AN EVALUATION OF RECENT APPROACHES FOR THE DESIGN OF BIOLOGICAL WASTE TREATMENT

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LIST OF SYMBOLS

Symbol			Chapter No.	Subtitle No.
A	=	Area of submerged contact plates in 1000 sq. ft.	3	3
A	=	A coefficient	3	9
A	=	(1+R - RC)	3	9
a	=	Completely-mixed fraction of total volume of aeration vessel	3	9
α	=	Percent or fraction of MLSS present in effluent	3	7, 8
B	=	A coefficient	3	9
B	=	(1+R) (1+R - RC + RCb)	3	9
b	=	Fraction of feed entering completely-mixed zone	3	9
C	=	Concentration factor of recycled and wasted sludge or separation factor	3	9
C	=	Concentration factor of recycled sludge ($C \geq 1$)	3	4
C	=	Concentration factor of wasted suspended solids	3	7
C	=	Materials balance factor for suspended solids at the final clarifier effluent	3	5
C_e	=	Effluent tracer concentration from system	3	9
C_i	=	Influent tracer concentration to system	3	9
C_o	=	Outlet tracer concentration from aeration tank	3	9
C_1	=	Tracer concentration in the stream entering the separator	3	9
D	=	Depth of tank in ft.	3	7
D	=	Dilution rate to the reactor - aeration tank	3	4
ΔS	=	Substrate concentration removed or substrate utilized	3	4, 5
ΔX	=	Increase in cell concentration	3	4

LIST OF SYMBOLS (Cont'd.)

Symbol			Chapter No.	Subtitle No.
ΔX	=	Net accumulation of concentration of biological solids	3	5
d'	=	Concentration of organic matter in effluent at zero organic loading	3	10
K	=	A coefficient varying from 0.48 to 0.82	3	4
K	=	Rate of substrate removal	3	10, 11
K_e	=	Endogenous respiration rate	3	5
K_s	=	Saturation constant of system in mg/l	3	4, 9
K_t	=	Endogenous respiration rate at T degrees C	3	5
K_0	=	Fraction of 5-day BOD oxidized	3	10
K_1	=	Rate of substrate removal in the minimal solids aeration tank, mg/l substrate removal/mg/l suspended solids/hr	3	14
K_2	=	Endogenous respiration constant	3	7
K_2	=	Rate of substrate removal in the extended or moderate aeration tank, mg/l substrate removal/mg/l suspended solids/hr	3	14
K_4	=	Coefficient of biological metabolism in declining growth environment in aeration tank	3	6
K_5	=	Coefficient of auto-oxidation of cellular active mass	3	6
K_5	=	Rate of biological metabolism	3	7
K_6	=	Coefficient of BOD oxidation	3	6
K_6	=	Biosynthesis constant	3	7
K_7	=	Endogenous respiration constant	3	7, 8
K_7	=	Endogenous respiration constant per day	3	8
K_8	=	Endogenous respiration of active mass at 20 degrees C per day	3	7, 8
K_9	=	Oxygen equivalent of utilizable effluent organic matter	3	7

LIST OF SYMBOLS (Cont'd.)

Symbol			Chapter No.	Subtitle No.
K_{20}	=	Endogenous respiration rate at 20 degrees C	3	5
K'	=	Relation constant between S_2 and L	3	10
L	=	Organic loading	3	10
L	=	Volumetric loading	3	9
L_v	=	Volumetric loading in lbs. BOD/day/1000 cu. ft.	3	2
L_0	=	Organic loading in lbs. BOD/day/lb/MLVSS	3	1
m	=	Effluent substrate concentration	3	10
m_s	=	Substrate removal per unit weight of biological mass for a given substrate concentration	3	10
m_{μ}	=	Exponential growth rate in 1/T	3	4, 5, 9
$m_{\mu m}$	=	Growth rate constant or maximum growth rate in 1/T	3	4, 9
OE	=	Oxygen equivalent of biological solids	3	8
OR	=	Overflow rate in final sedimentation tank, GPD/sq. ft.	3	7
θ	=	A constant (Average value = 1.075; Range .EG. 1.065 .EL. 1.085)	3	5
P	=	Purification efficiency in percent	3	3
Q	=	Inflow rate or waste water flow rate in MGD	2 3	3 1 - 11, 13, 14
Q_{rw}	=	(R+W)Q, where R is obtained from Eq. 50 and W from Eq. 51	3	5
Q_r	=	Flow of return sludge plus waste sludge	3	13
R	=	Recirculation ratio	2 3	3 1 - 9, 11-13
S	=	Concentration of influent BOD in mg/l	3	1, 2, 4
S	=	Concentration of oxidizable substance in mg/l	3	9
S	=	Substrate concentration	3	4
SDI	=	Sludge density index	3	7

LIST OF SYMBOLS (Cont'd.)

Symbol			Chapter No.	Subtitle No.
S_e	=	Substrate concentration in the effluent for steady state	3	9
S_i	=	Oxidizable substrate concentration in influent	3	9
S_0	=	Substrate concentration in influent	3	5
S_1	=	Inflow substrate concentration, mg/l	3	10, 14
S_1	=	Influent BOD or COD in mg/l or substrate concentration	3	3, 4 6 - 11
S_1	=	Influent substrate concentration in terms of mg/l COD	3	8
S_1	=	Substrate concentration in reactor	3	5
S_2	=	Effluent BOD or COD in mg/l or substrate concentration	3	3, 4, 6, 7, 8, 10, 11
S_2	=	Effluent substrate concentration in terms of mg/l COD	3	8
S_2	=	Substrate concentration in the minimal solids aeration tank and its effluent, mg/l	3	14
S_2	=	Substrate or organic matter concentration in effluent or recycled substrate concentration	3	5, 10
S_3	=	Substrate concentration in the extended or moderate aeration tank and in the final effluent, mg/l	3	14
t	=	Detention time in hours	3	1, 2, 3 8, 9, 10
t	=	Detention time	3	1 - 6, 11
t_a	=	Detention time of aeration tank in days	3	7
t_b	=	Detention time of final sedimentation tank in days	3	7
t_s	=	Detention time of stabilization unit in days	3	13
t_1	=	Detention time of minimal solids aeration tank, hr	3	14
t_2	=	Detention time of extended or moderate aeration tank, hr	3	14

LIST OF SYMBOLS (Cont'd.)

Symbol			Chapter No.	Subtitle No.
X_1	=	Inflow suspended solids concentration, mg/l	3	14
X_1	=	MLVSS or MLSS concentration in aeration tank	3	5
X_2	=	Concentration of microorganisms or suspended solids in the effluent	3	4
X_2	=	MLVSS or MLSS in final clarifier effluent	3	5
X_2	=	MLSS concentration in minimal solids aeration tank and in its effluent, mg/l	3	14
X_s	=	MLSS concentration in the extended or moderate aeration tank and in the final effluent, mg/l	3	14
X'	=	Concentration of MLVSS in mg/l	3	1
Y	=	Cell yield in mg/l organisms/mg/l oxidizable substrate	3	4, 5, 9, 11
Y_b	=	MG of solids/MG COD removed for batch systems	3	4
Y_c	=	MG of solids/MG COD removed for continuous systems	3	4
Y_0	=	5-day 20 degrees C BOD in lb/day	3	3
Z	=	Sludge index	3	12

CHAPTER 1

INTRODUCTION

1. Characteristics of Municipal Wastes and Some Other Organic Wastes

The wastes from households and related establishments are composited within the sewage system to produce relatively constant per capita amounts of suspended solids and organic matters. The organic waste subjected to biological changes is traditionally interpreted in terms of biochemical oxygen demand (BOD) at the end of five-day incubation at 20° C. However, on account of time consumption in the determination of five-day BOD, chemical oxygen demand (COD) has become accepted as a pollution parameter in the investigation of waste water. The drawback of the COD determination is the inability to depict the intensity of organic matters which are biologically degradable. Many researchers are still interested in using BOD as a pollution criterion.

The compositions of waste water are characteristic of the activity of the community from which the waste water is produced. Different types of communities will produce various types of wastes. In the United States, the average composition of domestic waste water can be characterized by the constituents and concentrations shown in Table I (1).

The average per capita BOD of combined sewage lies in the vicinity of 1.4 times the BOD of domestic sewage (1). In other words, the total five-day BOD in the combined municipal sewage would be $140 \text{ mg}/\ell \times 1.4 = 196 \text{ mg}/\ell$ (approximately $200 \text{ mg}/\ell$). The average

TABLE I

AVERAGE COMPOSITION OF DOMESTIC WASTE WATER

	Solids (mg/ℓ)			5-Day 20° C BOD (mg/ℓ)	COD (mg/ℓ)
State of Solids	Mineral	Organic	Total		
Suspended	65	170	235	110	108
Settleable	40	100	140	50	42
Non-settleable	25	70	95	60	66
Dissolved	210	210	420	30	42
Total	275	380	655	140	150

data for six years at Tucson, Arizona (2), illustrated similar statistical values. 243 mg/ℓ suspended solids and 206 mg/ℓ five-day BOD were obtained to represent pollution intensity for the waste water at Tucson.

The organic matters present in the domestic sewage contain 40% nitrogenous substances, 50% carbonhydrates, and 10% fat. Microbial requirements for nitrogen and phosphate are proportional to BOD loading. The maximum ratio of BOD:N:P is 150:5:1, while the minimum ratio is 90:5:1. Therefore, chemicals may have to be brought within range of the above proportions by mixing industrial wastes with domestic sewage or other suitable waste waters. In general, addition of required chemicals is applied at the inlet of an aerator. However, in the contact-stabilization process, Komolrit, Goel and Gaudy (3), recommended applying the nitrogenous chemical at the inlet of the stabilization reactor instead of at the inlet of the contact aeration tank in order to prevent the chemicals from leaking into the effluent of the final clarifier and to save unnecessary chemical wasting.

The characteristics of organic wastes depend much upon the specific industries. The concentrations of suspended solids and BOD loading etc. are listed in Table II (4).

2. Mechanisms of Waste Purification in the Activated Sludge Processes

The waste water is composed of suspended solids, colloidal matter and soluble organic substances. The settleable portions of the wastes are frequently removed prior to the aeration reactor. Other impurities are to be reduced or eliminated through various mechanisms during the detention of the waste water in the aeration tank. In order to

TABLE II

CHARACTERISTICS OF SOME TYPICAL ORGANIC WASTE

Types of Waste	Suspended Solids (ppm)	5-Day BOD 20°C (ppm)	pH
Sewage	100 - 300	100 - 300	
Pulp & Paper	75 - 300		7.6 - 9.5
Dairy	525 - 550	800 - 1500	5.3 - 7.8
Cannery	20 - 3500	240 - 6000	6.2 - 7.6
Packing House	650 - 930	900 - 2200	
Laundry	400 - 1000	300 - 1000	
Textile	300 - 2000	200 - 10,000	
Brewery	245 - 650	420 - 1200	5.5 - 7.4

more fully understand these mechanisms, the ecology of activated sludge will be described.

The principal and most numerous biological "workmen" in the aeration tank are saprophytic microorganisms including autotrophic bacteria and gelatinous masses constructed by bacteria with filamentous organisms such as Zooglea ramigera. Bacteria are responsible for the stabilization of the organic matter through biochemical processes. The suspended solids are removed by flocculation and enmeshment in the biological floc. The colloidal matter is absorbed on the biological surfaces. The protozoa present in the waste water assist in removal of dispersed microorganisms to produce a clarified effluent in the final sedimentation tank.

The hypothetic biosorption mechanism to remove soluble organic constitutents present in the wastes has been recently disproved by Krishnan and Gaudy (5), Siddiqi, Engelbrecht and Speece (6), and many other investigators. It was found that soluble organic substances would be removed from biological systems primarily under enzymatic reactions. The study of organic transport into cell interiors has been reported by Siddiqi, et al (6). It was concluded that intracellular enzymes, permeases, and extracellular enzymes were the three systems to operate the whole biochemical reactions between organic substances and enzymes. The intracellular enzymes comprise (a) hydrolases and (b) synthesis and respiration enzymes. Permeases facilitate the transport of exogenous substance into the cells. The extracellular hydrolases are secreted by the cells to hydrolyze long polymeric substrates into smaller units.

In the conventional activated sludge process, the sludge organisms are in contact with the waste for a period of six to eight hours. This period is sufficiently long to permit the organisms to synthesize inducible enzymes which may be required for the utilization of particular waste, but which are not present initially in sufficient quantity. On the other hand, in the contact stabilization process, the sludge is in contact with the waste in the contact unit for only a short period of time, usually 0.5 to 1.0 hour. The sludge is then separated from the waste and returned to the stabilization unit in which the assimilated organic compounds are used for biosynthesis and maintenance of microbial life under the reaction of intracellular enzymes with the assimilated substance. Therefore, satisfactory operation of the contact-stabilization activated sludge process can be achieved when the sludge organisms process a complete set of preformed enzyme systems at the time they are introduced to the waste water.

In the extended aeration process, rate of substrate removal per unit weight of activated sludge decreases as the aeration time prolongs. The loss of activity is due to inactivation of synthesis and respiration enzyme systems. The achievement of a high degree of purification is attributed to high concentrations of the mixed liquor suspended solids.

The transport of substrate into cells is the limiting step. Subsequent endogenous utilization of the substrate by hydrolysis, synthesis and respiration enzymes permits additional permeation of substrate into the cells. Recirculation of sludge to the aeration tank, of course, is intended to increase microbial populations and to decrease the organic loading on a unit weight of sludge. In addition to the above, the assimilated organic matter within cell cytoplasm are stabilized during detention in the final sedimentation tank prior to introduction into the inlet of the aerator to mix with incoming wastes. Such recirculated sludge possesses high ability for further permeation of exogenous soluble organic substance.

3. Demands for Various Modifications in the Activated Sludge Processes

The activated sludge process is a recent development in sewage and industrial waste treatment. It was devised and recognized as a useful method of sewage treatment by Lockett and Ardern in 1913. Since then many plants throughout the world have employed this process in its original form to produce a clear and non-putrescent effluent. In recent years, many modifications have been proposed attempting to increase loadings, reduce operation cost, improve efficiency or simplify operation. The original type of activated sludge process is called the "Conventional Process."

As the waste water passes through the conventional aeration tank, it acts as a plug flow. At the inlet of the aerator, high ratio of food to microorganism stimulates rapid cell growth and consequently demands a high intensity of dissolved oxygen. It seems impossible to maintain dissolved oxygen over 2 mg/ℓ at all times in order that anaerobic condition can be eliminated at any point in the oxidative treatment (7). Especially, with the use of detergents, it is becoming difficult to operate activated sludge systems with mixed liquor suspended solids below 3000 mg/ℓ, as severe foaming often occurs (7).

Conventional activated sludge has a tendency to bulk. From an economic viewpoint, BOD loadings limited to an average of 35 pounds per 1000 cu. ft. aeration tank would result in a high capital investment (8).

Since the oxygen supply to the aeration tank is not utilized at a uniform rate throughout the tank, the tapered aeration process was conceived to meet the oxygen required at any point. The air supply would be adjusted either by increasing the diffuser area at the inlet end or by using a larger amount of air at the inlet and smaller amounts along the tank (9).

In the conventional process, the entire load is introduced at the inlet of the aeration tank. It has been found that in the first quarter or third of the tank, the dissolved oxygen content of the mixed liquor is often zero. Apparently, although sufficient oxygen is supplied, the organisms simply cannot handle the load. They can work on the organic matter at a certain maximum rate and cannot be pushed further. With this idea, the step aeration process was then developed. The total organic loadings are distributed at several points along the tank, so the microorganisms are able to work at a more uniform rate and achieve more efficient purification. This would eliminate the zero dissolved oxygen areas, and a more uniform air supply can be applied (9).

The successful operation of high-rate activated sludge process depends on exploitation of the ability of bacteria to reproduce under exponential growth rate, commonly called "log growth phase." The high rate process attempts to overcome the limitations of plain aeration by using a small amount of return sludge which acts as an innoculum to reduce the lag period, to shorten the period of log growth phase, and to provide more surface for flocculation of new bacterial cells (10). Therefore, the "high-rate activated sludge treatment" is also termed "low-solids activated sludge treatment" (11). An important factor for operation is that return sludge should be in a highly active form to overcome the lag period. The temperature would determine, to a large extent, the amount of return sludge or return mixed liquor (9).

Examination of the conventional process indicated there was considerable mixing along the aeration tank by the diffused air. Therefore, it was proposed that wastes be introduced along the length of the aerator just above the air diffusers so as to take advantage of the maximum amount of mixing. In this process, there is very rapid and very complete mixing of the raw wastes with the activated sludge. More loads are put on the aeration tank and it produces an effluent with a lower oxygen demand than conventional activated sludge (12).

The small sewage treatment plant for simplifying operation and providing low construction cost sometimes employs an extended aeration process. Because a high efficiency of effluent is required, the mixed liquor of raw sewage and return sludge is aerated so that organic matters can be almost completely oxidized and removed. Total oxidation is another name for extended aeration. There is no need for primary sedimentation or sludge digestion. Therefore, screens and grit chambers, if necessary, are provided (10), (13). The effluent from the final clarifier is ready for introduction into a stream, but it is generally introduced into a polishing pond first because the plant may sometimes unload ash.

The contact aeration process is the one which needs solid plates suspended in the aeration tank as medium for bioflocculation. Microbial growth forms a zoogleal mass on the material submerged in the sewage. Excess growth may be shaken off periodically, or it may be allowed to slough off naturally and precipitate in the final clarifier.

The contact-stabilization process has developed from the processes of reaeration and biosorption. It was originally developed at Austin, Texas, by Ullrich and Smith and is used mainly for domestic sewage. The primary sedimentation tank is frequently eliminated in order that the suspended solids concentration of the sewage will be increased to enhance contact efficiency. Then a grit chamber is necessary to remove settleable inorganic matter. The sewage is mixed and aerated with previously aerated sludge for about 30 minutes to accomplish mixing and oxidative assimilation of soluble organic compounds. The real aeration for stabilization of endogenous substrate takes place in a separate aeration unit called a "stabilization tank" or "reaeration tank" (14).

There are many other modifications which are not described above. They may have particular value for the treatment of specific wastes.

4. Scope and Purpose of Present Study

Regarding activated sludge processes, biological activity towards removal of soluble organic compounds, to a large extent, is dependent on the characteristics of the waste water. The waste characteristics involve constituents as well as temperature, pH and other physical properties. It is believed that many factors, either chemical or physical, will have an effect on selection of predominating species among heterogeneous populations upon which the biochemical reactions are performed. An economic and safe design for the aeration tank and associated facilities can only be determined with pilot plant studies for the treatment of particular wastes. Nevertheless, the design approaches and parameters involved are frequently common to the design of various modifications.

The present study attempts to extend the developed equations for the design of various modifications in activated sludge processes. Major work is concentrated on the design of the aerator, i.e., volume of the aerator, detention time of waste flows, air supply, sludge recirculation and sludge wasting. A minor purpose of the present work is to provide information on design criteria. Design equations are obtained either empirically or theoretically. They are developed not only for the design of new plants but also for the operation and control of established plants.

CHAPTER 2

DESIGN CRITERIA

1. Design Variables in the Process Design of the Activated Sludge System

The primary variables involved in the design of the activated sludge aeration tank are organic loading, detention time, microbial concentrations, type of aeration equipment, quantity of air supply and tank configuration. Most of these variables are interrelated and cannot be separated easily.

The rate of oxygen transfer from gaseous phase to liquor phase is not only limited by temperature, pressure and solubility of oxygen in the liquor into which oxygen is supplied, but also upon the type of aeration employed for mixing and degree of agitation. In Europe, mechanical mixing by paddle, spray, link belt, etc., are quite well accepted. However, in the United States, air diffusion has been more widely employed to supply oxygen required for biological growth and to maintain aerobic conditions throughout the mixed liquor system.

Few reports, if any, have been published to relate tank configurations with respect to waste purification efficiency. Thus far, "Sewage Treatment Plant Design" (16) has been generally accepted as a guide for the design of aeration tanks. As stated (16), "most diffused-air aeration tanks in the United States have liquor depths of about 15 feet, but tanks less than 9 feet have been constructed." The ten-state standards (17) state that "ordinarily liquor depths will not be less than ten feet nor more than fifteen feet." In fact, the depth is governed in part by foundation and construction costs, in part by aeration efficiency and power costs, and in part by the size of the tanks (16). Modern design requires that the width of the tank should not be less than $1\frac{1}{2}$ times its depth, and widths as great as 2.15 times the depth provide no evidence of quiescent core during operation (18). The proportion of length to width has no specific limitation to be generally followed for the design of various modifications. A minimum free board of 8 inches has been used in the design; however, four or five times as much as 8 inches will be provided when a heavy froth occurs.

There exist diversified opinions as to the required detention time. Many people define detention time as the ratio of tank volume to waste flow. Nevertheless, recirculation of sludge flow to the aeration reactor is required for the proper operation of the activated sludge system. Thus, the time for a waste particle to remain in the aeration tank varies with recirculation ratio. It seems more meaningful to define detention time as the ratio of tank volume to the overall flow (raw waste flow plus recirculation flow) passing through the inlet of the reactor.

2. Loadings to the Aeration Tank for Various Modifications in the Activated Sludge Processes

Loadings applied to the aeration tank are generally expressed in terms of either "pounds BOD/day/pound mixed liquor volatile suspended solids" or "pounds BOD/day/1000 cu. ft. aeration tank." The former is called "Organic Loading" and the latter is called "Volumetric Loading." A satisfactory effluent can be obtained when receiving an appropriate loading intensity. The limitation of loading intensity for various modifications of activated sludge processes is listed in Table III.

The above information of appropriate loadings was presented by many investigators (8), (13), (15), (16), (19), (20). In the conventional process, if the BOD loading is higher than 0.5 lb./day/lb. MLVSS bulking may occur and result in lower purification efficiency (15), (16). For the design of the high-rate activated sludge process, BOD loadings as high as 2 to 4 lbs./lb. MLVSS have been reported (15). According to the experience at Austin, Texas, Ullrich (10) suggested that volumetric loading as high as 140-150 lb. BOD/day/ 1000 cubic feet is considered acceptable to apply on the contact unit in the contact-stabilization process.

Regarding the choice of design criteria between organic loading and volumetric loading, Borrough (7) recommended that the use of "organic loading" for design would lead to a design based on higher mixed liquor suspended solids.

3. Relationship Between Concentrations of Mixed Liquor Suspended Solids and Ratio of Sludge Recirculation

If it is assumed that the concentration of mixed liquor suspended solids in the aeration tank is X, the concentration of recirculated sludge is X_r and the suspended solid concentration of waste flow is X_o, then, the recirculation ratio, R, under flow rate Q will result in the following balanced relationship with suspended solid concentration in the aeration tank.

$$QX_o + RQX_r = (1 + R)QX \qquad (1)$$

TABLE III

ORGANIC LOADING AND VOLUMETRIC LOADING FOR VARIOUS MODIFICATIONS OF ACTIVATED SLUDGE PROCESSES

Modifications	Organic Loading lb. BOD/day/lb. MLVSS	Volumetric Loading lb. BOD/day/1000 cu. ft.
Conventional	0.2 - 0.5	20 - 50
Tapered Aeration	0.2 - 0.5	20 - 50
Step Aeration	0.2 - 0.5	50 - 75
High-Rate	0.5 - 2.0	125 - 200
Complete-Mixing	0.6	60
Extended Aeration	0.05 - 0.2	15 - 20
Contact-Stabilization	0.15 - 0.35	35 - 70

Since concentration of suspended solids applied at the inlet of the aeration tank is negligible compared with the concentration of recirculated sludge, and assuming that X_o equals zero, Equation 1 can be rewritten as follows:

$$R = \frac{X}{X_r - X} \qquad (2)$$

The composition of mixed liquor suspended solids consists of non-settleable suspended solids coming from the raw waste, cell growth due to oxidative assimilation of organic compounds and the return sludge. However, the concentration of the mixed liquor suspended solids are controlled principally by the recirculation of the settled sludge.

Past experience indicated (21) that minimum mixed liquor suspended solids of 1000 mg/ℓ would produce good flocculation and good separation. High solid concentrations generally produce better sludge separation; however, it is essential that the solid level should not become too large as it is difficult to remove large amounts of solids from the treated effluent. McKinney, et al.(21) also suggested that all activated sludge systems should possess at least 50% sludge recirculation capability with 100% sludge return for high-rate and extended aeration systems. Little value is gained with sludge return rates in excess of 100%, as solid separation systems are not designed to handle the high solid concentrations produced.

With a normal activated sludge, density is approximately 1.0 for the mixed liquor suspended solids. The return sludge concentrations are generally in the range of 8000 mg/ℓ to 10,000 mg/ℓ (21).

Based upon Equation 2, concentrations of the mixed liquor suspended solids related to the recirculation ratio at various concentrations of settled sludge are shown in Table IV.

It is apparent that the lower concentrations in the return sludge require more recirculation to maintain the desired concentrations of suspended solids in the mixed liquor. Table IV becomes more useful when it is graphically interpreted, as shown in Figure 1.

It is seen that when Equation 2 is rewritten in the form of $X/X_r = R/(1 + R)$, the degree of sludge recirculation governs the ratio of concentrations between mixed liquor suspended solids and return sludge. The relationship between recirculation ratio and the ratio of mixed liquor suspended solid concentration to return sludge concentration is tabulated in Table V. (The data listed in Table V can be graphically interpreted by Figure 2.)

4. Quantity of Air Supply Demanded

The particles of activated sludge in the aeration tank consist of thousands of cells, together with suspended or colloidal dispersed matter in the medium. The size of aggregates are around 0.5 to 2.0 mm in diameter, but the aggregates will remain dispersed under aeration.

Oxygen transfer potential is usually expressed as partial pressure in the gaseous phase, and concentration or activity in the liquor phase.

TABLE IV

RELATIONSHIP BETWEEN CONCENTRATIONS OF MIXED LIQUOR SUSPENDED SOLIDS AND RECIRCULATION RATIO AT VARIOUS CONCENTRATIONS OF RETURN SLUDGE

Concentrations of **MLSS** (X) mg/ℓ	Recirculation Ratio (R), %		
	X_r = 8000 mg/ℓ	X_r = 9000 mg/ℓ	X_r = 10000 mg/ℓ
1000	14.3	12.5	11.1
1500	23.1	20.0	17.6
2000	33.4	28.6	25.0
2500	45.5	38.5	33.4
3000	60.0	50.0	42.8
3500	77.8	63.5	53.8
4000	100	80.0	66.7
4500	128	100	81.8
5000	167	125	100

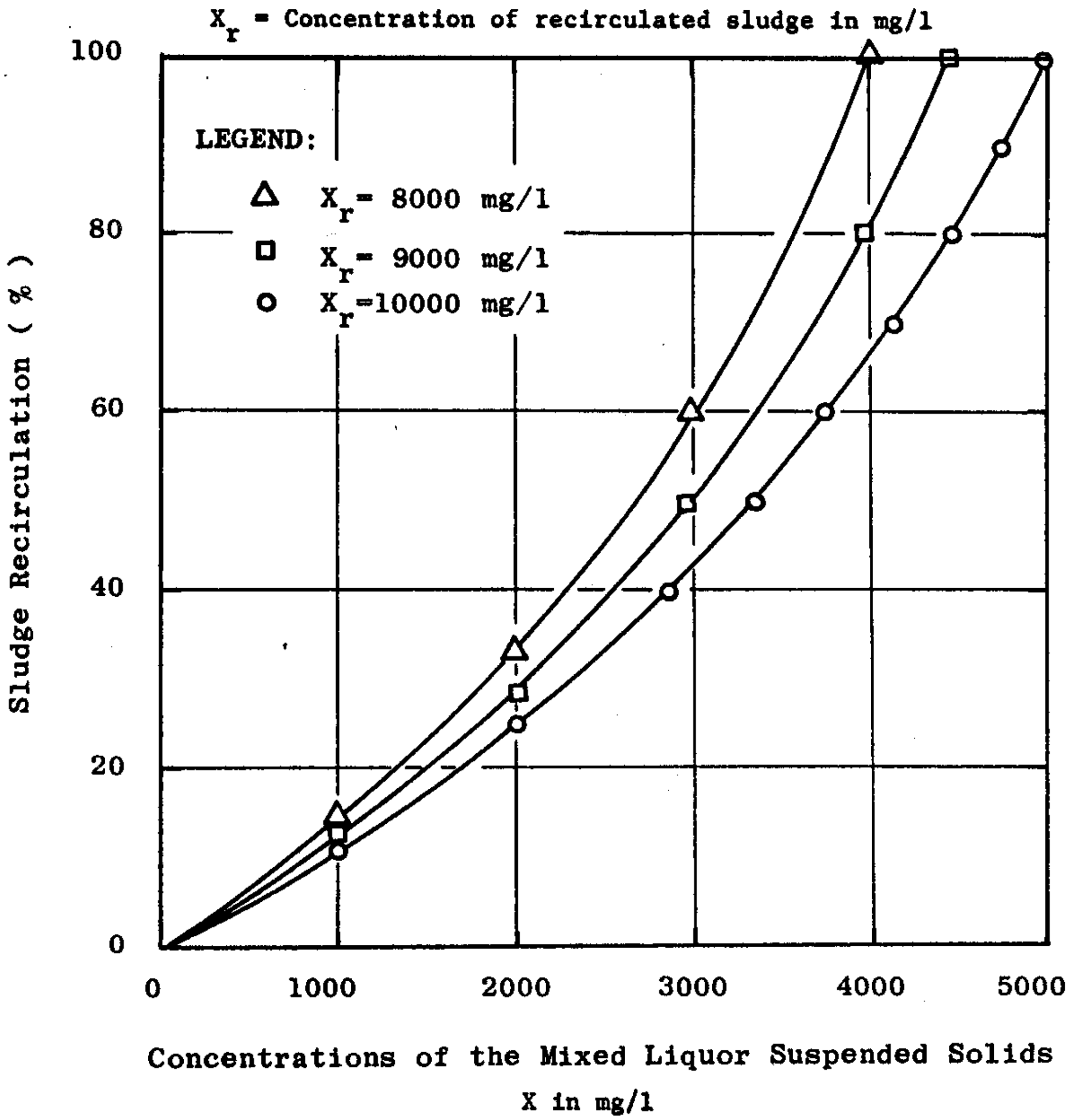


Figure 1

Relationship Between Concentrations of Mixed Liquor Suspended Solids and Sludge Recirculation at Various Concentrations of Return Sludge

TABLE V

RATIO OF MIXED LIQUOR SUSPENDED SOLID CONCENTRATION TO RETURN SLUDGE CONCENTRATION AT VARIOUS DEGREES OF RECIRCULATION

R (%)	X/X_r	R(%)	X/X_r
25	0.2	65	0.40
30	0.23	70	0.41
35	0.26	75	0.43
40	0.29	80	0.45
45	0.31	85	0.46
50	0.33	90	0.47
55	0.36	95	0.49
60	0.38	100	0.50

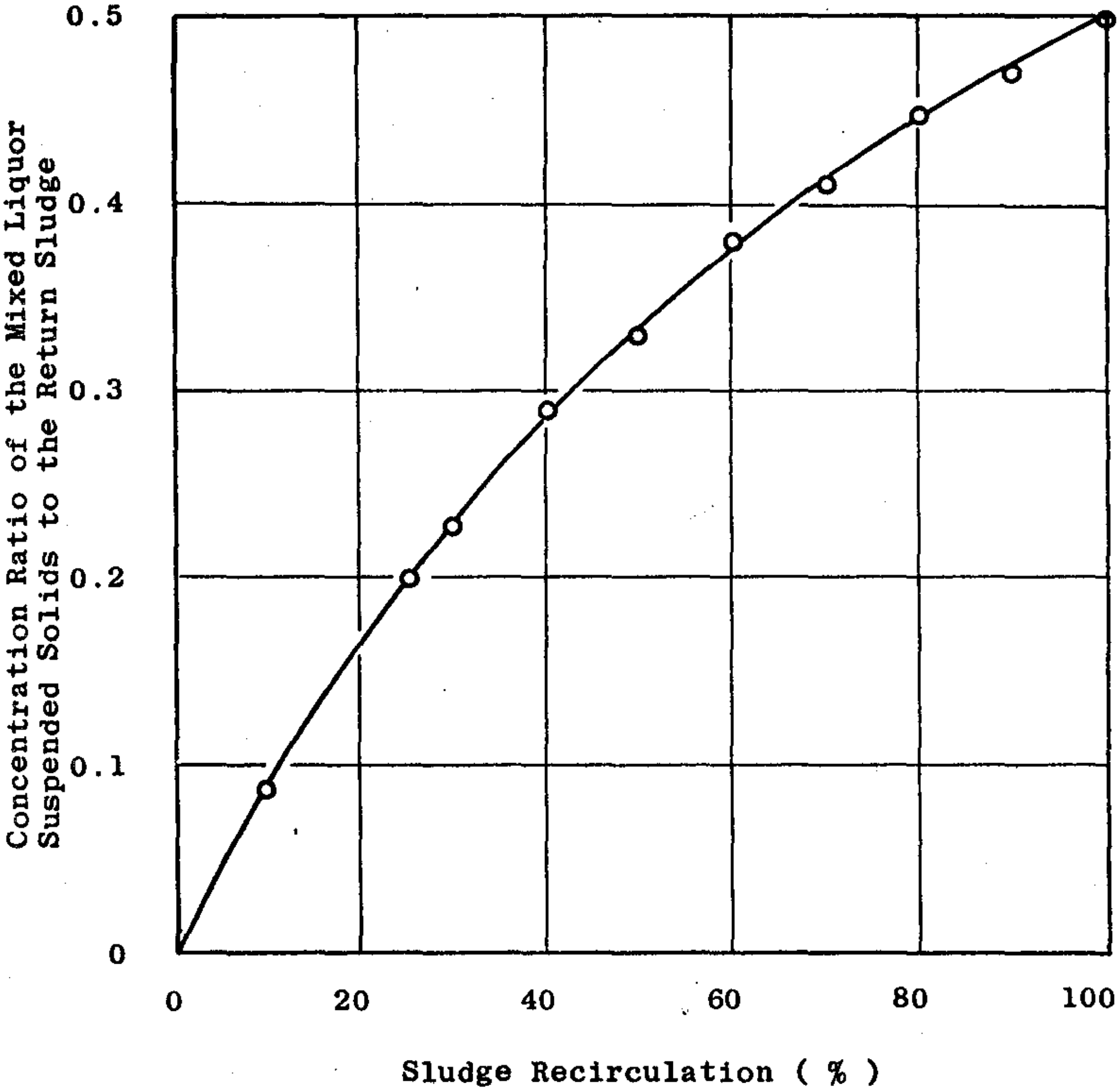


Figure 2

Concentration Ratio of Mixed Liquor Suspended Solids to Return Sludge at Various Percentages of Sludge Recirculation

Oxygen demand in waste water represents the concentration of pollutants. The conventional aeration tank, from a practical standpoint, is uniform in the concentrations of biological solids and soluble organic substance in each tank cross-section, but a decreasing gradient in the concentration of biological solids exists along the tank. The BOD demands are high at the inlet and low at the outlet. On the other hand, in the completely-mixed system, instantaneous mixing throughout the entire aeration tank exerts uniform organic loading and requires uniform oxygen demand. However, when nitrification is predominant in the system, the demand for oxygen would be higher than that of the organic matters present, because additional oxygen will be needed by the nitrifying bacteria.

It has been reported (20) that oxygen tension which affects metabolic rate lies below 0.5 mg/ℓ of dissolved oxygen. But differences in floc sizes, oxygen uptake rates and the degree of nitrification of activated sludge result in diversified reports about the minimum dissolved oxygen concentrations to be maintained under aerobic conditions (23). It has become widely accepted that a dissolved oxygen concentration of at least 2 mg/ℓ should be maintained in all parts of an aeration tank.

Since the primary purpose of the activated sludge process is the reduction of the oxygen demand of the sewage or waste, it appears logical to relate air requirements to BOD removed. As long as the BOD loading is not less than 0.3 lb/day/lb MLVSS, air supply generally averages about 500 to 700 cubic feet per pound of BOD removed. But at lower BOD to solids loadings, the average air requirements increased enormously, approaching 1200 to 1800 cubic feet per pound of BOD removed (16). In fact, the factors that influence air requirements are the altitude, temperature, dissolved oxygen present in the mixed liquor, tank dimensions, types, number, location and the arrangement of the diffusers (16).

For the design of the air flow system, McKinney and O'Brien (21) recommended a supply of 1000 cubic feet of air per pound BOD removed for normal activated sludge or 2000 cubic feet per pound BOD removed for extended aeration modification. One thousand cubic feet of air will supply approximately one pound of oxygen per pound of BOD removed. However, Westrick, Morand and Eye (24) reported that in the extended aeration process, any reduction in air below 4000 cubic feet/lb BOD removed would result in inefficient oxidation. Quantities of air supply needed for various activated sludge processes are shown in Table VI.

For a demand of one pound dissolved oxygen to stabilize the organic compounds in biological systems, amounts of air supply can be calculated as shown below (25).

At standard conditions:

Temperature = 15° C
Pressure = 76 mm Hg
Density of air = 1.226 mg/ℓ

$$= 1.226\ \text{mg}/\ell \left(\frac{2.205 \times 10^{-3}\ \text{lb/gm}}{0.03531\ \text{ft}^3/\ell}\right)$$

$$= 0.077\ \text{lb/ft}^3$$

In air, free oxygen is 23% of air by weight. Air required to supply one lb. of free oxygen $= 1 \times \frac{1}{0.23} \times \frac{1}{0.077}$

$$= 56.41\ \text{cu. ft.}$$

As stated by Smith (26), 9% of free oxygen is transferred and dissolved oxygen has to be maintained at 2 mg/ℓ level. Since saturated dissolved oxygen at standard conditions is 9.2 mg/ℓ, then the deficit of dissolved oxygen will be

$$9.2 \text{ mg}/\ell - 2 \text{ mg}/\ell = 7.2 \text{ mg}/\ell.$$

The factor which governs oxygen transfer is

$$0.09 \times \frac{7.2}{9.2} = 0.0702$$

Amounts of air needed to supply one pound of dissolved oxygen $= \frac{56.41}{0.0702} = 803.5$ cu. ft.

TABLE VI

QUANTITIES OF AIR SUPPLY NEEDED FOR VARIOUS ACTIVATED SLUDGE PROCESSES

Modifications	Air Supply cu. ft./lb. BOD removed	Remarks
Conventional	500 - 700	loading > 0.3 lb BOD/lb VSS
	1000	Normal
	1200 - 1800	loading < 0.3 lb BOD/lb VSS
High-rate	600	
Extended Aeration	2000 - 4000	
Contact Stabilization	1000	1/3 for contact unit 2/3 for stabilization unit

CHAPTER 3

DESIGN OF AERATION TANK

1. Design Based on Organic Loading

As the definition indicated, organic loading is representative of lbs. BOD applied per day per pound mixed liquor suspended solids present in the aeration tank. The mathematic expression can be interpreted in the following form.

$$L_o = \frac{8.34\ S\ Q}{8.34\ X'V} \tag{3}$$

Where,

L_o = Organic loading in lbs. BOD/day/lb MLVSS

S = Concentration of influent BOD in mg/ℓ

Q = Waste water flow rate in MGD

X' = Concentration of MLVSS in mg/ℓ

V = Volume of aeration tank in MG

That is,

$$L_o = \frac{S\ Q}{X'V} \tag{4}$$

$$V = \frac{S\ Q}{X'L_o} \tag{5}$$

Assuming the recirculation ratio is R

$$(1 + R)\ Q\ t = V$$

$$t = \frac{V}{(1+R)Q}$$

Substituting Eq. 5 in the above equation, there is obtained

$$t = \frac{S}{X'L_o(1+R)} \tag{6}$$

When 1 MGD of waste flow with 100 mg/ℓ 5-day BOD is applied to the aeration tank, the capacity of the aeration tank is hereafter called "unit volume of aeration tank" with symbol "V_u" to stand for it. As, S = 100 mg/ℓ, Q = 1 MGD, then Eq. 4 will become

$$V_u = \frac{100}{X' L_o} \qquad (7)$$

Equation 7 indicates that unit volume of aeration tank is inversely proportional to the product of organic loading and concentration of mixed liquor volatile suspended solids. As long as the products of X′ and L_o become constant, then the design capacity of the aeration tank is totally determined by the influent BOD concentration and waste water flow rate. Under a certain organic loading, L_o, unit volume of the tank is governed by the concentrations of mixed liquor volatile suspended solids. The interrelations among unit volume, organic loading and the concentrations of MLVSS are tabulated in Table VII and graphically expressed in Figure 3.

The design volume of the aeration tank will be expressed as:

$$V = V_u \, S \, Q/100 \qquad (8)$$

in which S represents influent BOD concentrations in mg/ℓ, Q represents inflow rate in MGD and V and V_u are in MG.*

2. Design Based on Volumetric Loading

By definition, the volumetric loading is expressed in terms of lb. BOD/day/1000 cu. ft., and the desired capacity of the tank can be formulated as Eq. 9.

Since 1000 cu. ft. = 7480 gallons

$$= 7.48 \times 10^{-3} \text{ MG}$$

*(Neither the effluent BOD nor the % BOD removal is given by this method.)

TABLE VII

UNIT VOLUMES OF AERATION TANK UNDER VARIOUS ORGANIC LOADINGS AND CONCENTRATIONS OF MLVSS

Organic Loading (lb BOD/day/ lb MLVSS)	Unit Volumes of Aeration Tank (Million Gallons)								
	X′=1000 mg/ℓ	X′=1500 mg/ℓ	X′=2000 mg/ℓ	X′=2500 mg/ℓ	X′=3000 mg/ℓ	X′=3500 mg/ℓ	X′=4000 mg/ℓ	X′=4500 mg/ℓ	X′=5000 mg/ℓ
0.1	1.000	0.667	0.500	0.400	0.333	0.286	0.250	0.222	0.200
0.2	0.500	0.333	0.250	0.200	0.167	0.143	0.125	0.111	0.100
0.3	0.333	0.222	0.167	0.133	0.111	0.095	0.083	0.074	0.067
0.4	0.250	0.167	0.125	0.100	0.083	0.072	0.063	0.057	0.050
0.5	0.200	0.133	0.100	0.080	0.067	0.059	0.050	0.044	0.040
0.6	0.167	0.111	0.083	0.067	0.056	0.048	0.041	0.037	0.033
0.7	0.143	0.095	0.072	0.057	0.048	0.041	0.036	0.032	0.028
0.8	0.125	0.083	0.063	0.050	0.042	0.036	0.031	0.028	0.025
0.9	0.111	0.074	0.056	0.045	0.037	0.032	0.028	0.025	0.022
1.0	0.100	0.067	0.050	0.040	0.033	0.029	0.025	0.022	0.020
1.5	0.067	0.045	0.033	0.027	0.022	0.019	0.017	0.015	0.013
2.0	0.050	0.033	0.025	0.020	0.017	0.014	0.013	0.011	0.010
2.5	0.040	0.027	0.020	0.016	0.013	0.011	0.010	0.009	0.008
3.0	0.033	0.022	0.016	0.013	0.011	0.010	0.008	0.007	0.007
3.5	0.029	0.019	0.014	0.012	0.010	0.008	0.007	0.006	0.006
4.0	0.025	0.017	0.013	0.010	0.008	0.007	0.006	0.005	0.004

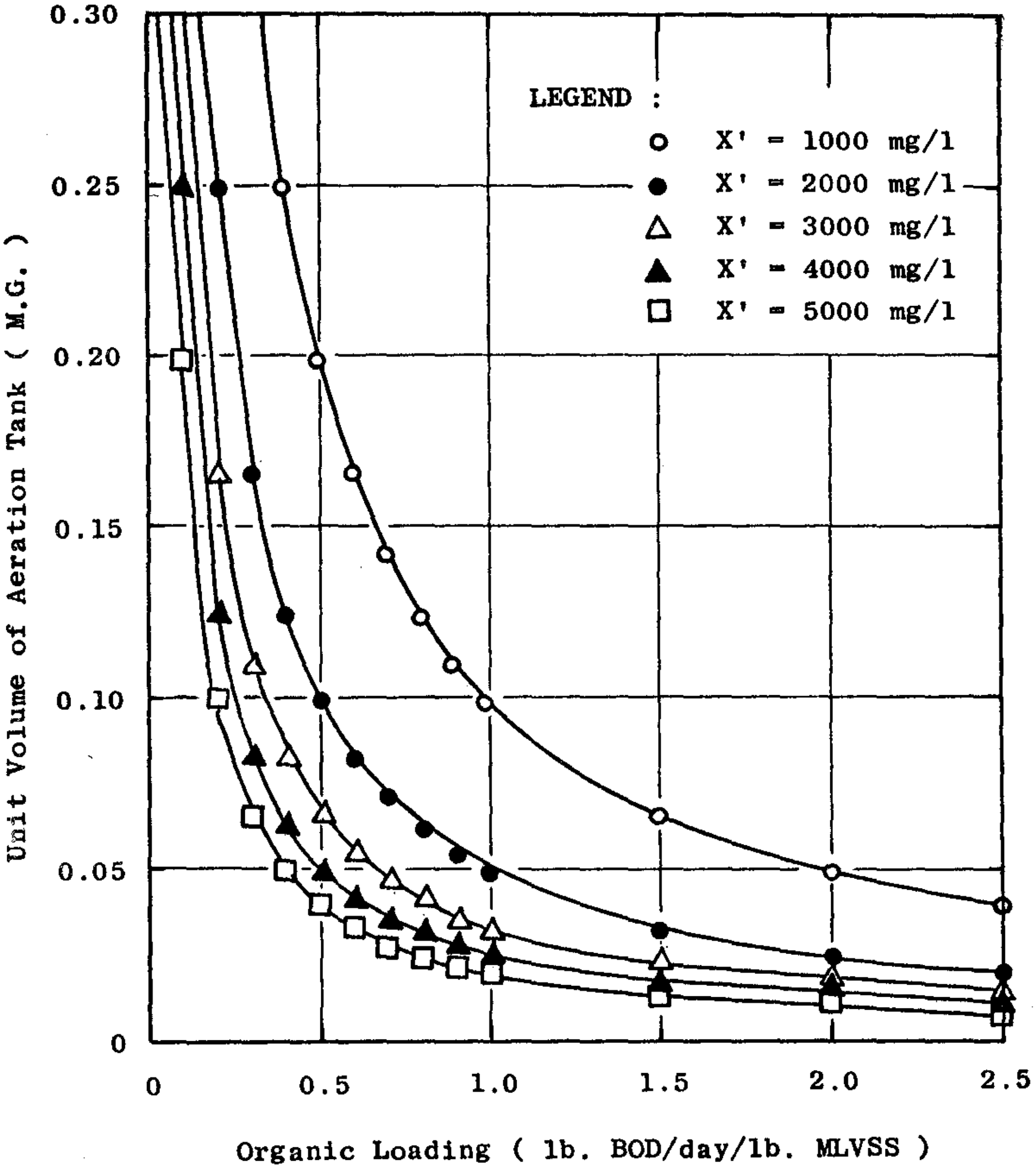


Figure 3

Unit Volumes of Aeration Tank Under Various Organic Loadings at Different Concentrations of Mixed Liquor Volatile Suspended Solids When Q = 1 MGD & S = 100 mg/l then,

$$L_v = \frac{8.34\ SQ}{V/(7.48 \times 10^{-3})} = 62.4 \times 10^{-3}\ SQ/V \tag{9}$$

$$V = 62.4 \times 10^{-3} SQ/L_v \tag{10}$$

Under operation with "R" recirculation ratio, detention time of incoming waste, "t" in days can be calculated from Eq. 11.

$$t = 62.4 \times 10^{-3}\ S/L_v\ (1 + R) \tag{11}$$

where,

V = Design volume of aeration tank in MG

S = Concentration of influent BOD in mg/ℓ

Q = Flow rate in MGD

L_v = Volumetric loading in lbs. BOD/day/1000 cu. ft.

It has been mentioned in the previous section that the unit volume of an aeration tank is the capacity of the tank provided for treating waste water with a concentration of 100 mg/ℓ influent BOD at a flow rate of 1 MGD. Equation 12 is a guideline for design on the basis of volumetric loading.

$$V_u = 6.24/L_v \tag{12}$$

Obviously, it is known from the above stated equation that unit volume of the aeration tank is inversely proportional to the volumetric loading, and the product of unit volume and volumetric loading results in a constant. In spite of the fact that allowable volumetric loadings were obtained empirically, there is doubt regarding the applicability of Eq. 12 to design an aeration tank for the treatment of wastes with certain organic concentrations but different in characteristics. In addition, the variables involved in Eq. 9 through 12 have nothing to do with the concentrations of suspended solids in the mixed liquor. Consequently, it is difficult to believe that sludge recirculation and concentrations of MLVSS become subordinary elements in reference to unit operation.

The design unit volumes under various volumetric loadings are tabulated in Table VIII, and graphically shown in Figure 4.

An effort is made to relate the organic loading with the volumetric loading as follows.

Since

$$V_u = \frac{100}{X' L_o} = \frac{6.24}{L_v}$$

$$\frac{L_o}{L_v} = \frac{16}{X'} \quad \text{or} \quad \frac{L_v}{L_o} = \frac{X'}{16} \tag{13}$$

When

$$L_r = 100\, L_o$$

$$X' = 1600 \text{ mg}/\ell$$

In other words, as the concentrations of MLVSS equal 1600 mg/ℓ, 0.1 lb. BOD/day/lb. MLVSS would be identical with 10 lb. BOD/day/1000 cu. ft. for the design of an aeration tank. When MLVSS is higher than 1600 mg/ℓ, L_v is larger than 100 L_o and the intensity of 0.1 lb. BOD/day/lb. MLVSS is indeed stronger than 10 lb. BOD/day/1000 cu. ft.

It is not unusual to maintain mixed liquor suspended solids at the level of 3200 mg/ℓ, except for the high-rate activated sludge process; then 0.1 lb. BOD/day/lb. MLVSS will be equivalent to the volumetric loading design criteria of 20 lb. BOD/day/1000 cu. ft. When a plot of volumetric loading against organic loading is made, it is seen from Figure 5 that the slope of each line is governed by the concentrations of MLVSS. Equation 13 depicts the slope of the line by dividing the

TABLE VIII

UNIT VOLUMES OF AERATION TANK UNDER VARIOUS VOLUMETRIC LOADINGS

L_v lb. BOD/day/10^3 cubic feet	V_u MG	L_v lb. BOD/day/10^3 cubic feet	V_u MG
10	0.624	90	0.069
20	0.312	100	0.062
30	0.208	120	0.052
40	0.156	140	0.045
50	0.125	160	0.039
60	0.104	180	0.035
70	0.089	200	0.031
80	0.078	300	0.021

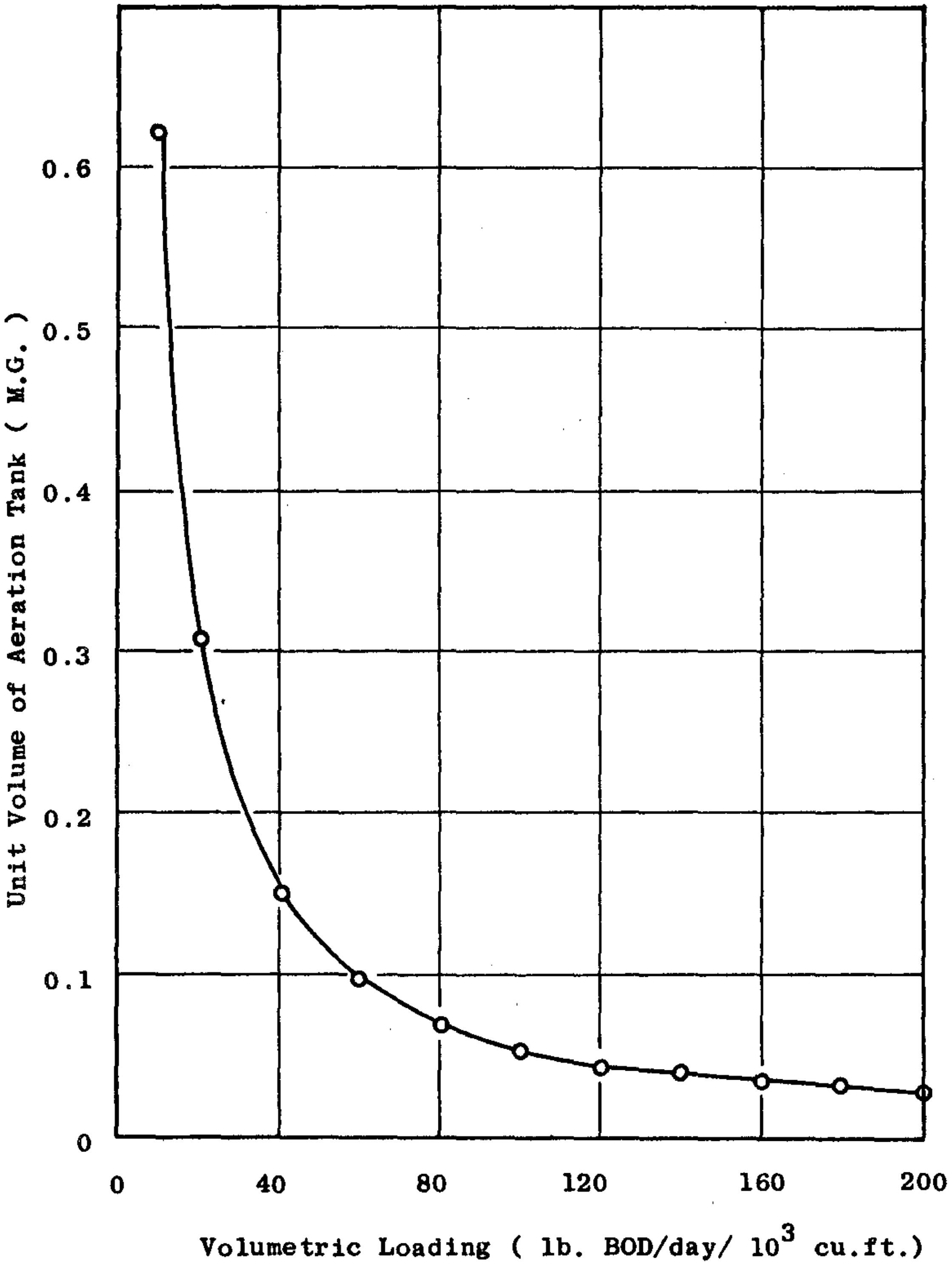


Figure 4

Unit Volumes of Aeration Tank Under Various Volumetric Loadings

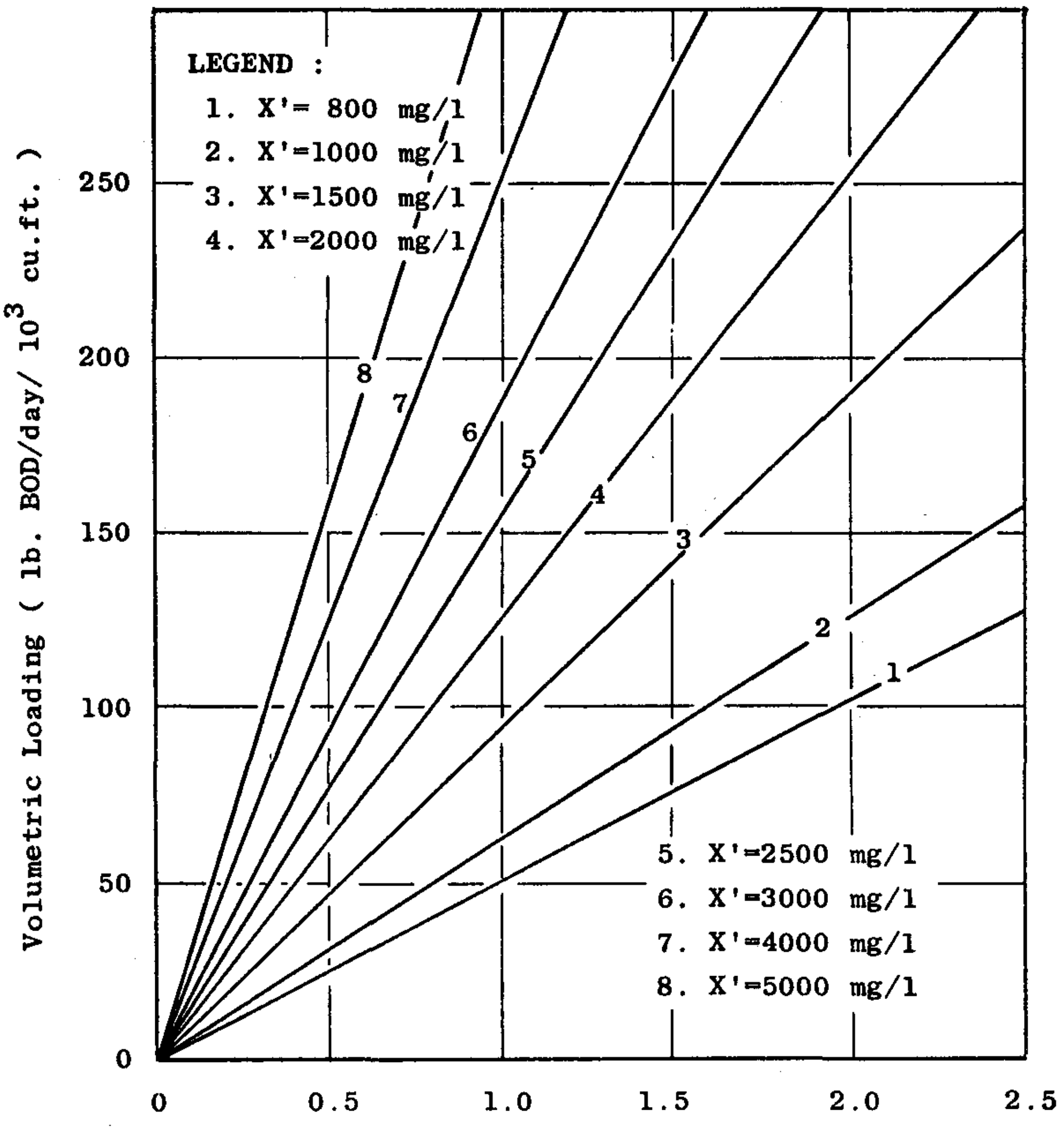


Figure 5

Relationship Between Organic Loadings and Volumetric Loadings at Various Concentrations of Mixed Liquor Volatile Suspended Solids mixed liquor volatile suspended solids concentrations by 16. The required capacity of an aeration tank can also be obtained from Eq. 8.

Whether the design is based on organic loading or on volumetric loading, loading intensity should not be arbitrarily chosen until pilot plant studies indicate a permissible effluent under such loadings, especially in the treatment of a particular industrial waste.

3. National Research Council Equations

(A) Activated sludge plants: The results of analyzing eighteen waste water treatment plants in the United States were generalized to an equation commonly applied for the design of diffused-air aeration tanks (26), (28). The original analytical data were taken from operation of the conventional activated sludge process; however, it is thought that application of this equation to the design of tapered aeration tanks or step aeration would be considered acceptable, since the equation was recommended by the National Research Council. The variables involved in the design processes cover purification efficiency, BOD loading, mixed liquor suspended solid concentrations and detention time of incoming waste water flow, and are shown in Eq. 14.

$$P = \frac{100}{1 + 0.03(Y_o/Wt)^{0.42}} \qquad (14)$$

where

P = Purification efficiency in percent

Y_o = 5-day 20° C BOD in lbs. per day

W = Weight of MLSS in 1000 lbs.

t = Detention time in hours

Solving Eq. 15 for t there is obtained

$$t = \frac{Y_o}{W}\left(\frac{0.03P}{100 - P}\right)^{2.38} \qquad (15)$$

Assume

Q = Inflow rate in MGD

S_1 = Influent BOD in mg/ℓ

S_2 = Effluent BOD in mg/ℓ

X = Concentration of MLSS in mg/ℓ

V = Volume of aeration tank in MG

R = Recirculation ratio

Then

$$t = \frac{8.34\ S_1\ Q}{8.34\ X\ V/1000}\left[\frac{0.03\left(\frac{S_1 - S_2}{S_1}\right) \times 100}{100 - \left(\frac{S_1 - S_2}{S_1}\right) \times 100}\right]^{2.38}$$

$$= \frac{1000\ S_1 Q}{X\ V}\left[0.03\left(\frac{S_1 - S_2}{S_2}\right)\right]^{2.38} \qquad (16)$$

Since

$$\frac{Q(1+R)}{24}\ t = V \qquad (17)$$

Substituting "V" in Eq. 16 from Eq. 17, Eq. 16 becomes

$$t^2 = \frac{24000\ S_1}{X(1+R)}\ (0.03)^{2.38}\left(\frac{S_1 - S_2}{S_2}\right)^{2.38}$$

$$t = 2.39\left[\frac{S_1}{X(1+R)}\right]^{0.5}\left(\frac{S_1 - S_2}{S_2}\right)^{1.19} \qquad (18)$$

and

$$V = 0.1\,Q\left[\frac{S_1(1+R)}{X}\right]^{0.5}\left(\frac{S_1 - S_2}{S_2}\right)^{1.19} \qquad (19)$$

(B) Contact Aeration Plants: The operational data of seven contact-aeration plants of U.S. military posts were analyzed and formulated in the following equation for the design of contact-aeration tanks

(27), (28).

$$P = \frac{100}{1 + 0.248\,(Y_o/At)^{0.746}} \tag{20}$$

Where

A = Area of submerged contact plates in 1000 sq. ft.

and the other terms are the same as for the activated sludge processes above.

Reformation of Eq. 20 obtains

$$t = \frac{Y}{A}\left(\frac{0.248P}{100-P}\right)^{1.34}$$

Where

Q = Inflow rate in MGD

S_1 = Influent BOD mg/ℓ

S_2 = Effluent BOD mg/ℓ

V = Volume of aeration tank MG

R = Recirculation ratio

$$t = \frac{8.34\,S_1 Q}{A}\left[\frac{0.248\left(\frac{S_1 - S_2}{S_1}\right)100}{100 - \left(\frac{S_1 - S_2}{S_1}\right)100}\right]^{1.34}$$

$$= \frac{1.275\,S_1 Q}{A}\left(\frac{S_1 - S_2}{S_2}\right)^{1.34} \tag{21}$$

Since

$$V = \frac{Q(1+R)}{24}\,t \tag{22}$$

Substituting "t" in Eq. 21 for Eq. 22, Eq. 22 becomes

$$V = \frac{Q(1+R)}{24}\;\frac{1.275\,S_1 Q}{A}\left(\frac{S_1 - S_2}{S_2}\right)^{1.34}$$

$$= 0.0532\,\frac{(1+R)\,S_1 Q^2}{A}\left(\frac{S_1 - S_2}{S_2}\right)^{1.34} \tag{23}$$

It was suggested (27) that the plates should be placed normal to the direction of displacement flow to constitute the contact medium. The dimensions of the plates are 4' x 8' x $\frac{1}{4}$'' and the spacings are $1\frac{1}{2}$'' on centers. After a comparison between Eq. 23 and Eq. 19, it was found that under a certain degree of recirculation, the plate area in the contact-aeration process plays the same role with equal importance as the concentrations of mixed liquor suspended solids in the diffused-air conventional process do. In order to maintain aerobic conditions in the contact-aeration tank, oxygen tension required must be as high as for the other processes.

4. Herbert's Equations for the Design of Completely-Mixed System (29)

The batch studies of Monod's data indicated the existence of the following relation:

$$\mu = \mu_m \frac{S}{K_s + S} \tag{24}$$

Where

μ = Exponential growth rate in time^{-1}

μ_m = Growth rate constant or maximum growth rate in time^{-1}

S = Substrate concentrations

K_s = Saturation constant

Assuming

X = Concentration of MLSS

and

t = Detention time

then at the exponential growth condition

$$\frac{dX}{dt} = \mu X$$

$$= \mu_m X \frac{S}{K_s + S} \tag{25}$$

By definition, cell yield, Y, equals the cell increase on the consumption of a unit weight of substrate. Therefore, "Y" can be interpreted mathematically as

$$Y = \frac{dX}{dS} \tag{26}$$

Rate of substrate removal is

$$\frac{dS}{dt} = \frac{dX}{dt}\,\frac{dS}{dX}$$

$$= \mu_m \frac{X}{Y} \left(\frac{S}{K_s + S}\right) \tag{27}$$

The flow diagram of continuous culture under recirculation is shown in Figure 6 (after Herbert).

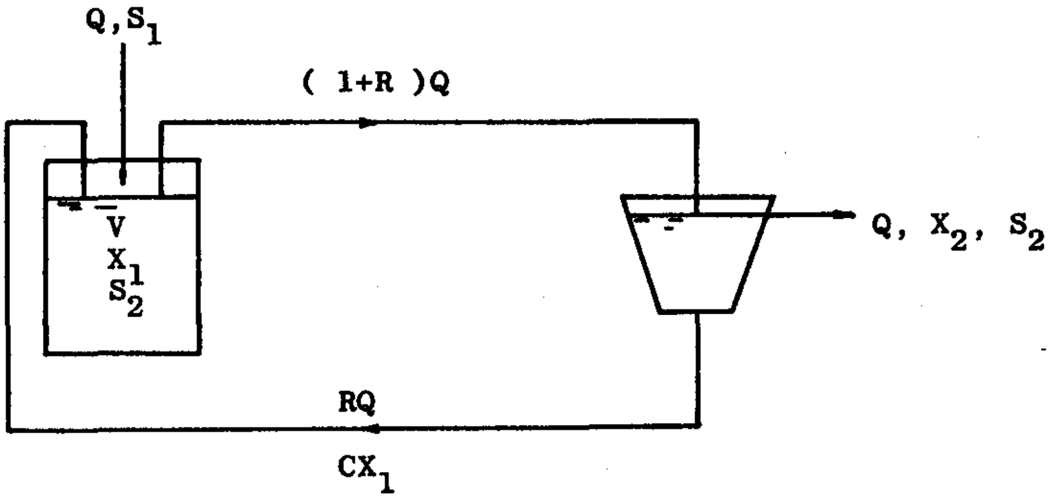


Figure 6

Flow Diagram of Completely-mixed Continuous Flow Activated Sludge Process with Recirculation (After Herbert)

Where

S_1 = Substrate concentrations in the influent

S_2 = Substrate concentrations in the effluent or in the aeration tank

X_1 = Concentration of microorganisms in the mixed liquor

X_2 = Concentration of microorganisms in the effluent

Q = Inflow rate

V = Volume of aeration tank

R = Recirculation ratio

C = Concentration factor of recycled sludge ($C \geq 1$)

Assume dilution rate to the reactor (aeration tank) = $D = 1/t$

$$D = \frac{(1+R)Q}{V} \tag{28}$$

Since

$$(1+R)QX_1 = QX_2 + RQCX_1$$

$$\frac{X_2}{X_1} = 1 + R - RC \tag{29}$$

$$\text{Cell feed-back fraction} = \frac{\text{Cells feed-back to reactor}}{\text{Cells issuing from reactor}}$$

$$= \frac{RQ \quad C \quad X_1}{(1+R)QX_1}$$

$$= \frac{RC}{1+R} \tag{30}$$

(A) Cell Balance in Reactor

Increase = Feedback – Outflow + Growth

$$\frac{dX_1}{dt} = R \quad D \quad CX_1 - (1+R)\,DX_1 + \mu_m X_1 \left(\frac{S_2}{K_s+S_2}\right)$$

At steady state,

$$\frac{dX_1}{dt} = 0$$

$$\mu_m \frac{S_2}{K_s + S_2} = (1+R - CR)D \quad (31)$$

(B) Substrate Balance in Reactor

Increase = Input + Feedback - Outflow - Consumption

$$\frac{dS_2}{dt} = DS_1 + DS_2 - (1+R)\ DS_1 - \frac{\mu_m X_1}{Y} \left(\frac{S_2}{K_s + S_2}\right)$$

At steady state,

$$\frac{dS_2}{dt} = 0$$

then

$$S_2 = \frac{K_s (1+R-CR)\ D}{\mu_m - (1+R-CR)\ D} \quad (32)$$

$$X_1 = \frac{Y}{(1+R-CR)} (S_1 - S_2) \quad (33)$$

$$X_2 = Y(S_1 - S_2) \quad (34)$$

Reformation of Eq. 32:

$$D = \frac{S_2 \mu_m}{(1+R-CR)(K_s + S_2)}$$

$$\frac{(1+R)\ Q}{V} = \frac{S_2 \mu_m}{(1+R-CR)(K_s + S_2)}$$

$$V = \frac{(1+R)\ Q}{S_2 \mu_m} (1+R-CR)(K_s + S_2) \quad (35)$$

$$t = \frac{V}{(1+R)Q}$$

$$= \frac{(1+R-CR)(K_s + S_2)}{S_2 \mu_m} \quad (36)$$

As the concentration of return sludge is X_r, then the concentration factor will be

$$C = \frac{X_r}{X_1}$$

and

$$X_1 = \frac{Y}{(1+R - \frac{X_r}{X_1} R)} (S_1 - S_2)$$

$$= \frac{Y(S_1 - S_2) + X_r R}{(1+R)} \qquad (37)$$

$$R = \frac{X_1 - Y(S_1 - S_2)}{X_r - X_1} \qquad (38)$$

$$V = \frac{(1+R)Q}{S_2 \mu_m} (1+R - \frac{X_r}{X_1} R)(K_s + S_2) \qquad (39)$$

$$t = \frac{(1+R - \frac{X_r}{X_1} R)(K_s + S_2)}{S_2 \mu_m} \qquad (40)$$

For the purpose of acquiring the acceptable quality in the effluent (S_2) under inflow rate (Q) at organic concentration (S_1), sludge recirculation and cell concentration in the mixed liquor must be controlled. However, cell yield (Y), saturation constant (K_s), and growth rate constant (μ_m) control the operation and design, since these variables are dependent upon waste characteristics and operation environment. Batch studies or once-through continuous flow studies are generally used to assess the values of Y, K_s, and μ_m.

(C) Batch Studies:

Since

$$\mu = \mu_m \frac{S}{K_s + S}$$

$$\frac{1}{\mu} = \frac{K_s + S}{\mu_m S}$$

$$\frac{1}{\mu} = \left(\frac{K_s}{\mu_m}\right) \frac{1}{S} + \frac{1}{\mu_m}$$

Plotting of the $1/\mu$ against $1/S$ is indeed an interpretation of the Lineweaver-Burk equation. It is shown in Figure 7

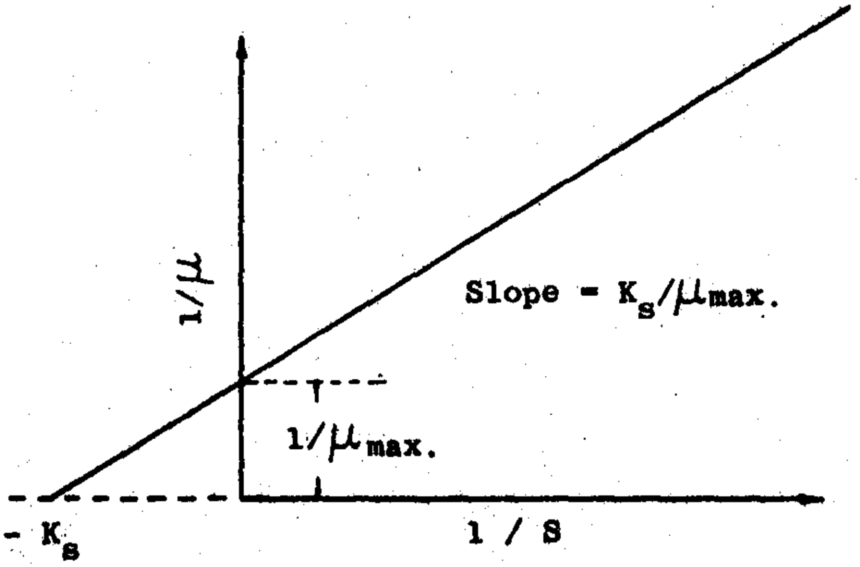


Figure 7

Plot of 1/μ vs 1/S Under Batch Studies

Cell yield (Y) is calculated by Eq. 41.

$$Y = \frac{\Delta X}{\Delta S} \tag{41}$$

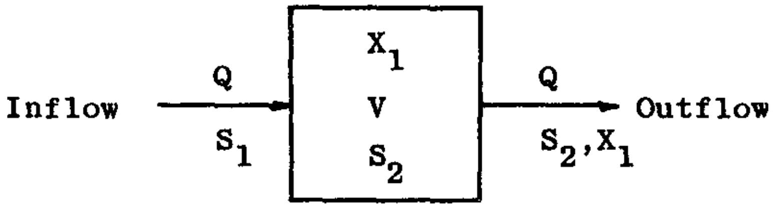
where

ΔX = Increase in cell concentration

ΔS = Concentration of substrate removed or substrate utilized

(D) Once-Through Continuous Flow Studies

The flow diagram of once-through continuous flow is shown in Figure 8.



AERATION TANK

Figure 8

Flow Diagram of Completely-mixed Once-through Continuous Flow System

(i) Cell Balance

Change = growth − loss in effluent

$$\frac{dX_1}{dt} = \mu X_1 - DX_1$$

$$= (\mu - D)X_1$$

At steady state,

$$\frac{dX_1}{dt} = 0$$

$$\mu = D = \frac{1}{t} \qquad (42)$$

(ii) Substrate Balance

Change = input - loss in effluent - consumption

$$\left(\frac{dS_2}{dt}\right) V = S_1 \ Q - S_2 Q - \frac{\mu X_1}{Y} \ V$$

$$\frac{dS_2}{dt} = \frac{S_1}{t} - \frac{S_2}{t} - \frac{\mu X_1}{Y}$$

At steady state,

$$\frac{dS_2}{dt} = 0$$

$$\frac{(S_1 - S_2)}{t} = \frac{\mu X_1}{Y} = \frac{X_1}{tY}$$

$$Y = \frac{X_1}{S_1 - S_2} \qquad (43)$$

Cell yield is the fraction of organic substance channeled into biosynthesis. It is generally expressed in terms of percent. Placak and Ruchhoft (30) reported sludge yields on carbohydrate wastes in the range of 65 to 85%. Hoover, et al.(31), using COD as a parameter, found that two-thirds of the carbon sources in dairy waste was assimilated into synthesis. Sawyer (32) tabulated data from many sources and concluded that sludge yields in the range of 50 to 60% can be expected. Gaudy and Engelbrecht (33) obtained a cell yield of 60%, using glucose as substrate. McKinney (34) has concluded that two-thirds of the ultimate oxygen demand of the organic matter being metabolized is converted into cellular masses. However, McWhorter and Heukelekian (35) have reported on studies using glucose as substrate wherein cell yields from 28 to 35% were obtained. McCarty (36) has reported that for glucose 0.55 grams of dry solids will be produced with the consumption of one gram COD. Long term studies by Rao and Gaudy (37) indicated a statistical range of yields from 48 to 82% from cells grown on glucose minimal medium using ammonium ion as the source of nitrogen. Rao, et al.(37) depicted the relationship between cell yield and COD removal expressed as:

$$Y = K(COD)$$

where

$$K = 0.65 \pm 0.17$$

From recent studies in forty-one batch experiments, Yu (38) has reported that the cell yield in the glucose systems averaged 47.9%, and in glycerol systems, 42.5%. The average yield in the glucose-glycerol combined system was 45.2%. The higher yield on glucose, as compared to glycerol, would indicate that one cannot predict cell yield on the basis of energy content of the substrate. Glycerol contains more oxidizable hydrogen than does glucose, and, if yield in aerobic systems could be adequately predicted on the basis of energy content of the substrate, one would have expected to observe greater yields on glycerol than on glucose.

The results obtained were in disagreement with the findings of Servizi and Bogan (39), (40). They found a correlation between sludge production and free energy of oxidation of the substrate based on yields with heterogeneous cultures for several compounds. If Servizi's and Bogan's equation for yield as a function of COD, Y = 0.37 COD is correct, then the reports from so many investigators regarding the yield problem must be wrong. The key point which causes much variation in the findings of Servizi and Bogan with those of others lies in the difference in using dimensions to express "yield." Servizi et al. expressed "yield" as the grams of cells grown per mole of organic substance consumed. If the yield is measured on this basis, a biologically degradable compound with higher molecular weight would be easily predicted to produce higher yields than a compound with lower molecular weight. McCarty (41) recognized the necessity of considering both the coupling of oxidation to phosphoralation and the coupling of ATP (Adenosine Triphosphate) production to synthetic reactions. McCarty concluded that reasonable predictions of yield can be based on free energy, energy transfer efficiency and energy requirements for synthesis and maintenance.

In the heterogeneous biological system, Rao and Gaudy (37) were aware that the different yields were correlated with macroscopically and microscopically detectable changes in predominance. Yu (38) also noticed that cell yields were quite predominance-dependent. Gaudy and Gaudy (42) added that factors other than efficiency of conversion of carbon source to cell material may govern selection in a mixed population, and that yields measured in short-term experiments may not be applicable in practice to activated sludges. The degree of environmental selection may be expected to vary with the particular treatment process employed and also with characteristics of the waste other than the amount of oxidizable carbon sources.

Using glucose as the sole carbon source for studies on the kinetic constants in the batch units and in the continuous flow apparatus, Gaudy, Ramanathan and Rao (43) reported that there was considerable variation in the values of K_s, μ_m, and Y at the various dilution rates. In general, μ_m values were lower for systems operated at the lower dilution rates and there was no discernible trend with dilution rate in the variation of K_s. The yield values obtained from batch experiments were consistently lower than those obtained directly from the continuous flow reactor. Gaudy, et al.(43) added that μ_m values of 0.5-0.6 hr^{-1} and K_s values of 75-125 mg/ℓ can be expected with yield coefficients of approximately 0.6. The best representation of the average values of the growth parameters are:

$\mu_m = 0.53\ hr^{-1}$

$K_s = 90\ mg/\ell$

Y_b = 0.45 mg solids/mg COD removed for batch systems

Y_c = 0.65 mg solids/mg COD removed for continuous systems.

5. Reynolds and Yang's Equations with Modifications for the Design of a Completely-Mixed System

Herbert's model of a completely-mixed continuous flow reactor has been recently modified by Reynolds and Yang (44). The difference between Herbert and Reynolds, et al, equations is in the consideration of microbial endogenous respiration. The flow sheet is diagrammatically shown in Figure 9.

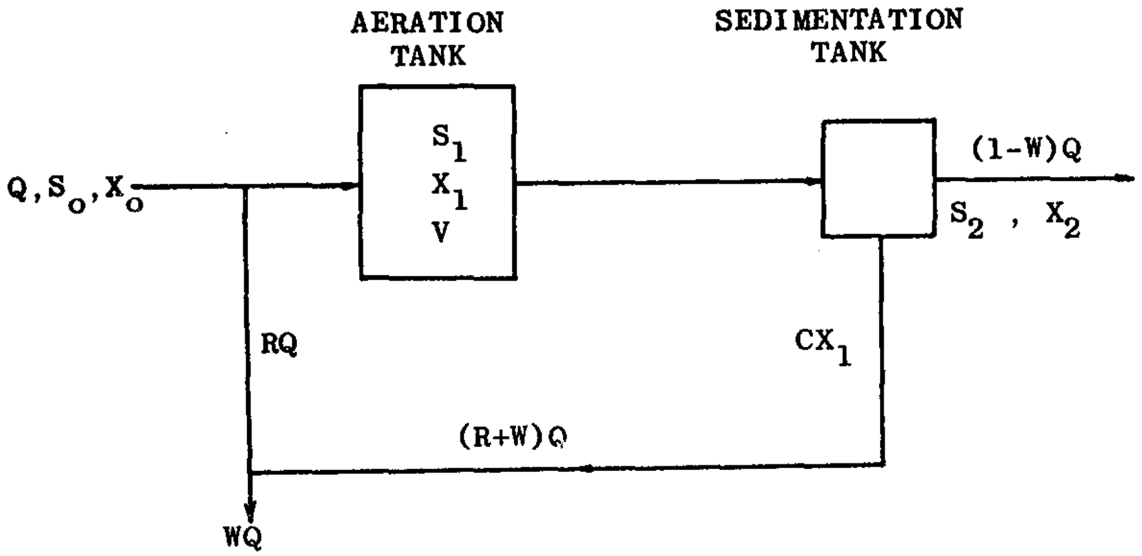


Figure 9

Flow Diagram of Completely-mixed Continuous Flow Activated Sludge Process (After Reynolds and Yang)

Where

Q = Inflow rate

R = Recirculation ratio

S_0 = Substrate concentration in the influent

S_1 = Substrate concentration in the reactor

S_2 = Substrate concentration in the final clarifier

W = Waste sludge factor

C = Material balance factor for suspended solids at the final clarifier

$$C = \frac{1 + R}{R + W} \tag{44}$$

X_1 = MLVSS or MLSS concentration in aeration tank

X_2 = MLVSS or MLSS in final clarifier effluent

X_0 = MLVSS or MLSS in influent

X_r = Concentration of settled sludge in mg/ℓ (return sludge or wasted sludge)

V = Volume of aeration tank

Assume

K_e = Endogenous respiration rate

t = Detention time

μ = Exponential growth rate

Y = Cell yield

It is noted that X_0 and X_2 are essentially equal to zero, compared with the concentration of suspended solids present in the mixed liquor, X_1.

(A) Cell Balance in the Aeration Tank

Increase = growth + feedback - endogenous respiration - outflow

$$V\left(\frac{dX_1}{dt}\right) = \mu X_1 V + RQCX_1 - X_1 K_e V - (1+R)QX_1$$

At steady state,

$$\frac{dX_1}{dt} = 0$$

$$X_1\left[\mu + \frac{RCQ}{V} - K_e - (1+R)\frac{Q}{V}\right] = 0$$

Since

$$X_1 \neq 0$$

Then,

$$\mu = K_e + (1+R)\frac{Q}{V} - RC\frac{Q}{V}$$

$$(1+R)\,Q\ \ t = V$$

$$\frac{Q}{V} = \frac{1}{(1+R)\,t}$$

$$\mu = K_e + \frac{1}{t} - \frac{RC}{(1+R)t}$$

$$\mu = K_e + \frac{(1+R) - RC}{(1+R)\ t} \qquad (45)$$

Because Reynolds, et al. did not consider the recirculation ratio for detention time calculation, their equation of growth rate was

$$\mu = K_e + \frac{(1+R) - RC}{t}$$

(B) Substrate Balance in the Aeration Tank

Change = input + feedback - consumption - outflow

$$V\left(\frac{dS_1}{dt}\right) = S_0Q + S_2RQ - \frac{\mu X_1}{Y}V - (1+R)\,QS_1$$

Essentially

$$S_1 = S_2$$

At steady state,

$$\frac{dS_1}{dt} = 0$$

$$S_0 = \frac{Q}{V} + RS_2\left(\frac{Q}{V}\right) - \mu\left(\frac{X_1}{Y}\right) - S_2\left(\frac{Q}{V}\right) - S_2R\left(\frac{Q}{V}\right) = 0$$

$$\mu\frac{X_1}{Y} = (S_0 - S_2)/(1+R)\ t$$

$$X_1 = \frac{Y(S_0 - S_2)}{\mu(1+R)t}$$

Substituting the value of "μ" from Eq. 45, there is obtained

$$X_1 = \frac{Y(S_0 - S_2)}{K_e\,t\,(1+R) + (1+R - RC)}$$

Since

$$C = \frac{1+R}{R+W}$$

Therefore,

$$X_1 = \frac{Y(S_0 - S_2)}{K_e\,t\,(1+R) + \left(1+R - R\,\frac{1+R}{R+W}\right)} \tag{46}$$

and

$$t = \frac{Y(S_0 - S_2) - X_1\left(1+R - R\,\frac{1+R}{R+W}\right)}{X_1K_e(1+R)} \tag{47}$$

$$V = \frac{Q\left[Y(S_0 - S_2) - X_1\left(1+R - R\,\frac{1+R}{R+W}\right)\right]}{X_1K_e} \tag{48}$$

Rewriting Eq. 48, there is obtained

$$1+R - R\,\frac{1+R}{R+W} = \frac{Y}{X_1}(S_0 - S_2) - \frac{V}{Q}\,K_e \tag{49}$$

Assume concentration of return sludge = X_r

$$C = \frac{1+R}{R+W} = \frac{X_r}{X_1}$$

Substituting $\frac{1+R}{R+W}$ for $\frac{X_r}{X_1}$, Eq. 49 will be

$$1 + R - R\frac{X_r}{X_1} = \frac{Y}{X_1}(S_0 - S_2) - \frac{V}{Q}K_e$$

$$R = \frac{X_1 Q + X_1 V K_e - Y(S_0 - S_2)}{Q(X_r - X_1)} \tag{50}$$

In addition,

$$\frac{1+R}{R+W} = \frac{X_r}{X_1}$$

$$W = \frac{X_1 - R(X_r - X_1)}{X_r}$$

$$= \frac{X_1}{X_r} - \frac{1}{X_r}\left[\frac{X_1 Q + X_1 V K_e - Y(S_0 - S_2)}{Q}\right]$$

$$= \frac{X_1 Q - X_1 Q - X_1 X K_e + Y(S_0 - S_2)}{QX_r}$$

$$= \frac{Y(S_0 - S_2) - X_1 V K_e}{QX_r} \tag{51}$$

The pumping rate required for return sludge as well as for wasted sludge will be,

$$Q_{rw} = (R+W)Q$$

Substituting the items of R and W from Eqs. 50 and 51,

$$Q_{rw} = \frac{X_1 Q + X_1 V K_e - Y(S_0 - S_2)}{(X_r - X_1)} + \frac{Y(S_0 - S_2) - X_1 V K_e}{X_r}$$

$$= \frac{X_r X_1 Q + X_r X_1 V K_e - X_r Y(S_0 - S_2) + Y(S_0 - S_2)(X_r - X_1 - X_r)}{X_r(X_r - X_1)}$$

$$= \frac{X_r X_1 Q + X_1 V K_e(X_r - X_r + X_1) + Y(S_0 - S_2)(X_r - X_1 - X_r)}{X_r(X_r - X_1)}$$

$$Q_{rw} = \frac{X_1\left[X_r Q + X_1 V K_e - Y(S_0 - S_2)\right]}{X_r(X_r - X_1)} \qquad (52)$$

The amount of dissolved oxygen required to supply a biological system includes oxygen uptake for substrate assimilation, biosynthesis and for endogenous respiration:

(C) Oxygen Uptake for Substrate Assimilation and Biosynthesis

$$= (1 - 1.42Y)\frac{(S_0 - S_2)}{t}$$

Here, it is assumed that the average molecular formula for activated sludge is $C_5H_7NO_2$ and the equivalent oxygen for 1 mg/ℓ biological solid is 1.42 mg/ℓ which has been widely accepted.

(D) Oxygen Uptake for Endogenous Respiration

$$= 1.42\ K_e X_1$$

Therefore, total oxygen demand

$$= (1 - 1.42Y)\frac{(S_0 - S_2)}{t} + 1.42\ K_e X_1 \qquad (53)$$

It is important to note that Y and K_e are, in a large measure, dependent on the specific waste to be treated under certain environmental conditions in operation of a biological system. Batch studies and once-through continuous flow studies are employed to determine the values of K_e and Y. The completely-mixed continuous flow process without sludge recirculation was studied and the flow diagram is shown in Figure 10.

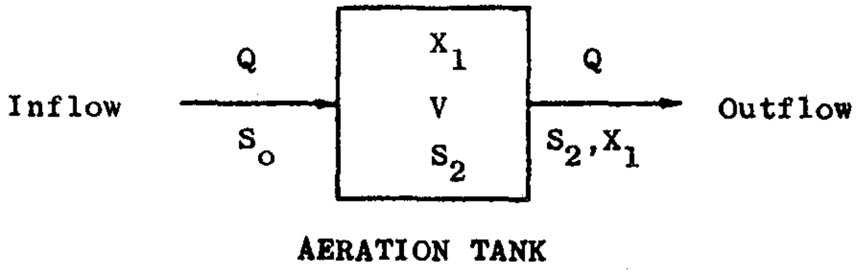


Figure 10

Flow Diagram of Completely-mixed Continous Flow System Without Sludge Recirculation

(E) Cell Balance in the Aeration Tank

Change = growth - endogenous respiration - loss of effluent

$$V\left(\frac{dX_1}{dt}\right) = \mu X_1 V - K_e X_1 V - X_1 Q$$

At steady state,

$$\frac{dX_1}{dt} = 0$$

$$\frac{1}{t} = \mu - K_e \qquad (54)$$

(F) Substrate Balance in the Aeration Tank

Change = input - consumption - loss in effluent

$$V\left(\frac{dS_2}{dt}\right) = S_0 Q - \mu \frac{X_1}{Y} V - S_2 Q$$

At steady state,

$$\frac{dS_2}{dt} = 0$$

$$\frac{S_0 - S_2}{t} = \mu \frac{X_1}{Y}$$

Since

$$\mu = K_e + \frac{1}{t}$$

$$\frac{S_0 - S_2}{X_1} = \frac{1}{Y}\left(K_e + \frac{1}{t}\right) t$$

$$= \frac{K_e}{Y} t + \frac{1}{Y} \qquad (55)$$

The graphic expression of Eq. 55 is shown below in Figure 11.

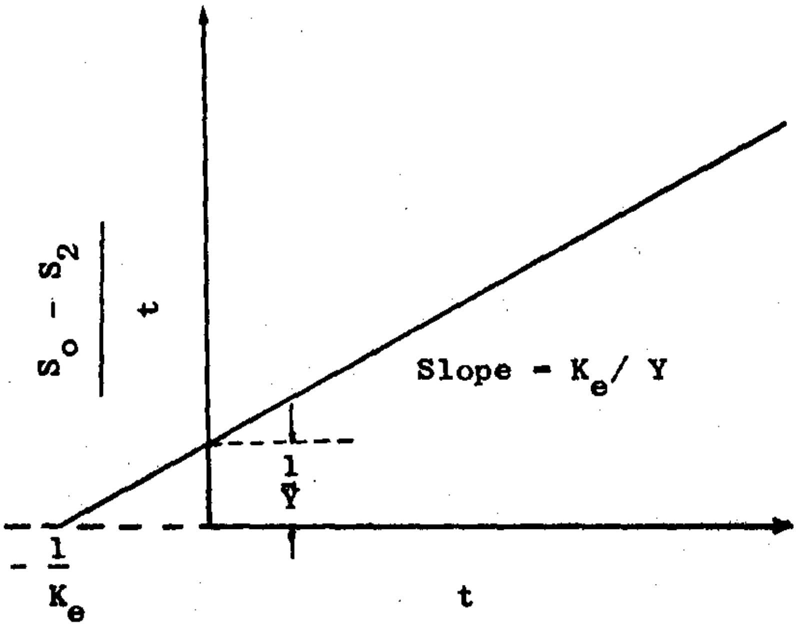


Figure 11

Plot of $(S_o - S_2)/X$, vs t

Batch Study:

$$\Delta X = Y\Delta S = K_e X \qquad (56)$$

Where,

ΔX = Net accumulation in biological solids concentration

X = Average biological solids concentration

ΔS = Substrate concentration removed

Y = Cell yield

$$\frac{\Delta X}{X} = Y\frac{\Delta S}{X} - K_e \qquad (57)$$

Eq. 57 is shown graphically in Figure 12.

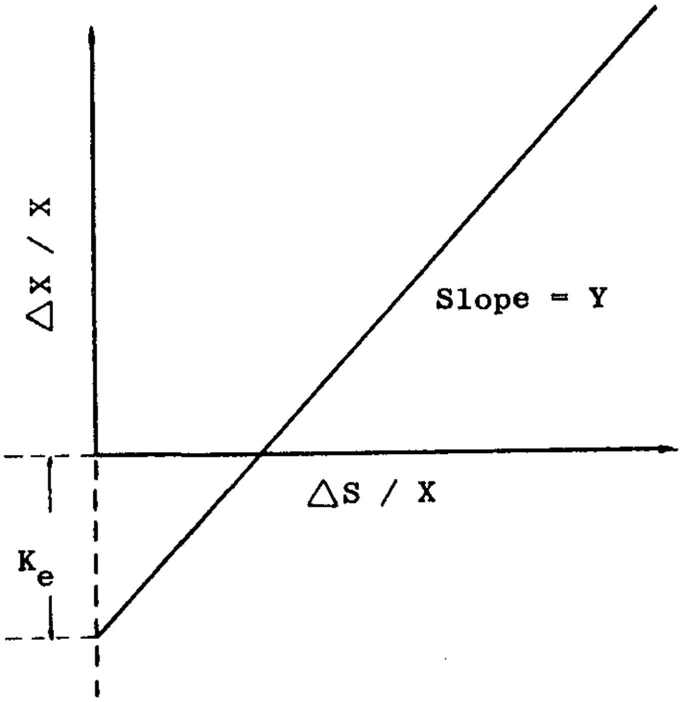


Figure 12
Plot of $\Delta X/X$ vs $\Delta S/X$

The coefficient of endogenous respiration in a biological system would be affected appreciably by the temperature (44). An equation, $K_T = K_{20}\theta^{(T-20)}$, was given to estimate K_e values at different temperatures. K_T represents the rate of endogenous respiration at a given temperature T other than 20° C. K_{20} represents the rate of endogenous respiration at 20° C. θ is a constant ranging from 1.065 to 1.085. The average value of θ will be 1.075.

$$K_T = K_{20}\theta^{(T-20)} \tag{58}$$

$$K_T/K_{20} = \theta^{(T-20)} \tag{59}$$

The ratio of K_T/K_{20} at various temperatures and θ values are tabulated in Table IX and graphically shown in Figure 13.

From Figure 13 and Table IX, it seems apparent that temperature effect on K_T/K_{20} is tremendously significant. Nevertheless, the variation of K_T and K_{20} at certain temperatures below 25°C is insignificant if $\theta = 1.075$ is chosen for estimation of endogenous respiration rate of a given biological system.

In order to obtain the required purification in the effluent, attention must be paid to the effect of endogenous respiration on purification efficiency. A rearranged form of Eq. 46 is shown in Eq. 60.

$$S_0 - S_2 = \frac{X_1}{Y}\left[(1+R - R\frac{1+R}{W+R}) + K_e t(1+R)\right] \tag{60}$$

The factor involved affecting waste elimination is the term,

$$\frac{X_1}{Y} K_e t(1+R).$$

The percent variation attributed to endogenous respiration in purification work can be calculated from Eq. 61.

TABLE IX

RATIO OF K_T TO K_{20} AT VARIOUS TEMPERATURES AND θ VALUES

T	K_T/K_{20}		
°C	$\theta = 1.065$	$\theta = 1.085$	$\theta = 1.075$
0	0.285	0.196	0.236
5	0.396	0.294	0.338
10	0.535	0.442	0.485
15	0.730	0.666	0.696
20	1.000	1.000	1.000
25	1.37	1.50	1.435
30	1.87	2.26	2.06
35	2.53	3.40	2.96
40	3.51	5.10	4.23

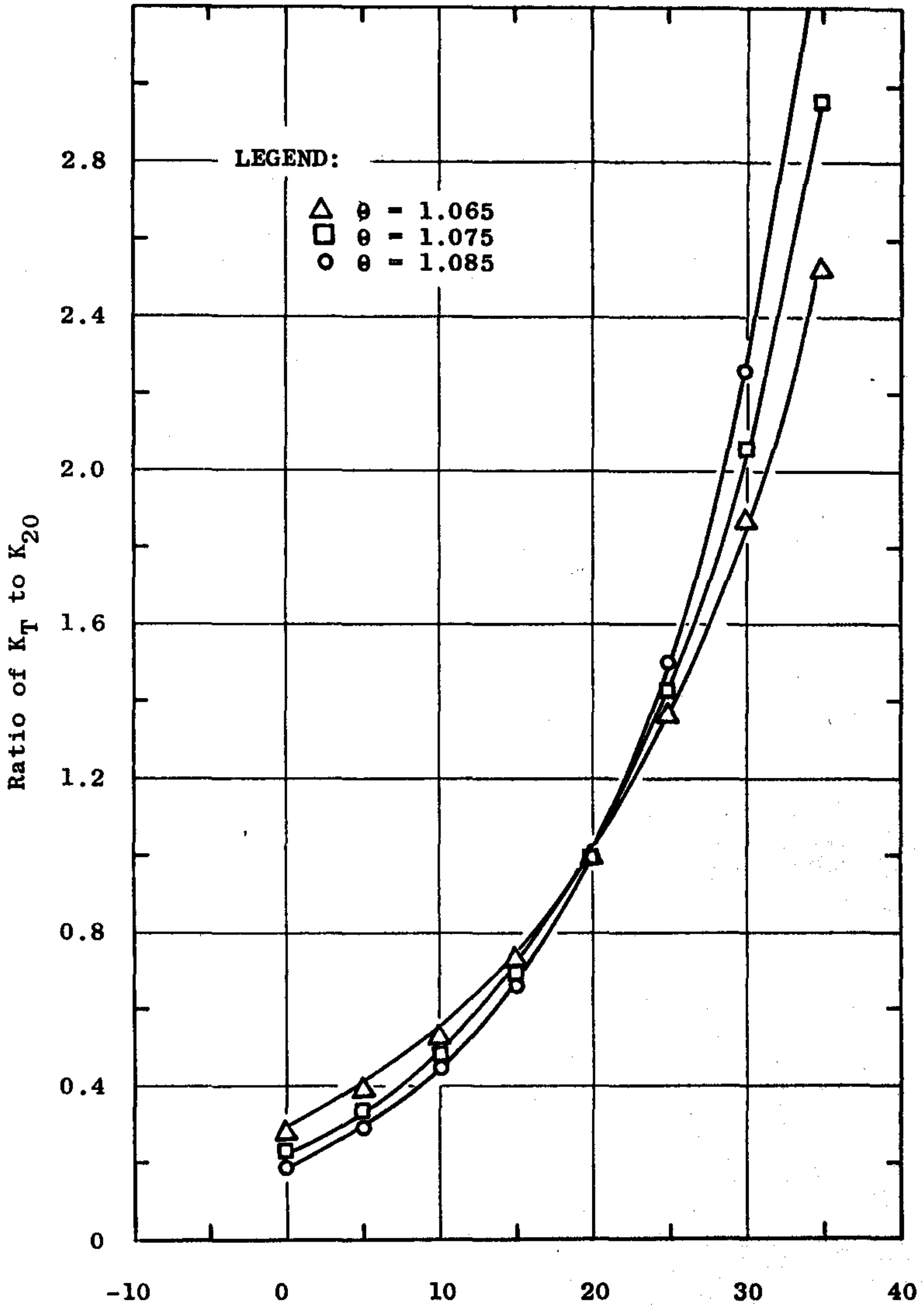


Figure 13

Ratio of K_T to K_{20} at Various Temperatures and θ Values

$$\text{Percent variation} = \frac{K_e t(1+R)}{(1+R - R\frac{1+R}{R+W}) + K_e t(1+R)} 100\% \qquad (61)$$

It is assumed that the activated sludge system is operated at a four-hour detention time with 10% flow rate of waste sludge per day, and the K_e value is assumed to be 0.16 day^{-1} (44). Then the percent variation in waste purification related with various sludge recirculation is shown in Figure 14, which indicates that endogenous respiration in governing waste purification becomes more important as the recirculation ratio increases.

It is noted in Eq. 48 that the biological mass undergoes endogenous respiration in a given volume of the aeration tank. The most effective concentration of mixed liquor can be derived from the following mathematical relation:

$$V = \frac{Q\left[Y(S_0 - S_2) - X_1(1+R - R\frac{1+R}{R+W})\right]}{X_1 K_e} \qquad (48)$$

Differentiating

$$\frac{dV}{dK_e} = -\frac{\theta}{2}\left[\frac{Y(S_0 - S_2)}{X_1 K_e^{\,2}} - \frac{1+R - R\left(\frac{1+R}{R+W}\right)}{K_e^{\,2}}\right]$$

For a given aeration tank at steady state,

$$dV/dK_e = 0$$

$$Y(S_0 - S_2) = X_1\left[1+R - R\left(\frac{1+R}{R+W}\right)\right]$$

$$X_1 = \frac{Y(S_0 - S_2)}{1+R - R\left(\frac{1+R}{R+W}\right)} \qquad (62)$$

Eq. 62 implies that the concentration of mixed liquor suspended solids at "X_1" under a given volume of aeration tank "V," the biological

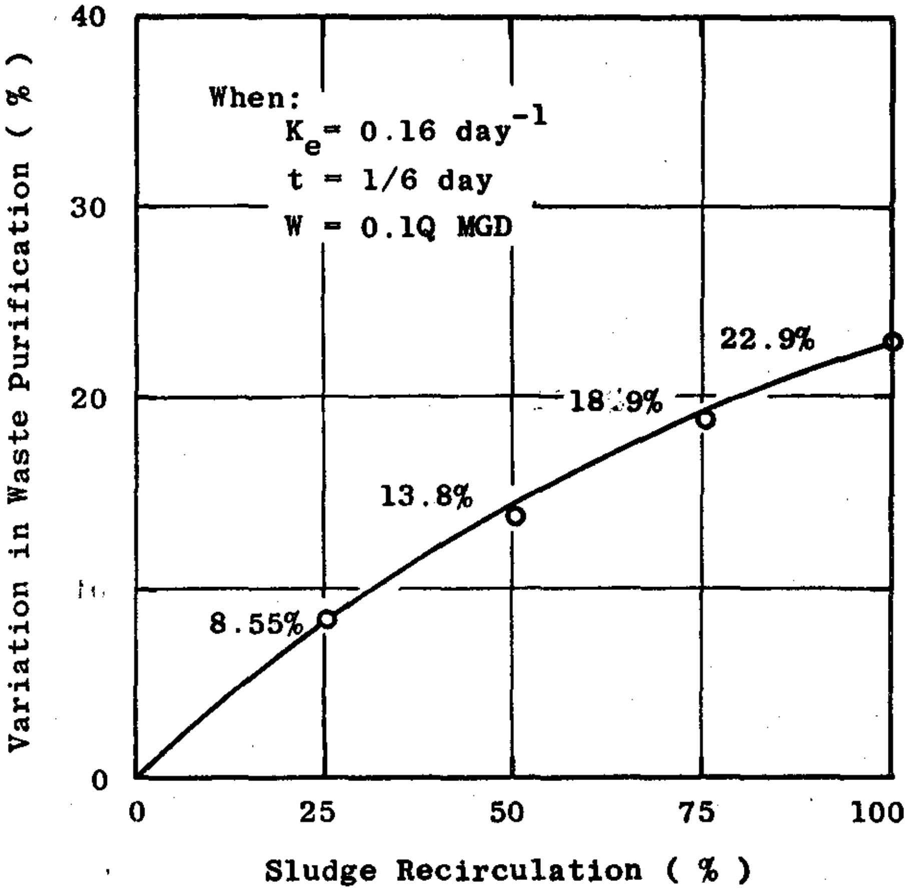


Figure 14

Percent Variation in Waste Purification at Various Sludge Recirculation Rates system will maintain a steady state when the endogenous respiration attains a minimum. A comparison between Eq. 62 and Eq. 46 indicates that the occurrence of endogenous respiration introduces a factor

$$\frac{1+R-R\left(\frac{1+R}{R+W}\right)}{K_e t(1+R)+(1+R-R\frac{1+R}{R+W})}$$

to determine the concentration of suspended solids in the mixed liquor. It has been mentioned that the difference between the work of Herbert (29) and that of Reynolds, et al.(44) lies in the fact that the latter considered the endogenous respiration and the former neglected it. As the rate of endogenous respiration approaches zero, then Eq. 62 will be identical with Eq. 33.

6. Smith's Equation with Modifications for the Design of a Completely-Mixed System

It has been well recognized that the continuous and homogeneous activated-sludge waste-water treatment system (completely-mixed activated sludge process) would provide an opportunity for significant cost reduction. Conventional activated sludge processes are designed to provide an aeration time of four hours or more to achieve 80 to 85% removal in normal domestic and commercial sewage. But the completely-mixed system can accomplish the same degree of treatment in an aeration period as short as one hour. Modifications of conventional systems such as step aeration or tapered aeration minimize certain environmental instability to an appreciable extent, but still provide a far less stable environment than does a homogeneous system.

Based on the fundamental idea of substrate balance taking place in the completely-mixed aeration tank, Smith (26), (53) sketched a flow sheet shown in Figure 15 to illustrate the process performance for which approaches of design are introduced as follows (45), (46), (26), and (53).

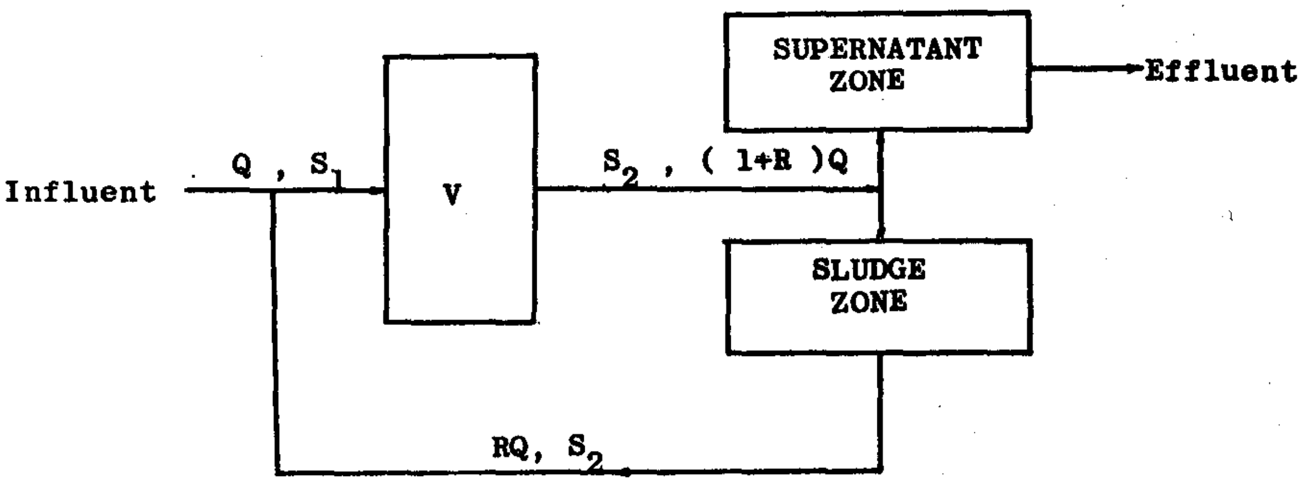


Figure 15

Flow Diagram of Completely-mixed Continuous Flow Activated Sludge Process

Substrate balance in the completely-mixed reactor:

$$QS_1 + RQS_2 = (Q + RQ)S_2 + K_4S_2V$$

$$\frac{S_2}{S_1} = \frac{1}{K_4(\frac{V}{Q}) + 1}$$

$$= \frac{1}{K_4(1+R)t+1} \qquad (63)$$

$$V = Q(1+R)\frac{(S_1 - S_2)}{S_2 K_4 (1+R)}$$

$$= \frac{Q(S_1 - S_2)}{K_4} \qquad (66)$$

Estimation of oxygen requirements:

Assuming

X_{ar} = Oxygen equivalent of active mass in return sludge

X_a = Oxygen equivalent of active mass in mixed liquor

then, BOD oxidation for energy will be at a rate proportional to BOD concentration in the aeration tank, or $K_6 S_2$ and the oxygen for auto-oxidation of the active mass in the aeration tank will be at a rate proportional to the active mass of mixed liquor suspended solids or $K_5 X_a$.

Where

K_5 = Coefficient of auto-oxidation of cellular active mass

K_6 = Coefficient of BOD oxidation

and

$$RQX_a + (Q + RQ)\ \Delta X_a = (Q + RQ)X_a$$

At steady state balance,

$$QS_1 + RQ(S_2 + X_{ar}) = (Q + RQ)(S_2 + X_a) + VK_6 X_a + VK_5 X_a$$

$$Q(S_1 - S_2) - Q(1+R)\ \Delta X_a = V(K_5 X_a + K_6 S_2)$$

Dividing by Q,

$$(S_1 - S_2) - (1+R)\ \Delta X_a = \frac{V}{Q}(K_5 X_a + K_6 S_2) \qquad (67)$$

The left side of Eq. 67 expresses oxygen requirement in terms of ultimate BOD removal and net change in active mass.

The right side of Eq. 67 expresses oxygen required in terms of the volume of the aeration tank, inflow rate, effluent and mixed liquor BOD and mixed liquor active mass.

The coefficients, K_4, K_5, and K_6 were determined in a municipal sewage system (Iowa City) by the personnel from the University of Iowa. The average value of these coefficients are listed below:

$$K_4 = 15 \text{ hr.}^{-1} \text{ at } 20^\circ \text{C}$$

Smith (53) has reported that the K_4 values in the sewage system at 20° C ranged from 21 hr^{-1} when $t = 1$ hour to 12 hr^{-1} when $t = 4$ hours. These correspond to total BOD removal of 85 and 98% respectively. It is reasoned that the K_4 value is dependent upon the time, since the nutritional environment in which metabolism occurs at the longer periods contains a larger fraction of less readily metabolizable components.

$K_5 = 0.027 \text{ mg}/\ell \text{ hr} \div \text{mg}/\ell$, if X_a is in units of oxygen
$K_5 = 0.038 \text{ mg}/\ell \text{ hr} \div \text{mg}/\ell$, if X_a is in active cell mass
$1 \text{ mg}/\ell$ cell mass $= 1.42 \text{ mg}/\ell$ oxygen equivalent
$K_6 = 3.8 \text{ mg}/\ell \text{ hr} \div \text{mg}/\ell$

The active mass can be estimated at from 25 to 30% of volatile suspended solids in the mixed liquor. The ratio of active mass to volatile suspended solids depends to a slight extent on the organic loading factor following the repression equation:

$$\frac{X_a}{\text{MLVSS}} = 0.111(\text{organic loading factor}) + 0.216 \qquad (68)$$

Smith obtained a ratio of active mass to volatile suspended solids to be approximately 0.27. Reynolds and Young (44) indicated that the

MLVSS of domestic activated sludge possessed 88% MLSS. Therefore, oxygen demand to supply the biological system will be

$$\text{Oxygen Requirements} = \frac{24V}{Q}\left[3.8\,S_2 + (0.038)\,(0.27)\,(0.88X)\right] \text{mg}/\ell$$

$$= \frac{24V}{Q}(3.8\,S_2 + 0.009X)\ \text{mg}/\ell \qquad (69)$$

Where

X = Concentration of mixed liquor suspended solids in mg/ℓ

V = Volume of aeration tank in MG

Q = Inflow rate in MGD

S_2 = Effluent or recycled substrate concentration in mg/ℓ

In the extended aeration activated sludge system, field study by Westrick, Morand and Eye (24) reported that 75% of the mixed liquor suspended solids are volatile. Then, oxygen requirements will be

$$\text{Oxygen Requirements} = \frac{24V}{Q}\left[3.8\,S_2 + (0.038)\,(0.27)\,(0.75X)\right] \text{mg}/\ell$$

$$= \frac{24V}{Q}(3.8\,S_2 + 0.0077X)\ \text{mg}/\ell \qquad (70)$$

Since

1 mg/ℓ = 8.34 lb/MG

At inflow rate Q MGD, the oxygen supply required per day will be

(i) $24V(3.8\,S_2 + 0.009X)$ lb/day for completely-mixed activated sludge process design

or

(ii) $24V(3.8\,S_2 + 0.0077X)$ lb/day for extended aeration activated sludge process design.

7. McKinney's Equations with Modifications for the Design of a Completely-Mixed System

Bacterial metabolism proceeds continuously in the sedimentation tank. This is especially true of endogenous metabolism. The recirculation of sludge from the sedimentation tank to the aeration tank permits the mixed liquor suspended solids to approach an equilibrium value that is essentially the same in the entire system, compensating for sludge concentration and sludge removal. Examination of the aeration-sedimentation system indicates that the two units combined with sludge recirculation can be considered a single unit.

(A) Entire Sludge Wasting in Effluent

The simplest aeration-sedimentation system is the one in which all of the excess suspended solids are lost in the effluent. This is the typical pattern of the extended-aeration activated sludge system. In order to obtain systematic symbols throughout the present study, the notations used by McKinney (47) and McKinney, et al.(48) have hereafter been changed. The schematic diagram of the aeration-sedimentation system with the entire sludge wasted in the effluent is shown in Figure 16.

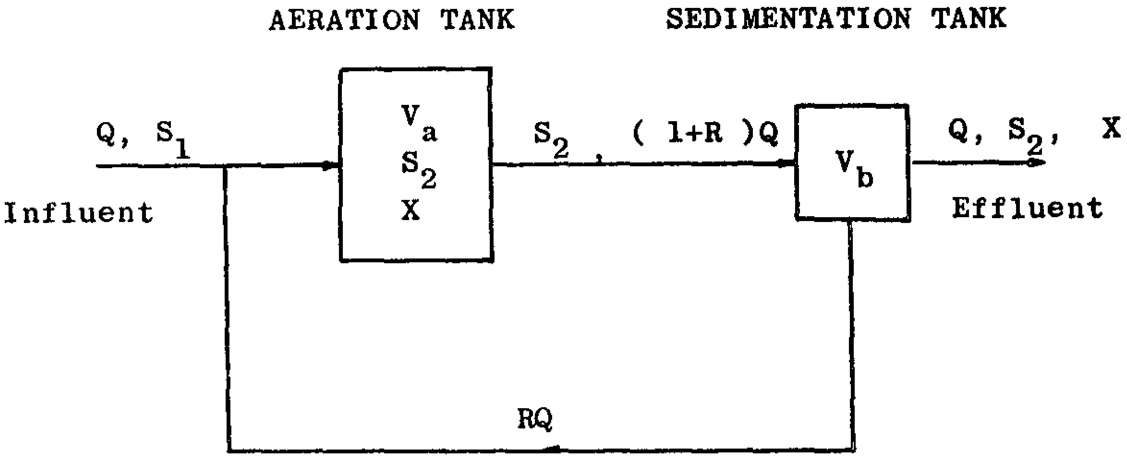


Figure 16

A Complete Mixing Activated Sludge System with Excess Sludge Discharge in Effluent (After McKinney)

The rate of change of the organic concentration in the system is given by

$$V' \frac{dS_2}{dt} = Q\ S_1 - S_2 - V'\ K_5\ S_2$$

in which V′ is the total volume (volume of aeration tank plus volume of final sedimentation tank) of the system. Assume t′ is the detention period in the entire system. Therefore,

$$\frac{dS_2}{dt} = \frac{Q}{V'} S_1 - \frac{Q}{V'} S_2 - K_5 S_2$$

When

$$\frac{dS}{dt} = 0$$

$$S_2 = \frac{S_1}{K_5 t' + 1} \tag{71}$$

Where

Q = Inflow rate

S_1 = Influent substrate concentration

S_2 = Effluent substrate concentration

K_5 = Rate of biological metabolism

Reformation of Eq. 71 obtains

$$t' = \frac{S_1 - S_2}{K_5 S_2}$$

where

t' = Detention time of aeration tank and final sedimentation tank together

Since,

$$V' = V_a + V_b$$

Therefore,

$$V_a + V_b = \frac{Q(S_1 - S_2)}{K_5 S_2} \tag{72}$$

or

$$V_a = \frac{Q(S_1 - S_2)}{K_5 S_2} - V_b \tag{73}$$

where

V_a = Volume of aeration tank

V_b = Volume of final sedimentation tank

Smith's approach in the derivation of the equation for the design of an aeration tank was based on the assumption that none of the biochemical activity occurred in the final sedimentation tank (26). On the contrary, McKinney postulated that microbial populations in the final sedimentation tank would proceed with activity similar to that in the aeration tank (47). Regardless of diversified opinion between Smith and McKinney, they both obtained an equal rate of biological metabolism with a value of 15 hr^{-1}, (K_4 in Eq. 66, K_5 in Eq. 73). Smith considered the design detention time "t" to be the time of passage of waste flow through the aeration tank, while McKinney considered the detention time as the time the waste flow resided in the whole biological system (aeration tank plus sedimentation tank). It seems quite reasonable that microbial activity would proceed, at least to a certain extent, in the final sedimentation tank as proposed by McKinney. Comparing Eq. 66 and Eq. 73, it is apparent that a more economic design will be obtained in case the calculation is based on Eq. 73.

The volume of final sedimentation tank is designed to produce as clear an effluent as possible. It is obvious that sludge settleability would play a determinative role in deciding on the surface flow rate used. While it has been suggested by McKinney, Symons, Shifrin and Vezina (50) that 150 gallons per day per square foot of surface flow rate be used for the design of final sedimentation tanks in the completely-mixed activated sludge process, Fair, Geyer, and Okun (1) stated that a loading value of 1200 gpd per square foot makes allowance for poorly settling sludge. But under ideal conditions, the surface loading of final settling tanks treating activated sludge may, therefore, be as high as 4000 gpd per square foot and the detention period as low as 0.47 hr. or 30 minutes in a tank ten feet deep.

Assume

O.R. = Overflow rate in final sedimentation tank in terms of gpd/sq. ft.

D = Depth of the tank in feet

Q = Inflow rate in MGD

t_a = Detention time of aeration tank in days

t_b = Detention time of final sedimentation tank in days

V_a = Volume of aeration tank in MG

V_b = Volume of final sedimentation tank in MG

then,

$$D = (\text{O. R.})(t_b)\left(\frac{1}{7.48}\right)$$

$$= (\text{O. R.})\left(\frac{V_b}{Q}\right)\left(\frac{1}{7.48}\right)$$

$$V_b = \frac{7.48\,DQ}{(\text{O. R.})} \tag{74}$$

Eq. 73 can be rewritten as

$$V_a = \frac{Q(S_1 - S_2)}{K_5 S_2} - \frac{7.48\,(D)\,(Q)}{(\text{O. R.})} \tag{75}$$

The detention time of raw waste in the aeration unit, t_a, can be calculated using the recirculation ratio, R, from Eq. 75.

$$t_a = \frac{V_a}{(1+R)Q}$$

$$= \frac{1}{(1+R)} \frac{(S_1 - S_2)}{K_5 S_2} - \frac{7.48\,(D)\,(Q)}{(O.\,R.)} \tag{76}$$

When,

$$t_a > 0$$

$$\frac{(S_1 - S_2)}{K_5 S_2} - \frac{7.48\,D}{(O.\,R.)} > 0$$

$$\frac{S_1 - S_2}{K_5 S_2} > \frac{7.48\,D}{(O.\,R.)}$$

$$\frac{(O.\,R.)}{7.48\,D} > \frac{K_5 S_2}{S_1 - S_2}$$

$$O.\,R. > 7.48\,D \left(\frac{K_5 S_2}{S_1 - S_2} \right) \tag{77}$$

and

$$(S_1 - S_2) > \frac{7.48\,D}{(O.\,R.)} (K_5 S_2)$$

$$S_1 > \frac{7.48\,D}{(O.\,R.)} (K_5 S_2) + S_2 \tag{78}$$

As stated by Fair, et al (1), a loading of 1200 gpd per square foot is allowable for poorly settling sludge in the design of final sedimentation tanks, and it is apparent that Eq. 77 implies its existence in the case of

$$(O.\,R.) > 7.48\,D \frac{K_5 S_2}{S_1 - S_2} > 1200$$

$$S_1 - S_2 < 7.48\,D \frac{K_5 S_2}{1200}$$

$$S_1 < \frac{7.48\,D\,K_5 S_2}{1200} + S_2 \tag{79}$$

Combining Eq. 78 and Eq. 79, gives

$$\left(\frac{7.48\,D\,K_5 S_2}{1200} + S_2 \right) > S_1 > \left[\frac{7.48\,D\,K_5 S_2}{(O.\,R.)} + S_2 \right] \tag{80}$$

in which O. R. 1200 gpd/sq. ft.

A demonstration is attempted to reveal the strength of influent waste permissible for treatment of the waste in the aeration tank of the completely-mixed activated sludge system according to these formulas. An example is given below based on the following assumption.

$D = 10$ ft. $\qquad$ O. R. = 1200 gpd/sq. ft.

$K_5 = 350\ \text{day}^{-1}$ $\qquad$ $S_2 = 15$ mg/ℓ BOD

From Eq. 76, for permissible substrate concentration in terms of BOD S_1 can be interpreted as

$$S_1 = K_5 S_2 \left[t_a (1+R) + \frac{7.48\,D}{(\text{O. R.})} \right] + S_2 \qquad (81)$$

When

$$t_a = 0, \; S_1 = 342\ \text{mg}/\ell$$

In spite of the fact that no plant has been designed with zero detention time in the aeration tank, it nevertheless demonstrates an important idea that the complete-mixing activated sludge processes are subject to strong waste treatment. Table X provided the information of acceptable influent 5-day BOD concentration to the aeration tank at various detention times and sludge recirculation in the completely-mixed activated sludge system based on the above assumptions.

A graphic interpretation of Table X is given in Figure 17. It is interesting to realize that permissible concentration of inflow waste, under conditions of a given detention time, is in proportion to sludge recirculation. In addition, under a given ratio of sludge recirculation, an increase of 225 mg/ℓ of inflow waste concentration is allowed if the detention time is extended by one hour (i.e., volume of aeration tank is increased).

TABLE X

ACCEPTABLE BOD CONCENTRATIONS TO THE AERATION TANK AT VARIOUS DETENTION TIMES AND SLUDGE RECIRCULATIONS

Recirculation Ratio (R)	Acceptable Influent BOD Concentration to the Aeration Tank, (mg/ℓ), S_1				
	t_a = 1/24 day	t_a = 2/24 day	t_a = 3/24 day	t_a = 4/24 day	t_a = 5/24 day
0	577	801	1026	1251	1475
0.25	633	857	1082	1307	1531
0.50	688	913	1140	1364	1588
0.75	746	970	1194	1420	1647
1.00	801	1026	1251	1475	1701

For: D = 10 ft. O.R. = 1200 gpd/sq. ft.

K_5 = 350 day^{-1} S_2 = 15 mg/ℓ BOD

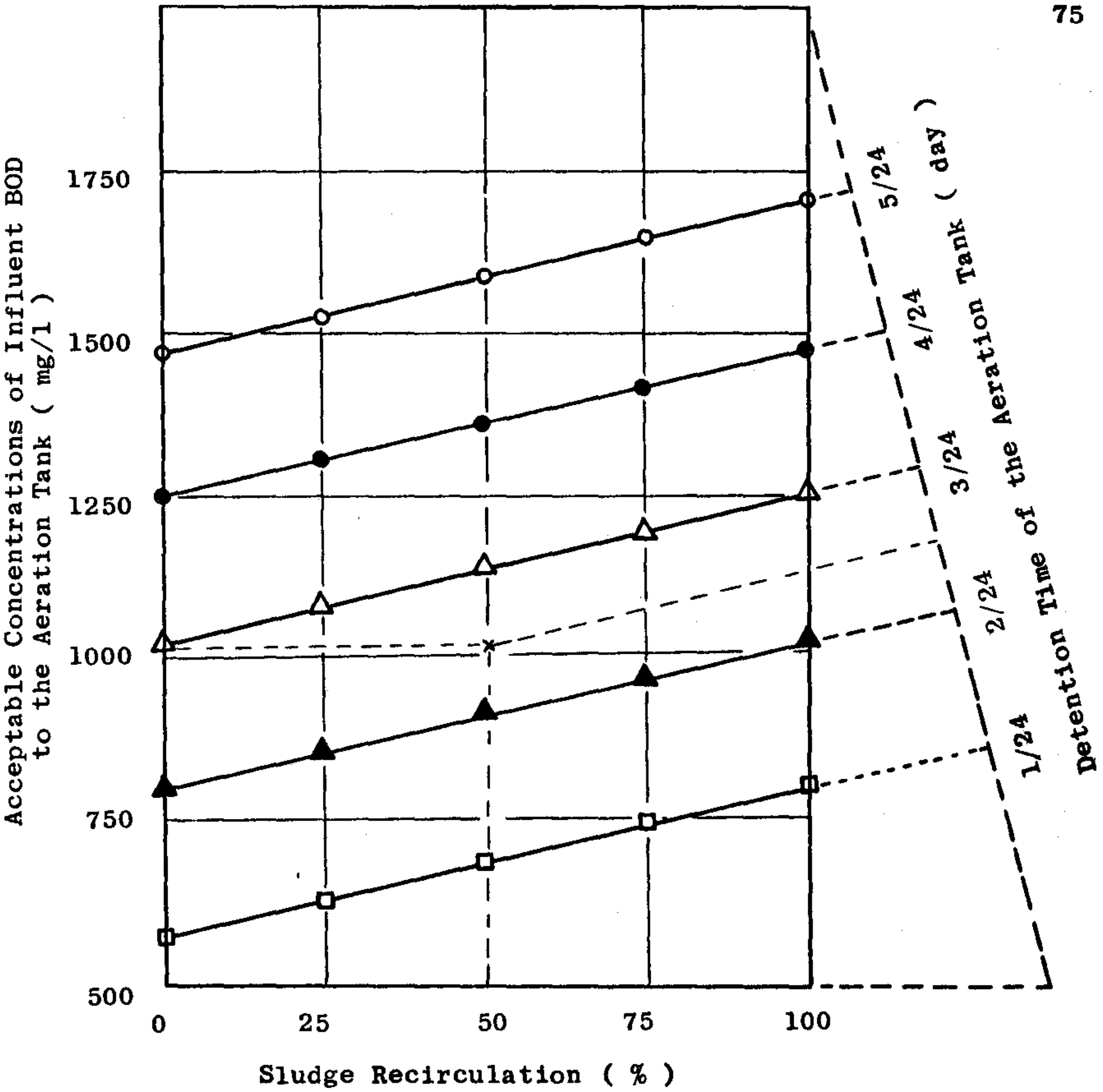


Figure 17

Acceptable Concentrations of Influent BOD to the Aeration Tank at Various Detention Times Under Different Sludge Recirculation Rates

It is noted that in Figure 17 all the lines are parallel. If the time scale (detention time) is drawn perpendicular to these lines, then any value of acceptable influent BOD concentration can be obtained by drawing a line at a desired position of time parallel to these lines and intersecting with the line perpendicular to the designed recirculation. As an example, if the aeration tank possesses 2.5 hours detention time (2.5/24 days) with 50% sludge recirculation, then 1026 mg/ℓ influent BOD are accepted.

The rate of change of active mass is given by

$$V' \frac{dX_a}{dt} = V' K_6 S_2 - V' K_7 X_a - Q \alpha X_a$$

$$\frac{dX_a}{dt} = K_6 S_2 - K_7 X_a - \frac{Q}{V'} dX_a$$

When

$$\frac{dX_a}{dt} = 0$$

$$X_a = \frac{K_6 S_2}{\frac{\alpha}{t'} + K_7} \qquad (82)$$

Where

K_6 = Biosynthesis constant

K_7 = Endogenous respiration constant

X_a = Active mass of sludge

α = Fraction of MLSS present in the effluent from aerator

The endogenous mass, X_e, increases in the system in proportion to the sludge lost from the system.

$$X_e = \frac{K_8 X_a t'}{\alpha} \qquad (83)$$

The maximum total mixed liquor suspended solids concentration that can be carried in the aeration tank may be calculated from

$$X_t = \frac{(SDI)\ 10000}{\frac{1}{R} + 1} \tag{84}$$

in which X_t is the total MLSS concentration, SDI is the sludge-density index, and R is the recirculation ratio. The volatile MLSS must be determined from the total MLSS for the particular type of waste to be treated. Domestic-sewage activated sludge is generally approximately 2/3 volatile solids, whereas pure chemical-wastes activated sludge may be as high as 90% volatile solids. The total volatile MLSS can be calculated from

$$X_v = X_a\left(1 + \frac{K_8 t'}{\alpha}\right) + X_m + X_i \tag{85}$$

Where

X_v = Total MLVSS
X_m = Metabolizable MLVSS
X_i = Inert MLVSS
K_8 = Endogenous respiration of active mass

If X_m is zero or close to zero, then Eq. 85 becomes

$$X_v = X_a\left(1 + \frac{K_8 t'}{\alpha}\right) + X_v \tag{86}$$

The inert volatile solids in the raw wastes, $(X_i)_r$, will build up in the aeration system until equilibrium is reached, at which point the inert, volatile solids in the influent will be

$$(X_i)_i = \alpha X_i \tag{87}$$

Substituting the value of X_i from Eq. 87 in Eq. 86

$$X_v = X_a\left(1 + \frac{K_8 t'}{\alpha}\right) + \frac{(X_i)_r}{\beta}$$

$$\alpha = \frac{X_a K_8 t' + (X_i)_r}{X_v - X_a} \qquad (88)$$

The 5-day BOD of the effluent can be calculated from Eq. 89.

$$\text{Effluent BOD} = S_2 + \alpha K_{10} X_a \qquad (89)$$

Where K_{10} is the endogenous metabolism constant at 20°C over a 5-day period.

The rate of oxygen demand can be calculated from Eq. 90.

$$O_2 = (K_9 S_2 + K_2 X_a)\, t' \qquad (90)$$

Where

K_2 = Endogenous respiration constant

K_9 = Oxygen equivalent of utilizable effluent organic matter

(B) Separate Sludge Wasting

The most common activated-sludge system has separate sludge wasting in order to produce an effluent with the least suspended solids. The schematic diagram of the separate sludge wasting system is shown in Figure 18.

Where,

Q = Inflow rate

W = Fraction of Q wasted

α = Fraction of MLSS present in the aeration tank

X = Concentration of MLSS in the aeration tank

S_1 = Substrate concentration in the influent

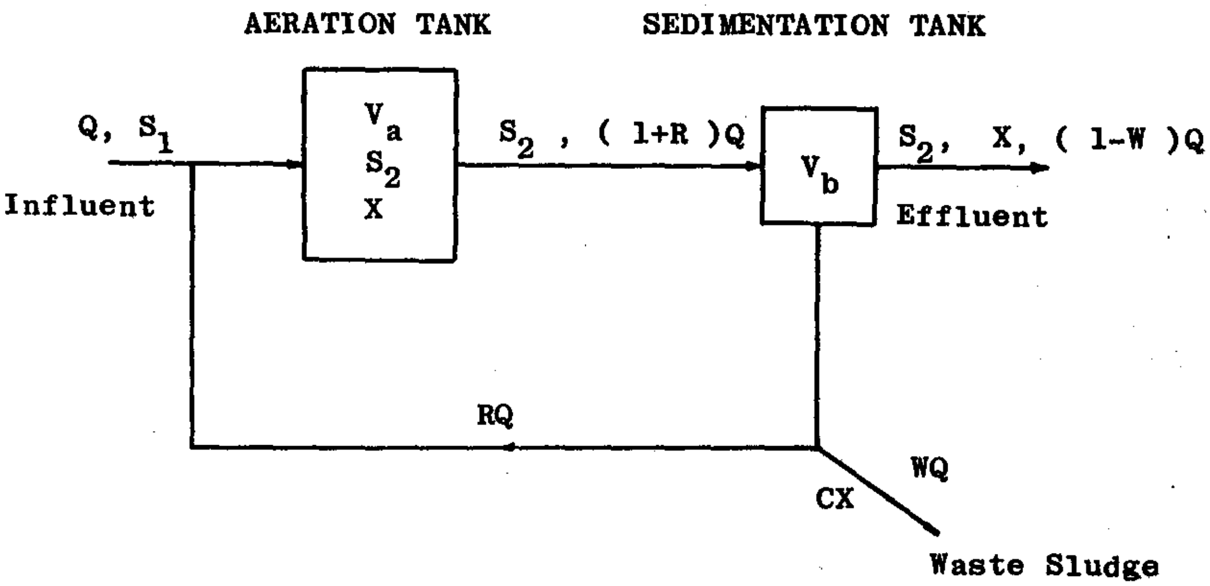


Figure 18
A Complete Mixing Activated Sludge With Separate Wasting of Excess Sludge (After McKinney)

S_2 = Substrate concentration in the effluent or in the aeration tank

R = Recirculation ratio

C = Concentration factor of wasted suspended solids

As long as the food to microorganism ratio stays below the critical value, the active mass in the system at equilibrium becomes related to both the effluent suspended solids and the sludge wasted. It is interpreted in Eq. 91.

$$X_a = \frac{K_6 S_2}{t'(\alpha - \alpha W + CW) + K_7} \qquad (91)$$

K_9 = 5 mg/ℓ O_2 utilization per mg/ℓ ultimate O_2 demand remaining per hour

K_9 = 7.5 mg/ℓ O_2 utilization per mg/ℓ 5-day BOD demand remaining per hour

K_9 = 120 mg/ℓ O_2 utilization per mg/ℓ ultimate O_2 demand remaining per day

K_9 = 180 mg/ℓ O_2 utilization per mg/ℓ 5-day BOD demand remaining per day

K_{10} = 0.6 for carbonaceous metabolism

K_{10} = 0.9 for carbonaceous and nitrogenous metabolism

α = 0.005 to 0.01 (If the sedimentation tank is very efficient, α = 0.005).

Sludge density index (SDI) for normal activated sludge averages 1.0.

Endogenous respiration occurs during the uptake of substrate, but when synthesis ceases the endogenous respiration becomes the only reaction.

Different groups of organic compounds had different energy-synthesis relationships. The carbohydrates had the lowest energy requirement per unit of substrate removed. The various cellular components of the microbial mass underwent endogenous respiration at different rates.

McKinney, et al.(48) stated that 1/3 of the substrates total oxygen demand would be used for energy, while 2/3 of the total oxygen demand would be converted into microbial protoplasm. The experimental data obtained revealed that the postulation was valid in domestic sewage, but an average of 0.25 ± 0.035 mg/ℓ O_2 uptake/mg substrate removed was obtained in the carbohydrate system, and an average of 0.48 ± 0.041 mg/ℓ O_2 uptake/mg substrate removed in non-carbohydrate system.

Since

$$(1+R)\ Q\ t = V$$

then,

$$V = \frac{S_1}{40}(1+R)\ Q \tag{95}$$

Where

S_1 = Influent BOD

Q = Inflow rate

R = Recirculation ratio

V = Volume of aeration tank

Based on 1 mg/ℓ biological solids equivalent to 1.42 mg/ℓ COD, Stack and Conway postulated the existence of Eq. 96.

$$\frac{\text{Wt. of } O_2 \text{ utilized}}{\text{Wt. of COD removed}} + 1.42\ \frac{\text{Wt. of volatile sludge produced}}{\text{Wt. of COD removed}} = 1 \tag{96}$$

Wt. of O_2 utilized

$$= \text{Wt. of COD removed}\left[1 - 1.42\ \frac{\text{Wt. of volatile sludge produced}}{\text{Wt. of COD removed}}\right]$$

$$= \text{Wt. of COD removed} - 1.42\ (\text{Wt. of volatile sludge produced}) \tag{97}$$

It was noted that with detention time less than four hours, the biological growth was mostly in the logarithmic phase; hence, the growth became well dispersed and the system would fail. For the industrial waste studies, high turbidity in the effluent was evident at detention time less than fifteen hours. Therefore, Stack, et al.(49) recommended selecting a minimum detention time of fifteen hours for the operation of the aeration tank in the activated sludge processes to treat industrial waste.

At a detention time of fifteen hours, the experimental data indicated an oxygen uptake rate of 110 mg/ℓ per hour and an oxygen demand of 0.7 lb/lb COD removed. A net sludge production of not more than 0.4 lb/lb COD removed can be obtained.

Eq. 96 and Eq. 97 are valid and applicable only when endogenous respiration in the biological system is neglected and the oxygen equivalent of 1 mg/ℓ biological solids in 1.42 mg/ℓ COD (i. e., average sludge composition), is $C_5H_7O_2N$. In addition, the portion of sludge which registers COD is not necessarily volatile matter.

Eq. 97 is to be modified to Eq. 98:

Wt. of O_2 utilized

= Wt. of COD removed − (oxygen equivalent of solids)

(Wt. of biological solids produced) + Oxygen consumed due to endogenous respiration (98)

According to Burkhead and McKinney (48) the average sludge compositions are waste water dependent. In other words, oxygen equivalent for unit concentration of microbial mass is not 1.42 for all kinds of activated sludge.

Endogenous respiration is a mechanism by which cells undergo oxidation of their own tissue for energy (50). Oxygen equivalents of biomass are calculated from the empirical formula of cell compositions after Burkhead and McKinney (48).

(1) Activated sludge harvested from carbohydrate wastes.

a. $C_5H_9O_3N + 5\,O_2 \longrightarrow 5\,CO_2 + 3\,H_2O + NH_3$

131 160

1 mg/ℓ x mg/ℓ

$$x = \frac{160}{131} = 1.22 \text{ mg}/\ell$$

b. $C_5H_{11}O_4N + 5\ O_2 \longrightarrow 5\ CO_2 + 4\ H_2O + NH_3$

159 160

1 mg/ℓ x mg/ℓ

$x = \frac{160}{159} = 1$ mg/ℓ

(2) Activated sludge harvested from non-carbohydrate wastes

a. $C_5H_7O_2N + 5\ O_2 \longrightarrow 5\ CO_2 + 2\ H_2O + NH_3$

113 160

1 mg/ℓ x mg/ℓ

$x = \frac{160}{113} = 1.42$ mg/ℓ

b. $C_5H_9O_3N + 5\ O_2 \longrightarrow 5\ CO_2 + 3\ H_2O + NH_3$

131 160

1 mg/ℓ x mg/ℓ

$x = \frac{160}{131} = 1.22$ mg/ℓ

(3) Activated sludge harvested from domestic sewage

$C_5H_{11}O_4N + 5\ O_2 \longrightarrow 5\ CO_2 + 4\ H_2O + NH_3$

159 160

1 mg/ℓ x mg/ℓ

$x = \frac{160}{159} = 1$ mg/ℓ

From Eq. 88

$$X_a = \frac{X_v - \frac{(X_i)_r}{\alpha}}{\left(1 + \frac{K_8 t'}{\alpha}\right)}$$

Total mass due to endogenous respiration per day = $K_7 X_{a_1}$

$$= K_7 \frac{X_v - \frac{(X_i)_i}{\alpha}}{\left(1 + \frac{K_8 t'}{\alpha}\right)} \tag{99}$$

Oxygen utilized per day (in pounds)

= Minimum oxygen supply per day (in pounds)

$$= 8.34\left[Q(S_1 - S_2) - (OE)(WQX_r) + K_7 Q \frac{X_v - \frac{(X_i)_r}{\alpha}}{1 + \left(\frac{K_8 t'}{\alpha}\right)}\right]$$

$$= 8.34\, Q\left[(S_1 - S_2) - (OE)(WX_r) + K_7 \frac{X_v - \frac{(X_i)_i}{\alpha}}{1 + \left(\frac{K_8 t'}{\alpha}\right)}\right] \tag{100}$$

Where

Q = Flow rate in MGD

S_1 = Influent substrate concentration in terms of mg/ℓ COD

S_2 = Effluent substrate concentration in terms of mg/ℓ COD

OE = Oxygen Equivalent of biological solids

W = Ratio of wasted sludge flow to inflow

X_r = Concentration of settled sludge in mg/ℓ (return sludge or wasted sludge)

K_7 = Endogenous respiration constant per day

K_8 = Endogenous respiration of active mass at 20°C per day

X_v = Concentration of MLVSS

X_a = Active mass of sludge

$(X_i)_r$ = Concentration of inert material of the incoming waste in mg/ℓ

α = Fraction of MLSS present in the effluent

t' = Detention time of combined aeration tank and sedimentation tank in days

In order to balance activated sludge concentration, it is generally considered that the wasted sludge is equal to the net synthesis of biological solids. Therefore, it is reasoned that the term "Wt. of biological solids produced" in Eq. 98 can be substituted for the "Wt. of sludge wasted" in Eq. 100.

9. Pipes, Grieves, and Milbury's Equations for the Design of a Completely-Mixed System

The previous models of biological growth and substrate reduction in the activated sludge system were derived upon the assumptions that the constituents persent in the aeration tank were completely mixed in one case, and no longitudinal mixing took place in the other. The former is called a "completely-mixed model," while the latter is called a "plug-flow model." The completely-mixed model assumes that the influent is uniformly and instantaneously distributed throughout the aeration tank as it enters. The plug-flow model is theoretically considered to be identical to the conventional activated sludge reactor. But neither of them adequately describes the real performance of the processes.

Recently, Pipes, Grieves and Milbury (51), (52) showed that a completely-mixed activated sludge reactor existed with short-circuiting based on laboratory bench scale studies by using sodium chloride and skimmed milk as tracers. They applied the idea to a biological system. The experimental apparatus was so designed that the inlet and outlet were placed at the same end of the reacting vessel and baffles divided 3/4 of the volume into interconnected cells (52). The ideal sketch of hydraulic flow sheet devised by Pipes, et al.(52) is shown in Figure 19.

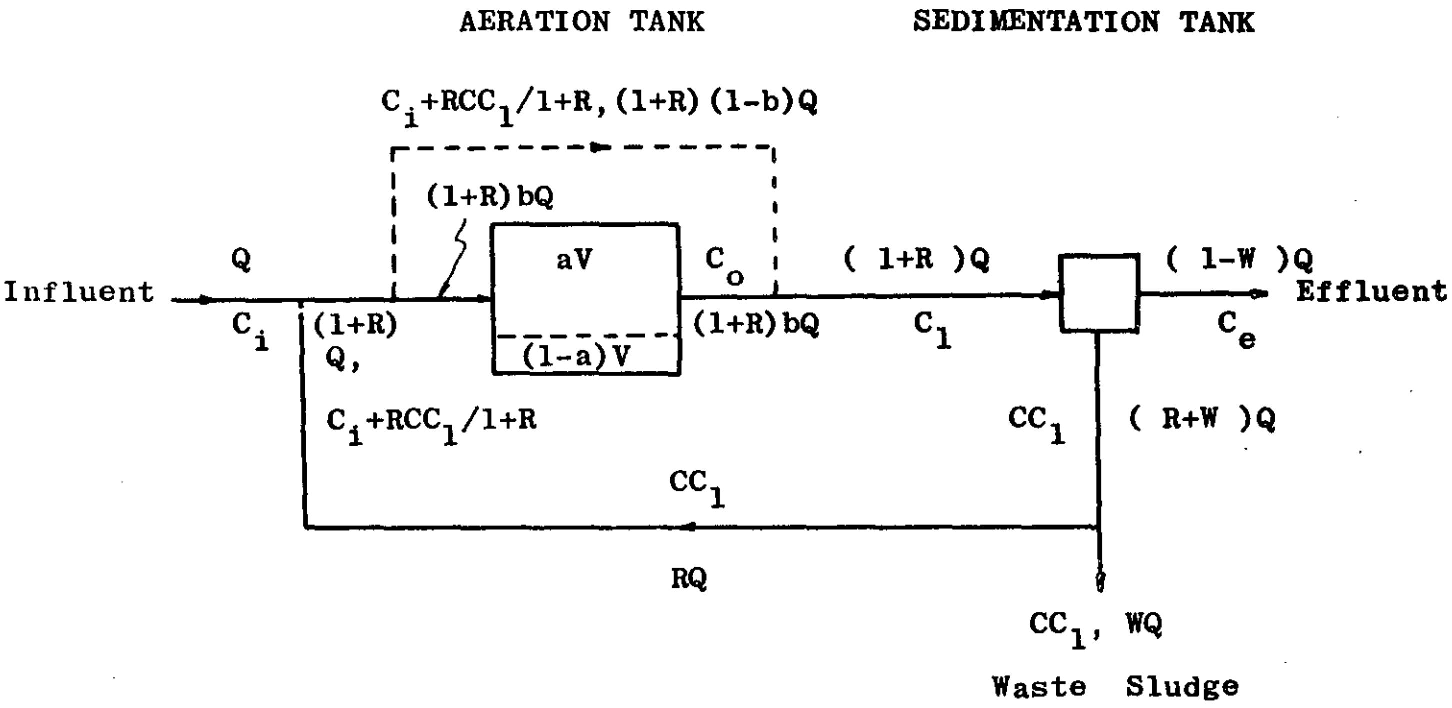


Figure 19

Hydraulic Flow Sheet of Completely-mixed Activated Sludge Process with Short-circuiting (After Pipes, Grieves, and Milburg)

The symbols employed by Pipes, et al.(51) have been changed somewhat and the nomenclature of the symbols in Figure 19 are given below.

a = Completely-mixed fraction of total volume of aeration vessel

b = Fraction of feed entering completely-mixed zone

C = Concentration factor of recycled and wasted sludge (or separation factor)

C_o = Outlet tracer concentration from aeration tank

C_e = Effluent tracer concentration from system

C_i = Influent tracer concentration to system

C_1 = Tracer concentration in the stream entering the separator

Q = Inflow rate

R = Recirculation ratio (fraction of system feed rate recycled to aeration unit)

W = Sludge wasting ratio (fraction of system feed rate wasted from recycled stream)

V = Volume of aeration tank

A mass balance applied to the process shows that C_1 is a function of C_i and C_o, and C_e is a function of C_1. These relationships are more specifically:

$$C_1 = \frac{(1+R)\,bC_o + (1-b)C_i}{1+R - RC + RbC} \qquad (101)$$

$$C_e = \frac{(1+R - RC' - WC')\,C_1}{1 - W} \qquad (102)$$

The concentration of tracer entering the aeration vessel is

$$\frac{C_i + RC\,C_1}{1+R}$$

and the rate of change in the concentration of tracer in the aeration tank is given by

$$\frac{dC_o}{dt} = \left(\frac{C_i + RC\,C_1}{1+R}\right)\left(\frac{(1+R)bQ}{aV}\right) - C_o\left(\frac{(1+R)bQ}{aV}\right) \qquad (103)$$

The tracer concentration in the aeration vessel is initially zero and the tracer is added to the influent to give a constant feed concentration, C_i.

When $C_o = 0$, and $t = 0$, Eq. 103 can be integrated to give

$$C_o = \frac{C_i}{A}\left[1-e\,(-AB\frac{bQ}{aV}\,t)\right] \qquad (104)$$

Where

$$A = (1+R - RC)$$

and

$$B = \frac{1+R}{1+R - RC + RCb}$$

The mathematical model of the completely-mixed activated sludge is an extension of the model of bacterial growth in continuous culture. Three fundamental equations describing microbial growth and metabolism are given below:

$$\frac{dX}{dt} = \mu X \tag{105}$$

$$\mu = \mu_m \frac{S}{K_s + S} \tag{106}$$

$$dX = - YdS \tag{107}$$

Where

X = Concentration of microbial population in mg/ℓ

S = Concentration of oxidizable substance in mg/ℓ

μ = Specific growth rate constant in hr^{-1}

μ_m = Maximum specific growth rate constant in hr^{-1}

K_s = Saturation constant of the system in mg/ℓ

Y = Cell yield in mg/ℓ organisms/mg/ℓ oxidizable substrate

t = Detention time in hours

Integration of Eq. 107

$$\int_{X_i}^{X} dX = -Y \int_{S_i}^{S} dS$$

$$(X - X_i) = Y(S_i - S) \tag{108}$$

Where X_i and S_i are the concentrations of microbial populations and oxidizable substrate in the influent. When X_i is neglected (usually

$X_i << X$) then,

$$X = Y(S_i - S) \tag{109}$$

Since it is the purpose of the activated sludge process to attain a very low concentration of oxidizable substrate in the effluent, it is logical to assume that the concentrations of substrate in the aeration tank, which controls the final effluent concentration, will be held to a low value.

When $S << K_s$, Eq. 106 can be written as

$$\mu = \frac{\mu_m S}{K_s} \tag{110}$$

Also, when $S << S_i$, Eq. 109 becomes

$$X = YS_i \tag{111}$$

By combining Eq. 105, 107, 110, and 111, there is obtained

$$\frac{dS}{dt} = \frac{-\mu_m S_i}{K_s} \quad S \tag{112}$$

If a substrate removal rate is defined by

$$K = \frac{\mu_m S_i}{K_s} \tag{113}$$

Eq. 112 becomes

$$\frac{dS}{dt} = -KS \tag{114}$$

Eq. 114 shows that under substrate-limiting conditions, substrate removal is analogous to the first order chemical reaction. If the substrate concentrations in Eq. 114 is in terms of BOD, then the equation is indeed the conventional BOD equation in differential form.

The effect of short-circuiting and stagnant zone upon the completely-mixed activated sludge process can be elucidated by a combination of the described models of microbial growth and substrate removal with the previously developed hydraulic model. Since the activated sludge process is operated with a final sedimentation tank, Eq. 103 can be modified to describe the change in substrate and biological solids concentrations with time. For the oxidizable substrate, Eq. 114, which takes into account the substrate removal in the aeration tank due to microbial growth, must be subtracted from Eq. 103. In addition, the separation factor, C, is equal to unity. The resulting equation is

$$\frac{dS}{dt} = \left(\frac{S_i + RS_1}{1+R}\right)\left(\frac{(1+R)bQ}{aV}\right) - \frac{S(1+R)bQ}{aV} - KS \tag{115}$$

Literally speaking, Eq. 115 implies

Change of substrate in aeration tank = Recycle and input
– Output – Substrate utilization.

So far as the microbial growth is concerned, the concentration in the influent to the system is negligible. Eq. 115, which also takes into account the growth of organisms, must be added to Eq. 103. The resulting equation is

$$\frac{dX}{dt} = \left(\frac{RCX_1}{1+R}\right)\left(\frac{(1+R)bQ}{aV}\right) - \frac{X(1+R)bQ}{aV} + \mu X \tag{116}$$

Again, literally speaking, Eq. 116 means that the change of microbial population in the aeration tank

= Recycle – Output + Growth.

If the values of S_i, Q, V, a, b, R, C, and W remain constant, then it can be assumed that the system will attain steady state operation when $dX/dt = 0$ and $dS/dt = 0$. Then, the concentrations of microbial populations and substrate can be computed.

$$S_e = \left(\frac{(1+R)b}{(1+Rb)}\right)\left(\frac{K_s}{\mu_m}\right)\left(\frac{bQ}{aV}\right)(AB) + \frac{(1-b)}{(1+Rb)}\,S_i \qquad (117)$$

$$X_e = \frac{W'}{A}\,Y(S_i - S_e) \qquad (118)$$

Where

$$A = 1 + R - RC$$

$$B = \frac{1+R}{1+R-RC+RCb}$$

$$W' = \frac{1+R-RC-WC}{1-W}$$

The volumetric loading is defined as

$$L = S_i\,\frac{Q}{V} \qquad (119)$$

By combining Eqs. 113, 117, and 119, we obtain

$$S_e = \left(\frac{(1+R)b}{(1+Rb)}\right)\left(\frac{L}{K}\right)\left(\frac{b}{a}\right)(AB) + \frac{(1-b)S_i}{(1+Rb)} \qquad (120)$$

Eq. 120 shows that the effluent substrate concentration is proportional to the volumetric loading only if no short-circuiting occurs. However, when short-circuiting occurs, the substrate concentration is increased by an amount which is proportional to the fraction of flow that is short-circuited.

In a hetergeneous microbial system, there is no good theoretical reason that μ_m, K_s, and Y should be constant. Nevertheless, the experimental results from many investigators indicated that these variables remained nearly constant, but their values did vary with the nature of the waste and with the temperature. In the derivation of the model, Grieves, et al.(51) stated that the assumptions have been made on a constant performance of influent flow, the effect of the final separation upon sludge concentration, the rate of sludge recirculation, and the rate of sludge wasting. In actual practice, these quantities vary significantly and the process never comes to true steady-state conditions. Gram (54) has considered this difficulty and concluded that a quasi-steady-state is attainable in practice with the results not varying too greatly from the theoretical predictions. Gram obtained the values of μ_m, K_s, and Y to be 0.1 hr^{-1}, 100 mg/ℓ, and 0.5 respectively.

Combining Eqs. 119 and 120, the volume of the designed aeration tank can be found from

$$V = \frac{\left[\frac{(1+R)b}{1+Rb}\right]\left(\frac{S_i Q}{K}\right)\left(\frac{b}{a}\right)(1+R-RC)\left(\frac{1+R}{1+R-RC-RCb}\right)}{S_e - \frac{(1-b)S_i}{(1+Rb)}} \qquad (121)$$

All of the equations stated above require the determination of constants "a" and "b" for a specific completely-mixed system in the presence of short-circuiting and stagnation. It is believed that the constants "a" and "b", in a large measure, depend upon tank configuration, degree of aeration and perhaps the inflow rate. Waste characteristics might also play a minor role for the determination of "a" and "b". The theoretical complete-mixing can be expressed as Eq. 122.

$$\frac{C_o}{C_i} = e^{-\frac{Qt}{V}} \qquad (122)$$

When short-circuiting occurs in the completely-mixed aeration tank, the tracer response for the partial complete-mixing illustrates the following relation (52):

$$\frac{C_o}{C_i} = b \; e^{-\frac{b}{a}\frac{Qt}{V}} \qquad (123)$$

$$\frac{1}{b}\frac{C'}{C_o} = e^{-\frac{b}{a}\frac{Qt}{V}}$$

$$\ln\frac{1}{b} + \ln\frac{C_o}{C_i} = -\frac{b}{a}\frac{Qt}{V}$$

$$\ln\frac{C_o}{C_i} = -\left(\frac{b}{a}\right)\frac{Qt}{V} + \ln b \qquad (124)$$

The constants "a" and "b" can be graphically determined by plotting

$$\ln\frac{C_o}{C_i}$$

as ordinate against $\frac{Qt}{V}$ as abscissa. Then the intercept is "b" and the slope is $-\left(\frac{b}{a}\right)$. The diagrammatic relation is shown in Figure 20.

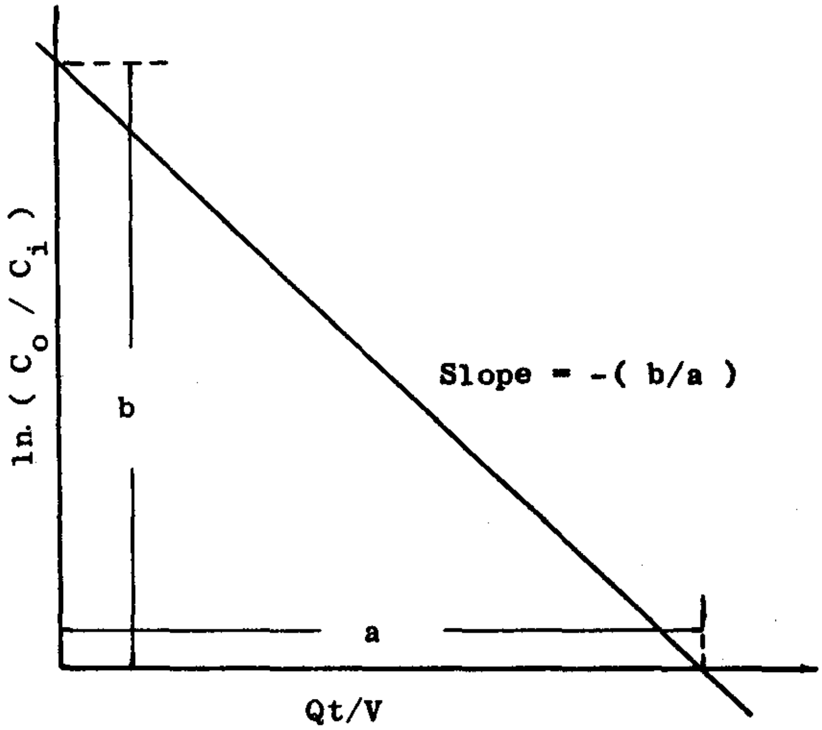


Figure 20

Diagram of Tracer Response for Partial Complete Mixing and Short-circuiting in the Aeration Tank

10. Busch's Equation with Modifications for the Design of a Completely-Mixed System

It is generally accepted that design criteria, other than effluent requirements, are dependent on the characteristics of the particular waste in question. Regarding the concentrations of organic matter discharging from the effluent of the theoretically completely-mixed activated sludge process, Grieves, et al.(51), as stated in the previous section, depicted its linear relationship with the organic loading applied to the system. Their statement substantiated the design approach of Busch reported in 1961 (55). Busch stated that when food concentration becomes limiting, the concentration of organic substance present in the effluent is linearly related with organic loading in the form of

$$S_2 = K' L + d' \qquad (125)$$

Where

S_2 = Concentration of organic matter in effluent

L = Organic loading

K' = Relation constant between S_2 and L

d' = Concentration of organic matter in effluent at zero organic loading

A graphic expression of Eq. 125 can be interpreted by Figure 21.

A pilot plant study of a particular waste water would provide the information of allowable organic loading to be applied to the activated sludge system under the attainment of desired effluent quality. Additional design criteria need to be formulated. Based on the concept of present study, it should be assumed that the rate of substrate utilization in a microbial system remains constant at steady-state, i.e.,

$$Q(S_1 - S_2) = V \quad K$$

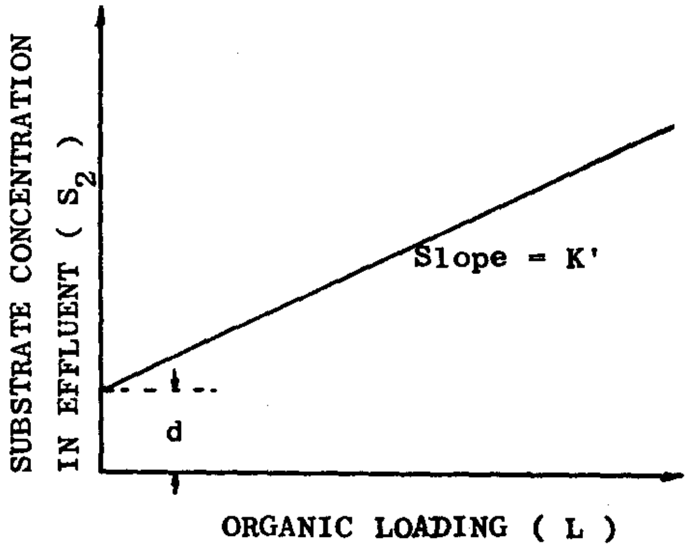


Figure 21
Relationship Between Effluent Substrate Concentrations and Organic Loadings in Completely-mixed Activated Sludge Process

or

$$V = \frac{Q}{K}(S_1 - S_2) \tag{126}$$

Where

Q = Inflow rate

S_1, S_2 = Influent and effluent substrate concentrations respectively

V = Volume of aeration tank

K = Rate of substrate removal

Under operation with sludge recirculation, R, then the detention time (t) of incoming waste can be calculated from Eq. 127.

$$t = (S_1 - S_2) / K(1+R) \tag{127}$$

Because the system is designed to operate at equilibrium, the rate of substrate removal represents the rate of oxygen uptake, $K_o K$, where K_o is the decimal fraction of the 5-day BOD oxidized. If the oxygen equivalent of biological solids is 1.414 (1.42 is generally accepted), the rate of oxygen uptake is given in Eq. 128.

$$K_o = \frac{(S_1 - S_2) - (1.414 \ \Delta X)}{(S_1 - S_2)} \qquad (128)$$

Eq. 128, in which S_1 and S_2 are expressed in terms of mg/ℓ COD and ΔX represents the net production of biological mass in mg/ℓ, introduced the factor of sludge age to be considered, since cell yields are affected by the age (38), (55).

It is important to note that the rate of substrate removal per unit mass of biological solids and the minimum oxygen requirement by unit mass are governed by substrate concentrations present in the biological system. Busch (55) related the above three variables and provided a graphic interpretation, as shown in Figure 22.

If it is assumed that the value of the substrate removal rate per unit weight of biological mass at the corresponding desired effluent substrate concentration is "m", then "m" can be interrelated with S_1, S_2, Q, X, and V by the following equation.

$$m = \frac{8.34 \ Q \ (S_1 - S_2)}{8.34 \ X \ V}$$

$$V = \frac{Q(S_1 - S_2)}{mX} \qquad (129)$$

As noted in Figure 22, the dimension of "m" is day^{-1}. When S_1, S_2, and X are expressed in terms of mg/ℓ, and the inflow rate in terms of MGD, the volume of the aeration tank will be expressed as MG.

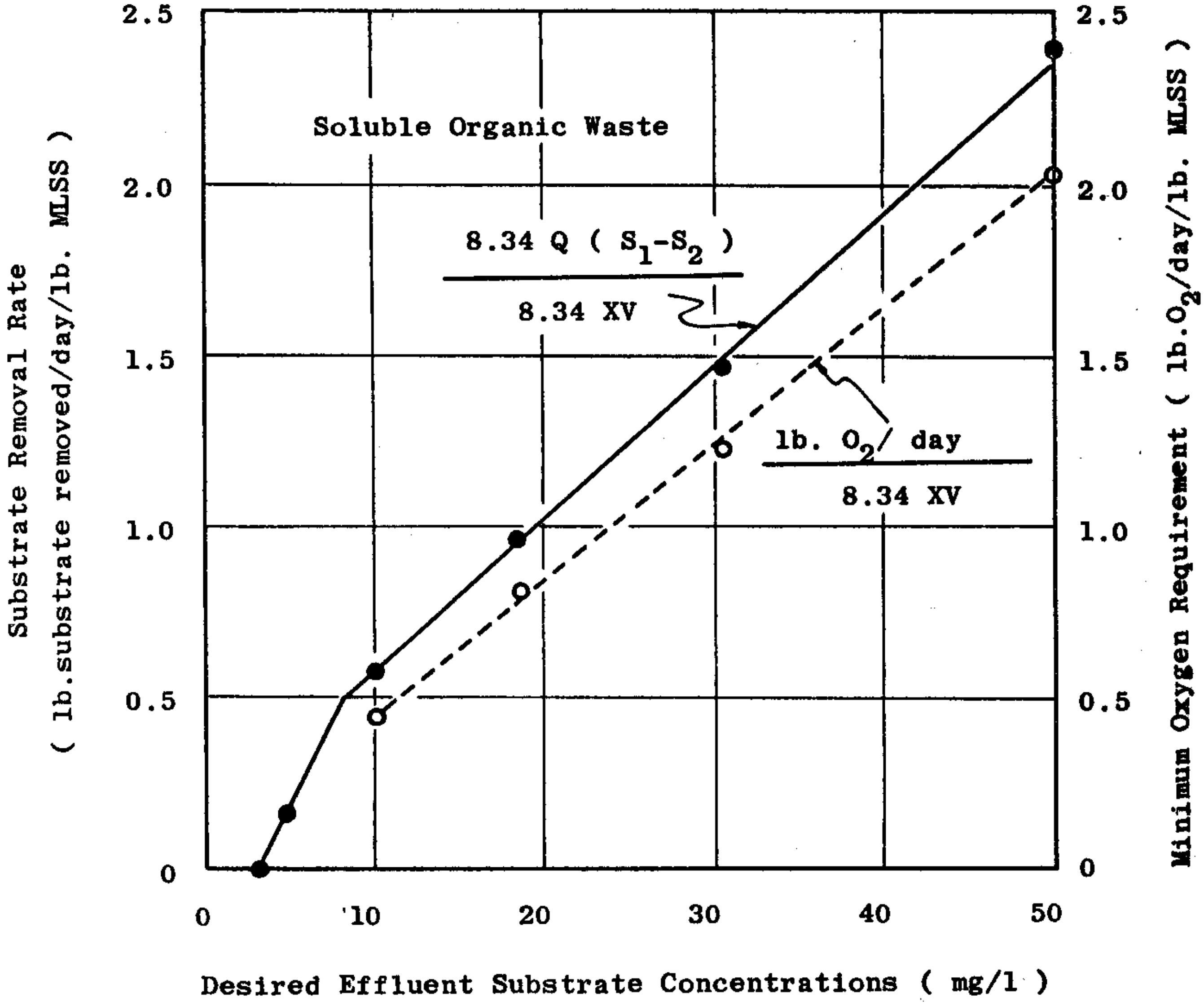


Figure 22

Diagram of Desired Effluent Substrate Concentration
Related to Substrate Removal Rate
and Minimum Oxygen Requirement
(After Busch)

Comparing Eq. 126 with Eq. 129, it is interesting to observe that the rate of substrate removal is proportional to the concentration of mixed liquor suspended solids present in the aeration tank, i.e., $K = mS$. Eq. 129 indicates that the volume of the aeration tank will become smaller as the concentrations of the mixed liquor suspended solids are maintained at a higher level.

If "n" is the value obtained from Figure 22 at the corresponding desired effluent substrate concentration, minimum oxygen requirement per day can be determined by the following equations.

$$n = \frac{\text{lb. } O_2/\text{day}}{8.34(X)(V)}$$

$$\text{lb. } O_2 \text{ per day} = 8.34\, n\, X\, V \tag{130}$$

11. Eckenfelder's Equations with Modifications for the Design of a Completely-Mixed System

The biological oxidation of organic wastes, Eckenfelder speculated (4), can be assumed to be a three-phase process. The demonstrated kinetics would depend upon the ratio of substrate concentration to biological solids concentration and also upon the nature of the constituted wastes.

As an initial high rate removal of organic substance on contact with microbial populations would occur when the initial substrate to sludge solids ratio exceeds two and substrate removal is independent of substrate concentration, Eckenfelder (4) stated that the substrate levels above 200 to 300 mg/ℓ would be applicable and the trend of substrate utilization follows first order increasing kinetics. In other words, biological growth in the presence of high substrate concentration proceeds exponentially. It is assumed that the initial concentration of biological solids in the aeration tank is X_o, the increase in solid concentration is ΔX due to substrate utilization, then the overall concentration in terms of biological solids will be

$$X = X_o + \Delta X \tag{131}$$

$$X = X_o + Y(S_1 - S_2) \tag{132}$$

Where Y represents cell yield or the fraction of substrate removed to be synthesized to sludge at any time, S_1 and S_2 represent substrate concentrations in the inlet and outlet of the aeration tank, respectively.

Since the microbial populations grow exponentially, the rate of change in biological solids concentration will be proportional to the concentration of biological solids present, i.e.,

$$\frac{dX_o}{dt} = \mu X_o \tag{133}$$

in which μ is the specific growth rate. Integration of Eq. 133 obtains

$$\ln \frac{X_o + X}{X_o} = \mu\, t \tag{134}$$

$$t = \frac{\ln \frac{Xo + X}{X_o}}{\mu}$$

$$= \frac{\ln\left[1 + \frac{Y(S_1 - S_2)}{X}\right]}{\mu} \tag{135}$$

$$V = (1+R)\, Q\, t$$

$$= \frac{Q(1+R) \ln\left[1 + \frac{Y(S_1 - S_2)}{X}\right]}{\mu} \tag{136}$$

In which

Q = Inflow rate

R = Recirculation ratio

V = Volume of aeration tank

t = Detention

When the initial substrate sludge ratio is less than two, and the substrate removal is independent of substrate concentration, microbial utilization of high concentration of organic waste would proceed at zero order kinetics (4).

$$\frac{S_1 - S_2}{t} = \frac{K \bar{X}}{Y} \qquad (137)$$

In Eq. 137, $\bar{X}$ represents the average concentration of biological solids and K represents the rate of substrate removal.

$$t = \frac{Y(S_1 - S_2)}{K \bar{X}} \qquad (138)$$

$$V = Q\,(1+R)\;t$$

$$= \frac{Q\,(1+R)\;Y(S_1 - S_2)}{K \bar{X}} \qquad (139)$$

At low concentrations of substrate with initial BOD less than 300 mg/ℓ, the biological growth rate and substrate removal rate will frequently be expressed by first order decreasing kinetics.

$$\frac{dS}{dt} = -\,K\,S \qquad (140)$$

Considering the effect of sludge solids, Eq. 140 becomes

$$\frac{dS}{dt} = -\,K\,\bar{X}\,S \qquad (141)$$

Integration of Eq. 140 obtains

$$\ln \frac{S_2}{S_1} = -\,K\,\bar{X}\;t$$

$$t = \frac{\ln \frac{S_1}{S_2}}{K \bar{X}} \tag{142}$$

$$V = Q(1+R) \frac{\ln \frac{S_1}{S_2}}{K \bar{X}} \tag{143}$$

12. Bloodgood's Equations with Modifications for the Design of an Activated Sludge Process

In 1944, studies of a sludge index were conducted, in which Bloodgood (56) found that the mixed liquor with exceptionally well settling ability exhibited a slightly higher value of sludge index at the end of 60 minutes than at the end of 30 minutes. For the mixed liquor of poor settling qualities, it was anticipated that the difference between the concentrations at 30 minutes and 60 minutes was even greater. It would seem, therefore, that the sludge index calculated on a 60-minute settling period would produce more satisfactory results.

It was assumed that the waste sludge, loss of solids by oxidation and the incoming raw solids were negligible in amount, and the biological solids in the mixed liquor were nearly equal to those in the return sludge. The sludge balance in the aeration tank is given below:

$$(Q + RQ)\,X = RQX_r$$

Where

Q = Inflow rate in MGD

R = Recirculation ratio

X = Concentration of MLSS in mg/ℓ

X_r = Concentration of return sludge (settling sludge) in mg/ℓ

If the sludge index of return sludge (settled sludge) is expressed in percent solids of the sludge, then

$$\frac{X_r}{10^4} = \frac{100}{Z}$$

$$Z = \frac{10^6}{X_r} \qquad (144)$$

Where "Z" is the sludge index

$$Z = \frac{10^6}{Q(1+R)X/RQ}$$

$$= \frac{10^6\, R}{(1+R)\, X} \qquad (145)$$

or

$$R = \frac{X\,Z}{10^6 - X\,Z} \qquad (146)$$

since,

$$V = (1+R)\ Q\ t$$

$$= (1 + \frac{X\,Z}{10^6 - X\,Z})\ Q\ t \qquad (147)$$

Recirculation of sludge, concentrations of mixed liquor suspended solids, and sludge index are expressed in Eqs. 145 or 146. A nomographic chart of the related variables shown in Figure 23 was given by Bloodgood (56) to serve as a convenient control for the operation of an activated sludge process.

The units of the variables in the chart are somewhat different from those used in the equations set forth above. According to the chart, return sludge is expressed in terms of percent instead of mg/ℓ. The return sludge is in terms of percent recirculation to the inflow rate instead of the ratio of the recirculated flow to influent flow, and the

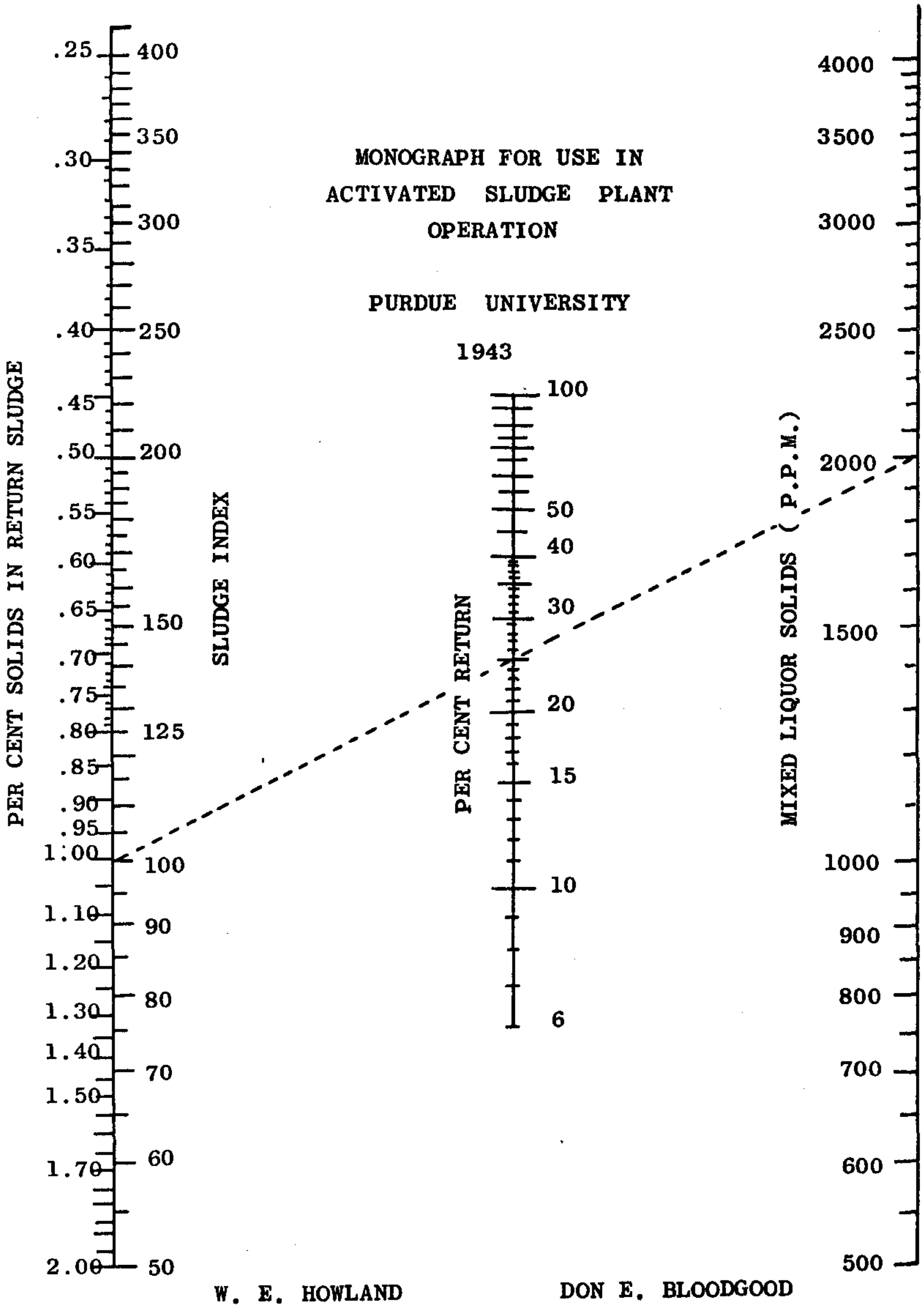


Figure 23

Nomograph for Use in Activated Sludge Plant Operation
(After Bloodgood and Howland)

concentration of mixed liquor is in terms of ppm instead of mg/ℓ. These differences in units can be converted in the following relation:

(1) Assuming the specific gravity of the mixed liquor is unity, then,

$$1 \text{ PPM} = 1 \text{ mg}/\ell$$

(2) Based on the above assumption,

$$1\% \text{ sludge} = 10{,}000 \text{ mg}/\ell$$

(3) Recirculation ratio =

$$\frac{\% \text{ Recirculation}}{100}$$

As indicated in Eq. 147, concentration of the mixed liquor suspended solids and the sludge index are involved in the design of an aeration tank under a given inflow rate and desired detention time. The drawback of this equation is the necessity of determining a proper detention time which would provide an acceptable effluent quality for a particular waste, and it is expected that the sludge index would vary to a certain extent depending upon the nature of the waste. Furthermore, the detention time desired and the sludge index exhibited would probably be governed by the modifications of the activated sludge employed.

13. Contact-Stabilization Activated Sludge Process

The contact-stabilization activated sludge process consists mainly of two aeration units between which a final sedimentation tank is provided for solids-liquor separation. The first aeration tank is generally called a "contact unit" to which the inflow waste water is introduced. The waste water is then contacted with the stabilized return sludge recirculated from the second aeration tank called a "stabilization unit."

The function of the contact unit is to accomplish oxidative assimilation. In other words, the introduced biologically degradable organic substances are temporarily stored in the cellular cytoplasm. Since a large volume of the microbial concentrations are recirculated and the populations are well stabilized, a short time is required to complete the work. After the mixed liquor passes through the contact unit, the biological activity continues to proceed and the remaining exogenous organic compounds are easily assimilated in the final sedimentation tank as well as in the stabilization unit. The endogenous assimilated compounds will be oxidized or stabilized as the microbial populations are retained in the clarifier and in the stabilization unit to which oxygen is supplied through aeration. The flow diagram of a contact-stabilization activated sludge process is shown in Figure 24.

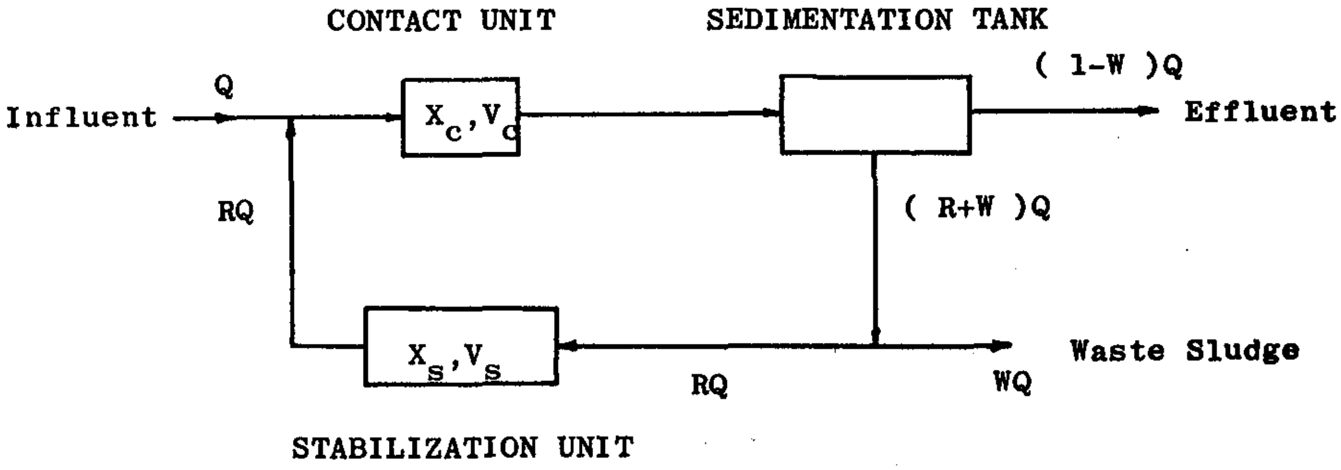


Figure 24

Flow Diagram of Contact-stabilization Activated Sludge Process

The contact period is determined by the capability of the process to produce a satisfactory effluent. In general, a contact time of $\frac{1}{2}$ to 1 hour will be enough to produce an acceptable effluent for most of the wastes. The organic matter in domestic waste water is primarily in suspended or colloidal form; hence, high overall removals are attained after only a short contact of waste water with biological mass. It has been stated that a longer contact period is essential to achieve high removal levels in the case of the soluble industrial waste.

In 1950, Keefer and Meisel (58) found that preaeration of activated sludge in a separated tank before it is added to the sewage or the inlet end of the step aeration tank may be more economical than mixing and aerating the unaerated sludge with sewage in the aeration tank. The work of Keefer, et al.(58) had already confirmed the idea of a modified activated sludge process at that time, but they did not know until the first successful operation was achieved in Austin, Texas (14).

The aeration period for the stabilization of a fixed amount of BOD is related to the concentration of biological solids. A higher concentration of biological solids maintained in the contact unit would require a shorter detention time. However, as high quality of the effluent is expected, the current design trends are directed toward a better balance between the stabilization aeration time and the contact time. In some contact-stabilization plants, the contact aeration time has been too short relative to the stabilization aeration time. The total aeration time is found from the sum of the contact aeration and stabilization aeration times.

Since the contact unit is purposely designed with a short detention time, the performance of this reactor would approach complete mixing.

In addition, the concentration of biological solids in the contact unit is maintained at a level as high as 4000 mg/ℓ to 5000 mg/ℓ, and it is reasonable to anticipate that removal of the remaining organic compounds in the final sedimentation tank would be very great. Thus, the design of the contact unit and the final sedimentation unit would follow the criteria and the equations proposed by McKinney (47) and McKinney, et al (48). (See Section 7)

The contact unit would assimilate about 50 mg/ℓ BOD/hr. at 4000 to 5000 mg/ℓ mixed liquor suspended solids. The higher concentration of the sludge maintained in the stabilization unit permits a greater rate of stabilization. The relative aeration period for the stabilization unit, as proposed by McKinney and O'Brien, can be calculated by the ratio of the sludge concentration in the stabilization unit to the mixed liquor suspended solids concentration in the contact unit times the detention period in the stabilization unit based on raw waste flow (59). The mathematical expressions are interpreted in Eq. 148.

$$\left(\frac{X_s}{X_c}\right)\left(\frac{V_s}{Q}\right) = t_s \tag{148}$$

and

$$Q = \left(\frac{X_s}{X_c}\right)\left(\frac{V_s}{t_s}\right) \tag{149}$$

Where

X_s = Concentration of sludge in the stabilization unit in mg/ℓ

X_c = Concentration of the MLSS in the contact unit in mg/ℓ

Q = Raw waste flow rate in MGD

V_s = Volume of stabilization unit in MG

t_s = Detention time of stabilization unit in days

The recirculation ratio, R, needed can be estimated from the following relationship, if the quantity of the incoming suspended solid is neglected.

$$X_s \ RQ = (1+R) \ Q \ X_c$$

$$R = \frac{X_c}{X_s - X_c} \qquad (150)$$

Since a large concentration of sludge has to be recycled, the sludge wasting ratio will be small compared with the sludge to be recycled. Thus, the approximate pumping rate required for return sludge as well as waste sludge can be estimated as

$$Q_r = R \ Q$$

$$= \frac{X_c}{X_s - X_c} \ \frac{X_s \ V_s}{X_c \ t_s}$$

$$= \frac{X_s \ V_s}{(X_s - X_c) \ t_s} \qquad (151)$$

Where Q_r = Flow of return sludge plus waste sludge

The overall pumping rate of the recycled system is not only governed by the volume of the stabilization unit, but also by the concentration of sludge in the stabilization unit as well as in the contact unit.

However, the design of the stabilization unit involves two unknown variables, i.e., V_s and t_s. It seems apparent that the design volume must be based on the chosen detention time. In a properly operating contact stabilization system, a detention time of two to six hours is generally accepted, depending upon the waste strength and characteristics. For low BOD wastes, such as domestic waste, a two to four hours stabilization period appears to be adequate (60).

When the waste removal fraction is too low to attain a permissible effluent quality, the contact period can be extended to obtain additional purification. Consequently, the required stabilization period will depend on the magnitude of initial BOD removal and on the contact period the unit provides. Increasing the contact period would permit a reduction of the stabilization period, but increasing the initial removal per unit of sludge mass increases the time required for stabilization.

14. Minimal Solids Aeration Activated Sludge Processes

Tenney, Johnson and Symons (61) proposed a modified activated sludge process utilizing two stages to treat strong wastes by maintaining a minimal solids aeration in the first stage and a moderate or extended solids aeration in the second stage. In the first stage, microbial populations multiply exponentially and a high tension of oxygen supply is required for rapid growth. In this stage, the soluble organic matters are appreciably reduced; however, microorganisms are well dispersed. In order to separate suspended solids from the liquor phase, an additional aeration tank, i.e., the second stage, is needed to stabilize the sludge and to assimilate the remaining utilizable organic substances discharge of the effluent of the first stage aeration tank. An oxygen tension in the second stage is normally demanded.

The major function operating in the first stage aeration tank is oxidative assimilation of the organic loading, while in the second stage, the major function is to flocculate, settle and destroy the volatile solids. The settling of the primary effluent solids can be enhanced by addition of a coagulant. The effluent of the second stage can be polished by treatment in a stabilization pond.

According to Tenney, et al (61), the minimal solids aeration modification under pilot plant studies indicates that substrate volumetric loadings are as high as 1690 lbs. COD per day per 1000 cubic feet of aeration tank or the organic loading 11 lbs. COD per day per lb. volatile solids. The flow sheet of minimal solids aeration activated sludge process is shown in Figure 25.

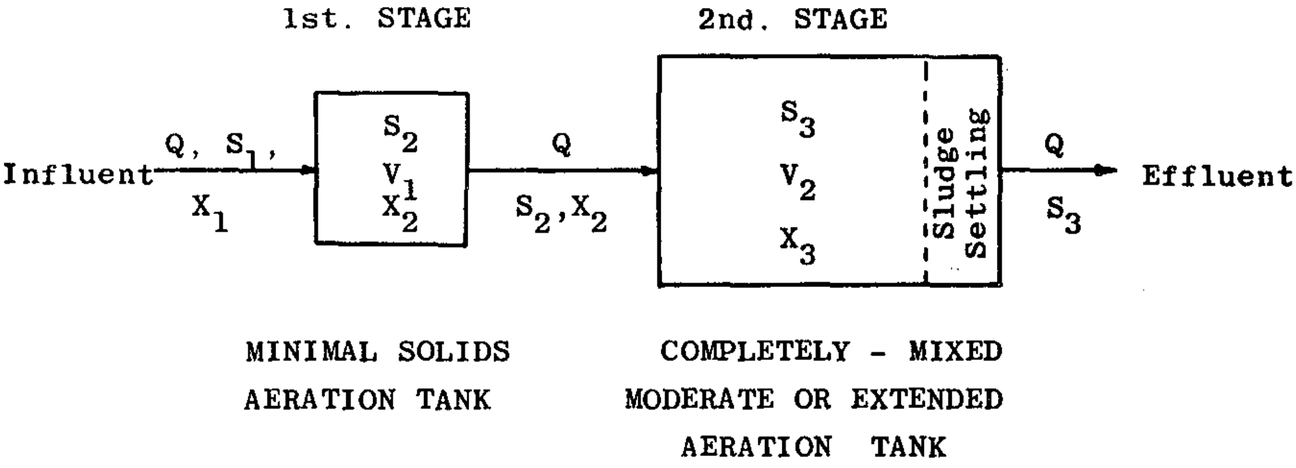


Figure 25

Flow Diagram of Minimal Solids Aeration Activated Sludge Process (After Tenney, Johnson, and Symons)

Concentrations of mixed liquor suspended solids present in two sequential stages of an aeration process are governed in large measure by sludge wasting and solids detention time (sludge age). The strength of the introduced organic waste also plays an equal importance in determination of the suspended solids concentration in the mixed liquor.

K_1 = Rate of substrate removal in the minimal solids aeration tank, mg/ℓ substrate removal/mg/ℓ suspended solids/hr

K_2 = Rate of substrate removal in the extended or moderate aeration tank, mg/ℓ substrate removal/mg/ℓ suspended solids/hr

V_1 = Volume of minimal solids aeration tank, MG

V_2 = Volume of extended or moderate aeration tank, MG

t_1 = Detention time of minimal solids aeration tank, hr

t_2 = Detention time of extended or moderate aeration tank, hr

Then,

(A) Design of minimal solids aeration tank:

$$\frac{(S_2 - S_1)}{t_1} = K_1(X_2 - X_1)$$

$$t_1 = \frac{(S_2 - S_1)}{K_1(X_2 - X_1)} \tag{152}$$

$$V_1 = Q\ t_1$$

$$= \frac{Q(S_2 - S_1)}{K_1(X_2 - X_1)} \tag{153}$$

(B) Design of moderate or extended aeration tank

$$\frac{(S_3 - S_2)}{t_2} = K_2(X_3 - X_2)$$

$$t_2 = \frac{(S_3 - S_2)}{K_2(X_3 - X_2)} \tag{154}$$

$$V_2 = Q\ t_2$$

$$= \frac{Q(S_3 - S_2)}{K_2(X_3 - X_2)} \tag{155}$$

Since the quality of the final effluent must satisfy these requirements, it is seen that the longer the detention time employed in the minimal solids aeration tank, the shorter the detention time needed for stabilization of the introduced organic compounds. On the contrary, if the detention time used in the first stage aeration tank is short, longer time is required for the young organisms to subside in the settling compartment and to purify the remaining organic matter discharged from the first stage aeration tank.

As described before, the advantage of this modification is to treat strong industrial wastes. It is also felt that installation of the second stage aeration tank provides sufficient flexibility to receive to a certain extent either qualitative or hydraulic shock loadings without immediate disruption of system performance. In other words, the second stage aeration tank might create "buffering ability" in case the shock loadings are applied. The advantage of the minimal solids aeration tank is not only to achieve rapid assimilation of the biodegradable organic matters, but also to remove appreciable amounts of phosphate or nitrogen which might be present in the raw wastes because of the demands of bacterial growth.

The employment of this process would be considered uneconomical for the municipal waste treatment. For a large community, installation of the extended aeration tank seems unappropriate and the extra installation of the minimal solids aeration tank appears to be unnecessary.

CHAPTER 4

SUMMARY OF EQUATIONS

Conventional Activated Sludge Processes

1. Organic Loading

$$L_o = SQ/X'V \tag{4}$$

$$V = SQ/X'L \tag{5}$$

$$t = S/X'L_o(1+R) \tag{6}$$

$$t = 24\,V/Q(1+R)$$

$$V_u = 100/X'L_o \tag{7}$$

$$V_u = V\,SQ/100 \tag{8}$$

$$P = 100(S - S_e)/S$$

2. Volumetric Loading

$$V = 62.4 \times 10^{-3} SQ/L_v \tag{10}$$

$$t = 62.4 \times 10^{-3} S/L_v(1+R) \tag{11}$$

$$V_u = 6.24/L_v \tag{12}$$

3. National Research Council Equations

A. Conventional Activated Sludge Processes

$$V = 0.1Q\left[S_1(1+R)/X\right]^{0.5} (S_1 - S_2/S_2)^{1.19} \tag{19}$$

$$t = 2.39\left[S_1/X(1+R)\right]^{0.5} (S_1 - S_2/S_2)^{1.19} \tag{18}$$

$$t = 24\,V/Q(1+R)$$

B. Contact-Aeration Processes

$$V = 0.0532 \frac{(1+R)S_1 Q^2}{A} \left(\frac{S_1 - S_2}{S_2}\right)^{1.34} \qquad (23)$$

$$t = \frac{1.275\ S_1 Q}{A} \left(\frac{S_1 - S_2}{S_2}\right)^{1.34} \qquad (21)$$

Completely-Mixed Activated Sludge Processes

1. Herbert's Equations with Modifications

$$V = \frac{(1+R)Q}{S_2 \mu_m} \left(1+R - \frac{X_r}{X_1} R\right)(K_s + S_2) \qquad (39)$$

$$t = \frac{\left(1+R - \frac{X_r}{X_1} R\right)(K_s + S_2)}{S_2 \mu_m} \qquad (40)$$

$$t = 24V/Q(1+R)$$

$$X_1 = \frac{Y(S_1 - S_2) + X_r R}{1+R} \qquad (37)$$

$$X_2 = Y(S_1 - S_2) \qquad (34)$$

$$S_2 = \frac{K_s\left(1+R - \frac{X_r}{X_1} R\right) D}{\mu_m - \left(1+R - \frac{X_r}{X_1} R\right) D} \qquad (32)$$

Where

$$D = \frac{(1+R)\,Q}{V} \qquad (28)$$

2. Reynolds and Yang's Equations with Modifications

$$V = \frac{Q\left[Y(S_0 - S_2) - X_1\left(1+R - R\frac{1+R}{R+W}\right)\right]}{X_1 K_e} \qquad (48)$$

$$t = \frac{Y(S_0 - S_2) - X_1\left(1+R - R\frac{1+R}{R+W}\right)}{X_1 K_e (1+R)} \qquad (47)$$

$$X_1 = \frac{Y(S_0 - S_2)}{K_e t(1+R) + (1+R - R\frac{1+R}{R+W})} \tag{46}$$

$$R = \frac{X_1 Q + X_1 V K_e - Y(S_0 - S_2)}{Q(X_r - X_1)} \tag{50}$$

$$W = \frac{Y(S_0 - S_2) - X_1 V K_e}{Q X_r} \tag{51}$$

3. Smith's Equations with Modifications

$$V = \frac{Q(S_1 - S_2)}{S_2 K_4} \tag{66}$$

$$t = \frac{(S_1 - S_2)}{S_2 K_4 (1+R)} \tag{65}$$

Oxygen Required

$$= (S_1 - S_2) - (1+R)\Delta X_a$$

$$= \frac{V}{Q}(K_5 X_a + K_6 S_2) \tag{67}$$

4. McKinney's Equations with Modifications

A. Entire Sludge Wasting in Effluent (Extended Aeration Processes)

$$V' = V_a + V_b$$

$$= \frac{Q(S_1 - S_2)}{K_5 S_2} \tag{72}$$

$$t' = \frac{S_1 - S_2}{K_5 S_2}$$

$$V_b = \frac{7.48\ DQ}{(O.\ R.)} \tag{74}$$

$$V_a = \frac{Q(S_1 - S_2)}{K_5\ S_2} - \frac{7.48\ D\ Q}{(O.R.)} \tag{75}$$

$$X_t = \frac{(SDI)\ 10000}{\frac{1}{R} + 1} \tag{84}$$

$$X_a = \frac{K_6 S_2}{\frac{\alpha}{t'} + K_7} \tag{82}$$

Oxygen Demand

$$= (K_9 S_2 + K_2 X_a)\ t'$$

B. Separate Sludge Wasting (Most Common Activated Sludge Processes)

$$V' = \frac{Q(S_1 - S_2)}{K_5 S_2} \tag{72}$$

$$t' = \frac{S_1 - S_2}{K_5 S_2}$$

$$V_b = \frac{7.48\ D\ Q}{(O.R.)} \tag{74}$$

$$V_a = \frac{Q(S_1 - S_2)}{K_5 S_2} - V_b \tag{75}$$

$$X_a = \frac{K_6 S_2}{t'(\alpha - \alpha W + C W) + K_7} \tag{91}$$

$$X = X_a (1 + \frac{K_8 t'}{\alpha + CW}) + \frac{(X_i)_i}{\alpha + CW} \tag{93}$$

5. Stack's and Conway's Equations with Modifications

$$t(hr) = \frac{S_1 (mg/\ell)}{40} \tag{94}$$

$$V = \frac{S_1 Q}{40} (1+R) \tag{95}$$

Oxygen Demand Per Day (in pound)

$$= 8.34\ Q \left[(S_1 - S_2) - (OE)\ W X_r + K_7 \frac{X_v - \frac{(X_i)_i}{\alpha}}{1 + \frac{K_8 t'}{\alpha}} \right] \tag{100}$$

6. Pipes', Grieves' and Milbury's Equations with Modifications

$$K = \frac{\mu_m S_i}{K_s} \qquad (113)$$

$$V = \frac{\left[\frac{(1+R)b}{1+Rb}\right]\left(\frac{S_i Q}{K}\right)\left(\frac{b}{a}\right)(1+R - RC)\left(\frac{1+R}{1+R - RC - R_C b}\right)}{S_c - \frac{(1-b)S_i}{(1+Rb)}} \qquad (121)$$

$$L = S_i \frac{Q}{V} \qquad (119)$$

$$\ln \frac{C_o}{C_i} = \left(\frac{b}{a}\right)$$

$$S_e = \left[\frac{(1+R)b}{1+Rb}\right]\left(\frac{L}{K}\right)\left(\frac{b}{a}\right)(1+R - RC)\left(\frac{1+R}{1+R - RC - R_C b}\right) + \frac{(1-b)S_i}{(1+R\,b)} \qquad (120)$$

7. Busch's Equations with Modifications

$$V = \frac{Q(S_1 - S_2)}{K} \qquad (126)$$

$$t = \frac{S_1 - S_2}{K(1+R)} \qquad (127)$$

$$V = \frac{Q(S_1 - S_2)}{mX} \qquad (129)$$

Oxygen Demand Per Day (in pound)

$$= 8.34\, n X V \qquad (130)$$

8. Eckenfelder's Equations with Modifications

$$V = \frac{Q(1+R)\ln\left[1 + \frac{Y(S_1 - S_2)}{X}\right]}{\mu} \qquad (136)$$

$$t = \frac{\ln\left[1 + \frac{Y(S_1 - S_2)}{X}\right]}{\mu} \tag{135}$$

$$V = \frac{Q(1+R)\ Y(S_1 - S_2)}{K\bar{X}} \tag{139}$$

$$t = \frac{Y(S_1 - S_2)}{K\bar{X}} \tag{138}$$

$$V = Q\,(1+R)\,\frac{\ln\frac{S_1}{S_2}}{K\bar{X}} \tag{143}$$

$$t = \frac{\ln\frac{S_1}{S_2}}{KX} \tag{142}$$

9. Bloodgood's Equations with Modifications

$$R = \frac{XZ}{10^6 - XZ} \tag{146}$$

$$V = \left(1 + \frac{XZ}{10^6 - XZ}\right) Q\,t \tag{147}$$

10. Contact-Stabilization Activated Sludge Process

A. Design of Contact Unit

Same as McKinney's Equations

B. Design of Stabilization Unit

$$R = \frac{X_c}{X_s - X_c} \tag{150}$$

$$t_s = \frac{X_s}{X_c}\ \frac{V_s}{Q} \tag{148}$$

or

$$V_s = \frac{Q X_c\ t_s}{X_s} \tag{148}$$

II. Minimal Solids Aeration Activated Sludge

A. Design of Minimal Solids Aeration Tank

$$V_1 = \frac{Q(S_2 - S_1)}{K_1(X_2 - X_1)} \quad \text{also} \quad V_1 = Qt_1 \qquad (153)$$

$$t_1 = \frac{(S_2 - S_1)}{K_1(X_2 - X_1)} \qquad (152)$$

B. Design of Moderate or Extended Aeration Tank

$$V_2 = \frac{Q(S_3 - S_2)}{K_2(X_3 - X_2)} \quad \text{also} \quad V_2 = Qt_2 \qquad (155)$$

$$t_2 = \frac{(S_3 - S_2)}{K_2(X_3 - X_2)} \qquad (154$$

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