Final Report

OWRR A-027-OKLA

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A DIMENSIONLESS PARAMETER STUDY OF GROUNDWATER RECHARGE IN OKLAHOMA

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Submitted to

OKLAHOMA WATER RESOURCES RESEARCH INSTITUTE

Submitted by

UNIVERSITY OF OKLAHOMA RESEARCH INSTITUTE
NORMAN, OKLAHOMA
June, 1971

ACKNOWLEDGMENT

The material presented in this report is the result of research work sponsored by the Oklahoma Water Resources Research Institute.

The study was conducted at The University of Oklahoma School of Civil Engineering and Environmental Science. The required data for this study were provided by Mr. Ray Duffy, Utility Superintendent of the City of Frederick, Oklahoma. The typing was performed by Mrs. Lizette Bagdadipour.

Norman, Oklahoma
June, 1971

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A DIMENSIONLESS PARAMETER STUDY OF GROUNDWATER RECHARGE IN OKLAHOMA

CHAPTER I

INTRODUCTION

The use of groundwater during this decade is being accelerated to meet the growing demands for water by industries, cities and towns, and farming communities throughout the world. This has created a multitude of practical problems, among which "mining of water" and the attendant depletion damage to the aquifer, the unpleasant law suits, and the use of lower quality impoundment water are but a few. Additionally, the engineering problems associated with groundwater have become more acute, and their assessment indicates that recharge is of paramount significance. This is not to say that the problems of sustained yields of wells and aquifers, the interference between wells and well fields, and the quality of groundwater are not important. It rather implies that before planning the utilization of the available groundwater, the recharge rate should be correctly estimated and become an integral part of the analysis and design process. Thus, the principle of conservation is duly considered, and hopefully solutions reflect compliance with it.

Groundwater is continuously being discharged from an aquifer through any of the following or combinations thereof: pumping, percolation into another aquifer, effluent seepage, evaporation, and transpiration by plants. The Water Resources Board of Oklahoma uses the coefficient method to account for losses due to evaporation and transpiration (1). To compute the exact amount of losses due to transpiration and evaporation is extremely difficult. Intensive studies and reliable data will be required to compute the exact amount of losses due to these factors. The biggest handicap in such a study would be gathering of extensive and reliable data under controlled conditions.

Under natural conditions an aquifer may receive water from one or more of several sources. Recharge may be effected from streams that introduce water from outside the catchment area of the aquifer, from percolation, from another aquifer, and from infiltration of precipitation. Under controlled conditions, the replenishment of groundwater could also be accomplished through artificial recharge.

Today (1971), in order to compute recharge of the groundwater by rainfall, the simple method of coefficient is most often used (1). In this method the recharge rate Q_r is computed by the following formula:

$$Q_r = C\ i\ A \quad . . . . (1)$$

where C is a coefficient relating to the infiltration of water through the ground, i is the rainfall intensity, and A is the area contributing to the recharge of the aquifer.

It could easily be seen that the method of coefficient described above does not incorporate all the parameters involved in computing the net amount of recharge. The attempt of this study, then, is to derive Q_r as a function of all the involved parameters in the form of dimensionless terms. The different parameters and dimensionless terms will be discussed later.

CHAPTER II

LITERATURE SURVEY

In order to discuss the purpose of this study more fully, it is necessary to review, briefly, some of the research results that have been obtained in this area as related to the use of dimensionless parameters.

The authors referred to in the bibliography, Yeh (2), Singh (3), Hunt (4), Ahmed and Sunada (5), and Esmaili and Scott (6), have all integrated different parameters to form dimensionless terms in order to solve complex equations encountered in various hydraulics problems. In their studies they show the twofold advantages of the use of these dimensionless terms. First, the use of dimensionless terms reduces the number of terms involved in their equations, and second, the equations are independent of the system of units being used.

In an attempt to apply the results of their work to an example, the majority of these researchers (2, 3, 4, 5) have introduced dimensionless variables in their governing equations in order to make numerical computations more convenient. On the other hand, Esmaili and Scott (6) developed numerical solutions to determine the aquifer characteristics and unsteady radial flow through injection wells in unconfined aquifers under constant drawdown and injection pressure conditions. Their approach is based on a numerical solution of the basic differential equation of flow which is transformed into an explicit form through the introduction of appropriate dimensionless parameters. Additionally, dimensionless graphical solutions are presented for the rate and accumulative volume of discharge and recharge. As depicted in Figure 1, recharge is plotted versus time for injection wells in unconfined aquifers. The dimensionless parameters are:

$$Y_1 = Q/KH_w^2 \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (2)$$

$$X_1 = H_o/H_w \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (3)$$

$$X_2 = TKH_w/SR_w^2 \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (4)$$

$$Y_1 = f(X_1, X_2) \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (5)$$

where, Q = recharge rate, cubic feet per hour

H_o = piezometric head at time zero, feet

H_w = piezometric head in the well, feet

T = time, hours

S = storage coefficient

K = coefficient of the hydraulic conductivity, feet per hour

R_w = well radius, feet

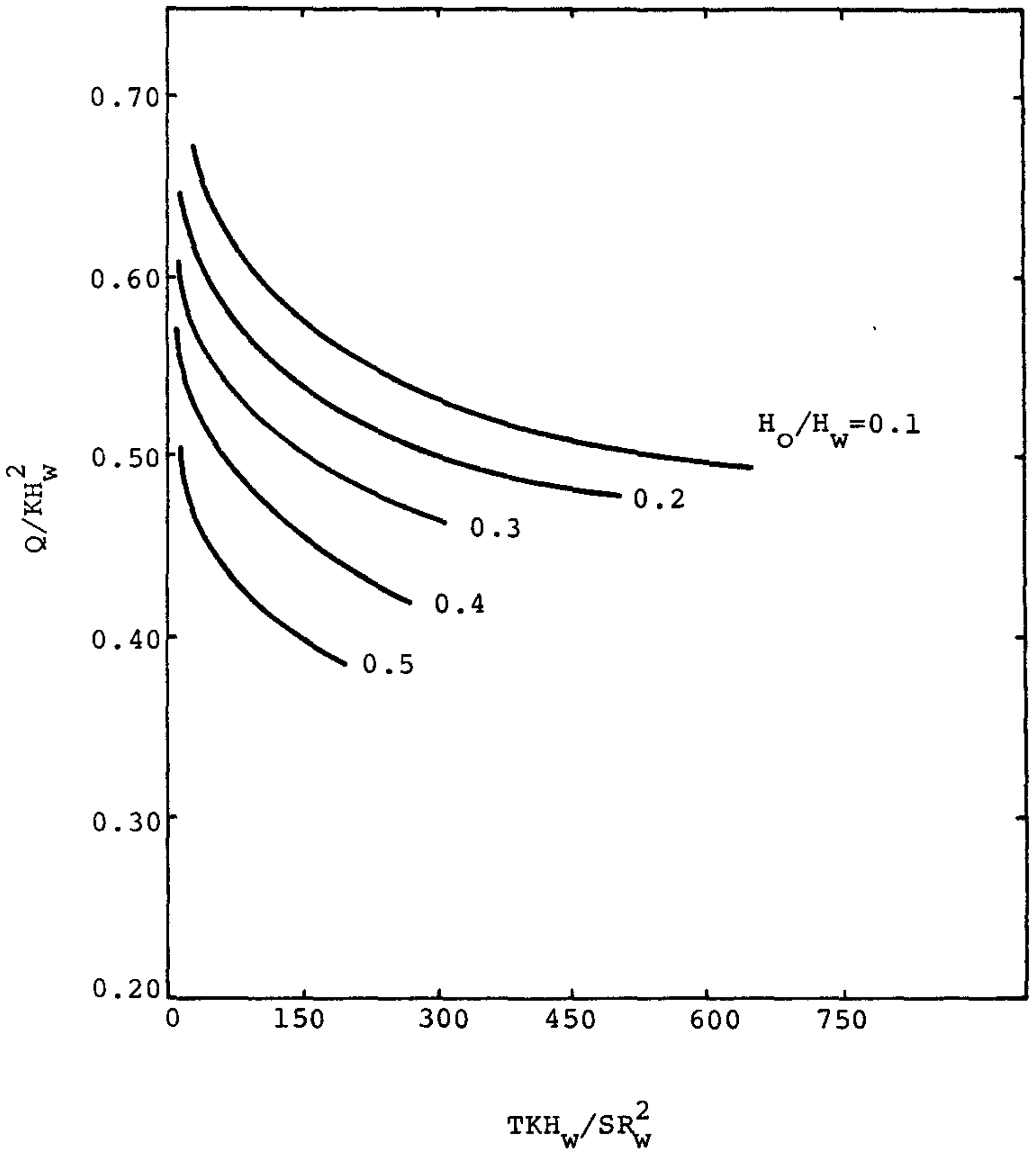

FIG.1.-GRAPH OF RECHARGE RATE VERSUS TIME FOR INJECTION WELLS IN UNCONFINED AQUIFERS (6)

Some of the above parameters are illustrated in Figure 2.

It should be noted that although the aforementioned work is in the field of groundwater, it does not deal with the problem of computing recharge due to rainfall; it is related to the present study only insofar as the use of dimensionless terms is concerned.

Thus, the method of approach presented herein can be considered as the first attempt to put groundwater recharge on a mathematically sound basis.

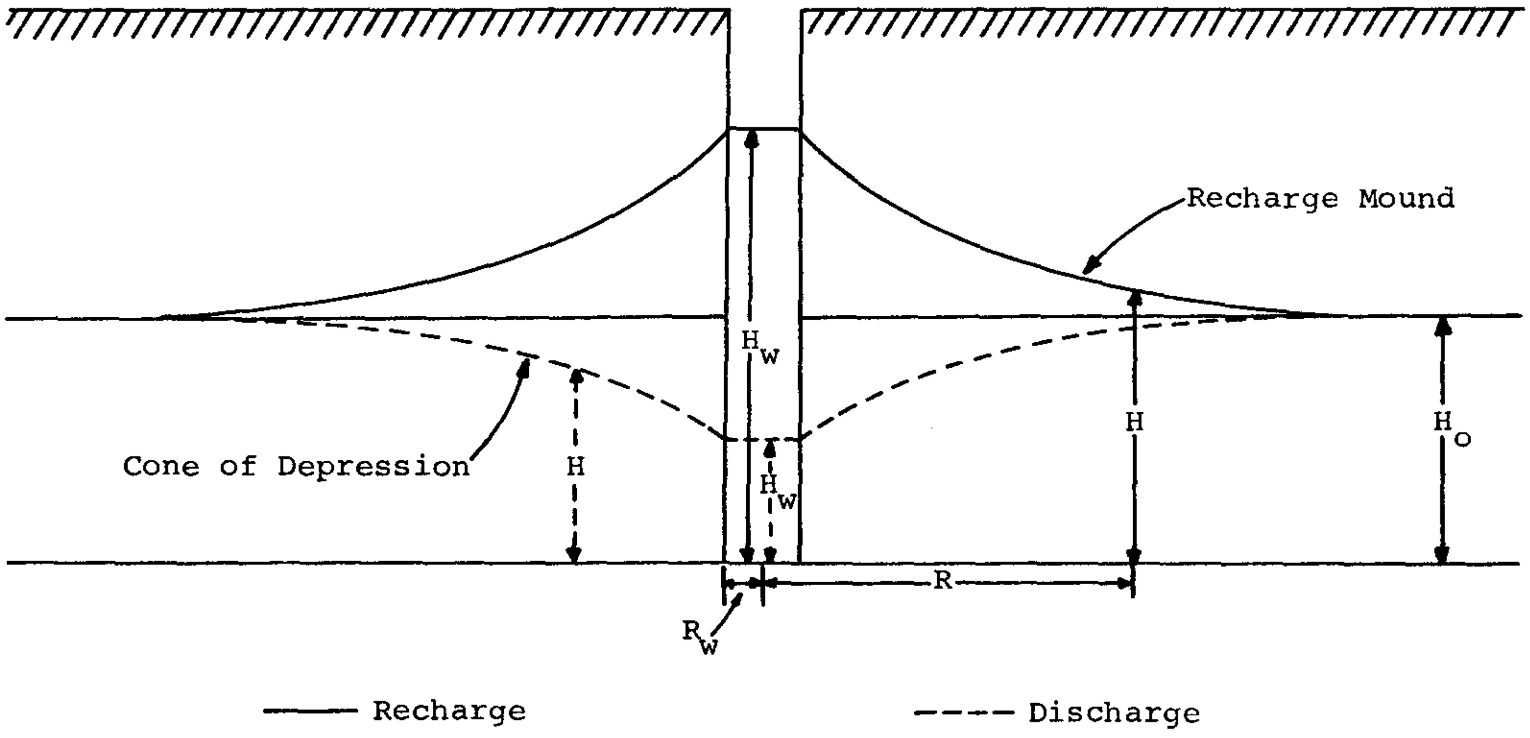

FIG.2.-DEFINITION SKETCH FOR UNSTEADY RADIAL FLOW IN UNCONFINED AQUIFERS

CHAPTER III

COLLECTION OF DATA

In order to obtain reliable data for this study, all possible channels were tried. With the use of the geological map of the State of Oklahoma, a number of cities located on different aquifers were chosen, and questionnaires were mailed out to the city managers of these cities inquiring about the availability of data concerning the wells located in their cities. The cities chosen were the following: Guymon, Woodward, Arnett, Watonga, Shawnee, Okemah, Pauls Valley, Sayre, and Frederick, all on terrace and alluvial deposits; Cordell, Anadarko, Duncan, Durant, Hugo, and Idabel, all on the sandstone aquifers; and Sulphur, located on the Arbuckle formations. The major groundwater aquifers in Oklahoma and the locations of the cities mentioned above are shown in Figure 3.

The response to these questionnaires was extremely poor as evidenced by the fact that only three replies were received, two of which stated that surface water was being used in their cities but that they did not have any records of the wells used in the past.

The third reply was from the City of Frederick in Tillman County located in the southwestern corner of Oklahoma.

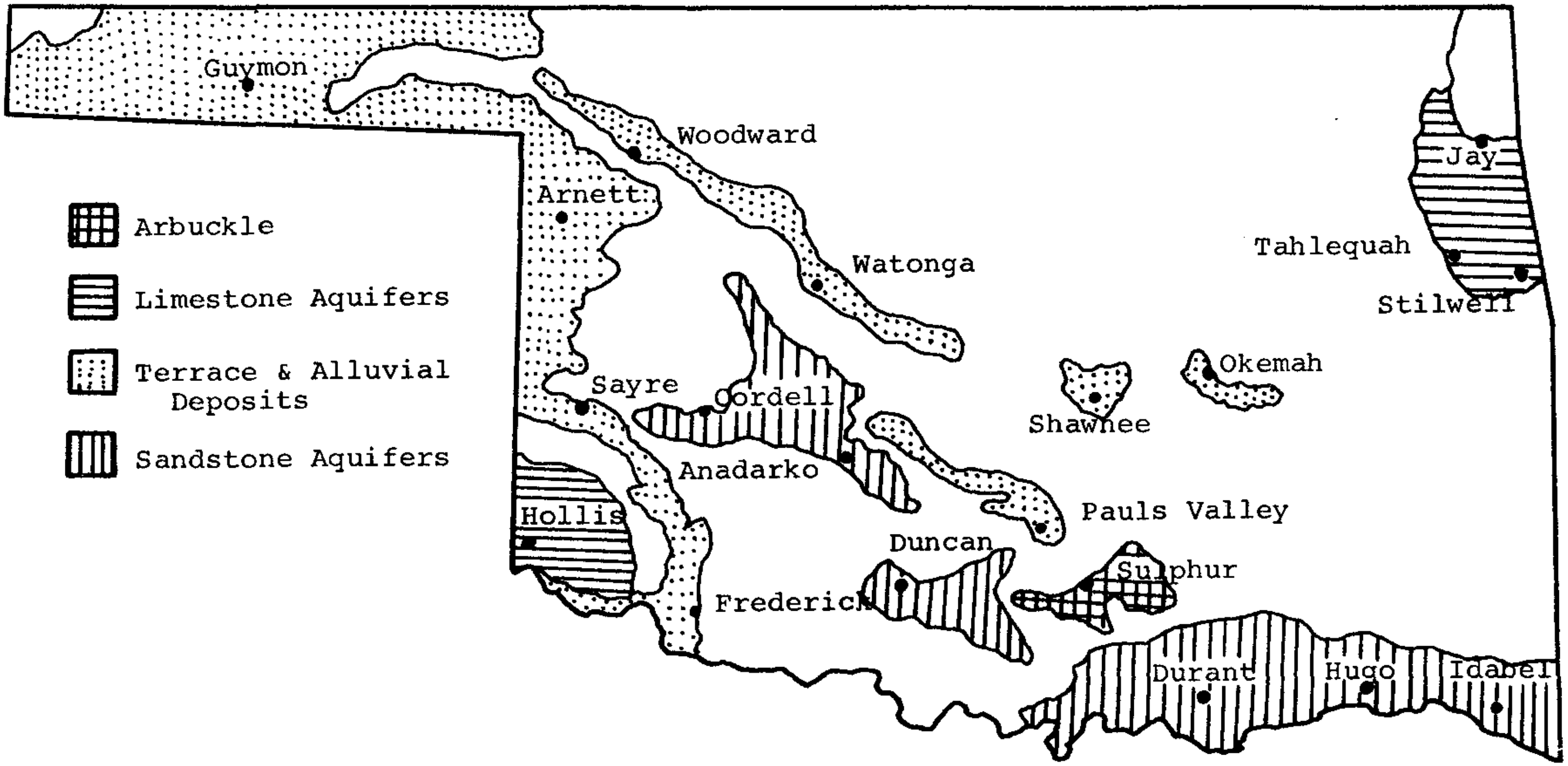

FIG. 3.-LOCATION OF THE CITIES CONTACTED ON THE DIFFERENT AQUIFERS IN OKLAHOMA

The data were reliable enough for the purposes of this study, and they related to a number of city-owned wells which are being used today. Thus, the data provided information concerning the static head readings of the wells, the depth of the wells, the amount of pumpage, soil characteristics, location of wells, and the rainfall intensities.

The amount of pumpage and location of these wells are indicated in Table 1. The static head readings of these wells are plotted in Figures 4 through 8. Table 2 shows the amount of rainfall for each month of 1969. It should be noted that the values obtained are for the year 1969 only, and for better conclusive results in a study of this type, such data for a longer period of time would be most desirable.

The data from the City of Norman were rather incomplete. Other attempts to inquire about the availability of reliable and sufficient data through the Oklahoma Water Resources Board and a number of private sources failed to produce any positive results. Lack of data rather than unwillingness of agencies and people in charge is the main reason for the inability to obtain the required data.

TABLE 1.-AMOUNT OF PUMPAGE FOR 1969 OF THE FIVE WELLS IN THE CITY OF FREDERICK, OKLAHOMA

Well Identification and Location	Month	Amount Pumped, gallons	Total Pumped, gallons
#A	Jan.	-------	
	Feb.	-------	
S.W. 1/4, Sec. 4	March	199,000	
T2S R18W	April	-------	
	May	81,000	
	June	2,427,000	
	July	3,284,000	
	Aug.	2,247,000	
	Sept.	994,000	
	Oct.	887,000	
	Nov.	790,000	
	Dec.	1,704,000	
			12,603,000
#B	Jan.	-------	
	Feb.	-------	
S.E. 1/2, S. 1/2,	March	457,000	
Sec. 5, T2S R18W	April	3,641,000	
	May	3,592,000	
	June	3,875,000	
	July	3,644,000	
	Aug.	2,153,000	
	Sept.	-------	
	Oct.	2,740,000	
	Nov.	2,697,000	
	Dec.	3,473,000	
			26,272,000
#C	Jan.	681,000	
	Feb.	156,000	
S.E. 1/4, S. 1/2,	March	134,000	
Sec. 5, T2S R18W	April	-------	
	May	-------	
	June	2,641,000	
	July	4,489,000	
	Aug.	2,974,000	
	Sept.	1,239,000	
	Oct.	1,068,000	
	Nov.	971,000	
	Dec.	1,436,000	
			15,789,000

TABLE 1.-CONT.

Well Identification and Location	Month	Amount Pumped, gallons	Total Pumped, gallons
#D	Jan.	17,000	
	Feb.	22,000	
S.E. 1/4, S. 1/2,	March	1,000	
Sec. 5, T2S R18W	April	-------	
	May	17,000	
	June	2,137,000	
	July	5,235,000	
	Aug.	2,782,000	
	Sept.	921,000	
	Oct.	-------	
	Nov.	737,000	
	Dec.	394,000	
			12,263,000
#E	Jan.	74,000	
	Feb.	86,000	
S.E. 1/4, S. 1/2,	March	1,000	
Sec. 5, T2S R18W	April	-------	
	May	60,000	
	June	2,044,000	
	July	2,658,000	
	Aug.	1,608,000	
	Sept.	490,000	
	Oct.	9,000	
	Nov.	411,000	
	Dec.	398,000	
			7,839,000

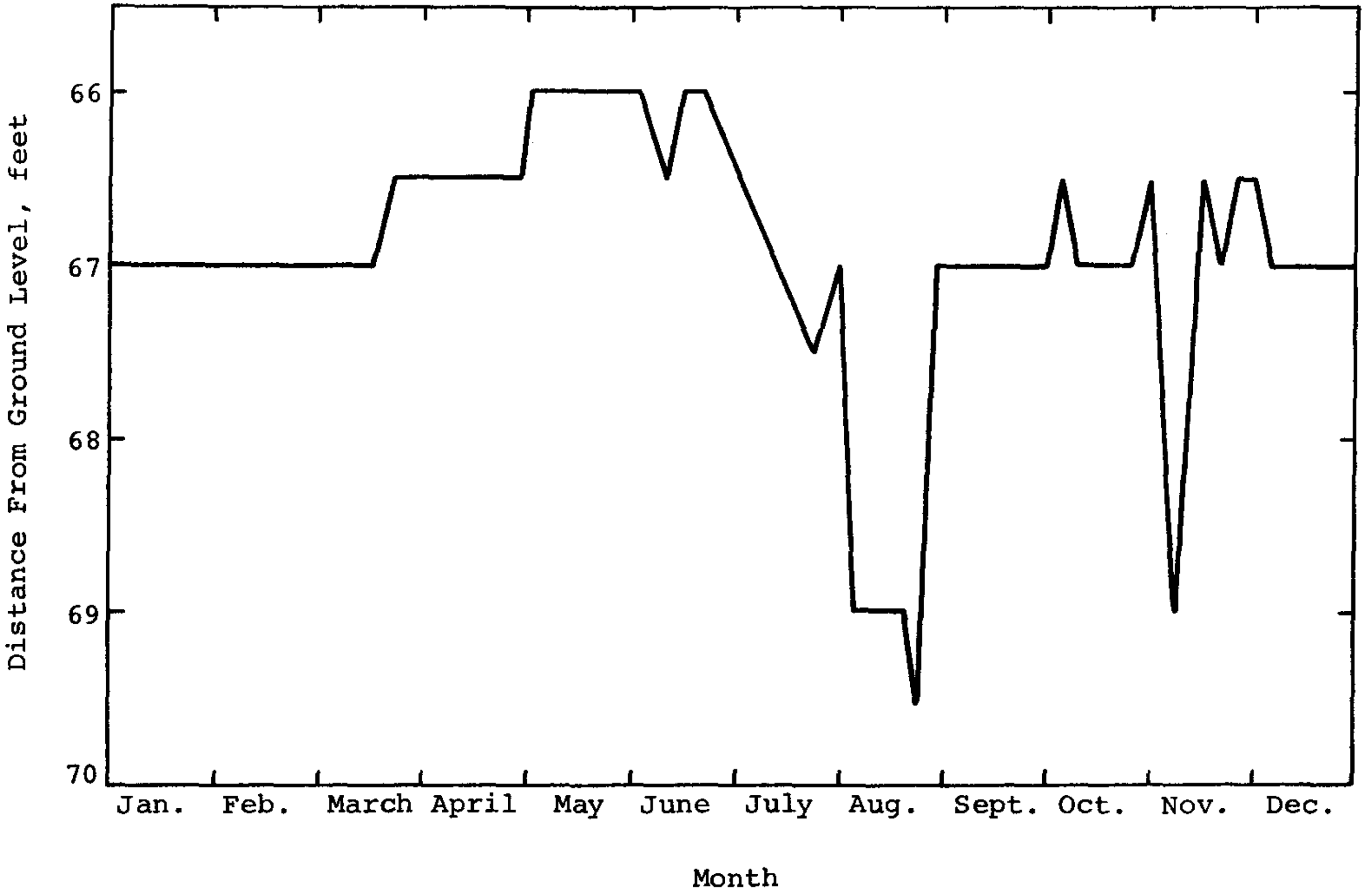

FIG.4.-STATIC LEVEL OF WELL # A FOR THE YEAR 1969

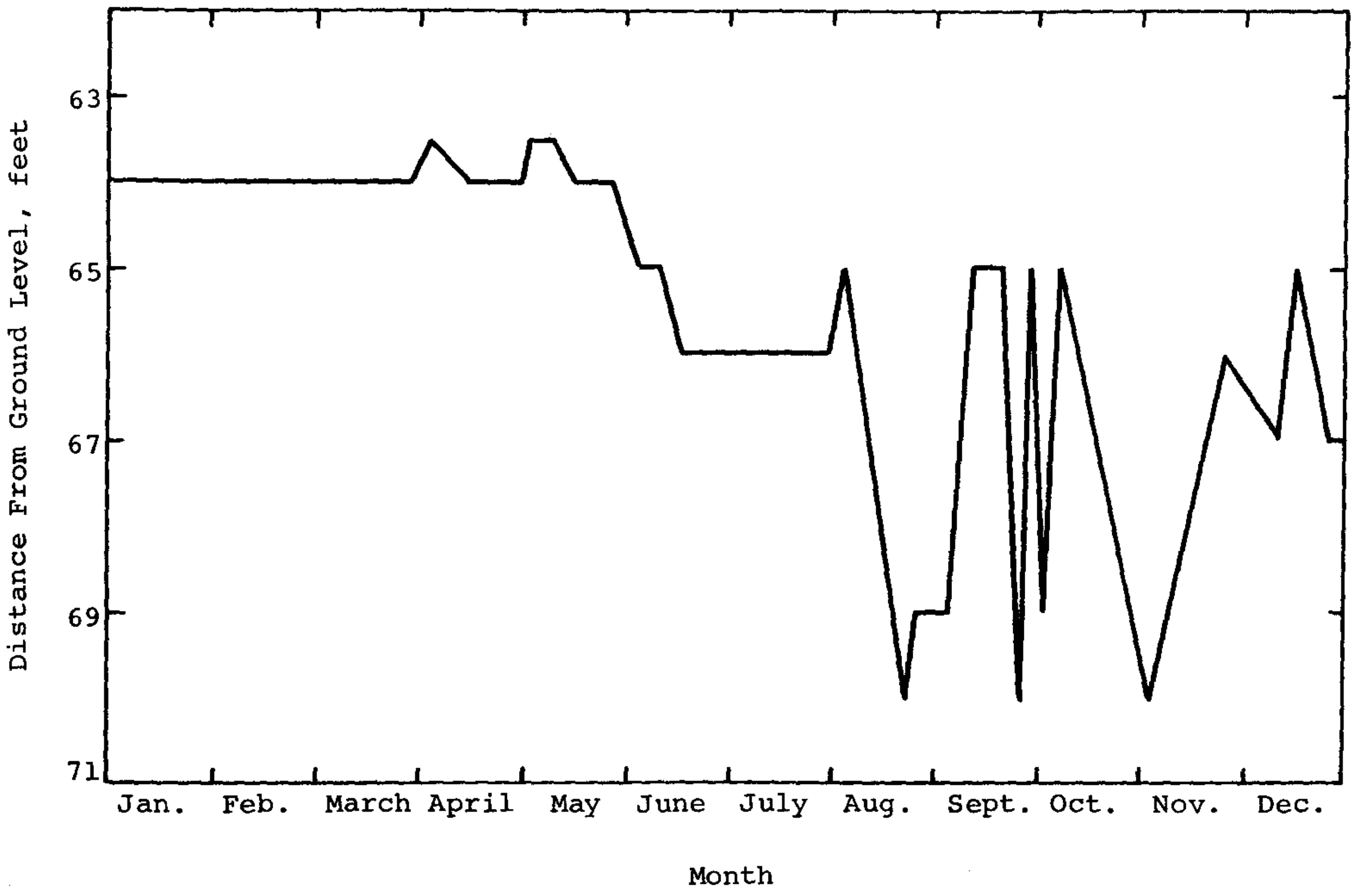

FIG.5.-STATIC LEVEL OF WELL # B FOR THE YEAR 1969

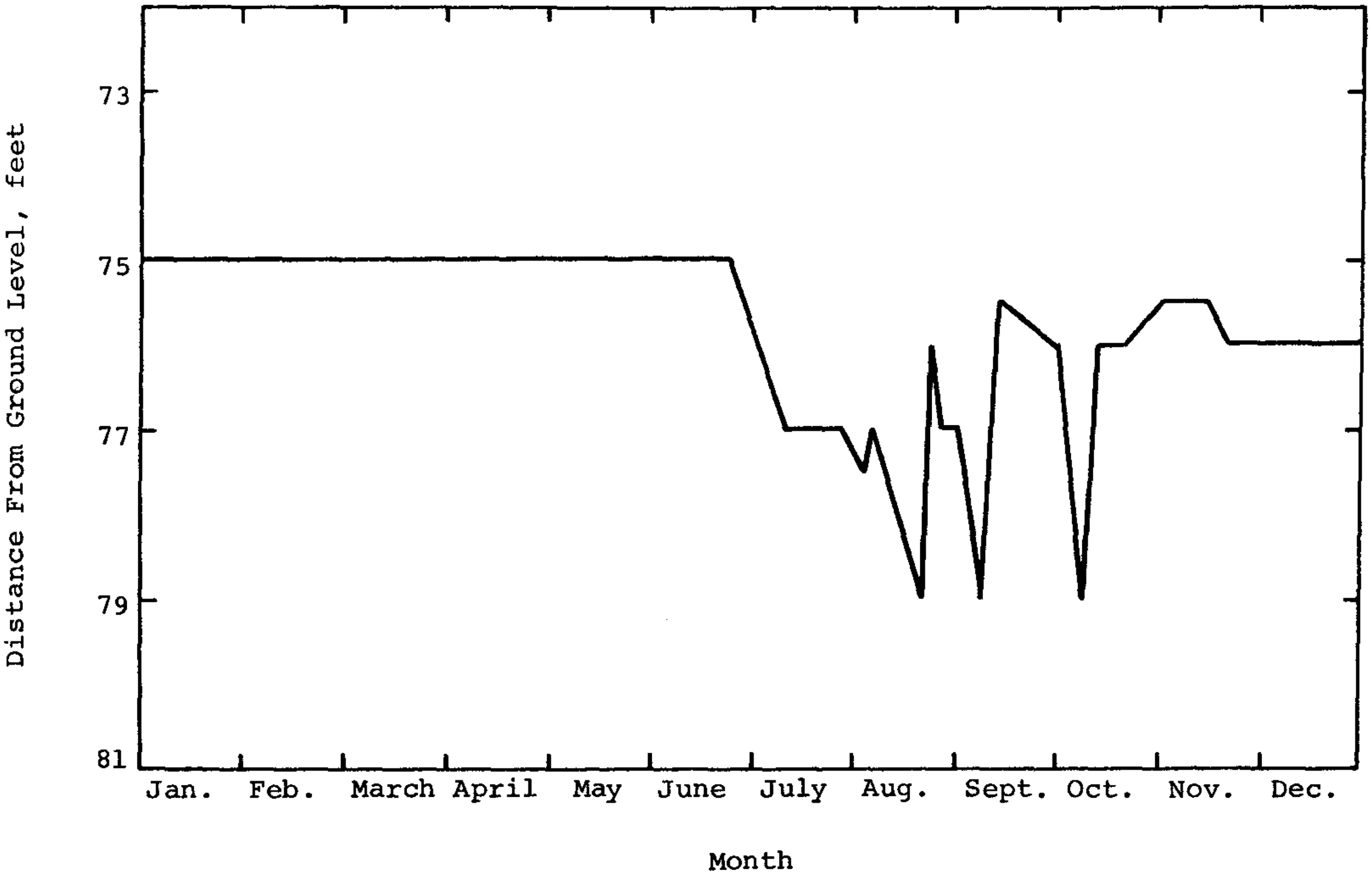

FIG.6.-STATIC LEVEL OF WELL # C FOR THE YEAR 1969

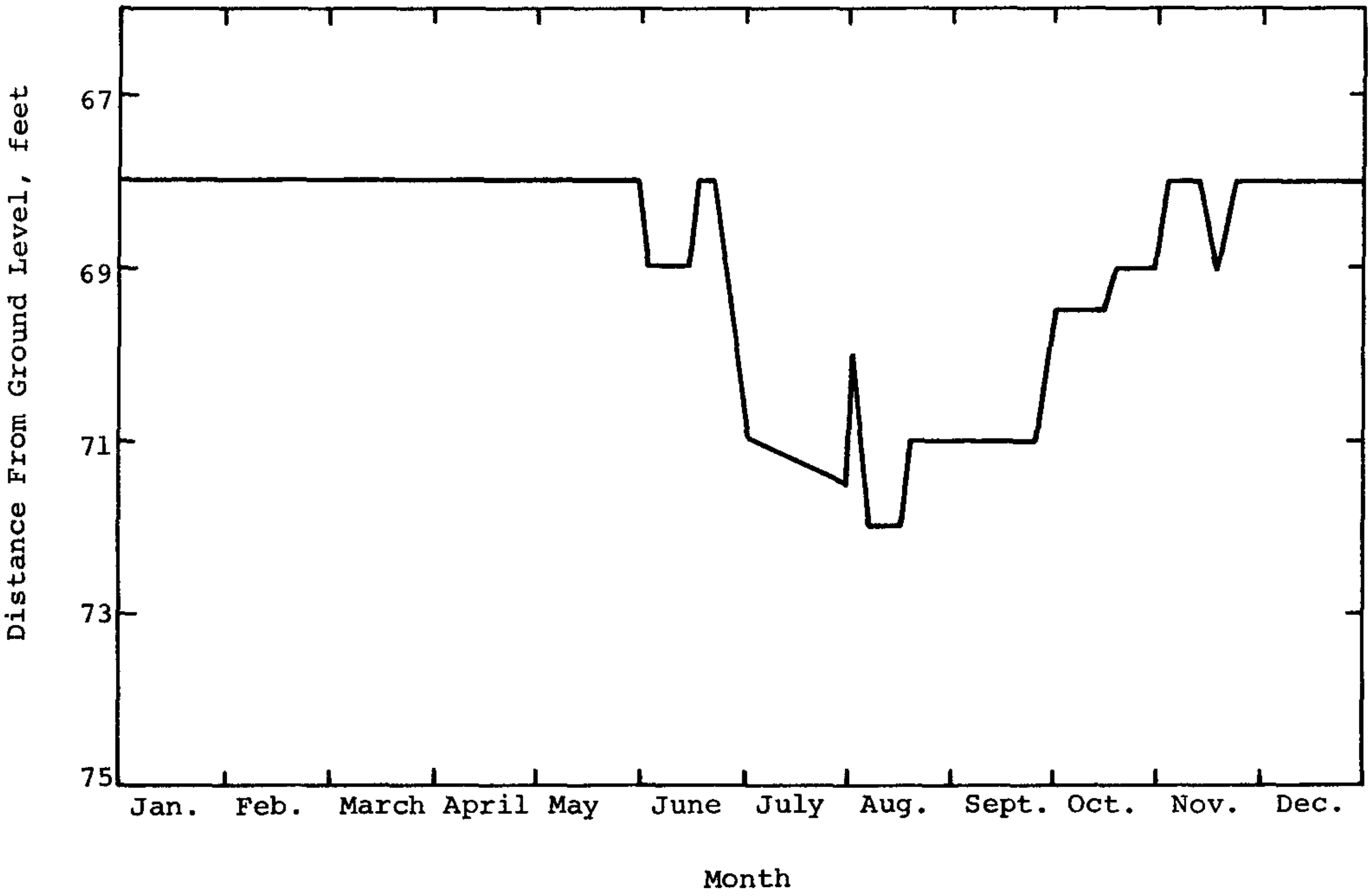

FIG.7.-STATIC LEVEL OF WELL # D FOR THE YEAR 1969

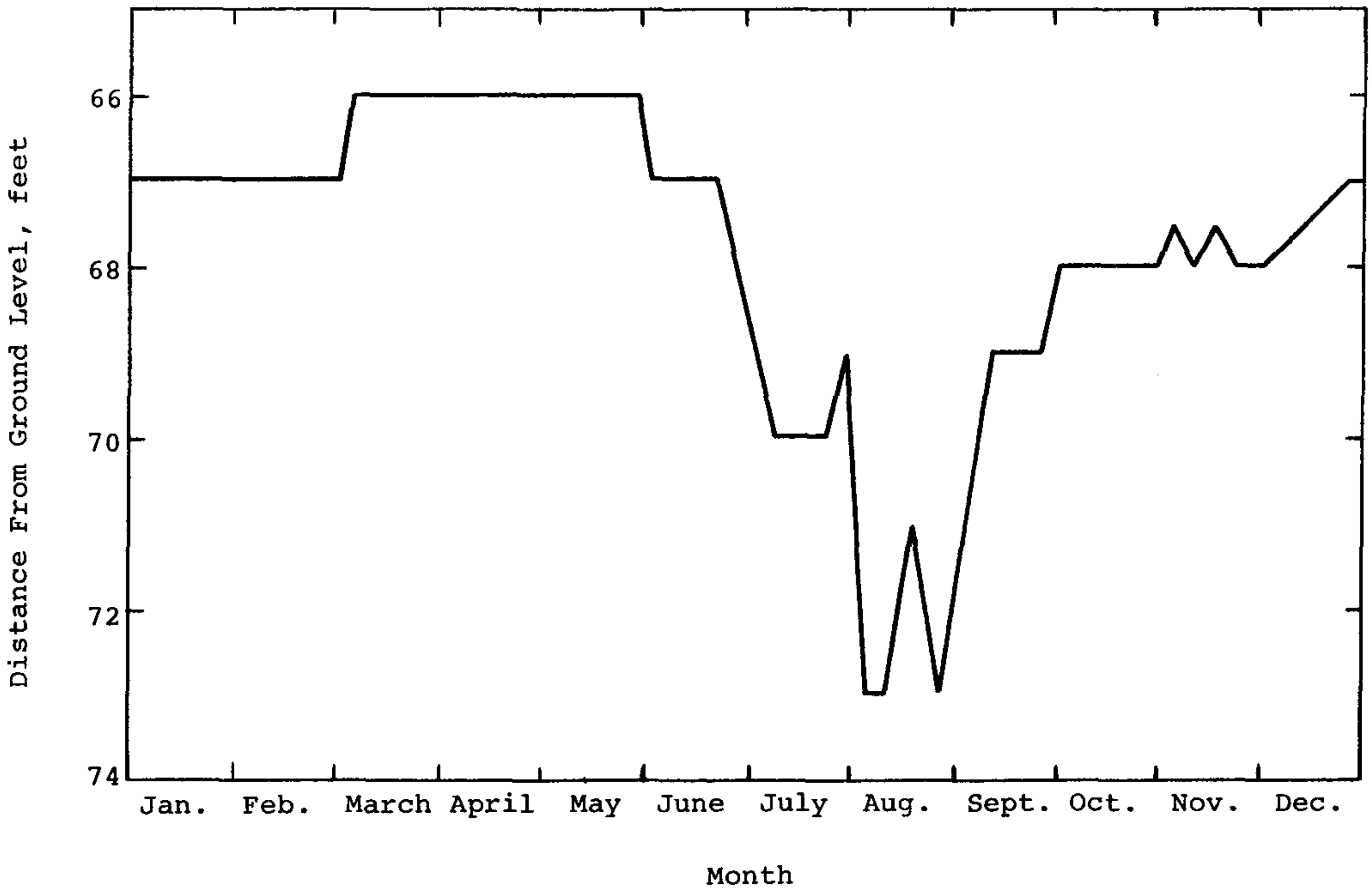

FIG.8.-STATIC LEVEL OF WELL # E FOR THE YEAR 1969

TABLE 2.-RAINFALL DATA FOR THE YEAR 1969, CITY OF FREDERICK, OKLAHOMA

Month	Amount of Rainfall, inches
January	0.22
February	2.46
March	1.98
April	0.91
May	4.37
June	2.62
July	1.90
August	3.95
September	8.43
October	3.12
November	0.32
December	0.69

CHAPTER IV

METHOD OF ANALYSIS

Pursuant to the objectives of this investigation, the analysis consisted of two phases. The initial phase encompassed the identification of all parameters that could possibly be involved in computing the net amount of recharge and also the most reasonable grouping of these parameters into dimensionless terms. In the second phase attempts were made to relate the dimensionless terms to one another through the use of multiple regression analysis. Detailed description of these two phases is presented below.

As depicted in Figure 9, the pertinent parameters in a study of this type are:

Q_r = recharge rate, cubic feet per unit time
Q = pumpage rate, cubic feet per unit time
A = the area contributing to recharge, square feet
i = rainfall intensity, feet per unit time
Q_i = influent flow, cubic feet per unit time
Q_e = effluent flow, cubic feet per unit time
h_o = piezometric head at time zero, feet

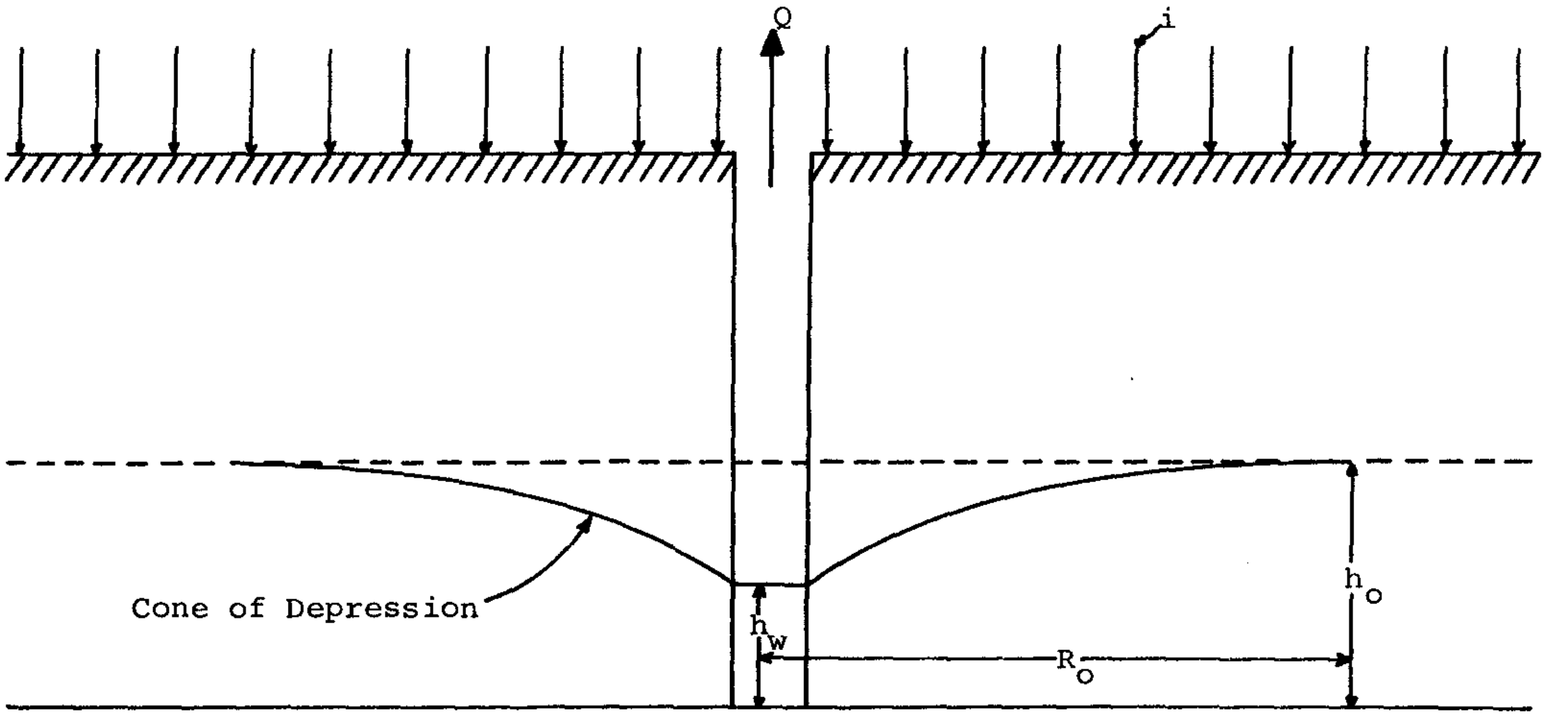

FIG.9.-DEFINITION SKETCH FOR UNSTEADY RADIAL FLOW IN UNCONFINED AQUIFERS WITH RECHARGE RATE "i"

h_w = piezometric head at time "t," feet
R_o = radial distance corresponding to h_o, feet
K = permeability, gallons per day per square foot

In the case of the five wells under study in Frederick, there is no apparent influent or effluent seepage. Figures 4 through 8 reveal that the static level of all the wells did not change during the month of January. The little amount of pumpage must have been, in all probability, compensated for by the amount of rainfall during that month. Therefore, it was concluded that there was either no influent or effluent seepage, or the amount was negligibly small. It appears, then, that the problem may be reduced to a simple functional relationship of the form:

$$Q_r = f(A,\ i,\ R_o,\ h_o,\ h_w,\ K,\ Q) \quad . \ . \ . \ . \ (6)$$

In order to reduce the number of parameters in Equation 6, the following dimensionless terms are introduced:

$$Y = Q_r/iA \quad \ldots\ldots\ldots\ldots\ldots \quad (7)$$
$$X = h_o/h_w \quad \ldots\ldots\ldots\ldots\ldots \quad (8)$$
$$Z = Q/KR_o^2 \quad \ldots\ldots\ldots\ldots\ldots \quad (9)$$

and thus, the problem becomes one of relating Y as a function of X and Z, or:

$$Y = f(X,Z) \quad . . . . (10)$$

In employing multiple regression analysis, Equation 10 assumes the form:

$$Y = B_o + B_1X_1 + B_2X_2 + \ldots + B_iX_i + e \quad . . . (11)$$

The problem of best fitting a hyper plane to a set of joint observations on a dependent variable can be accomplished by the least-squares principle which minimizes the residual sum of squares and provides an unbiased linear estimate with minimum variance of the parameter.

If the data do not fit a linear relationship directly, then there are two possibilities: (a) either an appropriate nonlinear fit directly to the data is attempted, or (b) an initial transformation of the data is made such that the relationship between the transformed data is almost linear, and the principle of least-squares can be applied. Transformations commonly used to reduce complex models to linear ones are logarithmic and reciprocal (8).

In this study the following four forms of regression equations were investigated:

$$Y = B_o + B_1X + B_2Z \quad . . . . (12)$$

$$\ln(Y) = B_o + B_1\ln(X) + B_2\ln(Z) \quad . . . . (13)$$

$$\ln(Y) = B_o + B_1X + B_2Z \quad . . . . (14)$$

$$1/\ln(Y) = B_o + B_1\ln(X) + B_2\ln(Z) \quad . \; . \; . \; . \; . \; . \; . \quad (15)$$

Since the same data were used to compute several forms of linear equations, the procedure employed for selecting the best form was to choose that form which gave the highest coefficient of determination, R^2, or the highest R, the coefficient of multiple correlation.

A significant F-value, i.e., the ratio of the regression mean square to the residual mean square, indicates that the regression coefficients explain more of the variation in data than expected by chance alone under identical conditions in a certain percentage of time. The sequential F-test using a 5 percent significance level was used to justify the acceptance of each variable into the regression equation. Reference to the work of Wetz made by Draper and Smith (8) suggests that an equation should be regarded satisfactory so long as the observed F-ratio does not exceed the selected percentage point of F-distribution by four times.

Residuals are defined as the difference between the observed and regression equation values of the dependent variables. There are certain basic assumptions made about the residuals when using least-squares regression analysis. These assumptions are that the residuals are independent, have a constant variance and zero mean, and if an F-test is used, they follow a normal distribution. A fitted model is regarded as correct if no evidence of violation of the above assumptions is found (9). The assumptions, constance variance and normality, were tested, and no evidence to the contrary was found. The method of least-squares necessarily gives a zero sum of residuals; therefore, no check could be made for this assumption. Also, since it is assumed that there are no observational errors, it could be seen that there is no correlation of the residuals for the chosen equation.

The values of the independent variables used in the regression equations are presented in Table 3. The values of the dependent variable Q_r were computed by the coefficient method, Equation 1. The contributing area of recharge for each well was found by the use of the Theisen method (10). C is given as 11.5 percent (11). The values of Q_r are also indicated in Table 3.

Then, through the use of the IBM 1130 Computer (12), the values of the dimensionless terms Y, X, and Z were computed and introduced into a subroutine program for computation of the regression coefficients, intercepts, R-values, and the F-values for Equations 12, 13, 14, and 15.

TABLE 3.-THE VALUES OF THE KNOWN PARAMETERS FOR THE WELLS IN THE CITY OF FREDERICK, OKLAHOMA

Well Identification	Well Property		Data Period, months	Parameters				
	Area, sq. ft.	R_o		i	h_o	h_w	Q	Q_r
#A	635,000	450	1	0.165	13.0	13.5	26,600	12,550
			6	1.050	13.0	13.5	362,000	80,000
			12	2.580	13.0	13.0	1,690,000	196,500
#B	635,000	450	1	0.165	14.0	14.4	611,000	12.550
			6	1.050	14.0	12.0	1,548,000	80,000
			12	2.580	14.0	11.0	3,520,000	196,500
#C	635,000	450	1	0.165	14.0	14.0	17,900	12,550
			6	1.050	14.0	13.2	483,000	80,000
			12	2.380	14.0	13.5	1,790,000	196,500
#D	282,600	300	1	0.158	17.5	16.0	700,000	5,380
			6	1.050	20.0	17.5	293,000	47,000
			12	2.580	20.0	20.0	1,649,000	84,500
#E	282,600	300	1	0.158	13.5	11.5	274,000	5,380
			6	1.050	16.0	11.5	303,000	47,000
			12	2.580	16.0	16.0	1,050,000	84,500

CHAPTER V

DISCUSSION OF RESULTS

The pertinent mathematical characteristics for Equations 12, 13, 14, and 15 which have been obtained by making use of the method of least-squares (7) are presented in Table 4.

The equation with the highest coefficient of multiple correlation is selected for best fitting the data. In this instance, R is maximum at 0.623 for Equation 12. By substituting the values of B_o, B_1, B_2, Y, X, and Z, Equation 12 assumes the form:

$$Q_r/iA = 0.055 + 0.068\ (h_o/h_w) - 3.778\ (Q/KR_o^2) \quad . \ . \ . \ (16)$$

The values of the F-test compared to 3.89, which is that for a 95 percent confidence interval and the same degrees of freedom as Equation 16, are not found to be significant (13).

Therefore, having obtained the different variables in Equation 16, the net recharge rate, Q_r, could be computed directly. Equation 16 could also be presented in a graphical form by plotting Q_r/iA as a function of Q/KR_o^2 for constant

TABLE 4.-VALUES FOR R, F, INTERCEPTS, AND COEFFICIENTS

Equation Number	R-Value	F-Value	B_o	B_1	B_2
12	0.623	3.802	0.055	0.068	-3.778
13	0.605	3.458	-2.592	0.494	-27.688
14	0.556	2.682	-2.252	0.576	-0.016
15	0.571	2.907	-0.439	-0.147	0.009

values of h_o/h_w, as shown in Figure 10.

Table 5 presents the residual values for Equation 16. These are the differences between the actual and estimated values of Q_r/iA for the three data periods and for each well as presented in Table 3. In Table 5 these sets of data are referred to as "Case Numbers." A close examination of the residual values reveals the negligible amount of error associated with estimating the values of Q_r/iA using Equation 16. Along with the high coefficient of multiple correlation obtained for Equation 16, the study of the residuals gives further indications of success in computing the recharge rate by this equation.

It should be noted that Equation 16 takes into account the amount of pumpage Q; consequently, it represents the net amount of recharge or discharge. A value of Q_r/iA less than zero will indicate that discharge is greater than recharge which means that a condition of mining has developed. This equation could also be used to compute the total amount of recharge by setting Q equal to zero.

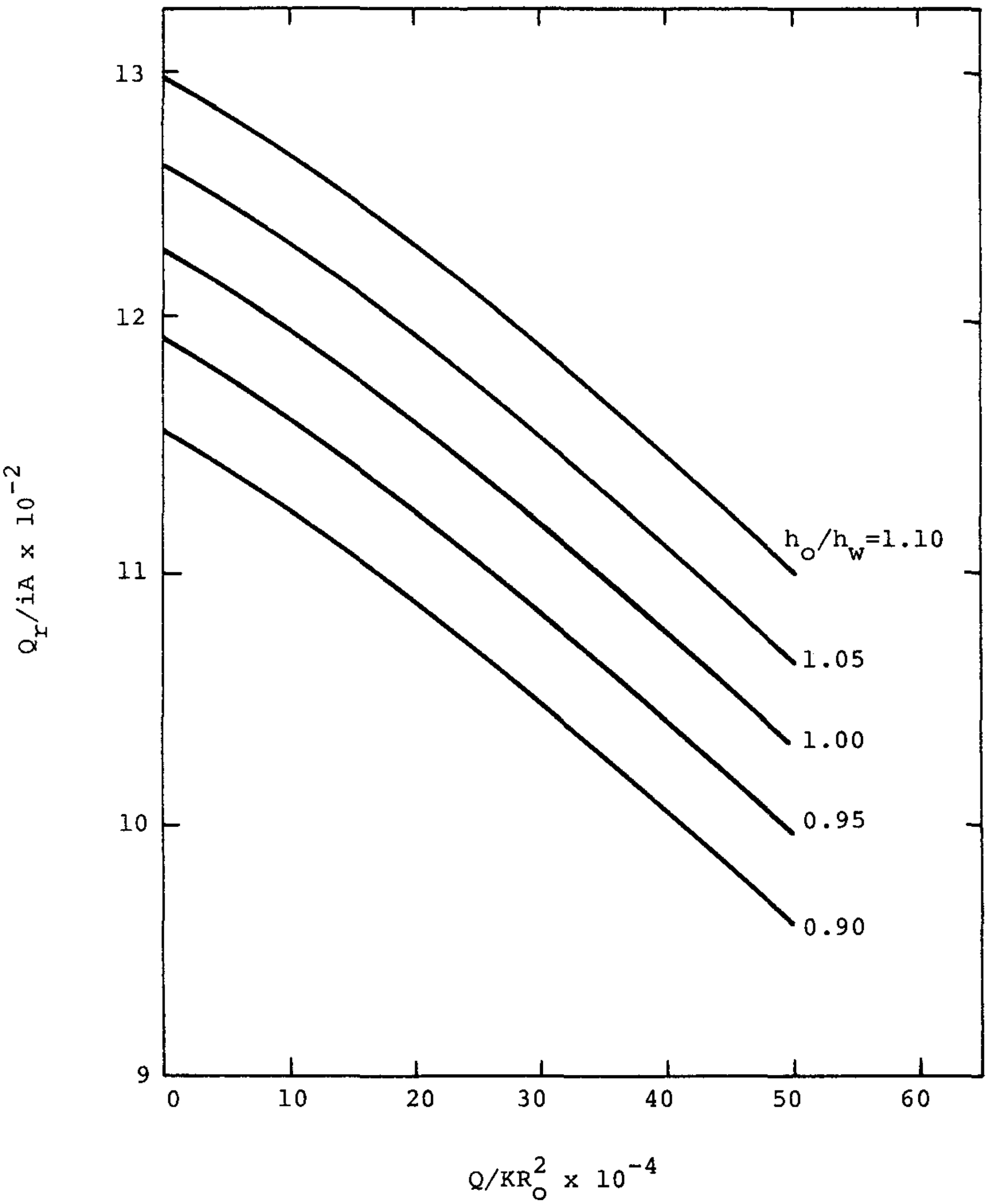

FIG.10.-GRAPHICAL REPRESENTATION OF EQUATION 16 FOR CONSTANT VALUES OF h_o/h_w

TABLE 5.-RESIDUAL VALUES FOR EQUATION 16

Case No.	Y-Value	Y-Estimate	Residual
1	0.11978	0.12070	-0.00092
2	0.11998	0.11942	0.00055
3	0.11994	0.11687	0.00306
4	0.11978	0.11910	0.00067
5	0.11998	0.12881	-0.00883
6	0.11994	0.12853	-0.00859
7	0.11978	0.12327	-0.00349
8	0.11998	0.12563	-0.00565
9	0.12016	0.11902	0.00113
10	0.10290	0.12372	-0.02082
11	0.15839	0.13058	0.02780
12	0.12008	0.10923	0.01084
13	0.12049	0.13287	-0.01238
14	0.15839	0.14750	0.01088
15	0.12008	0.11430	0.00577

CHAPTER VI

SUMMARY AND CONCLUSIONS

This study which was funded by the Oklahoma Water Resources Research Institute aimed at deriving a mathematical model relating recharge rate, Q_r, to all other factors which affect it. As a result of considering a number of possible approaches, it was decided to use dimensionless terms in deriving such a model. The literature survey which was conducted substantiated and justified the correctness of the decision.

The serious problem encountered in the course of this study has been that of obtaining sufficient and reliable data from the various sources contacted. The City of Frederick in Tillman County, Oklahoma, provided data reliable enough to be used in this study. The parameters of well and recharge hydraulics were grouped into three dimensionless terms which, in turn, were correlated to each other using multiple regression analysis.

On the basis of the limited data available and the results obtained, the following have been concluded:

1. Of the four different regression equations investigated, the one with the highest coefficient of multiple correlation is Equation 16 which has the form:

$$Q_r/iA = 0.055 + 0.068\ (h_o/h_w) - 3.778\ (Q/KR_o^2)$$

2. The mathematical model depicted by the aforementioned equation represents the net amount of discharge or recharge; it also gives a truer picture of the availability of groundwater than the conventional coefficient method heretofore used.

3. Since the equation takes into account the amount of pumpage, the amount of recharge may be computed by setting Q equal to zero.

4. A negative value of Q_r/iA indicates that discharge is greater than recharge, and it implies that a condition of "mining" has been established.

5. Although a predictive equation has been developed, it is not as comprehensive as desired. As more data become available, it would be possible to refine the mathematical model(s) so as to encompass any aquifer under any existing conditions.

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