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Sensitivity of Groundwater Flow Models to Vertical Variability Of Aquifer Constants

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SENSITIVITY OF GROUNDWATER FLOW MODELS TO VERTICAL VARIABILITY OF AQUIFER CONSTANTS

By

Richard N. DeVries and Douglas C. Kent

ABSTRACT

The Ogallala aquifer in the Oklahoma Panhandle is in need of better management because of increased groundwater demand which has caused declines in static water levels at an alarming rate. A groundwater management computer model was developed for the Ogallala aquifer in the Texas Panhandle and treats the aquifer as a homogeneous system. In this study, the computer model has been modified in order to evaluate the effect of vertical layering on semi-static water level changes which occur during the dewatering of a single unconfined aquifer. The modified model was applied to a study area near Guymon, Oklahoma, using both the homogeneous and the multi-layered cases. The aquifer is characterized by a saturated thickness of 400 feet. The accumulated drawdown values of the homogeneous and the multi-layered cases demonstrate that there is about 88 feet difference between the two cases before the base of the aquifer is encountered. Approximately 26 percent more time is required to dewater the layered aquifer. Thus, vertical variations of lithology in an aquifer such as the Ogallala should be considered when prediction is made relative to groundwater management.

A physical sand model was constructed using four layers of Ogallala aquifer material. Pumping tests and drainage tests were conducted. Average values of aquifer characteristics were obtained for the model under different saturated configurations. The results clearly indicate that the aquifer characteristics change as dewatering takes place.

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Keywords - *GROUNDWATER - *MANAGEMENT MODELS - GROUNDWATER FLOW MODELS - OGALLALA - SPECIFIC YIELD - TRANSMISSIBILITY - COEFFICIENT OF PERMEABILITY - HYDROGEOLOGY

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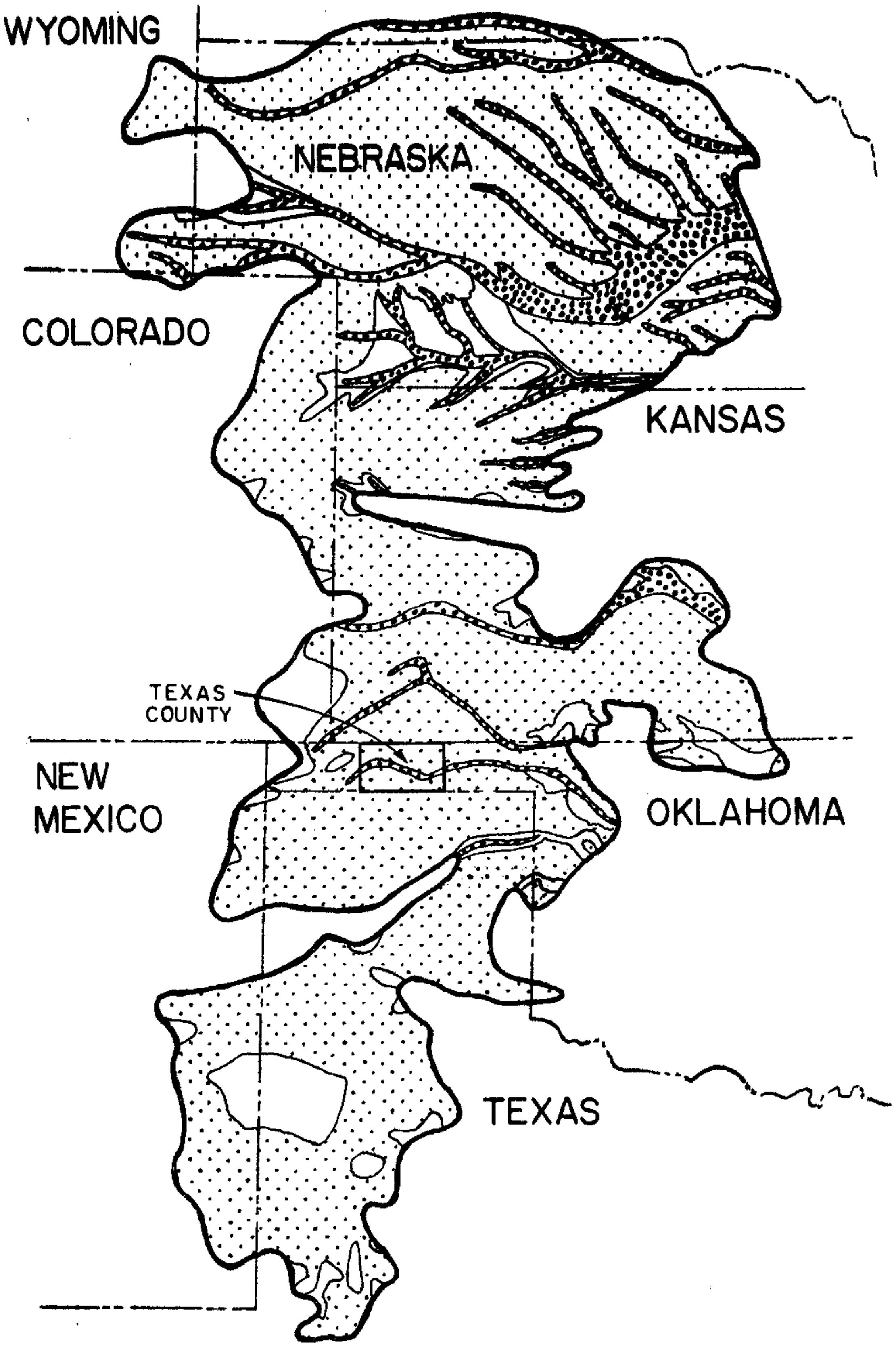
I. DESCRIPTION OF RESEARCH PERFORMED

INTRODUCTION

The Ogallala Formation has been used as an aquifer providing a groundwater resource to farm production and the agriculturally based industries in the High Plains Province of the United States. The formation extends from Nebraska to the Texas Panhandle. This study was restricted to the portion of the Ogallala Formation which occurs in the Oklahoma Panhandle. Although this aquifer occurs in Cimarron, Texas, and Beaver counties of the Oklahoma Panhandle, only Texas county is considered because of the availability and quality of data (see Figure 1).

Geologically, Pleistocene and Pliocene sediments crop out in the study area. The Ogallala Formation is of Pliocene age. However, because there is a lack of stratigraphic detail the name "Ogallala" was used in this study to include all Tertiary sediments. These sediments can occur either as unconsolidated or semiconsolidated sediments and are composed of discontinuous layers of sand, silt, clay, gravel, sandstone, caliche, limestone, conglomerate, and volcanic ash. Locally the units are tightly cemented by calcium carbonate while in other places, they are very poorly consolidated. These sediments are moderately permeable and provide a major source of ground water in the area. The saturated thickness ranges from 300 to 800 feet with an average thickness of 400 feet. Bedrock units of Mesozoic and Permian times subcrop under the Tertiary sediments. The bedrock within the study area is composed of vari-colored shale, sandstone, siltstone, and a limited occurrence of thin discontinuous gypsum beds. With the exception of Jurassic and Cretaceous sandstones in western Texas county, the bedrock is generally too fine grained and impermeable to transmit water. Thus, the bedrock surface forms an impermeable boundary at the base of the aquifer in the study area. The bedrock surface is characterized by moderate topographic relief with numerous local depressions which are considered to be bedrock valleys.

The Ogallala aquifer is being subjected to increased water withdrawals. These withdrawals far exceed the natural recharge, especially in the Southern High Plains area. The aquifer is being mined in this area and the resulting declines in static water level are becoming



THE HIGH PLAINS PROVINCE

FIGURE 1

critical. In order to predict these declines in the Texas Panhandle, a mathematical management model was developed by investigators of Texas Tech University's Civil Engineering and Mathematics departments and of the High Plains Underground Water Conservation District No. 1 at Lubbock, Texas (Sechrist, et al., 1970). McClain (1970) is using a similar approach to modeling the Ogallala Formation in Kansas. However, these investigators (Sechrist, et al., and McClain) are considering the Ogallala Formation as a homogeneous unit. Heterogeneous porous materials have also been considered by researchers such as Nelson and Cearlock (1967) as a homogeneous mass in which there is a statistical variation in the distribution of aquifer constants. They model the distribution of permeability irrespective of vertical variation in the aquifer and use fitting procedures to statistically determine lateral variations of permeability. A heterogeneous distribution of permeability has also been assumed by McMillan (1966) to be homogeneous with a specific range of variance.

Research by Frye (1970), Keys and Brown (1970), and Pearl (1970) has shown that the Ogallala Formation is neither homogeneous nor randomly heterogeneous but rather is discontinuously layered. The importance of considering layering as it would apply to groundwater flow models is evident in articles which have appeared since the beginning of the middle 1960's. The bulk of this research has been restricted to the analysis of multi-aquifers (several aquifers) or to aquitards between multi-aquifiers. Bredehoeft and Pinder (1968, 1970), Hantush (1967), and Neuman and Witherspoon (1969, 1969) have applied mathematical models in this manner to nonhomogeneous, anisotropic, and/or leaky artesian aquifers.

Freeze and Witherspoon (1966, 1967, 1968) evaluated the effects of layering within a single aquifer (with different values of permeability) on flow net configurations within the saturated zone using the finite difference technique and the digital computer. More recently Javandel and Witherspoon (1969) have extended the layered case to consider the temporal effects of layered aquifers on drawdown associated with pump tests and their analysis. Current research concerned with mathematical modeling of a single multi-layered aquifer is being conducted by Pinder, Bredehoeft, and Bennett. They are concerned with the determination of factors and relationships that govern permeability distribution (including layering) which in turn will be useful for predicting permeability distribution by indirect means. In addition, they are considering how this information can be applied to mathematical models. However, it is apparent that no attempt is being made to specifically relate the effects of layering on semi-static water-level changes which occur during the dewatering of a single unconfined aquifer over a long period or time.

Thus, this study is an evaluation of how the variation of lithology within an aquifer can affect the rate of dewatering. This variation is assumed to be a major factor contributing to the response of mathematical groundwater flow models. This would be particularly valuable when such models are used for predicting the time for a given water-level change to occur during the dewatering of an aquifer.

The determination of the relationship between aquifer constants and declines in static water levels would not only be useful in analysis of the Ogallala aquifer but also could be applied to layered alluvial aquifers (floodplain and terrace deposits, alluvial fan deposits) as well as to layered basin and coastal plain aquifers. Layered alluvial deposits are associated with many of the major streams in the State of Oklahoma.

Therefore, the major objective of this study was to compare the response of a modified version of the Texas Tech management model to multi-layered and homogeneous cases. This was accomplished by making modifications in the management model which would accommodate the multi-layered case and the assumption based on the use of weighted-average values to represent the hydraulic coefficients. Comparisons were subsequently made between the homogeneous and multi-layered case using hydrographs.

DESCRIPTION AND MEASUREMENT OF AQUIFER CHARACTERISTICS

In constructing the groundwater management model for the Ogallala aquifer in Texas County, hydrogeologic data were collected and analyzed. Data were evaluated in order that a basic set of assumptions could be determined and adaptations made in the mathematical model.

The well data for Texas County were provided by the United States Geological Survey and the Oklahoma Water Resources Board. An isometric map was prepared to show the three dimensional distribution of the lithologic characteristics of the Ogallala aquifer both in Texas County and in the test area, located northeast of Guymon, Oklahoma. (See Figure 2).

Layer codes (see Figure 3), were used to simplify log descriptions and provide uniformity in the data. This was achieved by identifying the principal grain sizes. Subsequently the codes were used to prepare isometric diagrams for Texas County and the test area (See Figure 3). Two maps were also used to represent other hydrogeologic aspects of the test area. These two additional maps include the water-level map (Figure 4) and the saturated thickness map (Figure 5).

The isometric diagram was prepared to show the lithologic characteristics of the Ogallala aquifer both in Texas County and in the test area. Preparation of this diagram involved the transformation of coded layer data into a visual three-dimensional diagram. The map grid was skewed to a 30-degree angle in order to provide a three-dimensional view of the groundwater system. A reference datum of 3300 feet above sea level was used with a vertical scale of 1 inch to 100 feet and a horizontal scale of 1 inch to 1 mile. Panel diagrams were not used to show correlation between wells because of the apparent discontinuous nature of the layers.

The water-level map was used to represent the water-table configuration of the test area. All water-level records for March, 1966, were taken from published data (from Hart, 1971). A contour interval of 10 feet was used. The saturated-thickness map was a modification of one prepared by Wood and Hart (1967). A contour interval of 100 feet was used to show the distribution of saturated thickness in the test area.

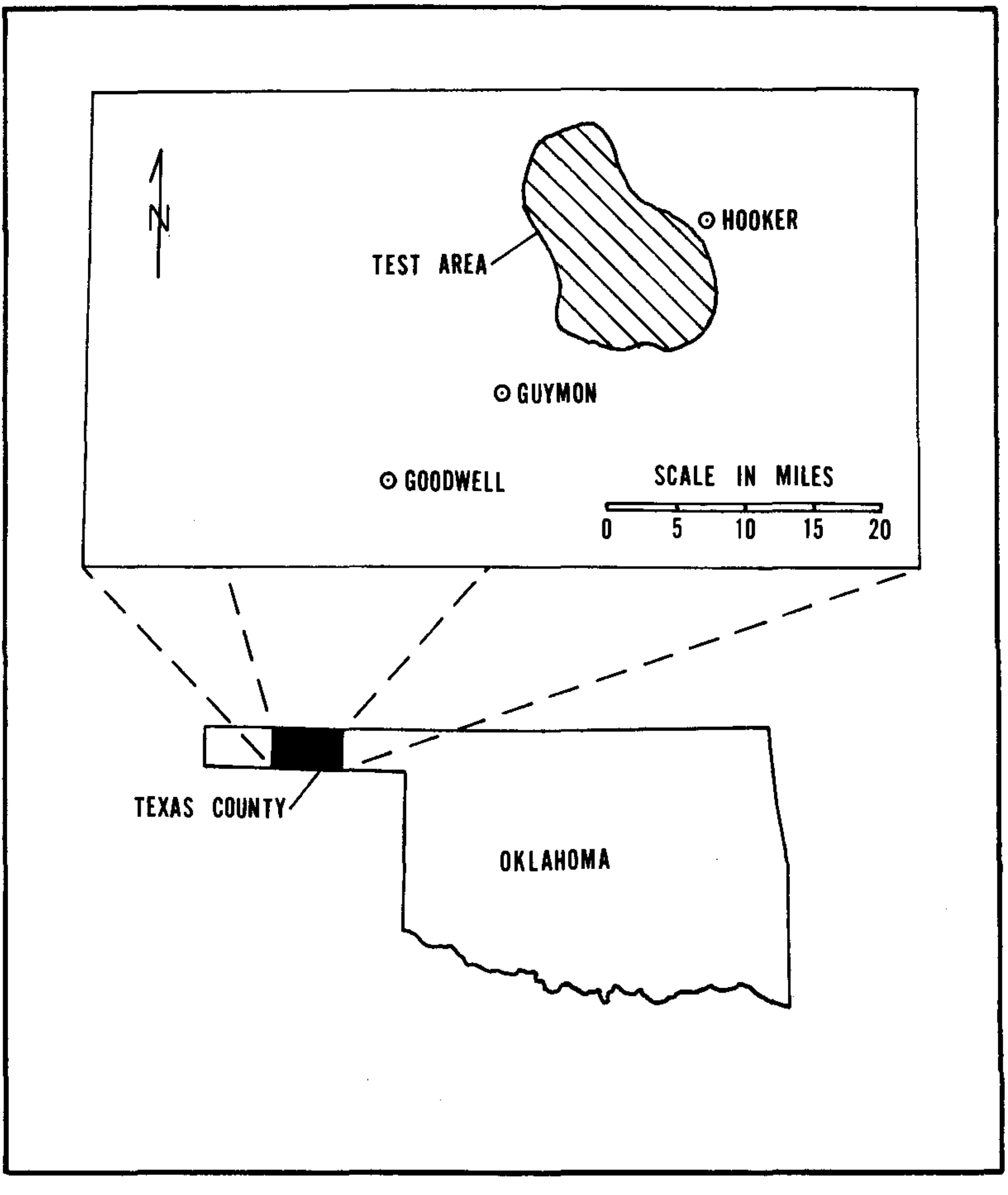

N
TEST AREA
HOOKER
GUYMON
GOODWELL
SCALE IN MILES
0
5
10
15
20
TEXAS COUNTY
OKLAHOMA

FIGURE 2

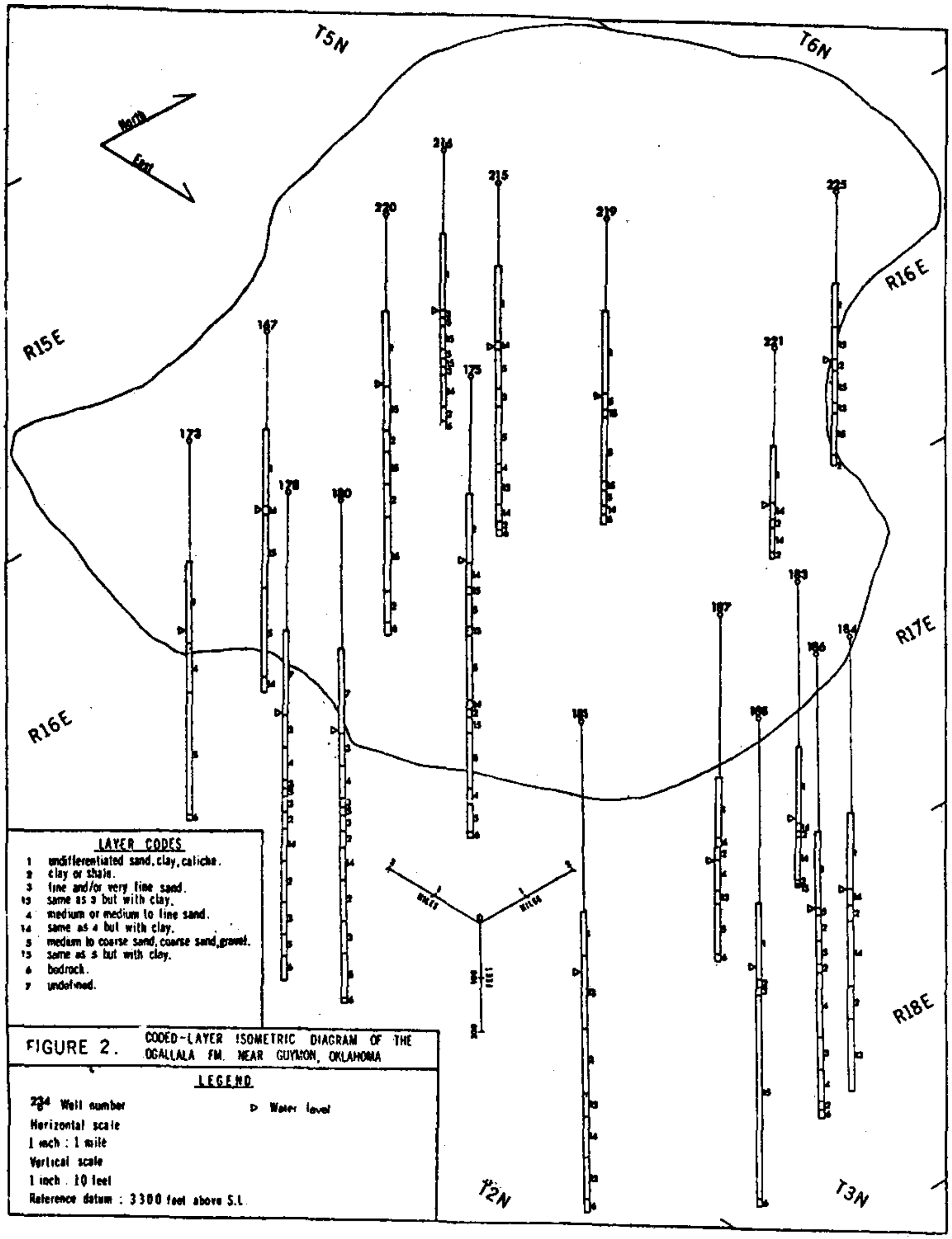


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Fig. 3.-Coded-layer isometric diagram of the Ogallala Formation near Guymon, Oklahoma

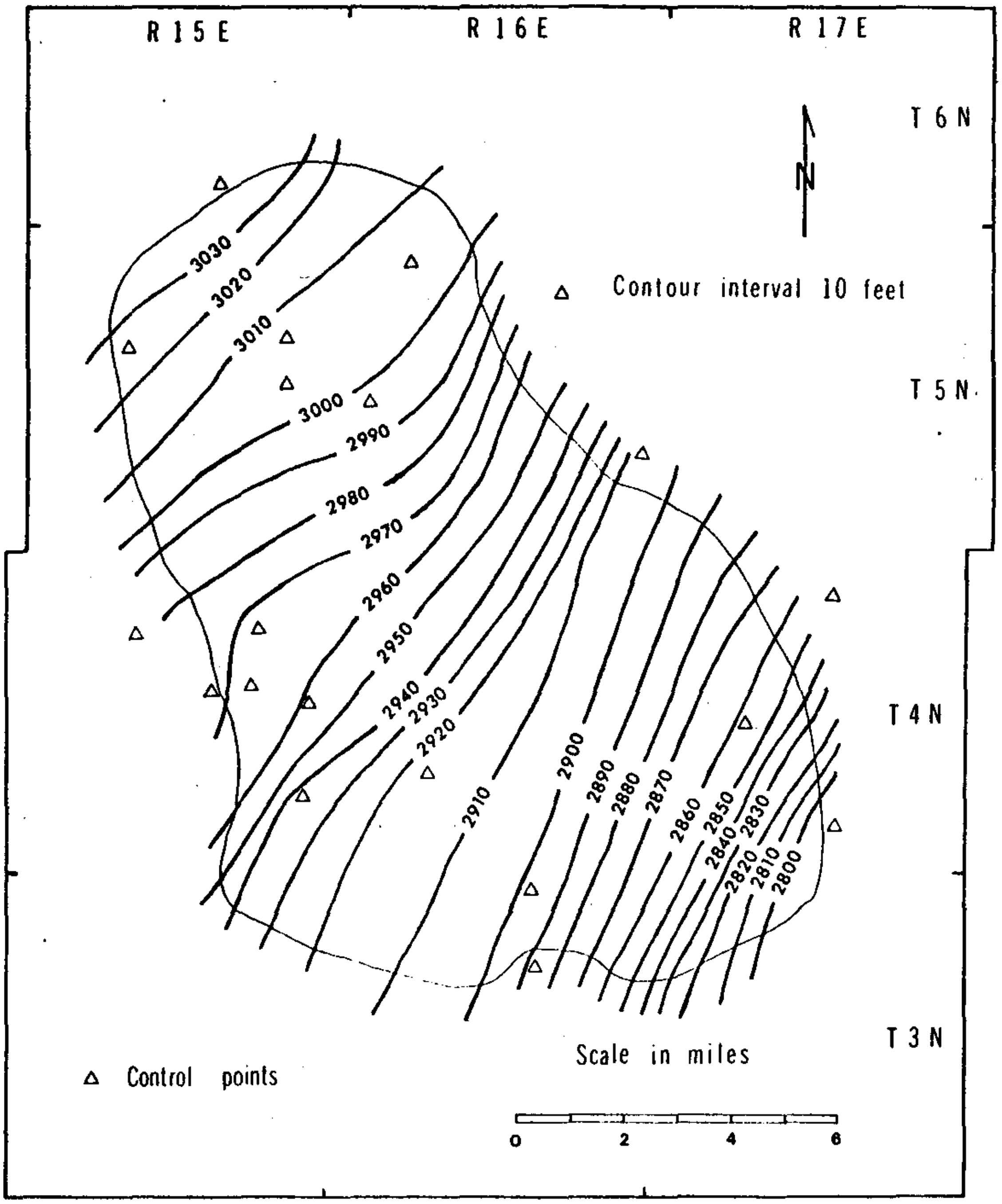

Fig. 4.-Water-level map of test area (after U.S.G.S.), March, 1966

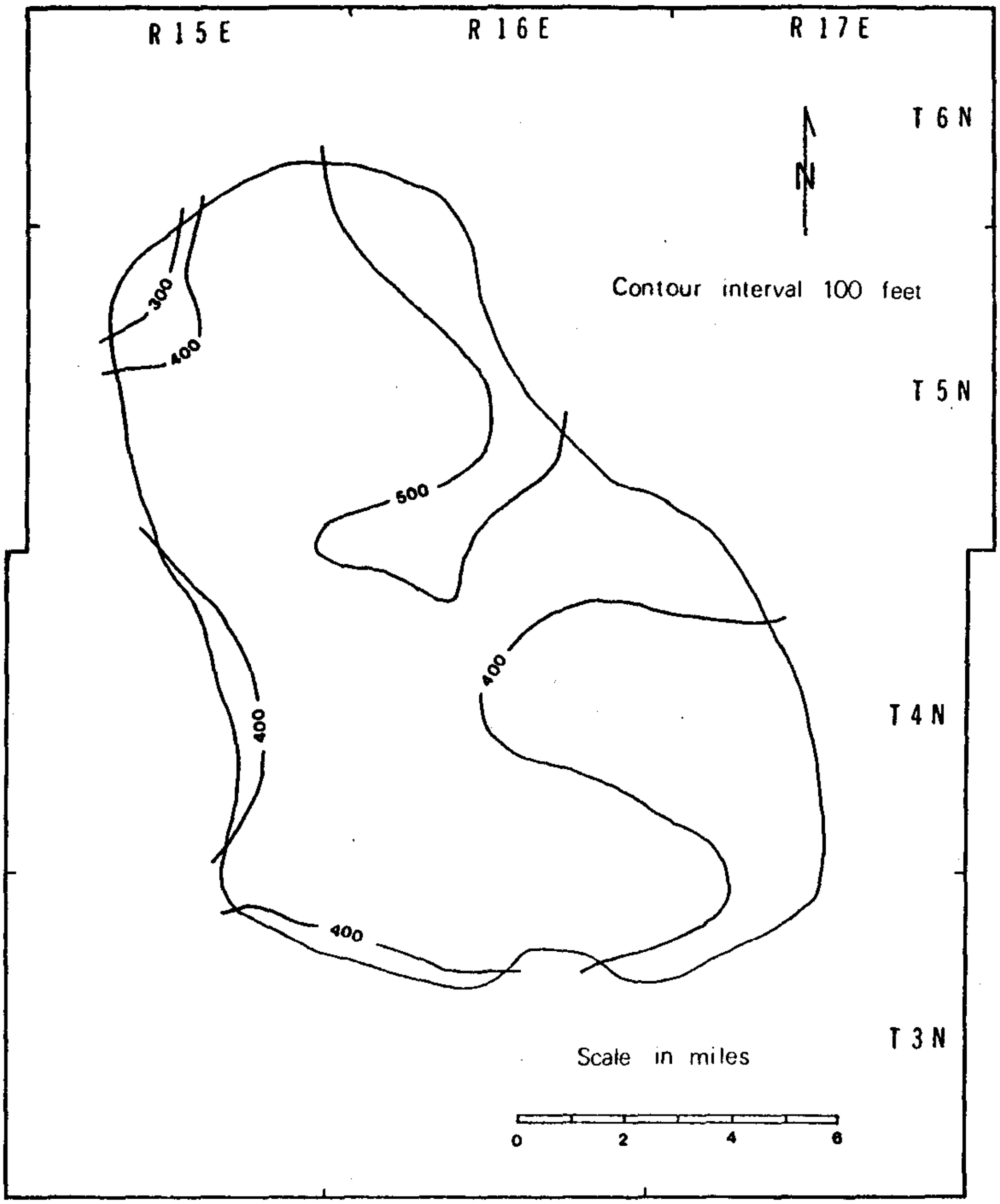

Fig. 5.-Saturated-thickness map of the Ogallala Formation in the test area (after U.S.G.S.), March, 1966

In order to develop an idealized conceptual model which would represent the layered character of the Ogallala Formation, a simple statistical study was made showing the frequency of occurrence of lithology type and layer sequence. In Texas County, 37 percent of the lithology is coarse sand, 25 percent is medium sand, and the remainder is fine sand and clay. (See Tables 1,2). By lumping thinner units together on the isometric map, a sequence of fine to coarse sediments from the water table to the base of the aquifer could be observed. There were 13 from a total of 17 wells within the test area which were representative of this sequence. A similar sequence was noted when 75 of the total number of wells (112) in Texas County were noted to have this sequence. Based on the frequency of occurrence of sequence and lithology type, it was concluded that the model should be described using medium and coarse sediments in a graded sequence with coarser sediments occurring at the base.

To simplify the model, four layers of uniform thickness were assumed. The thickness of each was 100 feet in order that the total thickness of 400 feet would correspond with the average thickness represented on the saturated-thickness map of the study area. (See Figure 5). It was also assumed that the layers were homogeneous when extended uniformly in the lateral direction. Although this assumption is an oversimplification, it was considered necessary for the Texas Tech model and before more complex models could be evaluated.

The distribution and character of the sediments were studied at an outcrop of the Ogallala Fromation at a location just west of Guymon. A photograph and generalized cross-sectional diagram of the Ogallala outcrop is shown in Figures 6 and 7. The outcrop consists of a buried sand and gravel channel fill overlain by later Ogallala deposits and the caliche caprock which is used as the upper boundary of the Ogallala Formation. It was assumed after examining the well logs that the channel and dissected deposits are similar to those which appear to occur at a much larger scale within the saturated zone of the Ogallala aquifer. Therefore, the graded sequence used to describe the mathematical model was characterized by the sand types at the outcrop (A, B, C, D, in Figure 7). The A, B, C, and D sands were classified as medium, coarse, and very coarse sands, respectively, on the basis of the median grain size.

Table -1-

Frequency of Occurrence of Sediment Types
within Texas County

Well No.	Accumulative Thickness of Sediment Types in Each Well: 2 (feet)	3(13) (feet)	4(14) (feet)	5(15) (feet)	Sum (feet)
5				150	150
6	50			145	195
7	45		30	90	165
8	64		16	219	299
10	215	15	55		285
13		21	52	39	112
16	19		27	131	177
17	10		80	165	225
19	40	32	48	80	200
20	27	24	36	72	159
21	49	23	62	106	240
23			160	50	210
26		150	30		180
27		120		120	240
29	34	9	242	32	317
30	23	23		103	153
31	40	205	62		307
33	45	163	56	203	467
34	80	5		111	196
35			125	90	215
38	225	15	45	75	360
41			217		217
47	40	70		150	260
48		90	30	90	210
50		60	10	54	124
52	64		69	140	273
108	75		130	80	285
109		146	30	105	281
110	66	77	185	20	348
111	55	90	166	49	360
112	20		120	140	280
113	120			150	270
114	100	60	240	40	440
116	55	14	116	80	265
117	50	210	30	80	370
118	135	68	140		343
119	35		175	103	315
121			150	180	330
125	85		120	15	220
131	235		40	140	415
134	37		6	77	120
135	40			140	180
137	3	136	5	50	194
146		220	30		250
147				180	180
149		90	10	270	370
151	30		90	125	245
152	30	15	90	120	255
154	60	18	69		147
156	98	164	58		320
159				180	190
160	160			240	400
197	83	108	6	132	329
56	30		230	100	360
58	60			120	180
59	80	20	20	180	300
62	40	236		40	316
65	20		85	120	225
71	16		117	83	216
72	45	235	20	60	360
76	120	50	80		250
79	30		212	60	302
80		120		135	255
81	30	40		226	296
82	20	127	288		435
83	13		168	50	231
84			80	216	296
87	32		176		208
90	284	10		106	400
92	103	10	104		217
94	90	30	190		310
96	45	30		165	240
97		60	80	120	260
98	75		105	105	285
100	45		30	140	215
101	25		90	190	305
102	46			176	222
104	70		20	212	302
107	91	25		124	240
162	35		330		365
163		40	230		370
164		145		230	375
167			44	300	344
168	30	45	30	300	405
169	45		205	15	265
170	20			260	280
171	60			220	280
172	120			250	370
173			92	228	320
175	15	15	90	382	502
178	105	165	120	55	445
180	105	165	120	100	490
181		375	75		450
182		320	30	45	395
183	80	10	6	25	121
184	90	135	150		375
185	80	120	330		480
186	75	60	180	60	375
187	35	25	74	96	230
188	15	15		375	405
190	179	7	26	69	281
192			158	37	195
194	65	35	10	30	140
195	15		30	90	135
196			75	90	165
235	25		15	100	140
200	60	45	30	105	240
201	105	33	22	209	369
202	80		40	75	195
203		170	30	30	230
204	45	30	90	105	270
205	30		120	75	225
207	90	75		90	255
208				220	220
209	100		100	50	250
210		50	127	10	187
211		20	20	320	360
215	15	90	60	180	345
216	30	30	60	90	210
217	70	15		10	95
219			15	210	225
220	155			280	435
221	30		75		105
225	40	20		200	260
226	270		75	15	360
227	80			130	210
228	72	86	96	26	280
229	76	54	10	10	150
230	214	7	77	21	319
232	150	170		45	365
234	38	40	48	50	176
236		30	75	45	150
238	90		75	90	255
239	15			180	195
240	24	29	178	167	398
241	85		12	231	328
242	36	23	21	107	187
244	30		45	112	187
245		70		120	190
249	17	18	190	41	249
252		30	18	90	138
253		50	100	50	200
254	14	88	87	41	230
255		40	120	60	220
256		135	45	60	240
260	120		165	75	360
262	40	240	80		360
263	60	260	20	40	380
264	20	340		80	440
265	120	240			360
268	15		135	109	259
269	227		54	81	361
272	130	15	80	60	285
Total thickness of each sediment type for all wells	7,991	7,654	10,568	15,595	41,808
% of total thickness	19.11	18.30	25.27	37.32	100

Table -2-

Frequency of Occurrence of Sediment Types within Study Area

Well No.	2* (feet)	3 & 13* (feet)	4 & 14* (feet)	5 & 15* (feet)	
167	0	0	44	300	
173	0	0	92	228	
175	15	15	90	232	
178	105	165	120	55	
180	105	105	120	100	
181	0	375	75	0	
183	80	10	6	25	
184	90	150	150	0	
186	75	60	180	60	
187	35	25	74	96	
188	15	15	0	375	
215	15	90	60	180	
216	30	30	60	90	
219	0	0	15	210	
220	155	0	0	280	
221	30	0	75	0	
225	20	20	0	200	
Total thickness of each sediment type for all wells	770	1045	1161	2437	Σ = 5413 ft.
% of total thickness	14.3%	19.3%	21.4%	45%	Σ = 100%

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*Codes are defined in Figure 7

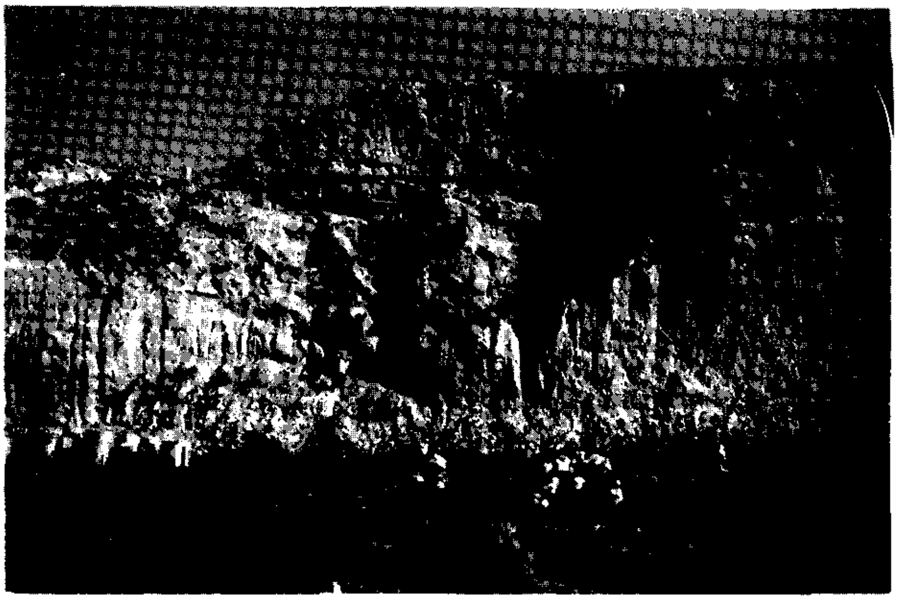
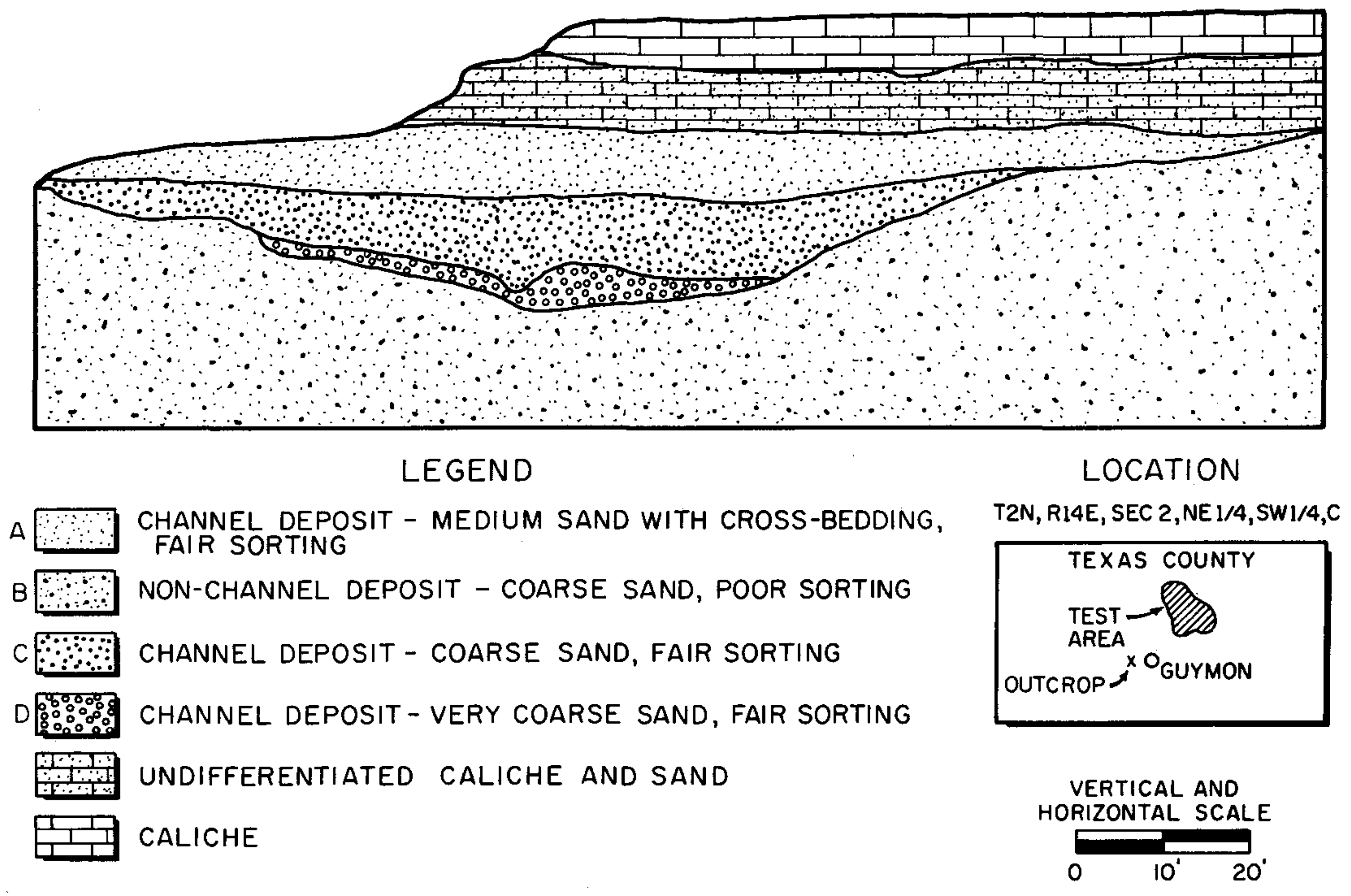


Figure 6. View of Ogallala Formation outcrop near Guymon, Oklahoma



CROSS-SECTIONAL DIAGRAM OF OGALLALA FORMATION OUTCROP NEAR GUYMON, OKLA.

FIGURE 7

Poor to moderate sorting was characteristic of these sands. The overall average coefficient of permeability of 400 gpd/ft^2 was used to represent the four layers. This average was the same as the value which was derived from pump test analysis and used in the original Texas Tech model. The coefficient of permeability values used for each of the sands A, B, C, D, were 150, 236, 380, and 835 gpd/ft^2 respectively. Specific yield values were estimated based on each sand type. An average specific yield value of 0.15 was used which corresponds to the value used previously in the Texas Tech model. Specific yield values of 0.07, 0.11, 0.17 and 0.25 were assigned to sands A through D respectively. Samples of each identified layer at the outcrop (A, B, C, and D in Figure 7) were collected for analysis and subsequently used in a sand model which will be discussed later.

ADAPTATION OF AQUIFER CHARACTERISTICS TO MANAGEMENT MODEL

Introduction

Because preliminary geologic and hydrologic data were used, a simplified conceptual model was considered. Therefore, the basic assumptions used in the modified version of the management model were the following: (1) The aquifer is multilayered and is ideally represented by four uniform sand layers of equal thickness. (2) Each layer is horizontally homogeneous. (3) The bedrock topography underlying the Ogallala aquifer is considered to be relatively smooth and slopes approximately 14 feet per mile in a southeasterly direction. (4) The bedrock and water-table surfaces are approximately parallel and are used as the lower and upper boundaries respectively. (5) Weighted average values of permeability and specific yield assigned at each time step are close approximations for that particular time period. (6) There is no recharge or discharge through the bedrock. (7) Natural recharge and discharge at the boundary of the study area are equal. (8) The total saturated thickness is subject to pump withdrawal in any one time step.

A hypothetical grid of well nodes was designed and subsequently adapted to the study area. Within this area, 24 nodes were assigned having fixed coordinates. In addition, 17 nodes were located around the perimeter of the area and were used as an aid in defining the boundary conditions. A separate computer program was used to divide the 24 internal nodes of the study area into a polygonal grid system using the Thiessen Method (see Figure 8). The amount of groundwater withdrawal from each polygon was based on the area of the polygon and on an assumed constant rate of discharge per unit area of the polygon. The computations used to obtain the adjusted pump withdrawal at each node is shown in Table 3.

Computer Programming of Management Model

The basic program used in this study was originally written by Weber (1968) and later revised by Sechrist, Claborn, Rayner, and Wells (1970). The latter program includes the Crank-Nicholson finite difference method and the Gaus-Seidel iteration procedure. In all preceding uses of the program, the homogeneous case was assumed. In this study, where the multi-layered case was also considered, vertical variations of permeability and specific yield values were introduced into the program as a sub-program, (Figure 9).

New weighted-average values of the coefficient of permeability and specific yield are computed in a sub-program between timesteps. The two hydraulic coefficients are averaged using the following equations:

$$K = \frac{\sum_{i=1}^{n} K_i M_i}{\sum_{i=1}^{n} M_i} \qquad S_y = \frac{\sum_{i=1}^{n} S_i M_i}{\sum_{i=1}^{n} M_i}$$

where

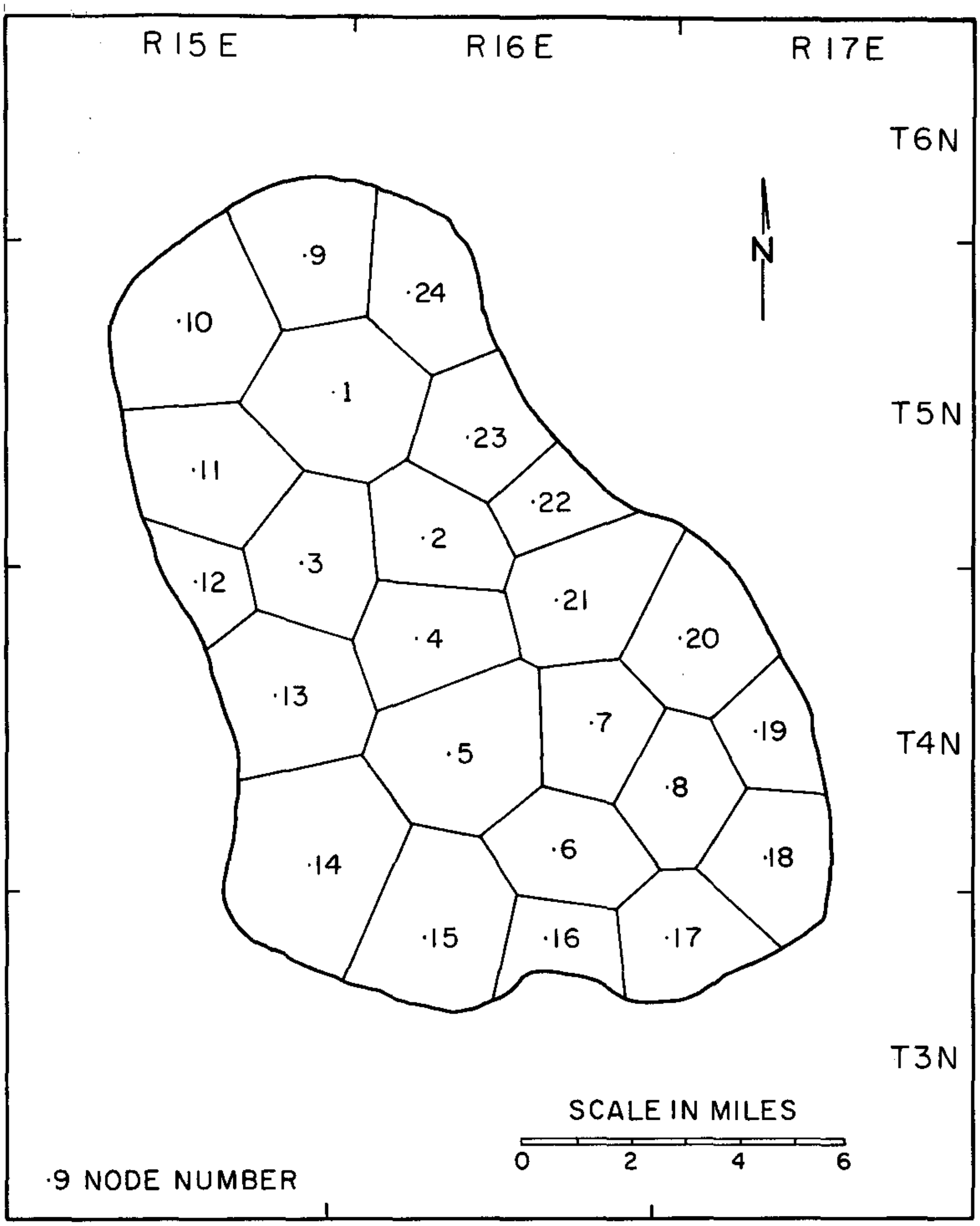
n = the number of layers

K_i = the coefficient of permeability value of ith layer

S_i = the specific yield value of ith layer

M_i = the saturated thickness of ith layer

The validity of the values used in this approach was subsequently tested using a physical sand model and will be the subject of the next part of this report.



POLYGON DISTRIBUTION IN TEST AREA

FIGURE 8

Table -3-

Data Used For Computation Of

Simulated Pump Withdrawal

At Each Node

Node	Area (acres)	Seasonal Withdrawal at each node(acre feet) (1.77 acre feet/ acre/ season)	Seasonal Boundary Discharge/ Recharge (acre feet)	Total Adjusted Seasonal Withdrawal at each node (acre feet)
1	4915.20	8699.9	0.0	8699.9
2	2867.20	5074.9	0.0	5074.9
3	3635.20	6463.3	0.0	6463.3
4	3532.80	6253.1	0.0	6253.1
5	4940.80	8745.2	0.0	8745.2
6	3225.60	5709.3	0.0	5709.3
7	2790.40	4939.0	0.0	4939.0
8	2816.00	4984.3	0.0	4984.0
9	4172.80	7385.9	-5011.3	2374.6
10	7219.20	12778.0	-7338.6	5439.4
11	4582.40	8110.8	-2094.5	6016.3
12	1766.40	3126.5	-1737.7	1388.8
13	4710.40	8337.4	-3452.1	4885.3
14	6016.00	10648.3	-7105.8	3542.5
15	4633.60	8201.5	404.0	8605.5
16	1638.40	2900.0	1344.0	4244.0
17	3276.80	5799.9	5384.0	11183.9
18	3614.40	6397.5	7808.0	14205.5
19	2611.20	4621.8	4864.0	9485.8
20	4608.00	8156.2	1904.0	10060.2
21	4070.40	7204.6	0.0	7204.6
22	1996.80	3534.3	788.0	4322.3
23	3072.00	5437.4	1976.0	7233.4
24	3763.20	6660.9	2448.0	9108.9
TOTAL	90,475.20	160,140.7	0.0	160,140.7

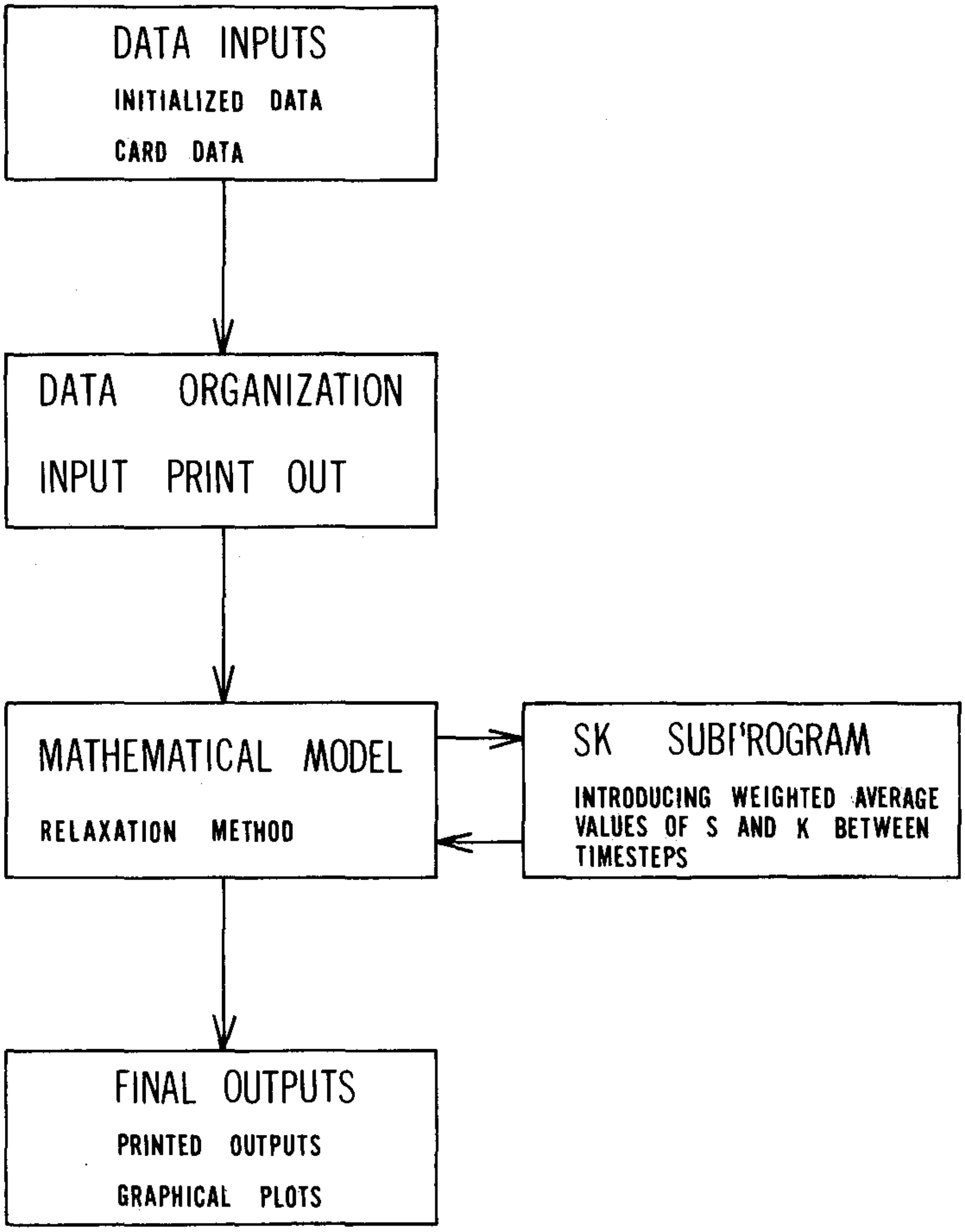

DATA INPUTS
INITIALIZED DATA
CARD DATA
DATA ORGANIZATION
INPUT PRINT OUT
MATHEMATICAL MODEL
RELAXATION METHOD
SK SUBPROGRAM
INTRODUCING WEIGHTED AVERAGE VALUES OF S AND K BETWEEN TIMESTEPS
FINAL OUTPUTS
PRINTED OUTPUTS
GRAPHICAL PLOTS

FIGURE 9

Final output from the computer program of the mathematical model was in the form of printed output and included average coefficients of permeability and specific yield, water-level elevations and accumulative drawdown values for each timestep. The results were also electronically plotted in the forms of accumulative drawdown curves for each node.

Results of the Mathematical Model

Comparison was made of accumulative drawdown curves representing nodes in both homogeneous and multi-layered cases. In addition, a sensitivity analysis of the coefficient of permeability and specific yield was conducted.

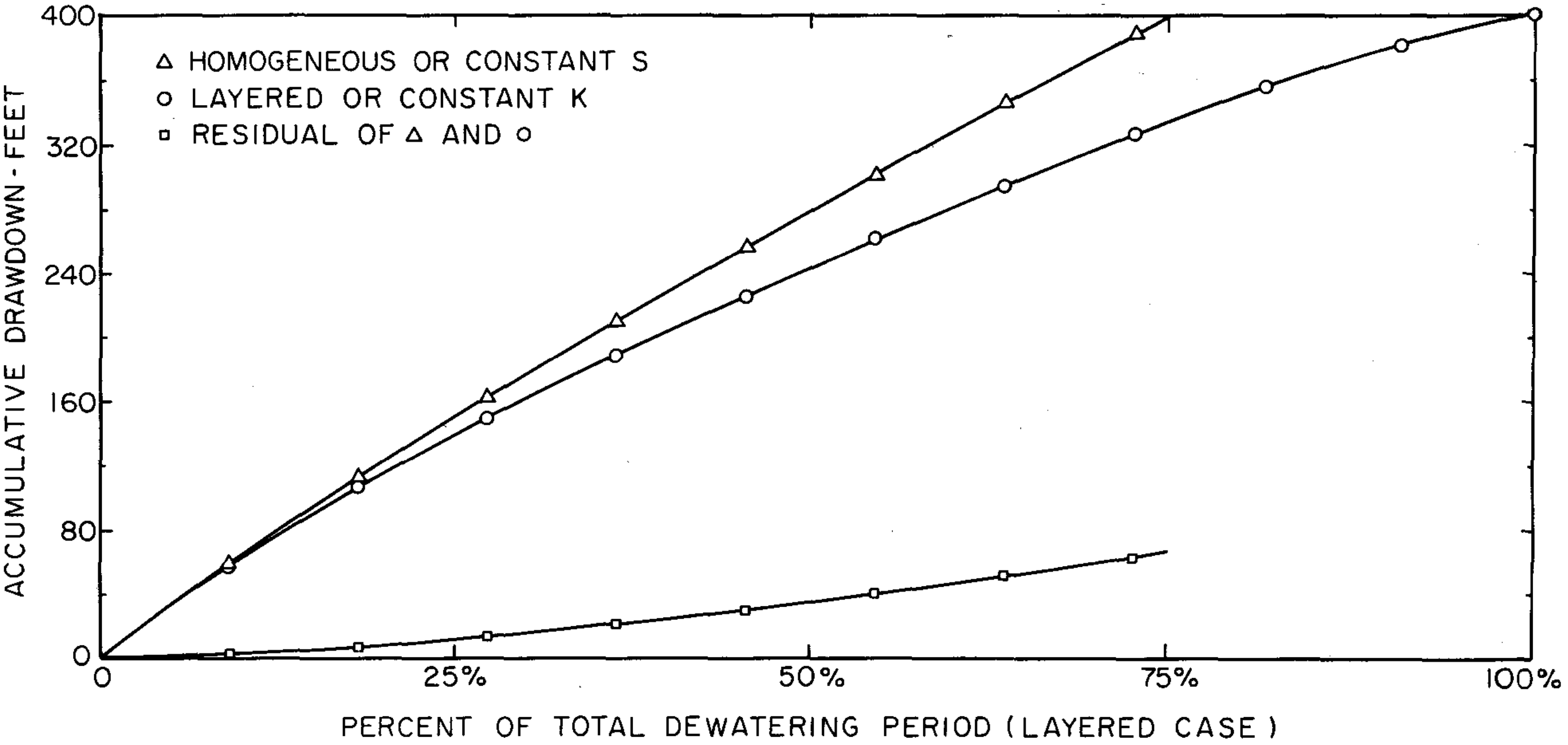
Accumulative drawdown curves representing the same nodes for both homogeneous and multi-layered cases were overlain on one another and a residual curve was drawn which represented the difference between the two curves. This was repeated for all 24 nodes (See Table 4). Accumulative drawdown curves for a representative node are shown in Figure 10. A significant difference between the homogeneous and the multilayered cases can be noted. The difference is clearly indicated by the residual curve. It can also be noted that the length of time for dewatering of the layered aquifer is approximately 26 percent longer than the comparable dewatering of the homogeneous aquifer. A maximum difference between accumulative drawdown curves for the two cases was 20 percent of the original saturated thickness. This maximum difference occurred at the time when a polygon was completely dewatered using the homogeneous case (see Figure 10).

The sensitivity of the model to the coefficients of permeability and specific yield was evaluated by keeping the initial average value of either of the two coefficients constant throughout the period of dewatering while using the multi-layered case. When specific yield was varied, it was noted that the water-level changes were clearly different in the two cases (see Figure 10). Conversely, the model response was identical to that of the homogeneous case if specific yield was held constant. It was concluded from these results that the model is insensitive to changes of the average coefficient of permeability over the same period of time. However, it should be noted that the model response for either case is different from the above when the initial average values of either coefficient are changed.

Table -4-

Accumulative Drawdown
Relationships Between Homogeneous and Layered Cases

Node Number	Extended Period of Dewatering Caused by Layering (%)	% of Aquifer Depth Remaining Saturated For Layered Case When Dry For Homogeneous Case (%)	Depth Remaining Saturated For Layered Case When Dry For Homogeneous Case (feet)
1	27.4	23.0	92
2	24.6	21.4	86
3	32.0	25.7	103
4	25.0	21.7	87
5	27.7	23.9	93
6	24.6	19.8	79
7	27.7	21.2	85
8	23.7	18.7	75
9	33.5	27.8	111
10	24.7	22.8	91
11	29.0	25.0	100
12	29.5	27.0	108
13	26.2	23.8	95
14	28.0	25.0	100
15	24.6	22.0	88
16	24.4	20.2	81
17	20.0	18.0	72
18	17.5	15.5	62
19	18.0	16.5	66
20	25.0	18.7	75
21	27.0	22.0	88
22	24.0	22.0	88
23	27.6	23.0	92
24	29.2	23.5	94
Average for all Nodes	25.8%	20.3%	88 Ft.



ACCUMULATIVE DRAWDOWN FOR A REPRESENTATIVE NODE IN TEST AREA

FIGURE 10

ADAPTATION OF AQUIFER CHARACTERISTICS TO SAND MODEL

Introduction

Using the aquifer material collected near Guymon, a physical sand model was constructed. A photo of the model is shown in Figure 11. The model consists of an inner tank 3' in diameter, and 4' diameter outer tank. The inner tank was perforated to allow water from the annular area to flow into or out of the tank. Aquifer material previously described as A, B, C, and D sands, (see Figure 7), were wetted and compacted into the inner tank in one foot layers respectively. A 1-1/4" Johnson well screen with four 10" sections was fabricated and used for the pumped well. Additional observation wells were made from 1" plastic pipe. Water-level observations were made using direct open hole measurements and water manometers.

Pump Test

The model was first constructed during early 1972, and pumping tests run during the Spring. The data from these tests are shown in Tables 5, 6, 7, and 8. Because of a requirement that the model be moved to another area in the laboratory, the original model was dismantled and reconstructed during the Summer of 1972. The reconstructed model was again pump tested and also allowed to gravity drain.

The configuration of the model was so designed that the pumping tests data could be evaluated using the Theis Non-equilibrium Equation. A constant head level could be maintained in the annular area between tanks and drawdowns in the pumped and observation well were observed.

Pump Test Results

Plots of the drawdown versus time for the four layers are shown on Figure 12. These data reflect a test procedure where the water level in the annular area was held constant at the upper level of each sand layer. The resulting average values of transmissibility and specific yield are shown in Table 9. The values obtained in these tests are

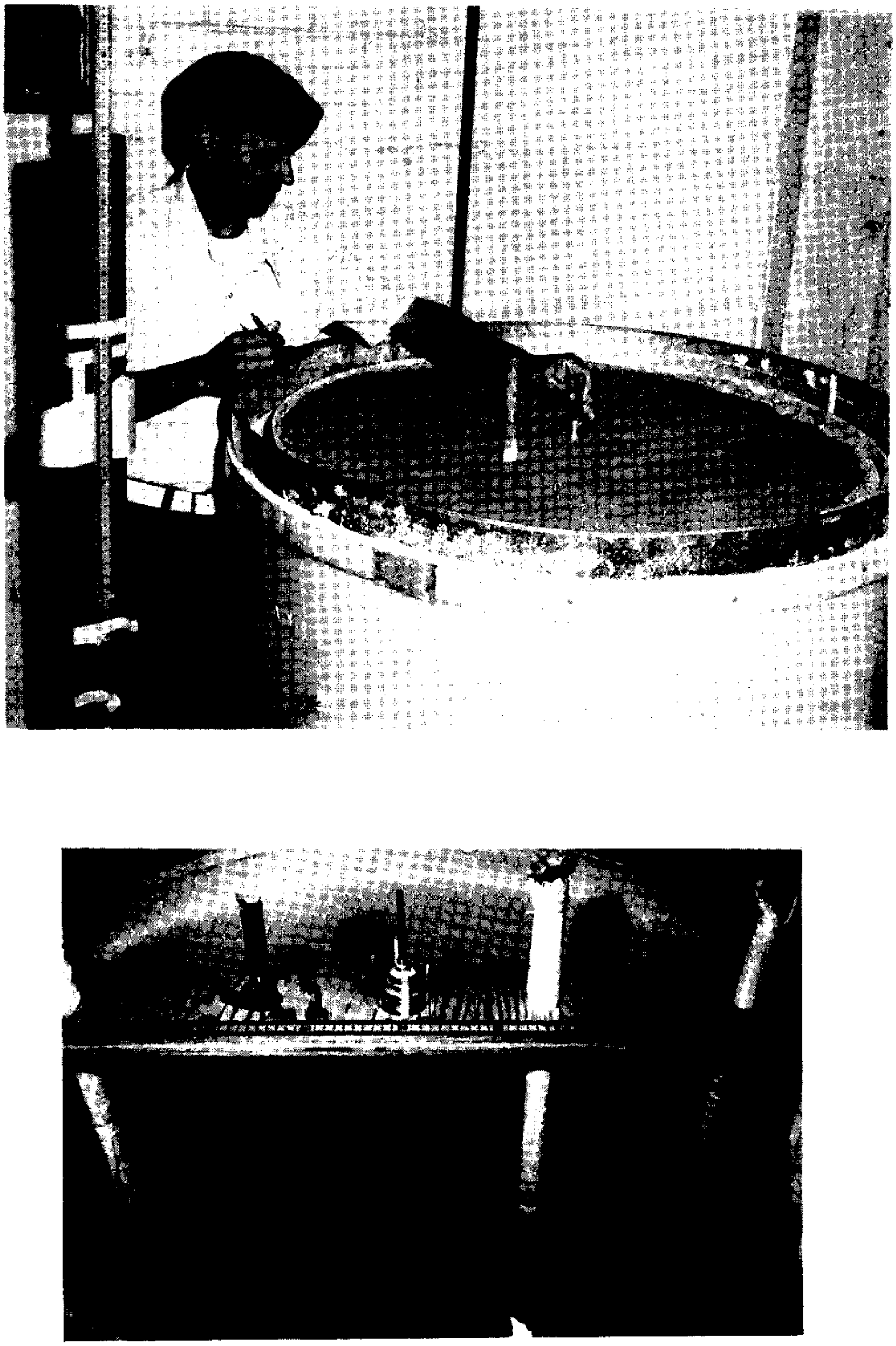


Figure 11. Physical Sand Model

Table -5-

Pump Test Data Layer A

$\bar{Q}$ = 46.41 ml/sec

time t = sec	drawdown s = cm	time t = sec	drawdown s = cm
0	0.0	210	10.6
20	0.9	215	10.8
25	1.3	240	11.4
30	1.7	270	11.8
35	2.0	300	12.5
40	2.2	330	12.8
45	2.7	360	13.1
50	3.1	390	13.4
55	3.5	420	13.7
60	3.8	450	13.8
65	4.2	480	14.0
70	4.5	510	14.1
75	4.9	540	14.25
80	5.3	570	14.35
85	5.5	600	14.45
90	5.9	630	14.5
95	6.15	660	14.6
100	6.4	690	14.7
105	6.7	720	14.75
110	6.9	750	14.8
115	7.1	780	14.95
120	7.4	810	15.05
125	7.7	840	15.1
130	7.9	870	15.2
135	8.2	900	15.3
140	8.4	930*	15.4
145	8.6	960	15.4
150	8.8	990	15.5
155	9.0	1000	15.4
160	9.3		
165	9.4		
170	9.5		
175	9.6		
180	9.8		
185	10.0		
190	10.2		
195	10.3		
205	10.5		

* equilibrium pt.

Table -6-

Pump Test Data Layer B

$\bar{Q}$ = 49.24 ml/sec

time t = sec	drawdown s = cm
0	0.0
5	4.0
10	9.7
15	13.3
20	15.4
25	16.7
30	17.8
35	18.5
40	19.0
45	19.4
50	19.65
55	19.80
60	20.00
65	20.1
70	20.2
75	20.3
80	20.4
85	20.4
90	20.6
95	20.6
100	20.9
105	20.9
110	21.05
115	21.05
120	20.2
210	20.7
240	20.5
270	20.6
300*	20.7
330	20.7
360	20.7
390	20.7
480	20.7
510	20.7

* equilibrium pt.

Table -7-

Pump Test Data Layer C

$Q = 50.44$ ml/sec

time t = sec	drawdown s = cm
0	0.0
5	3.71
10	9.82
15	14.43
20	18.24
25	21.3
30	24.35
35	25.32
40	26.79
45	27.86
50	28.75
55	29.5
60	30.37
65	30.74
70	31.21
75	31.38
80	31.55*
85	31.50
120	31.50

* equilibrium pt.

Table -8-

Pump Test Data Layer D

$\bar{Q}$ = 19.6 ml/sec

time t = sec	drawdown s = cm
0	0.0
10	8.4
15	11.3
20	12.8
25	13.2
30	13.4
35	13.9
40	14.0
60	14.1
80	14.3
100	14.4
120	15.5*
140	15.5

* equilibrium pt.

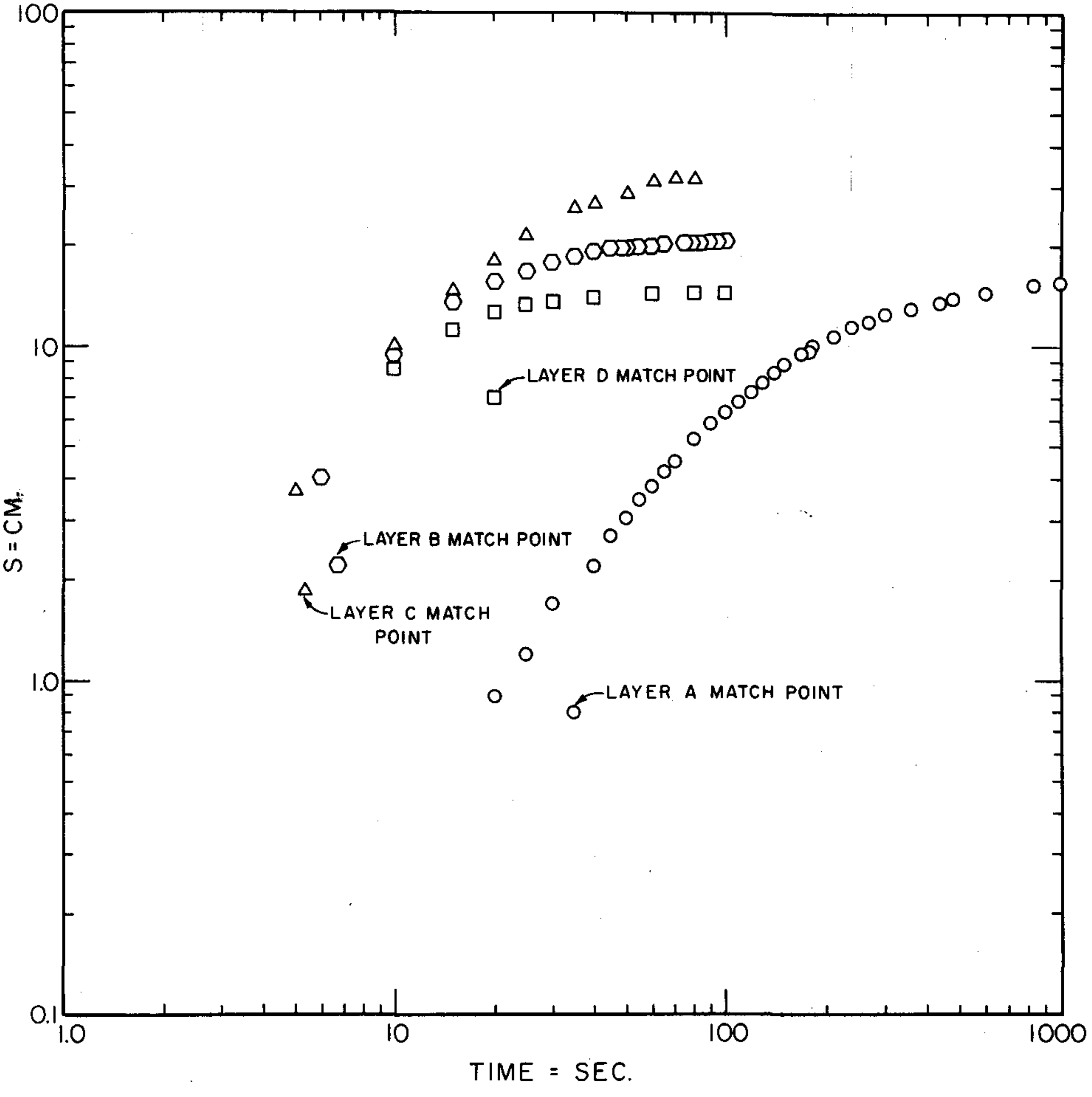


Figure 12. SAND MODEL PUMPING TEST.

Table -9-

Resulting Average Values of Transmissibility and Specific Yield

Active Layers	T (gal/day/ft)	S_y (dimensionless)	K (gal/day/ft^2)
A,B,C,D	325	0.28	80
B,C,D	125	0.05	40
C,D	150	0.02	75
D	155	0.007	155

considerably lower than those obtained in actual field tests. They are reported here though to confirm that the aquifer characteristics do, in fact change as dewatering takes place. No attempt was made to mathematically establish a set of characteristics curves (S, T, K) plotted versus depth from the results of the sand model tests. Additional runs are being made on the model as part of a class project and it is anticipated that a mathematical relationship can be derived.

Laboratory Measurements

Several samples for permeability analysis were obtained from each packed layer in the sand model by using 1-1/4" x 2" stainless steel tubes. Larger plastic tubes (length 7.7 cm; diameter, 6.49 cm) were also used to collect larger samples. In addition, samples were collected from the sand model for determination of grain-size distribution.

Standard methods of falling head and constant head analysis were used for the determination of the coefficient of permeability. The permeameter used was a soil test model K-670. It is equipped with a 6 x 24" reservoir tank for the permeability fluid (distilled water), a control regulator to maintain constant pressure, and a calibrated pipette to measure the change in head. Both falling head and constant head methods were used sequentially without having to remove the sample. The sample tubes were placed directly into the sample chamber in order to avoid any unnecessary disturbance of the textured properties of the samples. The following relationships were used for computing the coefficient of permeability:

$$\text{Constant Head } K = \frac{QL}{AH}$$

where

K = coefficient of permeability, cm/sec
Q = rate of discharge, cm^3/sec
l = length of sample, cm
A = area of sample, cm^2
H = pressure head, cm.

$$\text{Falling Head } K = \frac{2.3aL}{AT} \log_{10} \frac{H_0}{H}$$

where

K = coefficient of permeability, cm/sec
a = cross sectional area of pipette, cm^2
L = length of sample, cm
A = area of sample, cm^2
T = time of test, sec
H_0 = pressure head at beginning of test, cm
H = pressure head at end of test, cm.

The grain-size distribution of each sediment type was determined using visual-accumulation analysis. A 180 cm tube was used. The analysis included tracking the accumulation of grains which are sorted by size due to their fall velocities as described by Stoke's Law. The accumulative curves for each sediment type are shown in Figure 13.

Drainage tests were also conducted on samples collected in plastic tubes from the sand model. Each sample was saturated under a vacuum, weighed, and subsequently permitted to drain for 360 hours. For no apparent reason, no appreciable drainage occurred in the sample representing sediment type B.

Drainage Tests Using the Sand Model

Drainage tests were conducted on each sediment type (A, B, C, and D) using the sand model. The annular area was drained incrementally with each increment corresponding to the layer thickness of each sediment type. Each layer was permitted to drain radially into the annular area where the volume of drained water was collected and measured. The layers were permitted to drain until no further drainage occurred. The bulk volume of each layer was measured and computed. Corrections were made for the drained fluid volume in the annulus of the tank model and in wells which penetrated the layers in the sand model. The following relationship was used to compute the specific yield:

$$S_y = \frac{\text{Volume drained from layer}}{\text{Bulk Volume of layer}}$$

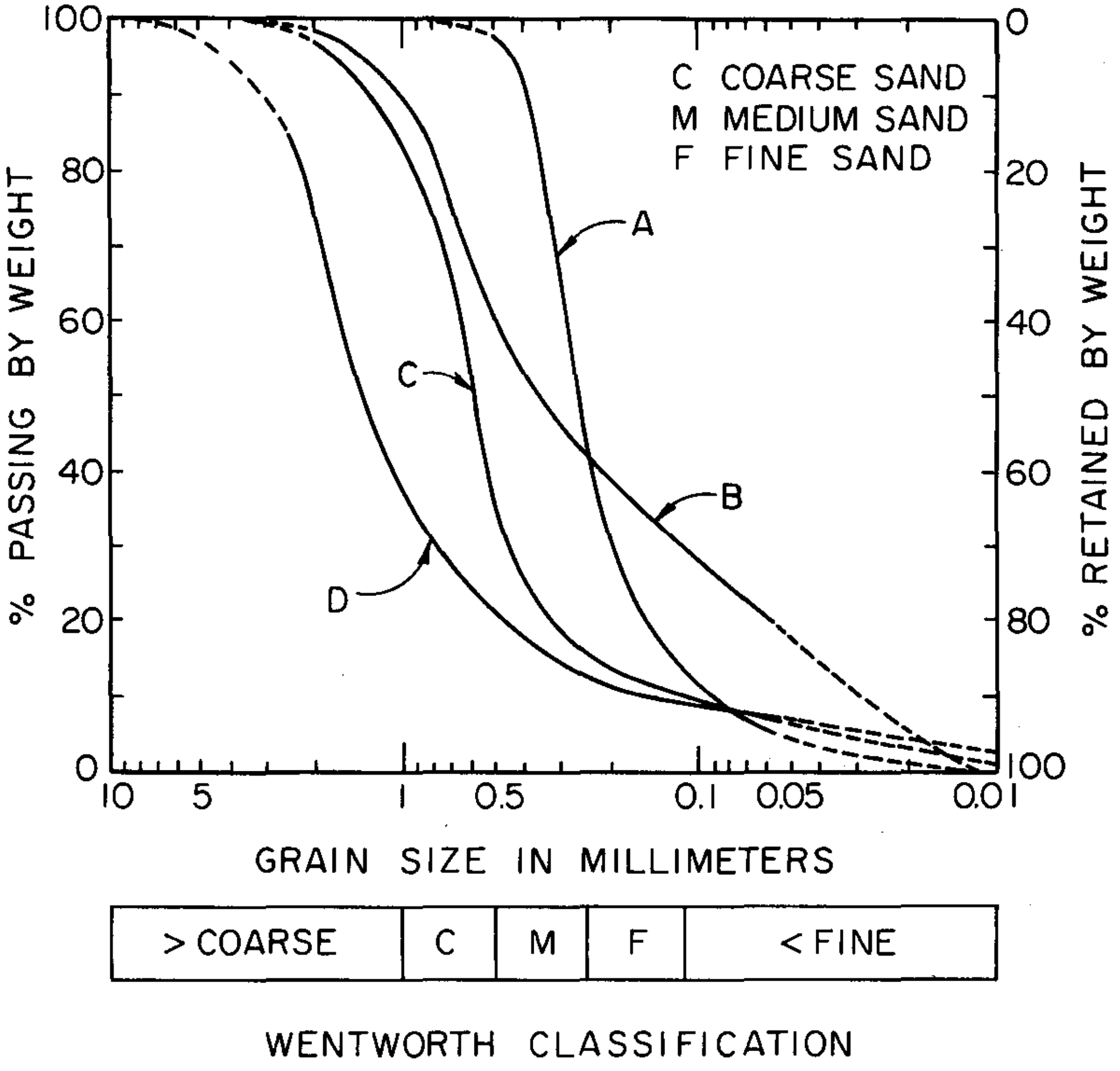


Figure 13. CUMULATIVE CURVE FOR GRAIN SIZE.

Results of Laboratory Measurements

Results of the permeability tests are shown in Table 10. The average of the four sediment types, comprising the four layers in the sand model, is 69 gpd/ft^2. This is considerably less than the value of 400 gpd/ft^2 used for the original and modified version (in this paper) of the Texas Tech Mathematical Model.

A summary of the statistics associated with textural characteristics of each sediment type is shown in Table 11. Only one of the four sediment types can be considered to have good sorting. The others were characterized by fair to poor sorting. The poor sorting was probably the main reason for the relatively low permeability coefficient values. If the 400 gpd/ft^2 is a reasonable estimate for the average permeability then it can be assumed that the samples used were more poorly sorted than those found within the saturated zone.

The drainage tests of the samples in the plastic tubes provided values of specific yield which were generally smaller than those obtained from drainage tests using the sand model. The results of these two tests are summarized in Tables 12 and 13 and in Figure 14. The average specific yields determined from Tables 12 and 13 are .09 and .12 respectively. The average specific yield obtained from the results using the plastic tubes is probably low because of the smaller sample size and loss of sediment type B sample. However, it is assumed that the drainage tests using the sand model are more accurate because of the larger volume of the sediment drained. The average of .12 corresponds reasonably well with the average of .15 used in both the original and modified versions (used in this study) of the mathematical model.

II. SUMMARY AND CONCLUSIONS

Results from the mathematical model indicate that a significant difference can be obtained when comparing the homogeneous and multi-layered approaches to aquifer management of the Ogallala aquifer. The length of dewatering time is approximately 25 percent greater in the multi-layered case. Therefore, it can be concluded that layering in the Ogallala aquifer should be considered in any management model

Table -10-

Laboratory Determinations of the Coefficient of Permeability

Sample Type	No. of Samples Run for Each Type	Constant Head		Falling Head		Overall Average for Constant and Falling Head	
		cm/sec	gal/day/ft^2	cm/sec	gal/day/ft^2	cm/sec	gal/day/ft^2
A	6	0.0012	24	0.0014	29	0.0013	27
B	7	0.0012	25	0.0018	38	0.0015	32
C	7	0.0044	93	0.0048	103	0.0046	98
D	6	0.0056	118	0.0056	122	0.0056	120

Table -11-

Grain Size Analysis

Sediment type	Sorting Coefficient (D_{60}/D_{10})	Degree of Sorting	Median Grain size (D_{50})
A	3.2	(Good)	.27
B	16.7	(Poor)	.36
C	5.4	(Fair)	.59
D	10.0	(Poor)	1.5

Table -12-

Drainage Tests of Core Samples of Sediment Types

A		C		D	
S_y	time t = Hrs	S_y	time t = Hrs	S_y	time t = Hr
0.019	2	----	----	----	----
0.02	12	0.002	12	0.004	12
0.02	47	0.007	47	0.045	47
0.021	96	0.018	96	0.059	96
0.026	360	0.04	360	0.19	360

Table -13-

Drainage Tests
Using Sand Model

Sediment type	Final Time	Specific Yield (S_y)	Specific Yield at 30 hrs	Specific Yield at 60 hrs
A	60 hrs	.024	.017	.024
B	65 hrs	.081	.072	.080
C	180 hrs	.134	.080	.099
D	190 hrs	.232	.185	.220

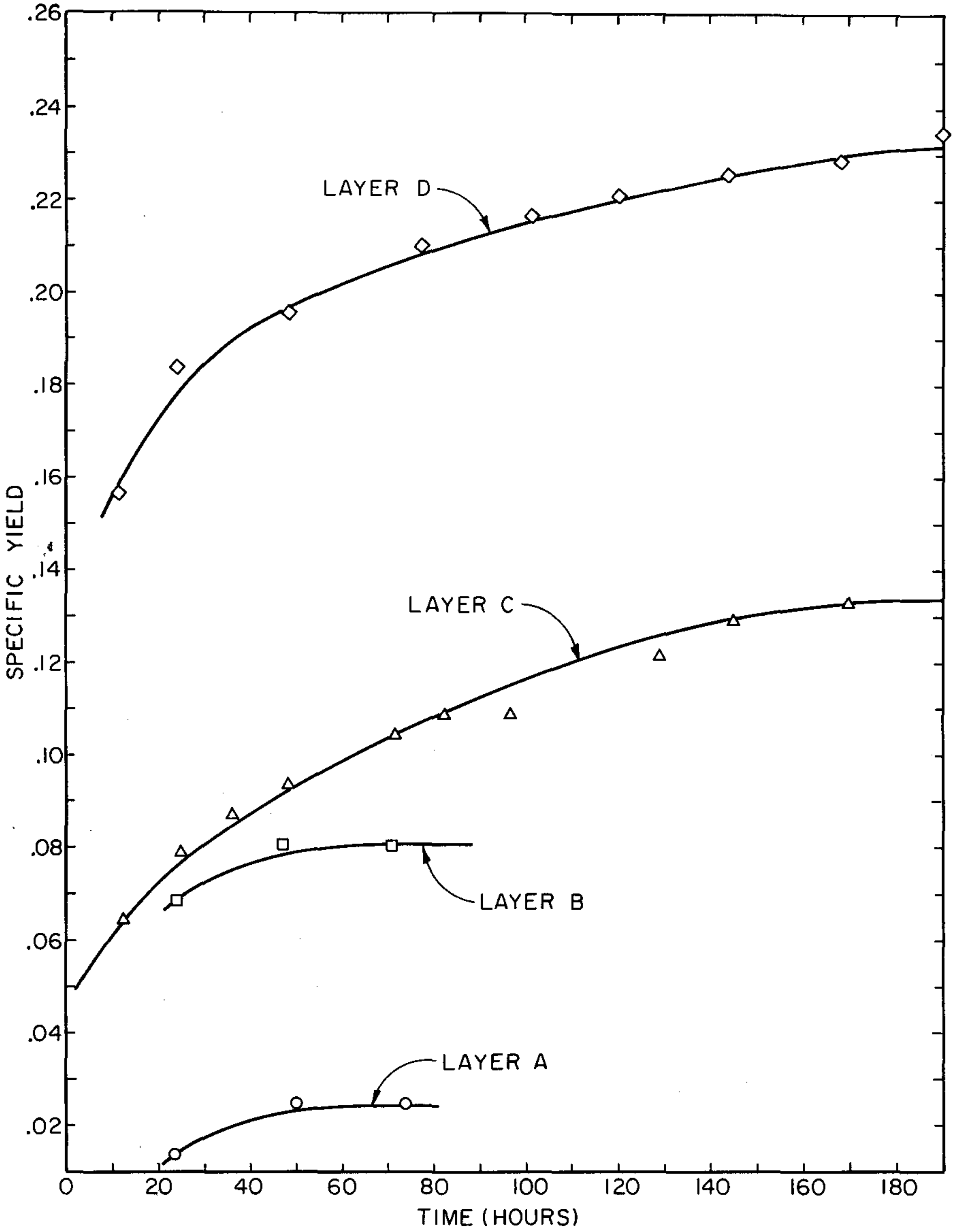


Figure 14. SPECIFIC YIELD - TIME RELATIONSHIPS DETERMINED FROM DRAINAGE TESTS.

which will be used for management of the groundwater resource. However, the assumption that the layers are considered to be of equal thickness and laterally homogeneous is as previously stated, an over-simplification. Before more complex layering can be considered, additional data and other types of mathematical models should be evaluated.

Results of the physical sand model and laboratory analysis indicate that the average coefficient of permeability of the four layers were 80 and 70 gal/day/ft^2, respectively. The average specific yield for the four layers determined from drainage tests was 0.12. The coefficient of permeability is considerably less than the 400 gal/day/ft^2 used in the mathematical model. The specific yield value is similar to that used in the mathematical model. If the mathematical model is to be used for future prediction pruposes, it is recommended that the average values of the coefficient of permeability and specific yield given above be used. However, it is concluded that the hydraulic characteristics do change as the dewatering of the layered aquifer progresses. This confirms the conclusions made from the modified mathematical model analysis.

Ongoing research involves the investigation of the feasibility of combining the results of this study with an economic model, for the prediction of future management alternatives for the Ogallala aquifer in Oklahoma.

III. PROJECT RELATED PUBLICATIONS

1. Loo, Walter Wei-To, "The Influence of Vertical Variation in Lithology on a Mathematical Management Model For the Ogallala Aquifer, Texas County, Oklahoma," unpublished Masters Thesis, Department of Geology, Oklahoma State University, Stillwater, Oklahoma, 1972.

2. DeVries, R. N., Kent, D. C., and Loo, W. W., "A Groundwater Management Model of a Multilayered Aquifer," Proceedings of the 23rd Annual Oklahoma Industrial Waste and Advanced Water Conference, Oklahoma State University, Stillwater, Oklahoma, 1972.

3. DeVries, R. N., and Kent, D. C., "Sensitivity of Groundwater Flow Models to Vertical Variability of Aquifer Constants," Presented at the Eighth American Water Resources Conference, St. Louis, Missouri, 1972.

IV. PROJECT PERSONNEL

The following personnel received financial support from this project:

1. R. N. DeVries, Associate Professor of Civil Engineering, Co-Principal Investigator
2. D. C. Kent, Associate Professor of Geology, Co-Principal Investigator
3. W. W. Loo, Graduate Student
4. E. Liao, Graduate Student
5. J. Chowning, Graduate Student
6. M. Hannon, Graduate Student
7. J. A. Tatchio, Secretary

SELECTED BIBLIOGRAPHY

Bredehoeft, J. D., and Pinder, G. F., 1968, A Numerical Technique for Aquifer Analysis, Proc., National Symp. Anal. Water-Resources Systems., 121 p.

Bredehoeft, J. D., and Pinder, G. F., 1970, Digital Analysis of Areal Flow in Multiaquifer Groundwater Systems: A Quasi Three-Dimensional Model, Water Resources Research, Vol. 6, No. 3, p. 883-888.

Freeze, R. A., and Witherspoon, P. A., 1966, Theoretical Analysis of Regional Groundwater Flow: 1. Analytical and Numerical Solutions to the Mathematical Model, Water Resources Research, Vol. 2, No.4, p. 614-656.

Freeze, R A., and Witherspoon, 1967, Theoretical Analysis of Regional and Groundwater Flow: 2. Effect of Water Table Configuration and Subsurface Permeability Variation, Water Resources Research, Vol. 3, No 2, p. 623-634

Freeze, R. A., and Witherspoon, P. A., 1968, Theoretical Analysis of Regional Groundwater Flow: 3. Quantitative Interpretations, Water Resources Research, Vol. 4, No. 3, p. 581-590.

Frye, J. C., 1970, The Ogallala Formation -- A Review, Ogallala Aquifer Symposium, International Center for Arid and Semi-Arid Land Studies, Special Report No. 39, p. 5-14.

Hansen, H. J., 1971, Common Stratigraphic Boundaries Associated with Coastal Plain Aquifers, Groundwater, Vol. 9, No. 1, p. 5-12.

Hantush, M. S., 1967, Flow of Groundwater in Relatively Thick Leaky Aquifers, Water Resources Research, Vol. 3, No. 2, p. 583-590

Hart, D. L., Jr 1971, Records of Water Level Measurements in Wells in the Oklahoma Panhandle 1966-70, U.S.G.S. Publication.

Javandel, I., and Witherspoon, P. A., 1969, A Method of Analyzing Transient Fluid in Multilayered Aquifers, Water Resources Research, Vol. 5, No. 4, p 856-869.

Jones, O R., and Schneider, A. D., 1970, Comparison of Methods of Determining the Specific Yield of the Ogallala, Ogallala Aquifer Symposium, International Center for Arid and Semi-Arid Land Studies, Special Report No. 39, p 118-130.

Keys, W. S., and Brown, R F., 1970, The Use of Well Logging in Recharge Studies of the Ogallala Formation, Ogallala Aquifer Symposium, International Center for Arid and Semi-Arid Land Studies, Special Report No 39, p 31-48.

Lamirand, T. J., 1971, Application of a Management Model to the Ogallala Groundwater Aquifer of the Oklahoma Panhandle, Master of Science Thesis, Oklahoma State University.

Levings, G. W., 1971, A Groundwater Reconnaissance Study of the Upper Sugar Creek Watershed, Caddo County, Oklahoma, Master of Science Thesis, Oklahoma State University.

Linsley, R. K., Jr., Kohler, M. A., and Paulhus, J. L., 1958, Hydrology for Engineers, McGraw-Hill Publishing Company, Inc.

McClain, T. J., 1970, Digital Simulation of the Ogallala Aquifer in Sherman County, Kansas, Kansas State Geological Survey, Open-File Report.

McMillan, W. D., 1966, Theoretical Analysis of Groundwater Basin Operations, California University, Water Resources Center Contribution, No. 114, 167 p.

Nelson, R. W., and Cearlock, D. B., 1967, Analysis and Predictive Methods for Groundwater Flow in Large Heterogeneous Systems, Symp. Amer. Water Resources Association, Proc. Ser. No. 4 18 p.

Neuman, S. P., and Witherspoon, P. A., 1969, Theory of Flow in a Confined Two Aquifer System, Water Resources Research, Vol. 5, No. 4, p. 803 816. *Also see last entry.

Pearl, R. H., 1970, Method for Estimating Average Coefficient of Permeability Using Hydrogeologic Field Data, Ogallala Aquifer Symposium, International Center for Arid and Semi-Arid Land Studies, Special Report No. 39, p. 131-144.

Sechrist, A., Claborn, B. J., Rayner, F., and Wells, D. M., 1970, Mathematical Management Model -- Unconfined Aquifer, Final Report, O.W.R.R. Project C-1537 (No. 1993) (1), 148 p.

Weber, E. M., Peters, H. J., and Frankel, J. L., 1968, California's Digital Computer Approach to Groundwater Basin Management Studies, Symposium on Use of Analog and Digital Computers in Hydrology, Tucson, Arizona, International Association Sci., Hydrology, Publication No. 80, Vol. 1, p. 215-223.

Wood, P. R., and Hart, D. L., Jr., 1967, Availability of Groundwater in Texas County, Oklahoma, U.S.G.S. Hydrologic Investigations Atlas HA-250.

Neuman, S. P.*, and Witherspoon, P. A., 1969, Applicability of Current Theories of Flow in Leaky Aquifers, Water Resources Research, Vol. 5, No. 4, p. 817-829.

APPENDIX A

```
C***********************************************************************C
C                                                                       C
C       A MODIFIED TEXAS TECH GROUNDWATER MANAGEMENT MODEL              C
C                    OKLAHOMA STATE UNIVERSITY                          C
C                           IBM 360/65 MODEL                            C
C                                                                       C
C***********************************************************************C
C                                                                       C
C       THIS IS A PROGRAM TO MODEL FLOW WITHIN A MULTILAYERED AQUIFER   C
C       WEIGHED AVERAGE VALUE OF PERMEABILITY & SPECIFIC YIELD FOR      C
C       REMAINING SATURATED THICKNESS WILL BE ASSIGNED AT EACH TIMESTEP C
C       BASIC ASSUMPTIONS:                                              C
C                       1 THE AQUIFER IS MULTILAYERED AND IS IDEALLY    C
C                         REPRESENTED BY 4 UNIFORM LAYERS OF EQUAL THICKNESSC
C                       2 EACH LAYER IS HORIZONTALLY HOMOGENEOUS        C
C                       3 WEIGHTED AVERAGE VALUES OF PERMEABILITY AND   C
C                         SPECIFIC YIELD ASSIGNED AT EACH TIMESTEP IS A C
C                         CLOSE APPROXIMATION FOR THE AQUIFER DURING THAT C
C                         PARTICULAR TIME PERIOD                        C
C                       4 THE BEDROCK TOPOGRAPHY IS RELATIVELY SMOOTH AND C
C                         IS SLANTED ABOUT 14 FEET PER MILE             C
C                       5 THE BEDROCK AND WATER TABLE SURFACES ARE APPROX. C
C                         PARALLEL                                      C
C                       6 THERE IS NO RECHARGE OR DISCHARGE THROUGH THE C
C                         BEDROCK                                       C
C                       7 RECHARGE AND DISCHARGE AT SURFACE OR BOUNDARY OF C
C                         STUDY AREA ARE ACCOUNTED FOR BY ADJUSTMENTS ON C
C                         WITHDRAWAL FROM NODES                         C
C                                                                       C
C***********************************************************************C
C                                                                       C
C                                 ARRAYS                                C
C                                                                       C
C       A(I)= AS(I)*SY   ACRES                                          C
C       AQ(I)= WITHDRAWAL FROM NODE(I)  ACRE-FT/TIMESTEP                C
C       AQS(I)= NET SURFACE FLOW OF NODE(I) AT TIMESTEP  ACRE-FT        C
C       AS(I)= AREA OF A POLYGON(I)  ACRES                              C
C       B(I)= ELEVATION OF BEDROCK AT POLYGON INTERFACE(I)  FEET        C
C       BL(I)= BEDROCK ELEVATION AT NODE(I)  FEET                       C
C       COEFF(I)= PERMEABILITY OF SATURATED MATERIAL UNDER NODE(I) AT   C
C                  A TIMESTEP  ACRE-FT/TIMESTEP/SQ.FT                   C
C       D(I)= THICKNESS OF AQUIFER AT POLYGON INTERFACE(I) FEET         C
C       DH(I,J)= ACCUMULATIVE DRAWDOWN AT NODE(I) AND                   C
C                AT TIME TSTEP(1,I)  FEET                               C
C       EL(I)= ELEVATION OF TOP OF LAYER(I)  FEET (DATUM= 0.0)          C
C       H(I)= WATER TABLE ELEVATION AT POLYGON(I)  FEET                 C
C       HINIT(I)= INITIAL WATER LEVEL AT NODE(I)  FEET                  C
C       HO(I)= INITIAL WATER TABLE ELEVATION AT NODE(I)  FEET           C
C       HS(I,J)= WATER TABLE ELEVATION OF NODE(I) AT TSTEP(1,J)  FEET   C
C       NODE1(I),NODE2(I)= FLOWPATH EXISTS BETWEEN CENTER NODE1(I)      C
C                          AND ADJACENT NODE NODE2(I)                   C
C        N1(I),N2(I)= SAME AS NODE1,NODE2                               C
C       NWELLC(I)= WELL NUMBER OF NODE(I)                               C
C       P(I)= CONSTANT FOR FLOWPATH(I) SO THAT FLOW CAN BE CALCULATED   C
C       PL(I)= PERMEABILITY OF LAYER(I) ACRE-FT/TIMESTEP/SQ.FT          C
C       Q(I)= FLOW FROM ONE POLYGON TO ANOTHER DURING ONE TIMESTEP      C
C               ACRE-FT                                                 C
C       QS(I)= VOLUME OF WATER ABOVE GROUND SURFACE OF NODE(I)          C
```

```
C                  ACRE-FT                                                      C
C           RELAX(I)=  STORAGE  CHANGE  AT  NODE(I)  PER  TIMESTEP              C
C                  ACRE-FT/TIMESTEP                                             C
C           RES(I)=  RESIDUAL  ERROR  AT  NODE(I)  AFTER  BALANCING  ALL  FLOWS C
C                            (BY  FINITE  DIFFERENCING)  PER  TIMESTEP          C
C                            (COMPARISON  OF  VOLUME  OF  DRAFT  WITH  VOLUME   C
C                             REPRESENTED  BY  DRAWDOWN  FOR  EACH  NODE  &     C
C                            TIMESTEP)   ACRE-FT/TIMESTEP                       C
C           S(I)=  NET  WITHDRAWAL  AT  NODE(I)  FOR  A  TIME  STEP   ACRE-FT   C
C           SCL(I)=  STORAGE  COEFFICIENT  OF  LAYER(I)                         C
C           SL(I)=  SURFACE  ELEVATION  AT  NODE(I)   FEET                      C
C           SLX(I)=  SURFACE  ELEVATION  AT  NODE(I)   FEET                     C
C           TSTEP(1,I)=  TIME  CORRESPOND  TO  A  TIMESTEP   CALENDER  YEAR     C
C           XNODE(I)=  X  COORDINATE  VALUE  OF  NODE(I)   MILES                C
C           Y(I)=  WIDTH  OF  FACE/DISTANCE  BETWEEN  NODES                     C
C           YNODE(I)=  Y  COORDINATE  VALUE  OF  NODE(I)   MILES                C
C                                                                               C
C*******************************************************************************C
C                                                                               C
C                                 VARIABLE  NAMES                               C
C                                                                               C
C           AH=  AVERAGE  WATER  TABLE  ELEVATION  FOR A PARTICULAR  TIMESTEP FEETC
C           AT=  AVERAGE  SAT.  THICKNESS  OF  AQUIFER  AT  A  PARTICULAR TIMESTEPC
C           BEL=  ZERO  DATUM  &  TOP  OF  BEDROCK                              C
C           COEFFA=  FIELD  PERMEABILITY  FOR  A  PARTICULAR  TIMESTEP          C
C           DELTA=  TIMESTEP  PERIOD    YEAR                                    C
C           ERROR=  CLOSURE  ALLOWANCE  FOR  A  TIMESTEP   ACRE-FT              C
C           ITER=  NUMBER  OF ITERATIONS  DONE                                  C
C           LIST=  NUMBER  OF  YEARS  OF  STUDY                                 C
C           LMAX=  NUMBER  OF  FLOWPATHS  AT  POLYGON  INTERFACES               C
C           MAJOR=  NUMBER  OF  TIMESTEPS  WITHIN  A  YEAR                      C
C           MESS=  ERROR  MESSAGE                                               C
C           MINOR=  NUMBER  OF  MINOR  TIMESTEPS  WITHIN  MAJOR                 C
C           MM=  TOTAL  NUMBER  OF  TIMESTEPS  TO  BE  PERFORMED                C
C           MMAX=  NUMBER  OF  WELLS  UNDER  STUDY                              C
C           SK=  NAME  OF  SUBPROGRAM  TO  COMPUTE  AVERAGE  COEFFA  &  SY      C
C               FOR  A  TIMESTEP                                                C
C           SY=  STORAGE COEFFICIENT  OR  SPECIFIC  YIELD                       C
C                FOR  A  PARTICULAR  TIME  PERIOD                               C
C           TIME=  INITIAL  TIMESTEP     CALENDER YEAR                          C
C           TIME2=  FIRST  TIMESTEP    CALENDER YEAR                            C
C           ALL  OTHER  INTEGER  &  REAL  VARIABLE  NAMES=  COUNTERS            C
C                                                                               C
C*******************************************************************************C
      DIMENSION AS(24),H(24),PL(4),SCL(4),AQ(24),EL(4),A(24)
      DIMENSION S(24),COEFF(24),QS(24),HO(24),Y(106),Q(106),SLX(24)
      DIMENSION NODE1(106),NODE2(106),B(106),D(106),P(106)
      DIMENSION N1(106),N2(106)
      DIMENSION HINIT(24)
      DIMENSION BL(24),SL(24),RELAX(24),RES(24)
      DIMENSION XNODE(44),YNODE(44),NWELLC(44),AQS(24),DRY(24)
      DIMENSION TSTEP(1,400),DH(24,400),HS(24,400)
      DIMENSION WR(24),VOL(24),SAQ(24)
      DIMENSION WC(24),WNN(24)
C*******************************************************************************C
C                                                                               C
C           INITIALIZED INPUT DATA                                              C
C           CARD INPUT DATA                                                     C
```

```
C         DATA PREPARATION FOR MODEL                                            C
C                                                                               C
C*******************************************************************************C
      PL(1)=0.0420225
      PL(2)=0.066154
      PL(3)=0.10561655
      PL(4)=0.23392525
      SCL(1)=0.07
      SCL(2)=0.11
      SCL(3)=0.17
      SCL(4)=0.25
      EL(1)=400.0
      EL(2)=300.0
      EL(3)=200.0
      EL(4)=100.0
      BEL=0.0
      KK=1
      KKK=1
      KA=1
      KB=1
      ITER=0
      NYAW=1
      DATA LMAX,MMAX /106,24/
      READ(5,101)LIST,MAJOR,MINOR
      MM=LIST*MAJOR
      READ(5,102)ERROR,TIME
      DO 1211 I=1,MMAX
      READ(5,1210) XNODE(I),YNODE(I),NWELLC(I)
 1211 CONTINUE
      DO 131 M=1,MMAX
  131 READ(5,14)N1(M),AQ(M)
C
C  CHECK DATA FOR CORRECT ORDER
C
      DO 140 M=1,MMAX
      IF(N1(M)-NWELLC(M))139,140,139
  139 MESS=1
      II=N1(M)
      III=M
      JJ=NWELLC(M)
      JJJ=M
C
C  MESSAGE=1 FEED DATA FOR A WELL NOT IN CLASS C OR OUT OF ORDER
C
      GO TO 10000
  140 CONTINUE
      DELTA=1./FLOAT(MAJOR*MINOR)
      DO 15 M=1,MMAX
   15 AQ(M)=-AQ(M)
      READ(5,100)(NODE1(L),NODE2(L),Y(L),L=1,LMAX)
      READ(5,104)(N1(M),BL(M),SL(M),AS(M),H(M),M=1,MMAX)
      CALL        SK(AS,H,PL,SCL,BL,COEFFA,BEL,KK,TIME,ITER,EL,A)
C
C  CHECK FOR OUT OF ORDER CARDS
C
      DO 105 M=1,MMAX
      IF(N1(M)-NWELLC(M))106,105,106
  106 MESS=2
```

```
      II=N1(M)
      III=M
      JJ=NWELLC(M)
      JJJ=M
C
C  MESSAGE=2 PHYSICAL WELL DATA FOR A WELL NOT IN CLASS C OR OUT OF ORDER
C  WELL WAS READ
C
      GO TO 10000
  105 CONTINUE
      DO 103 M=1,MMAX
  103 COEFF(M)=COEFFA
  999 DO 998 M=1,MMAX
  998 SLX(M)=SL(M)
C
C  IDENTIFY THE POSITION IN THE NWELLC ARRAY OF THE WELL NUMBERS IN THE NODE1
C  AND NODE2 ARRAYS.  STORE THIS POSITION NUMBER IN N1 AND N2
C
      DO 1400 M=1,LMAX
      IF(M-1)990,990,985
  985 IF(NODE1(M)-NODE1(M-1))990,986,990
  986 N1(M)=N1(M-1)
      GO TO 1105
  990 DO 1000 L=1,MMAX
      IF(NODE1(M)-NWELLC(L))1000,1100,1000
 1000 CONTINUE
      MESS=4
      II=NODE1(M)
      III=M
      JJ=NWELLC(L)
      JJJ=L
C
C  MESSAGE=4 NODE1 WAS NOT FOUND IN THE CLASS C WELLS
C
      GO TO 10000
 1100 N1(M)=L
 1105 DO 1200 L=1,MMAX
      IF(NODE2(M)-NWELLC(L))1200,1300,1200
 1200 CONTINUE
      MESS=5
      II=NODE2(M)
      III=M
      JJ=NWELLC(L)
      JJJ=L
C
C  MESSAGE=5 NODE2 WAS NOT FOUND IN THE CLASS C WELLS
C
 1300 N2(M)=L
 1400 CONTINUE
      DO 108 L=1,LMAX
      M=N1(L)
      N=N2(L)
      B(L)=(BL(M)+BL(N))*.5
  989 D(L)=(SLX(M)+SLX(N))/2.-B(L)
      P(L)=Y(L)*COEFFA
  108 CONTINUE
C**********************************************************************C
C                                                                      C
```

```
C         OUTPUT OF INITIAL CONDITION DATA & HEADINGS                        C
C                                                                            C
C****************************************************************************C
      WRITE(6,200)
      WRITE(6,670)
      WRITE(6,201)(M,NWELLC(M),A(M),SL(M),BL(M),H(M),M=1,MMAX)
      WRITE(6,202)
      WRITE(6,203)(L,NODE1(L),NODE2(L),P(L),B(L),D(L),L=1,LMAX)
      WRITE(6,204) LIST,MAJOR,MINOR,ERROR,COEFFA
      TIME2=TIME+FLOAT(LIST)
      WRITE(6,205) TIME,TIME2
      DO 666 I=1,MMAX
      SAQ(I)=AQ(I)*0.01
  666 HINIT(I)=H(I)
      DO 255 I=1,MMAX
  255 WR(I)=AQ(I)
      DO 150 L=1,LMAX
      B(L)=2.*B(L)
  150 P(L)=.5*P(L)
C****************************************************************************C
C                                                                            C
C         START OF MATH MODEL             START OF MATH MODEL                C
C                                                                            C
C****************************************************************************C
      DC10=0.0
      DO 600 LISTS=1,LIST
      DC1=0.0
      DO 601 M=1,MMAX
      DRY(M)=0.
  601 AQS(M)= 0.
      JDRY =0
      DO 500 MAJORS=1,MAJOR
      ITER=0
      DO 400 MINORS=1,MINOR
    1 TIME=TIME+DELTA
      DO 2 M=1,MMAX
      HO(M)=AMAX1(BL(M),H(M))
    2 SAQ(M)=SAQ(M)+AQS(M)
      IDRY = 0
    3 DO 4 M=1,MMAX
      RELAX(M)=A(M)/DELTA
      S(M)=RELAX(M)*(AMAX1(BL(M),H(M))-HO(M))
    4 RES(M)=SAQ(M)-S(M)
      ITER=ITER+1
      IF(ITER.GT.30)WRITE(6,598)
      IF(ITER.GT.30)MM=IFIX(TIME-1966.0)*4
      IF(ITER.GT.30)GO TO 900
      DO 5 L=1,LMAX
      N=N1(L)
      M=N2(L)
      Y(L)=P(L)*AMAX1(0.,H(M)+H(N)-B(L))
C
C  PREVENT FLOW FROM A DRY POLYGON
C
      IF (H(N)-H(M))701,703,711
C
C  FLOW FROM M TO N, M MUST NOT BE DRY
C
```

```
  701 IF(H(M)-BL(M))703,703,705
  703 Q(L)=0.
      GO TO 770
C
C  FLOW FROM N TO M, N MUST NOT BE DRY
C
  711 IF(H(N)-BL(N))703,703,705
  705 CONTINUE
      Q(L)=Y(L)*(H(M)-H(N))
  770 CONTINUE
      RELAX(M)=RELAX(M)+Y(L)
      RELAX(N)=RELAX(N)+Y(L)
      RES(M)=RES(M)-Q(L)
    5 RES(N)=RES(N)+Q(L)
    8 DO 12 M=1,MMAX
      RELAX(M)=1.0/RELAX(M)
      H(M)=AMAX1((H(M)+RELAX(M)*RES(M)),BL(M))
      IF(QS(M))12,9,9
    9 IF(H(M)-SL(M))11,11,10
   10 QS(M)=RES(M)
      RES(M)=0.
      H(M)=SL(M)
      GO TO 12
   11 QS(M)=0.
   12 CONTINUE
      DO 13 M=1,MMAX
      IF(ERROR-ABS(RES(M)))33,13,13
   33 IF(H(M)-BL(M))3,34,3
   34 IDRY=IDRY+1
   13 CONTINUE
      IF(IDRY)400,400,390
  390 DO 395 M=1,MMAX
      IF(H(M)-BL(M))391,391,395
  391 JDRY=1
      DRY(M)=DRY(M)+RES(M)
  395 CONTINUE
  400 CONTINUE
      TSTEP(1,KB)=TIME
      KB=KB+1
C******************************************************************************C
C                                                                              C
C        CALL FOR COEFFA & SY FOR A NEW TIMESTEP                               C
C                                                                              C
C******************************************************************************C
      CALL       SK(AS,H,PL,SCL,BL,COEFFA,BEL,KK,TIME,ITER,EL,A)
      IF(ITER.EQ.-1)MM=IFIX(TIME-1966.0)*4
      IF(ITER.EQ.-1)GO TO 900
  549 DO 548 I=1,MMAX
  548 HS(I,KKK)=H(I)
      DO 550 I=1,MMAX
  550 DH(I,KA)=HINIT(I)-H(I)
      KA=KA+1
      KKK=KKK+1
      QST=0.
      DO 403 M=1,MMAX
      IF(QS(M))403,403,401
  401 WRITE(6,402)M,QS(M)
      QST=QST+QS(M)
```

```
  403 CONTINUE
      DO 256 I=1,MMAX
      IF(KK.EQ.1)SAQ(I)=WR(I)*1.00
  256 CONTINUE
      KK=KK+1
      DO 250 I=1,MMAX
      IF(KK.EQ.2)SAQ(I)=WR(I)*1.00
      IF(KK.EQ.3)SAQ(I)=WR(I)*1.00
      IF(KK.EQ.4)SAQ(I)=WR(I)*1.00
  250 CONTINUE
  500 CONTINUE
      DO 254 I=1,MMAX
      VOL(I)=A(I)*(H(I)-BL(I))
      IF(VOL(I).GE.-AQ(I))WR(I)=AQ(I)
      IF(VOL(I).LT.-AQ(I))WR(I)=-VOL(I)
      Z=-AQ(I)
      DC1=DC1+WR(I)
      WC(I)=VOL(I)*160.0/AS(I)
      WNN(I)=ABS(WR(I)/WC(I))
      IF(WNN(I).LT.1.0)WNN(I)=1.0
      IF(VOL(I).GE.-AQ(I))WRITE(6,260)WNN(I),I,VOL(I),Z
      IF(VOL(I).LT.-AQ(I))WRITE(6,261)WNN(I),I,VOL(I),Z
  254 AQ(I)=WR(I)
      IF(JDRY) 5010, 5010, 5000
 5000 DO 5005 M=1,MMAX
      IF(DRY(M))5003,5005,5003
 5003 DRY(M)=AQ(M)-DRY(M)*DELTA
      WRITE(6,5002)M,NWELLC(M),AQ(M),DRY(M)
 5005 CONTINUE
 5010 CONTINUE
      DC10=DC10+DC1
      IF(MOD(LISTS,NYAW).NE.0)GO TO 600
      WRITE(6,252)NYAW,DC10
      DC10=0.0
  600 CONTINUE
C******************************************************************************C
C                                                                              C
C        END OF MATH MODEL             END OF MATH MODEL                       C
C                                                                              C
C******************************************************************************C
C                                                                              C
C        FINAL OUTPUT :      WATER LEVEL VS TIMESTEP                           C
C                            ACCUMULATIVE DRAWDOWN VS TIMESTEP                 C
C                            (PRINTED OUTPUT & PLOTS)                          C
C                                                                              C
C******************************************************************************C
  900 WRITE(6,301)
      WRITE(6,667)
      LL=1
      DO 510 L=10,MM,10
      WRITE(6,545)((TSTEP(I,J),I=1,1),J=LL,L)
      DO 529 I=1,3
  529 WRITE(6,543)
      DO 511 I=1,24
  511 WRITE(6,544)I,(HS(I,J),J=LL,L)
      DO 530 I=1,3
  530 WRITE(6,543)
  510 LL=LL+10
```

```
0240               WRITE(6,301)
0241               WRITE(6,668)
0242               NN=1
0243               DO 512 N=10,MM,10
0244               WRITE(6,545)((TSTEP(I,J),I=1,1),J=NN,N)
0245               DO 675 I=1,3
0246           675 WRITE(6,543)
0247               DO 513 I=1,MMAX
0248           513 WRITE(6,542)I,(DH(I,J),J=NN,N)
0249               DO 514 I=1,3
0250           514 WRITE(6, 543)
0251           512 NN=NN+10
0252               WRITE(6,301)
0253               CALL PLOTS
0254               CALL APLOT(TSTEP,DH,MM)
          C**********************************************************************C
          C                                                                      C
          C         FORMAT  STATEMENTS             FORMAT  STATEMENTS            C
          C                                                                      C
          C**********************************************************************C
0255            14 FORMAT(I7,F8.0)
0256           100 FORMAT(2(I7,1X,I7,1X,F10.2,1X),I7,1X,I7,1X,F10.2)
0257           101 FORMAT(3I10)
0258           102 FORMAT(3F13.4)
0259           104 FORMAT(I7,5X,F7.0,6X,F7.0,6X,F11.0,11X,F11.0)
0260           200 FORMAT('      MATHEMATICAL  MODEL  OF  GROUNDWATER  FLOW  IN  A  MU
              QLTILAYER  CASE'///'      24  INTERIOR  NODES  AND  106  POLYGON  CO
              QNTACT  FACES'///)
0261           201 FORMAT(I4,11X,I7,10X,4HAS= ,F8.1,4X,4HSL= ,F8.1,4X,4HBL = ,F8.1,4X
              Q,8HHINIT = ,F8.1)
0262           202 FORMAT(//////7H BRANCH,4X,20HBETWEEN WELL NUMBERS 4X,19HPSEUDO-PER
              2MEABILITY,3X,17H BOTTOM ELEVATION,6X,10H THICKNESS//)
0263           203 FORMAT(I6,9X,I7,3X,I7,7X,3HK= ,F8.4,10X,3HB= ,F8.1,9X,3HD= ,F8.1)
0264           204 FORMAT(/////7H LIST =I6/8H MAJOR =I5/8H MINOR =I5/8H ERROR =F8.2/
              19H COEFFA =F7.4)
0265           205 FORMAT(/////16H SIMULATION FROM ,F8.2,3H TO,F8.2)
0266           252 FORMAT(I7,'      YEAR ADJUSTED WITHDRAWAL = ',F15.2)
0267           253 FORMAT('      WELL ',I5,'    HAS HIT BOTTOM')
0268           260 FORMAT(F10.0,'      WELLS ARE NEEDED AT NODE',I10,'      NODE CAPACI
              QTY=',F10.1,'   DEMAND=',F10.1,'   DEMAND < NODE CAPACITY')
0269           261 FORMAT(F10.0,'      WELLS ARE NEEDED AT NODE',I10,'      NODE CAPACI
              QTY=',F10.1,'   DEMAND=',F10.1,'   DEMAND > NODE CAPACITY')
0270           301 FORMAT(1H1)
0271           402 FORMAT(5H NODE,I4,3X,4HQS= F8.1)
0272           405 FORMAT(/22H TOTAL SURFACE FLOW = F10.1)
0273           542 FORMAT(I10,10F10.5)
0274           543 FORMAT(1H0)
0275           544 FORMAT(I10,10F10.2)
0276           545 FORMAT('  WELL NO ',10F10.2)
0277           598 FORMAT('            OVERFLOW          ')
0278           667 FORMAT('          DISPLAY  OF  WATER  LEVEL  CORRESPOND  TO  TIME  ST
              QEP'///)
0279           668 FORMAT('          DISPLAY  OF  ACCUMULATIVE  DRAWDOWN  VS  TIMESTEP'/
              Q//)
0280           670 FORMAT(' NODE              WELL NO                      AREA    SURFACE EL
              QEV    BEDROCK ELEV     WATER LEVEL'///)
0281          1210 FORMAT(2F10.2,I5)
0282          5002 FORMAT(1H 4HNODE I5,16H STATE WELL NO. I9,31H WITH EXPECTED WITHDR
```

```
                  1RAWALS OF F10.0,19H ACRE FT REDUCED TO F10.0, 8H ACRE FT)
 0283       11000 FORMAT(1H ,19HTROUBLE AT MESSAGE  I4,9HWELL NO. I7,11H SUBSCRIPT
                  1I4,14H AND WELL NO. I7,11H SUBSCRIPT I4)
 0284       11100 FORMAT(1H ,10F12.1)
 0285             STOP
 0286       10000 WRITE(6,11000) MESS,II,III,JJ,JJJ
 0287             STOP
 0288             END
```

```
      SUBROUTINE SK(AS,H,PL,SCL,BL,COEFFA,BEL,KK,TIME,ITER,EL,A)
      DIMENSION AS(24),H(24),PL( 4),SCL(24),BL(24),EL(4),A(24)
      NLA=5
C
C        NL INDICATES THE NUMBER OF LAYERS USED IN THE MODEL
C
      NL=4
      AVH=0.0
      DO 1 I=1,24
C
C        CHECK IF WATER LEVEL HIT BEDROCK , IF SO, STOP OPERATION
C
C     IF(H(I).LE.BL(I))WRITE(6,9)I,H(I),TIME
C   9 FORMAT('   WELL   NO   ',I5,'   HAS   REACHED   BOTTOM   AT
C    Q2F10.2)
C     IF(H(I).LE.BL(I))RETURN
    1 AVH=AVH+(H(I)-BL(I))
      AH=AVH/24.0
      AT=AH-BEL
C
C        COMPUTATIONS OF WEIGHED AVERAGE VALUES OF K & SY
C
      IF(NL.EQ.1)GO TO 23
      DO 20 I=2,NLA
      IF(I.EQ.NLA)GO TO 23
      IF (AH.GE.EL(I)) ML=I
      IF(AH. GE. EL(I)) GO TO 21
   20 CONTINUE
   23 IF(AH.GE.BEL)COEFFA=PL(NL)
      IF(AH.GE.BEL)SY=SCL(NL)
      GO TO 10
   21 COEFFA=PL(ML-1)*(AH-EL(ML))+PL(NL)*(EL(NL)-BEL)
      SY=SCL(ML-1)*(AH-EL(ML))+SCL(NL)*(EL(NL)-BEL)
      IF(ML.EQ.NL)COEFFA=COEFFA/AT
      IF(ML.EQ.NL)SY=SY/AT
      IF(ML.EQ.NL) GO TO 10
      COEF=0.0
      SPY=0.0
      NNN=NL-ML
      DO 22 I=1,NNN
      COEF=PL(ML)*(EL(ML)-EL(ML+1))+COEF
      SPY=SCL(ML)*(EL(ML)-EL(ML+1))+SPY
   22 ML =ML+1
      COEFFA =(COEF+COEFFA)/AT
      SY=(SPY+SY)/AT
C
C        COMPUTE WITHDRAWAL FROM NODES FOR A TIMESTEP
C
   10 DO 3 I=1,24
    3 A (I)=AS(I)*SY
      IF(ITER.EQ.0)RETURN
      IF(KK.GT.4)KK=1
      WRITE (6,100)TIME,KK,AH,COEFFA,SY,ITER
  100 FORMAT(1H0,'    TIME=',F10.2,'    SEASON=',I5,'    AVE  SAT  THICK
     QNESS=',F10.2/'    AVE  PERMEABILITY=',F10.6,'    AVE  SY=',F10.6,'
     Q   ITERATION=',I5)
      RETURN
      END
```

```
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      SUBROUTINE APLOT(TSTEP,DH,MM)
      DIMENSION TSTEP(1,400),DH(24,400),X(400),Y(400)
C
C     NP INDICATES NUMBER OF HYDROGRAPHS TO BE PLOTTED
C
      NP=24
      J=MM-1
      J1=J+1
      J2=J+2
      XIN=FLOAT(J/40)+2.0
      XAX=0.0
      DO 1 I=1,J
    1 X(I)=TSTEP(1,I)
      X(J1)=1966.25
      X(J2)=10.0
C
C     START OF PLOTTING
C
      DO 2 I=1,NP
      DO 3 L=1,J
    3 Y(L)=DH(I,L)
      Y(J1)=0.0
      Y(J2)=40.0
      CALL PLOTC(XAX,-11.0,-3)
      XAX=0.0
      CALL PLOTC(XAX,0.5,-3)
      CALL AXIS(0.0,0.0,'TIME YEAR',-9,XIN,0.0,X(J1),X(J2))
      CALL AXIS(0.0,0.0,'ACCUMULATIVE DRAWDOWN FT',24,10.0,90.0,Y(J1),Y
     Q(J2))
      CALL LINE(X,Y,J,1,0,64)
      FPN=FLOAT(I)
      CALL SYMBOL(1.0,9.0,0.35,'WELL ',0.0,5)
      CALL NUMBER(999.0,999.0,0.35,FPN,0.0,-1)
      CALL PLOTC(XAX,-11.0,-3)
      XAX=XIN+3
    2 CONTINUE
      RETURN
      END
```