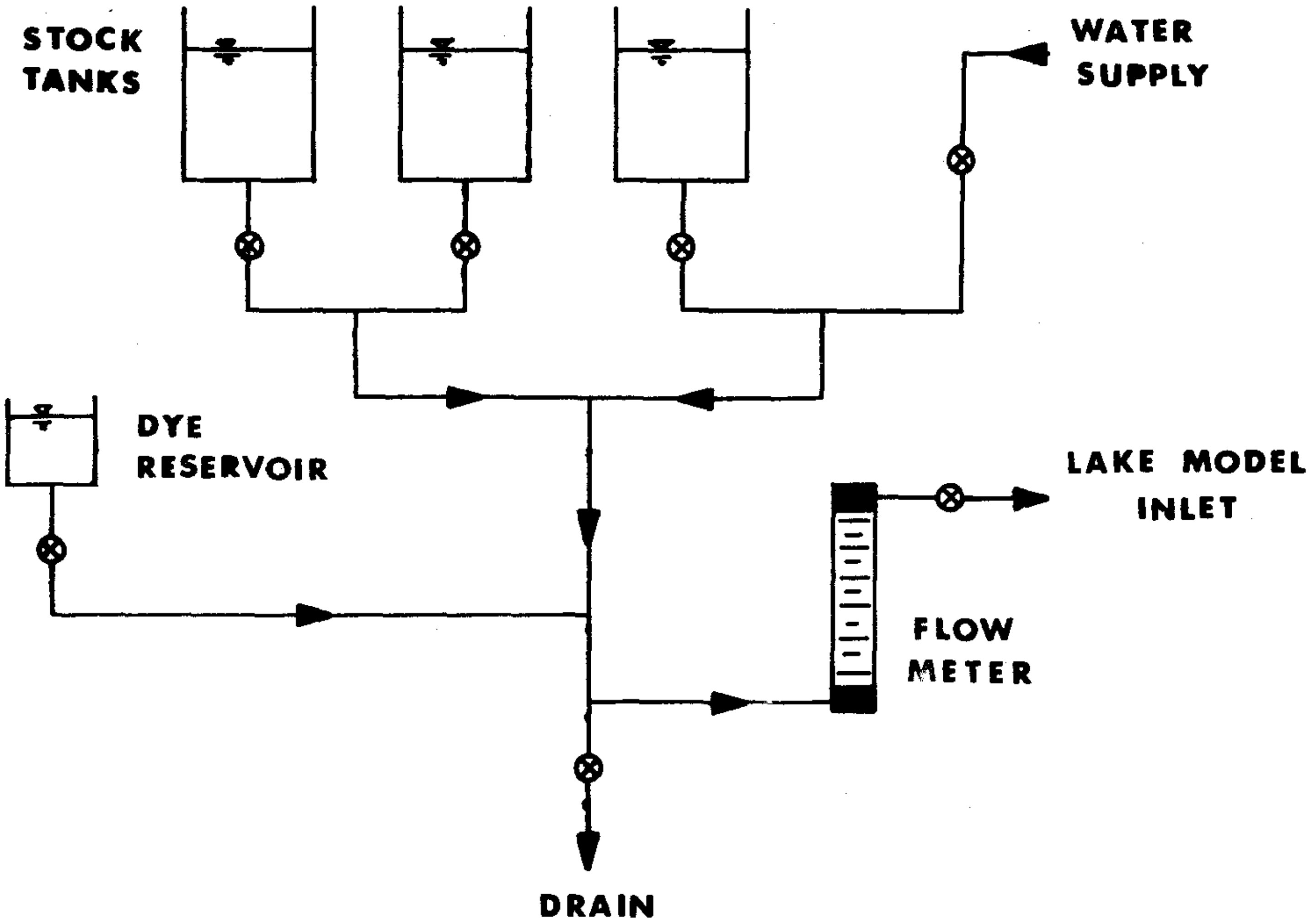

Figure 3. Schematic of Hydraulic System

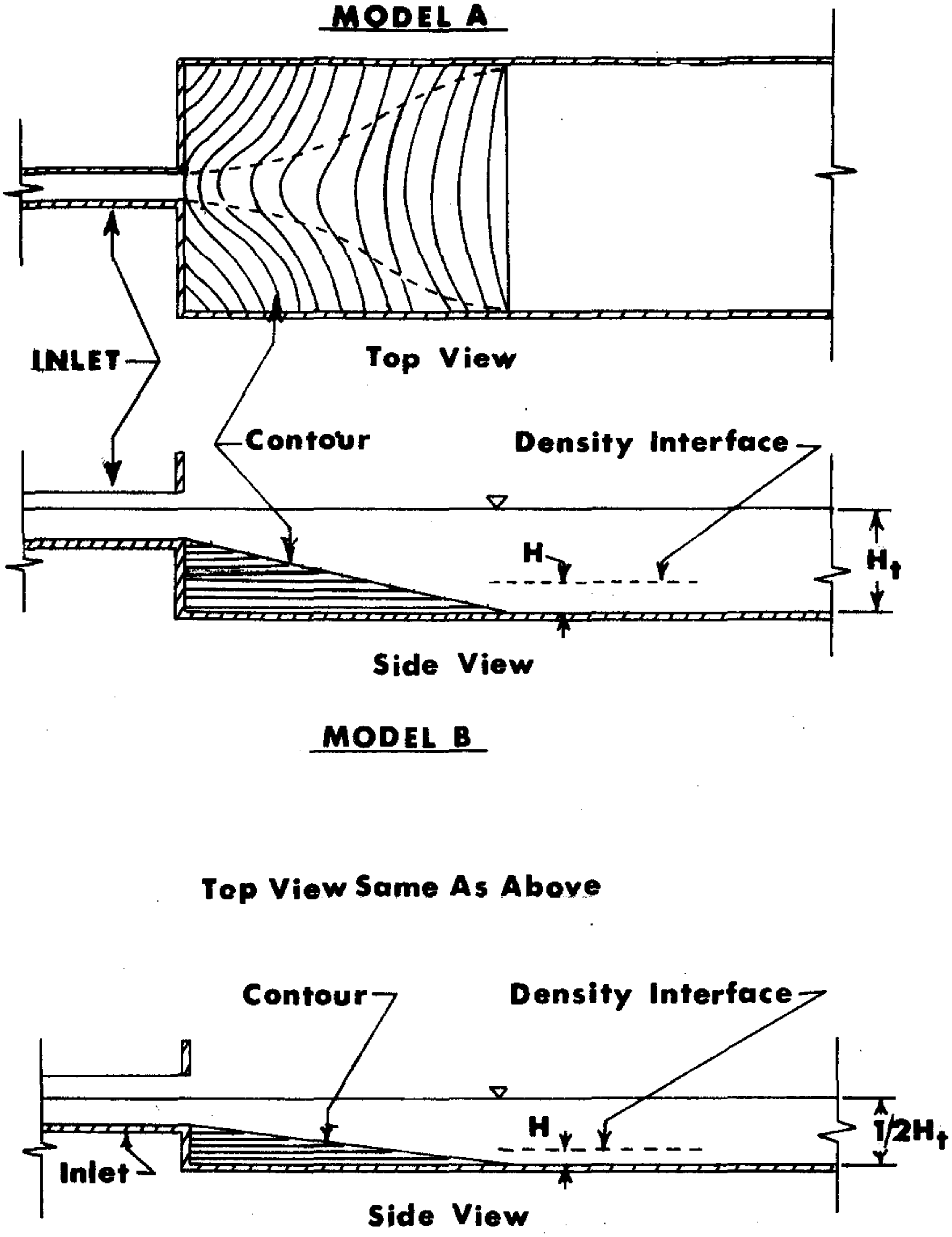

Figure 4. Cross Section Showing the Lake Model's Inlet, Contour, and Test Section.

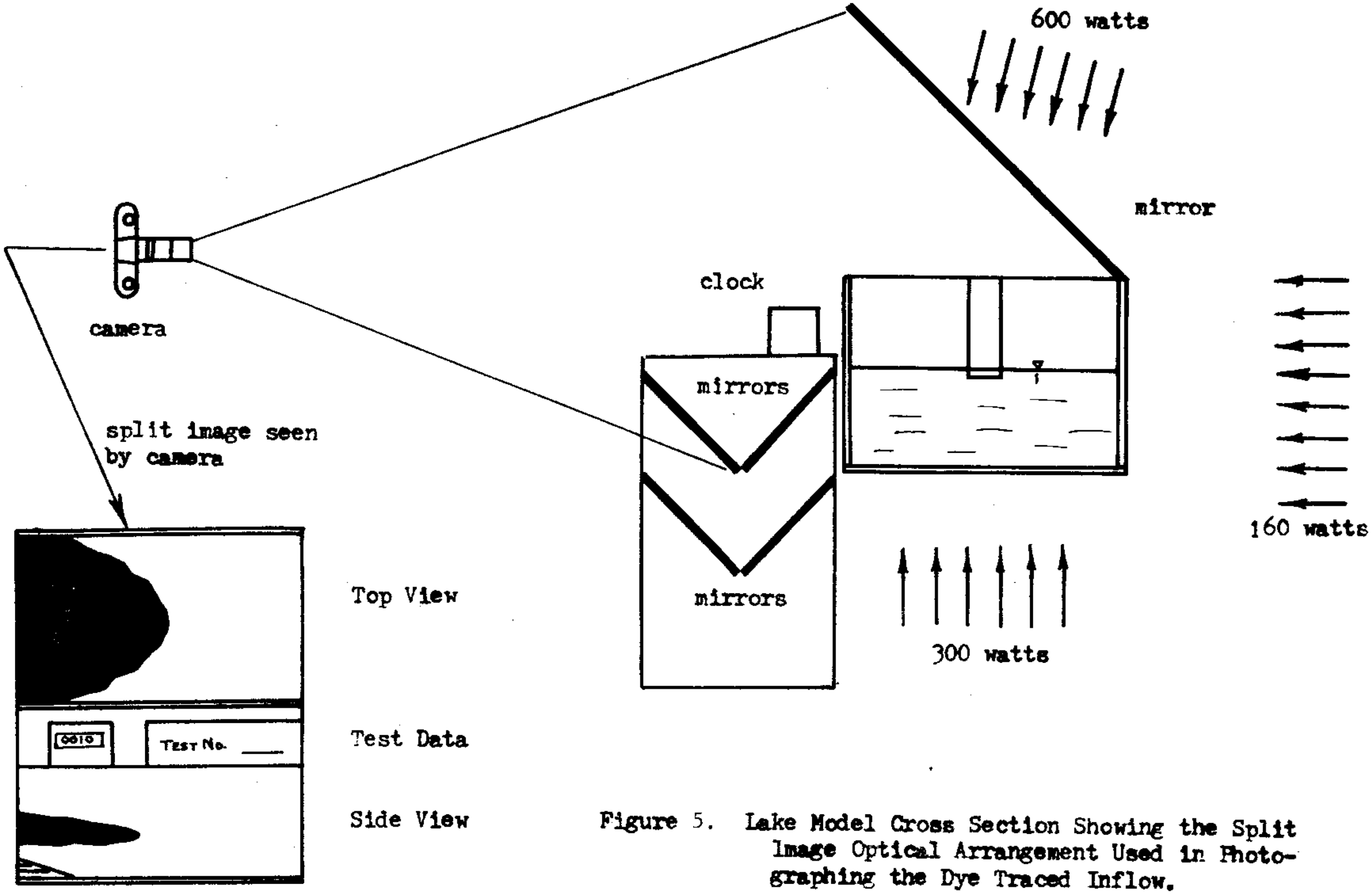

Figure 5. Lake Model Cross Section Showing the Split Image Optical Arrangement Used in Photographing the Dye Traced Inflow.

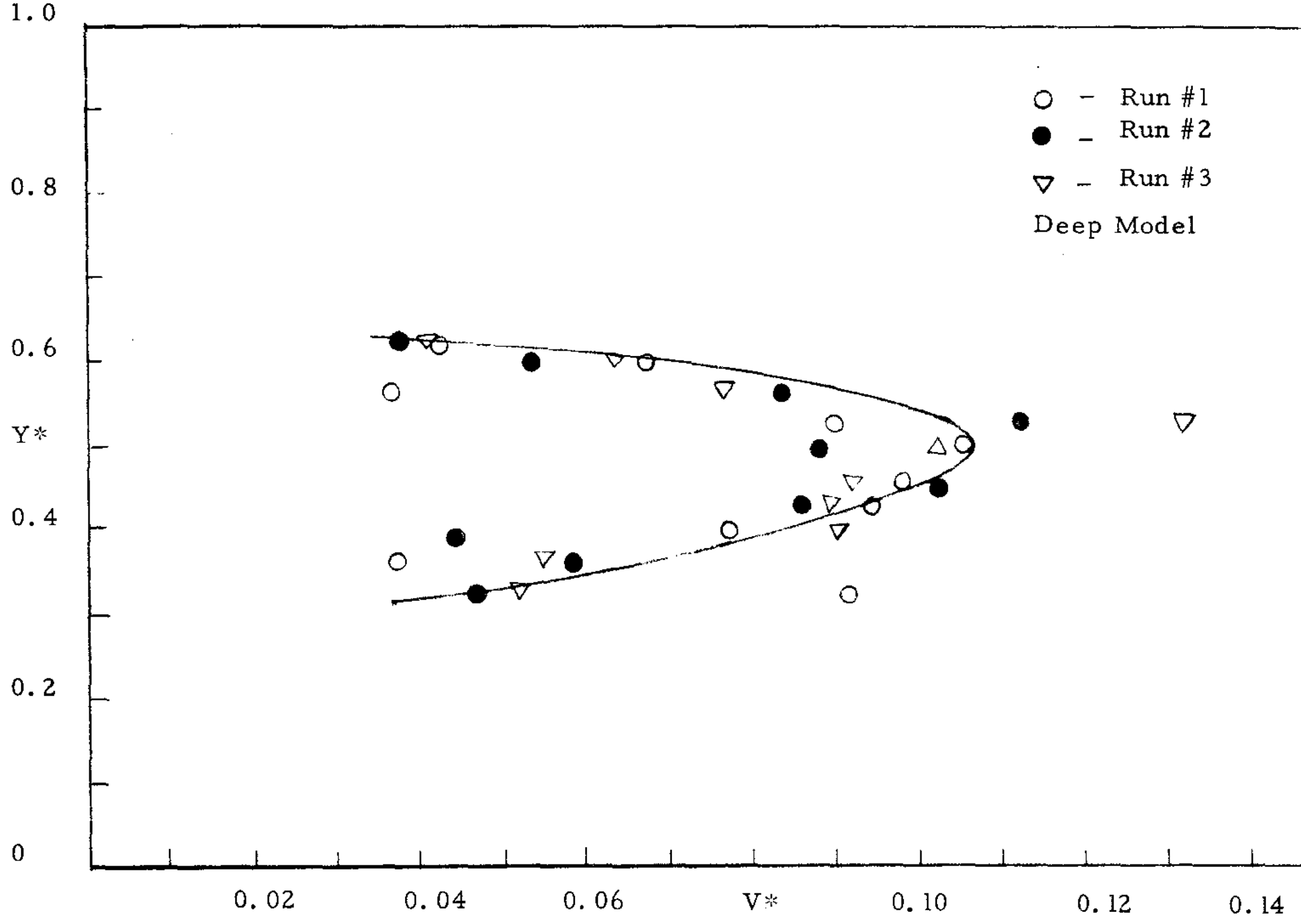

Figure 6. Typical Velocity Profile Developed From Successive Runs of the Same Test

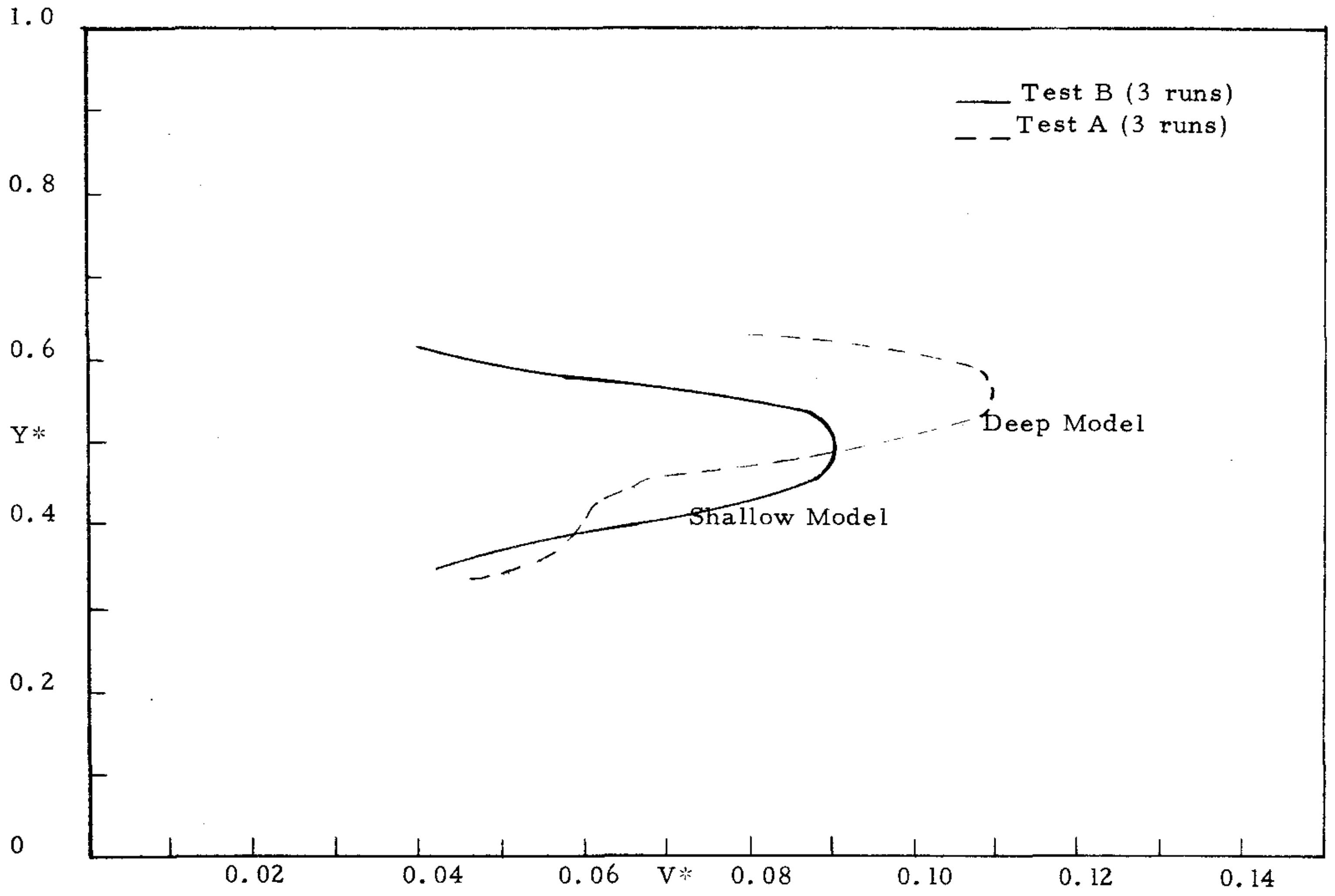

Figure 7. Mean Velocity Profiles for the Two Tests at X = 34 inches.

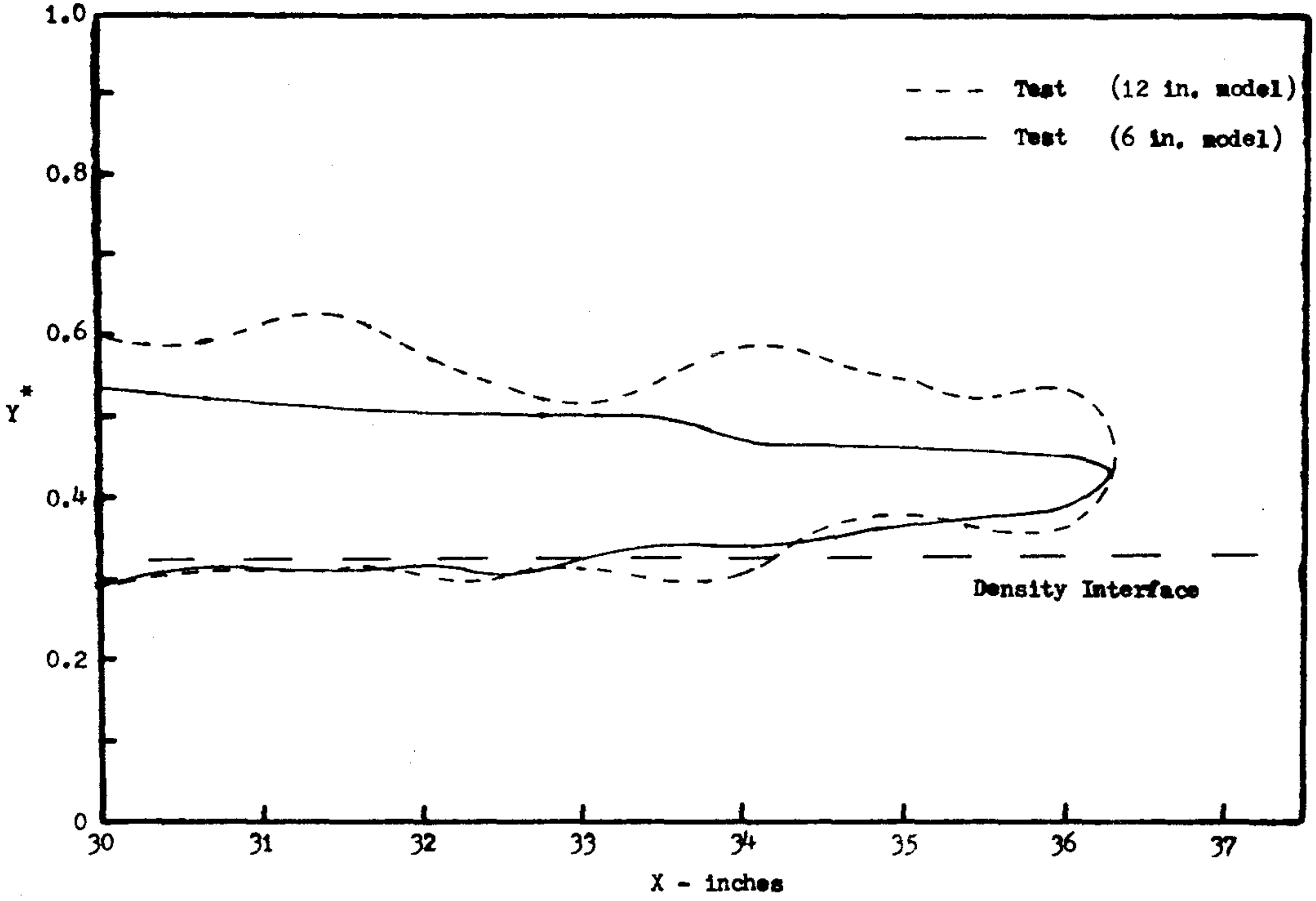

Figure 8. Dye Front Profiles, a Comparison Between the Same Test in Both Models.

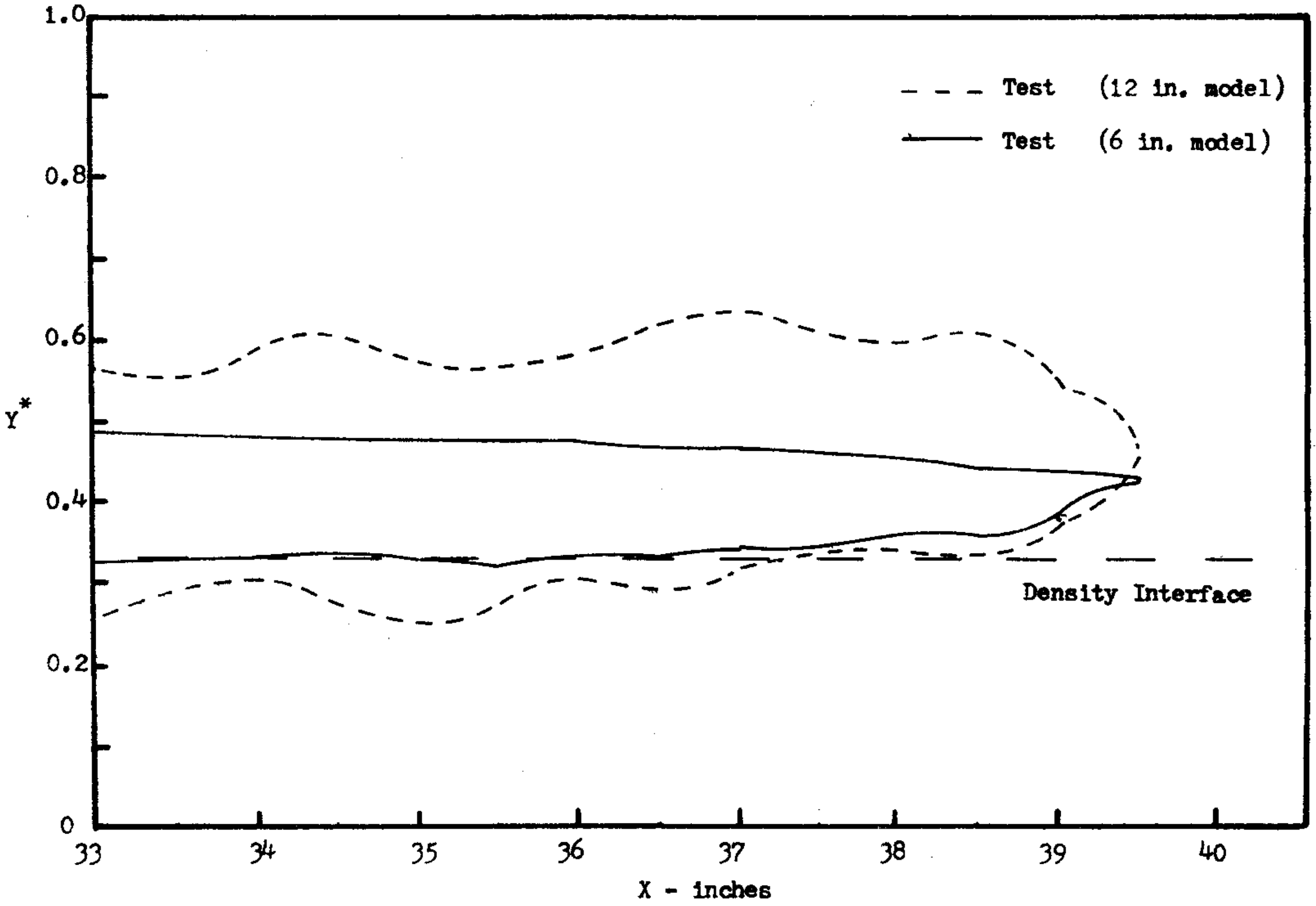

Figure 9. Dye Front Profiles, a Comparison Between the Same Test in Both Models.

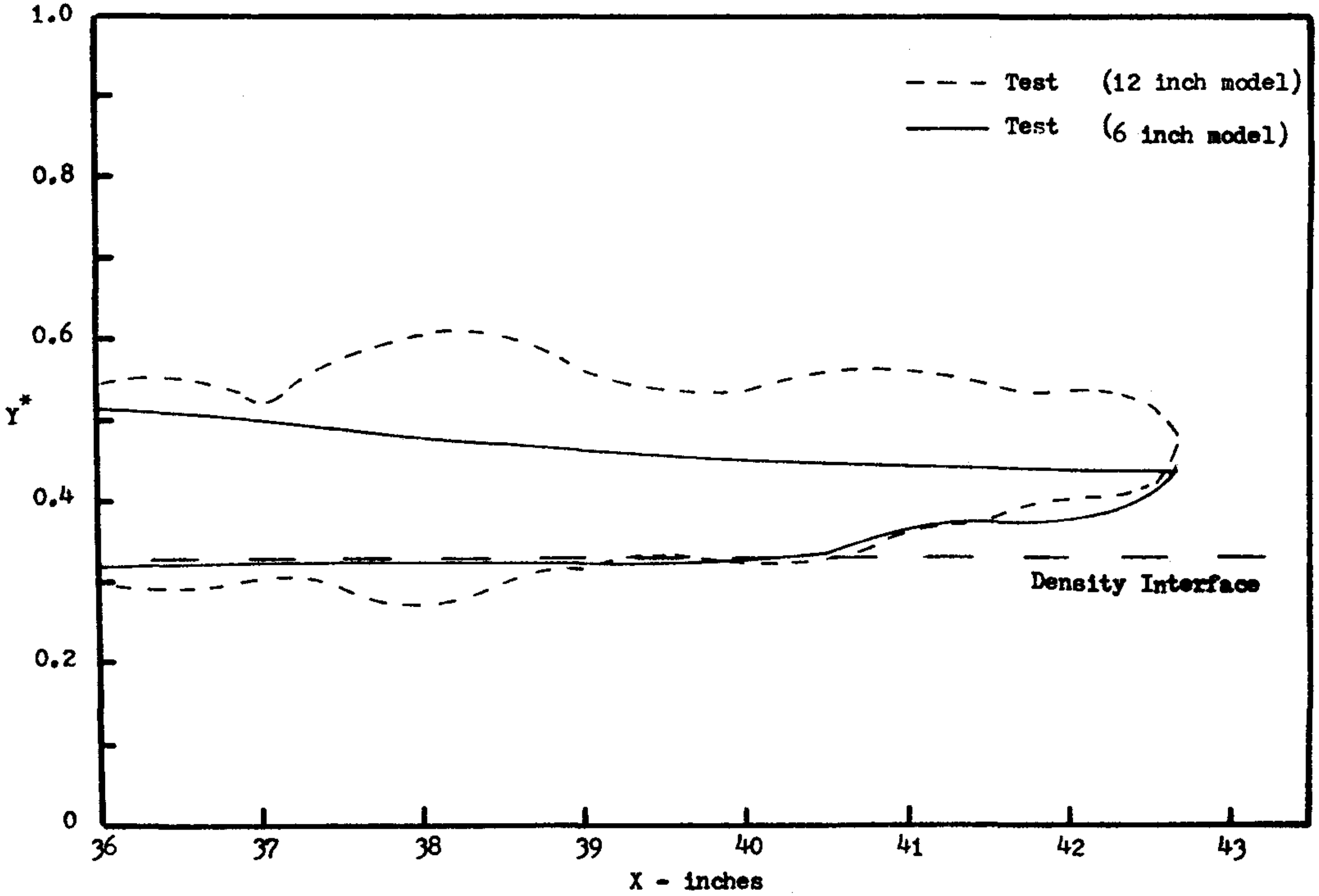

Figure 10. Dye Front Profiles, a Comparison Between the Same Test in Both Models.

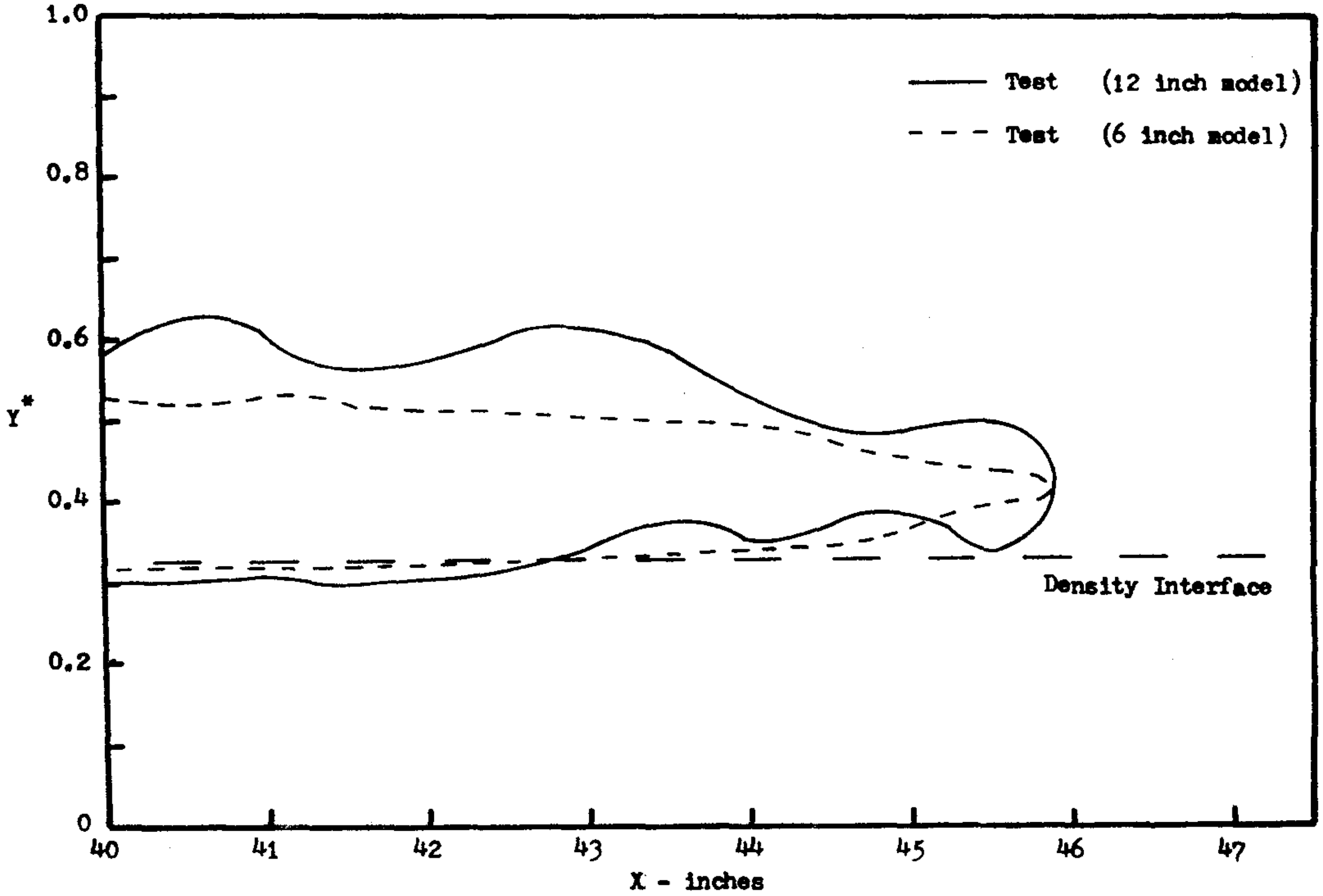

Figure 11. Dye Front Profiles, a Comparison Between the Same Test in Both Models.

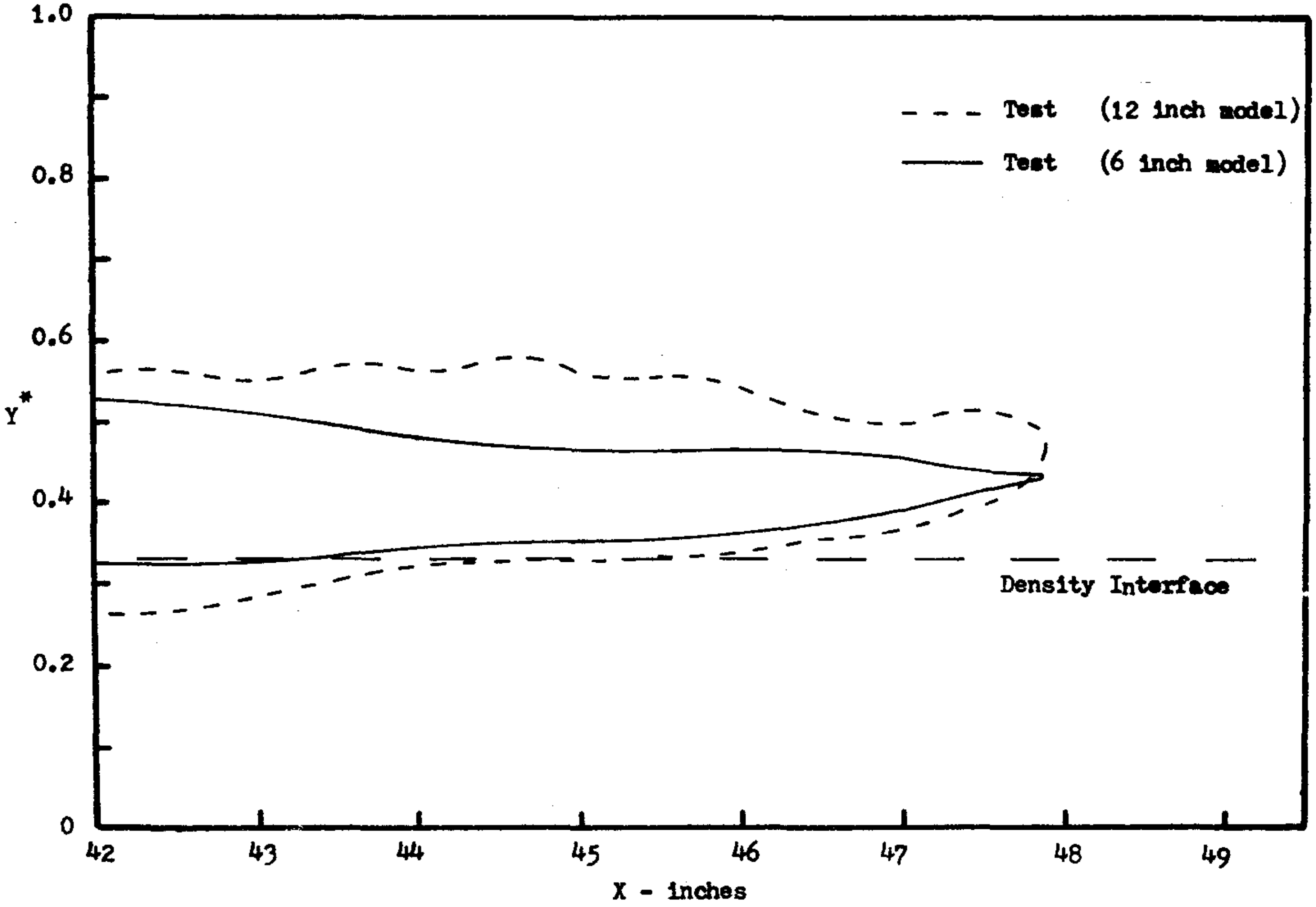

Figure 12. Dye Front Profiles, a Comparison Between the Same Test in Both Models.

APPENDIX B

COMPUTER PROGRAM LISTING

APPENDIX B

COMPUTER PROGRAM LISTING

This program is utilized with the Hewlett-Packard 2908B desk computer.

```
00   PRT "DELTA TIME"; PRT "CALCULATIONS"; 80-A; SPC 2 ⌿
01   ENT "TIME(MIN.)R 80=", R30; ENT R81, R82, R83, R84, R85, R86,
     R87, R88, R89 ⌿
02   PRT "DELTA TIME ="; SPC 1 ⌿
03   A + 1 → A ⌿
04   1 * (RA - R(A - 1)) - R(A - 10) ⌿
05   FXD 0; PRT A - 79 ⌿
06   FLT; PRT R(A - 10); SPC 1 ⌿
07   IF A = 89; GTO 9 ⌿
08   GTO 3 ⌿
09   SPC 5; ENT "Y POSITION =", R50; PRT "FOR POSITION Y =", R50;
     SPC 5⌿
10   16 → A; PRT "DELTA X"; PRT "CALCULATIONS"; SPC 2 ⌿
11   ENT "X POSITION(IN.)R16=", R16; ENT R17, R18, R19, R20, R21,
     R22, R23, R24, R25 ⌿
12   PRT "DELTA X ="; SPC 1 ⌿
13   A + 1 → A ⌿
14   RA - R(A - 1) - R(A - 10) ⌿
15   FXD 0; PRT A - 15⌿
16   FLT ; PRT R(A - 10); SPC 1 ⌿
17   IF A = 25; GTO 19 ⌿
18   GTO 13 ⌿
19   SPC 5 ⌿
20   6 → A; PRT "VELOCITY"; PRT "CALCULATIONS"; SPC 2 ⌿
21   70 → B ⌿
22   PRT "VELOCITY ="; SPC 1 ⌿
23   A + 1 → A ⌿
24   B + 1 → B ⌿
25   RA/RB → R(B - 41) ⌿
26   FXD 0; PRT A - 5 ⌿
27   FLT ; PRT R(B - 41); SPC 1 ⌿
28   IF A = 15; GTO 30 ⌿
29   GTO 23 ⌿
30   SPC 5 ⌿
31   PRT "AVERAGE VELOCITY"; PRT "COORDINATES"; SPEC 2⌿
32   6 → A⌿
33   A + 1 - A ⌿
```

```
34   R(A + 9) + RA/2 → X ⊬
35   FXD 0; PRT A - 5⊬
36   FLT; PRT X; SPC 1⊬
37   IF A = 15; GTO 39⊬
38   GTO 33⊬
39   SPC 5; 29 → A; ENT R51⊬
40   PRT "DIMENSIONLESS"; PRT "VELOCITY AT"; PRT "POINTS ABOVE";
     SPC 2⊬
41   PRT "DIMENSIONLESS"; PRT "VELOCITY ="; SPC 2⊬
42   A + 1 → A⊬
43   RA/R51 - R(A + 10)⊬
44   FXD 0' PRT A → 28⊬
45   FLT; PRT R(A + 10); SPC 1⊬
46   IF A = 38; GTO 48⊬
47   GTO 42⊬
48   SPC 5; PRT "FOR NEW Y VALUE,"; PRT "PUSH RUN PROGRAM";
     PRT "THEN ENTER 1"⊬
49   SPC 3; PRT "FOR FINAL VEL"; PRT "CALCUL., PUSH"; PRT "RUN
     PROGRAM AND"; PRT "ENTER 2"⊬
50   DSP "READ TAPE"; STP ⊬
51   ENT A⊬
52   IF A = 1; GTO 9⊬
53   SPC 5⊬
54   PRT "FINAL VELOCITY"; PRT "CALCULATIONS"; SPC 2⊬
55   ENT R94, R95, R96, R97, R98⊬
56   ENT "Y POSITION =", Y⊬
57   (R95 - R94)*((R96 - R97)/(R98 - R97)) + R94 → C⊬
58   ENT "NON DIM PARAM =", Z⊬
59   Y/Z → R52 ⊬
60   PRT "Y*=", R52⊬
61   PRT "V*=", C⊬
62   SPC 1⊬
63   GTO 55⊬
64   STP ⊬
65   END⊬ R272
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