# ANALYSIS OF IRRIGATION PUMPING AND APPLICATION EFFICIENCY IN THE CENTRAL OGALLALA FORMATION

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TABLE OF CONTENTS

Page

Page

Table Page

Table Page

Table Page

## LIST OF FIGURES

# ANALYSIS OF IRRIGATION PUMPING AND APPLICATION EFFICIENCY IN THE CENTRAL OGALLALA FORMATION

## INTRODUCTION

The Ogallala Formation is a major underground aquifer providing high quality irrigation water to farm operators throughout much of the Great Plains (Figure 1). The Central Ogallala Formation is created by penetration to bedrock of the Canadian River in the northern high plains of Texas and the Arkansas River in southwestern Kansas. The Central Ogallala includes the three Oklahoma Panhandle counties, portions of eight counties in southwestern Kansas, two counties in southeastern Colorado and eight counties in the northern high plains of Texas. This 47,500 square mile area overlies a closed container of water. Natural rainfall averages 10 to 15 inches annually across the region. Thus, recharge from natural percolation is negligible. The underground water supply, estimated to be 50 million acre-feet for the three Oklahoma Panhandle counties, is being "mined" by the actions of individual irrigators.

Declines in the water table reduce irrigation well yields and increase the feet of lift to the land surface. The effect of the declining water table is to increase the cost of pumping and, other things equal, to reduce the profitability of irrigated crop production and the economic life of the water supply. Because the aquifer is not

Figure 1. Approximate Delineation of the Ogallala Aquifer

farming exceed net returns per acre from irrigation, the aquifer is exhausted from an economic standpoint.

Faced with declining net returns from irrigated production, producers have become more interested in irrigation alternatives which use less water and energy. Irrigation scheduling, or applying water in accordance with soil water and the stage of plant development, may offer an opportunity to reduce water and energy use while maintaining the level of crop yields. If so, the level of net returns may be maintained and the economic life of the aquifer increased.

This study focuses on identification of irrigation technologies and strategies which improve pumping and application efficiency for irrigated grain sorghum in the central Ogallala region and extend the economic life of the underground aquifer. Stochastic efficiency concepts are used to identify risk efficient irrigation technologies for farm operators. Optimal control theory procedures are used to derive optimal irrigation schedules for grain sorghum based on soil water level and the stage of plant development.

The primary objective of this study is to identify irrigation technologies which will improve irrigation pumping and application efficiency in the central Ogallala Region. More specifically, the objectives are:

1. To evaluate existing data on soils and water resource situations in the Central Ogallala Formation and estimate the number of acres of irrigable clay and sandy loam soils in relation to the supply of water available for irrigation.
2. To determine the relationships between available soil water, atmospheric conditions, stage of plant growth and

uniform, producers in different portions of the region are affected quite differently by declines in the water table. Where the feet of saturated thickness is small and irrigation development intensive, producers are rapidly exhausting their water supply. Where depth to water and feet of lift are both substantial, producers may be facing economic exhaustion rather than physical exhaustion.

## THE PROBLEM AND OBJECTIVES OF THE PROJECT

With the development of irrigation pumping and water application technology, and the existence of a relatively plentiful and cheap supply of natural gas, irrigated agriculture in the Oklahoma Panhandle expanded significantly during the 1960's and 1970's. Irrigated acreage in the three Oklahoma Panhandle Counties (Beaver, Cimarron, and Texas) increased from 11,500 acres in 1950 to 405,680 acres in 1979 to nearly 417,000 in 1981 (Schwab). The Ogallala aquifer, the source of groundwater for irrigated agriculture in the Oklahoma Panhandle, is a confined aquifer receiving very little recharge. With continued expansion of irrigated crop acreage, water withdrawals from the aquifer far exceed recharge. The result of this imbalance of withdrawals and recharge is a continued and accelerated decline in the water table.

Rising energy costs, particularly the price of natural gas which is the primary fuel source for irrigation in the area, have combined with declining water levels to increase the cost of intensive irrigation. With prices received for agricultural products at relatively low levels, the economic life of the aquifer for irrigated production is being reduced. When net returns per acre from dryland development, and the timing and amount of moisture from rainfall and irrigation applications for grain sorghum, a key crop in the area.

3. To investigate alternative irrigation pumping and distribution technologies currently available, or in the development stages, and to evaluate their potential for improving irrigation pumping and application efficiency in the Central Ogallala Formation.
4. To evaluate the potential of existing and emerging technology for improving irrigation pumping and application efficiency, supporting net farm income and lengthening the economic life of the scarce underground water resource in the Central Ogallala Formation.

The remainder of this report is divided into several major sections. First, the model used in the analysis is developed and discussed in detail, including presentation of the procedures used to verify and validate the model. Then, stochastic efficiency and optimal control concepts are developed. The following section contains a discussion of the water conservation strategies analyzed in the study. The results of each set of alternatives evaluated are then presented. The final sections contain the summary and conclusions, publications resulting from the project and a selected bibliography.

## MODEL DEVELOPMENT

A farm level decision model was constructed for the analysis of irrigation pumping and application efficiency in the Central Ogallala Formation. While the methodological approach used in this study would be applicable for other key crops in the study area, the major focus of the analysis is on a single crop, grain sorghum. Thus, in developing the firm level decision model, considerable effort was placed on modifying a grain sorghum plant growth simulator, which was developed by Arkin, Vanderlip and Ritchie, so that the plant growth model would perform satisfactorily under soils and climatic conditions appropriate for the Central Ogallala Formation.

The relationships between the grain sorghum plant growth model and the firm's internal and external environment are presented in Figure 2. Other major components include Farm Resources, Environment, Dryland and Irrigation Scenarios, Production Costs and Decisions, Irrigation Decisions, Input and Output Prices and Net Returns. The interactions of these components compose a closed simulation model in which past environmental conditions and irrigations affect current and future irrigation decisions and net returns. In the following sections of this report, the components of the model are discussed in detail.

## FARM RESOURCES COMPONENT

The Farm Resources Component represents a quarter section of Richfield clay loam soil, a predominant soil in the central Oklahoma Panhandle. A typical irrigated farm in the area would contain a number of irrigated quarter sections of land. Of each 160 irrigable acres, approximately 155 acres could be irrigated with a single surface irrigation system. This analysis is limited to a single quarter section, but the implications can be aggregated for a typical farm or the region.

Dryland and irrigated grain sorghum are both simulated as

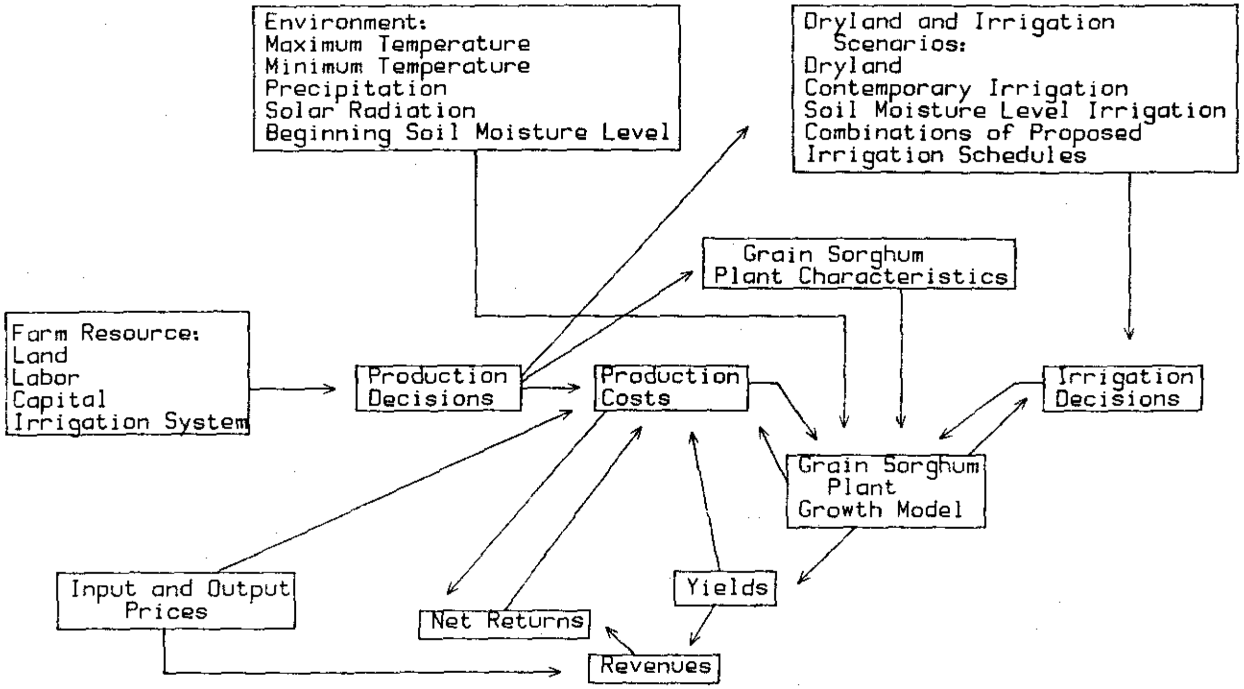

Figure 2. Organization and Structure of the Firm Level Model

designated in the Production Decisions Component. For irrigated grain sorghum, a total of 155 acres are irrigated from a single surface irrigation system with a 900 gallon per minute well yield. A well depth of 350 feet is assumed with water being lifted approximately 250 feet to the surface. The Oklahoma State University Irrigation Cost Generator (Kletke, Harris, and Mapp) is used to derive fixed and variable irrigation costs. Natural gas, the predominant irrigation fuel in the region, is priced at $1.50 per thousand cubic feet. Per acre non-irrigation costs and costs for dryland grain sorghum production are determined using the Oklahoma State University Crop Budget Generator for the Oklahoma Panhandle area (Oklahoma Crop and Livestock Budgets).

From the dryland and irrigated enterprise budgets certain assumptions are made about the farm firm. Custom harvesting is used for both irrigated and dryland situations. Machinery complements typical of the area are used for dryland and irrigated production and provide the basis for estimating fixed and variable machinery costs. Non-irrigation labor costs are determined from the enterprise budgets, and irrigation labor costs are derived from the irrigation cost generator. Enterprise cost information, exclusive of variable irrigation costs, enters the Production Costs Component and is eventually used in determining the level of net return.

## PRODUCTION DECISION COMPONENT

In the Production Decision Component, one of a number of grain sorghum production possibilities is determined. Grain sorghum may be produced under dryland conditions or under one of a number of possible irrigation schedules. Information from the Farm Resources Component flows into the Production Decision Component and the production decision influences actions in the Production Cost Component, Dryland and Irrigation Scenario Component and the Grain Sorghum Plant Characteristics Component. The type of production selected influences the irrigation scheduling and plant population decisions.

## DRYLAND AND IRRIGATION SCENARIO COMPONENT

The Production Decision Component determines the type of production to be analyzed and directly affects the Dryland and Irrigation Scenario Component. If dryland production is to be simulated, then the irrigation components are not used. If irrigated production is to be simulated, this component of the simulator determines the irrigation schedule to be analyzed. If contemporary irrigation practices (applying two acre feet per acre in a series of regularly scheduled irrigations) are to be examined, irrigations are initiated on specific calendar dates. If irrigations are to be scheduled based on extractable soil water and/or the stage of grain sorghum growth and development, a feedback loop connects the Grain Sorghum Plant Growth Model Component and the Irrigation Decisions Component.

## IRRIGATION DECISION COMPONENT

From Figure 2 there is a feedback loop between the Grain Sorghum Plant Growth Model Component and the Irrigation Decisions Component. Information on daily extractable soil moisture levels and stages of plant growth are transmitted to the Irrigation Decisions Component.

If the values for the extractable soil moisture level and stage of plant growth meet or are below a critical plant stress value, the Irrigation Decision Component initiates an irrigation application. A specified quantity of irrigation water for the calendar day is transmitted to the Grain Sorghum Plant Growth Model as input for the development of the grain sorghum plant. In some instances the Grain Sorghum Plant Growth Model derives a soil moisture level and stage of plant growth requiring an application of groundwater which the Irrigation Decisions Block cannot complete. The decision to irrigate must incorporate the pumping limits of the irrigation delivery system. The time required to apply water to 155 acres is calculated to insure that another irrigation can not be started before the previous application has been completed. For this analysis a pumping efficiency of 66.67 percent is used to derive the quantity of water that can be delivered to the plant. Net pumpage quantities are used by the grain sorghum plant growth model to determine plant yield. Gross quantities of water pumped are used to determine the application time. Thus, gross pumpage is calculated by multiplying net pumpage by a factor of 1.5.

For some of the irrigation schedules examined, the quantity of groundwater applied varies from 1 to 3 inches, depending on the level appropriate to maximize net returns to the producer. Therefore, a feedback loop is established between the Net Returns and Irrigation Decision Components. The quantity of groundwater pumped affects both the costs of irrigation or the Production Costs Component and the yields derived by the grain sorghum plant or the Yields Component. The cumulative effects of the interactions between various components of the firm level irrigated grain sorghum model through the feedback loops determine the quantity of groundwater applied and the level of net returns to the irrigated producer.

## GRAIN SORGHUM PLANT GROWTH MODEL COMPONENT

Early modeling research in agricultural production used statistical analysis to develop plant growth equations. Multiple regression models can provide a basis for estimating the timing of irrigations, however, data are seldom available on all of the biological factors important to crop growth. Jensen, Wright, and Pratt use daily climate, crop, and soil data to estimate daily soil moisture depletion. This type of computer modeling has provided the basis for the development of the soil moisture-plant growth models. With the successful modeling of photosynthesis by Duncan (1966), interest in further development of crop models was enhanced.

Stapleton and Myers modeled the growth of cotton, and Baker and Harrocks developed a model of corn grain production. Baker and Harrocks concluded that plant modeling should be on a daily basis rather than for a monthly or seasonal time period. Simulation models used to derive daily plant growth provide more realistic results than single regression equation with seasonal parameters. Soymod I developed by Curry, Baker, and Streeter was a computer model to simulate the growth of the soybean plant throughout the season. Duncan (1975) also developed a corn model, referred to as SIMAIZ, and lettuce growth was simulated for greenhouse operations by Sorbie and Curry. Few of these models have been used in economic analysis of irrigation strategies by producers.

The grain sorghum plant growth model developed by Arkin, Vanderlip, and Ritchie is used in this analysis to derive daily growth of the grain sorghum plant and to determine the effects on the plant of various irrigation decision strategies. The model is modified to simulate Central Ogallala growing conditions on a Richfield clay loam soil. The model depicts the growth of a single grain sorghum plant in a population of plants through time by linking climatalogical factors and plant growth equations. The model assumes five stages of growth for the grain sorghum plant: Stage 1, emergence to differentiation (floral initiation); Stage 2, differentiation to end of leaf growth; Stage 3, end of leaf growth to anthesis (half-bloom); Stage 4, anthesis to physiological maturity; and Stage 5, physiological maturity and beyond. Input data for the model comes from the Environment and Grain Sorghum Plant Characteristics Components. The model begins with the input of weather data on May 1 and ends at physiological maturity, which varies from year to year depending on the weather data. From May 1 until planting of grain sorghum on June 15, the model uses the climatic data to calculate the current day's soil moisture. Preplant irrigation applications are assumed to begin between May 25 and June 1 to permit time for a 3 inch application prior to planting on June 15.

After planting, the seedling emergence routine is initiated to determine the dates of germination and emergence. Both germination and emergence are a function of accumulated heat units, with germination being affected by available soil water and emergence by planting depth.

After emergence, leaf appearance and growth are derived daily based on accumulated heat units. For example, when a total of 50 heat units above the base temperature of 7°(45°F) is derived, a new leaf appears. Calculation of daily leaf growth is a function of the difference between average air temperature and a base temperature of 12°C (54°F).

The current area of a particular leaf is the leaf area from last period plus the current rate of leaf expansion. The current leaf area is compared to the maximum area for the particular leaf and, if leaf growth equals or exceeds the maximum area for the particular leaf, the growth of the leaf is completed. For each leaf beyond the eleventh leaf, a corresponding leaf starting with leaf one is lost. The remaining leaf area for the plant is calculated by subtracting the leaf area of the fallen leaf. This process continues for each successive leaf to determine the total leaf area for a plant.

After the leaf area calculation, intercepted photosynthetically active radiation and potential photosynthesis are calculated for a particular Calendar day.[1] The model derives the fraction of sunlight transmitted by the sorghum canopy as a fraction of the daily calculated leaf area index and row spacing. The model derives intercepted radiation which is used to derive potential net photosynthesis. An evaporation subroutine calculates the potential evaporation from the modeled soil-plant system for a particular Calendar day.

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[1] A Calendar day is defined as a day in the year in which there are 365 days with the first day being January 1 and the 365th day being December 31. In a leap year, there are 366 calendar days.

Potential photosynthesis is converted into dry matter after net photosynthesis has been derived. Net photosynthesis is expressed as:

$$(1) \quad P = (P \cdot \varepsilon_1 \cdot \varepsilon_2) - N$$

where P is net photosynthesis, $\varepsilon_1$ and $\varepsilon_2$ are efficiency parameters for temperature and soil moisture respectively, and N is nighttime respiration loss. The efficiency parameters, $\varepsilon_1$ and $\varepsilon_2$ , are dimensionaless parameters ranging from 0 to 1. Equation (1) states that a limiting environmental element would proportionately reduce the photosynthetic rate regardless of other variables. Each of the two efficiency parameters represents a particular environmental constraint on the photosynthetic rate.

The mean ambient temperature for a particular Calendar day is used, and the temperature efficiency coefficient $\varepsilon_1$ is derived from Figure 3. Photosynthesis is assumed completely inactive for temperatures between $0^\circ$C and $5^\circ$C ($41^\circ$F) and for temperatures above $45^\circ$C ($113^\circ$F). There is no temperature stress or $\varepsilon_1$ equals 1.0 for temperatures which range from $25^\circ$C ($77^\circ$F) to $40^\circ$C ($104^\circ$F).

Reductions in net photosynthesis because of insufficient soil moisture are derived from the water stress efficiency coefficient, $\varepsilon_2$. The model derives soil moisture levels daily from the equation:

$$(2) \quad SW_t = SW_{t-1} - ET_t + RAIN_t + IR_t$$

where:

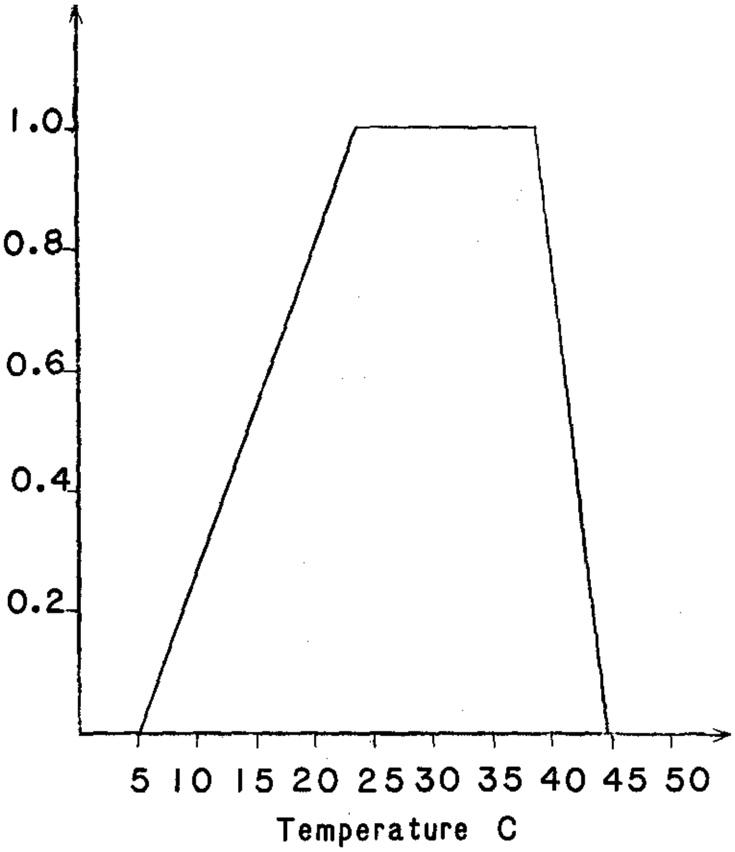

Figure 3. The effect of the Crop Efficiency Parameter $(\varepsilon_1)$ Corresponding to Plant Temperature on Net Photosynthesis.

$SW_t$ is extractable soil water in period t, $SW_{t-1}$ is extractable soil water in period t-1, $ET_t$ is evapotranspiration in period t, $RAIN_t$ is precipitation in period t, and $IR_t$ is quantity of irrigation water applied in period t.

Evaporation is calculated for the plant and soil. These two components are added together to derive total daily evapotranspiration or $ET_t$ which is used in equation (2). The daily extractable soil water level ($SW_t$) is divided by the upper limit extractable soil water level (UL) to derive the daily extractable soil moisture ratio ($SW_t$/UL) which is used in Figure 4 to derive plant water stress. From Figure 4 the effects of the extractable soil moisture level are most critical beyond the 40 percent range. Night respiration is derived from an equation by McCree for grain sorghum.

The daily extractable soil moisture ratio ($^{SW}$t/UL) and stage of plant growth are transmitted to the Irrigation Decisions Component. If an irrigation scenario is based on extractable soil moisture and stage of plant growth, an irrigation application is initiated if these criteria are met.[2] The quantity of groundwater applied ($IR_t$) is added to equation (2) and the development of the grain sorghum plant is continued. If a contemporary irrigation practice is simulated, the extractable soil moisture ratio and stage of plant growth are ignored. Irrigations for the contemporary irrigation practice commence on specified Calendar days.

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[2] Irrigation strategies evaluated in this analysis are discussed in detail in subsequent sections of this report.

Net photosynthesis is converted daily into dry matter which must be allocated to different parts of the plant. The fraction of dry matter being allocated to specific parts of the plant, which varies according to plant development, is important because the majority of dry matter is used to develop leaves. When the grain sorghum plant reaches Stage 3, the development of leaves slows and eventually stops with dryweight reversed to the development of roots, culm, and the head of the grain sorghum plant. When Stage 4 of plant development is reached, all of the dryweight goes to development of the grain head. The routine is continued until physiological maturity is reached. By completing this cycle for a number of years of weather data, a series of yields or replications is generated.

The grain sorghum plant growth model derives the different growth stages of the plant and determines the effects of climatological stress on plant production. The model derives production of a single grain sorghum plant which, when multiplied by the number of plants per acre, gives the grain sorghum yield per acre in hundredweights. The yield and price are used to calculate gross returns. The quantity of groundwater pumped is used to determine irrigation costs. These are combined in the Net Returns Component to derive net returns for each irrigation scenario.

## GRAIN SORGHUM PLANT CHARACTERISTIC COMPONENT

In order to initiate the model certain input data describing the plant characteristics must be developed. For this analysis, the grain sorghum plant is assumed to have 17 leaves and each leaf has a maximum area of 0.88, 2.30, 7.60, 12.30, 22.80, 42.50, 69.50, 113.00, 170.80,

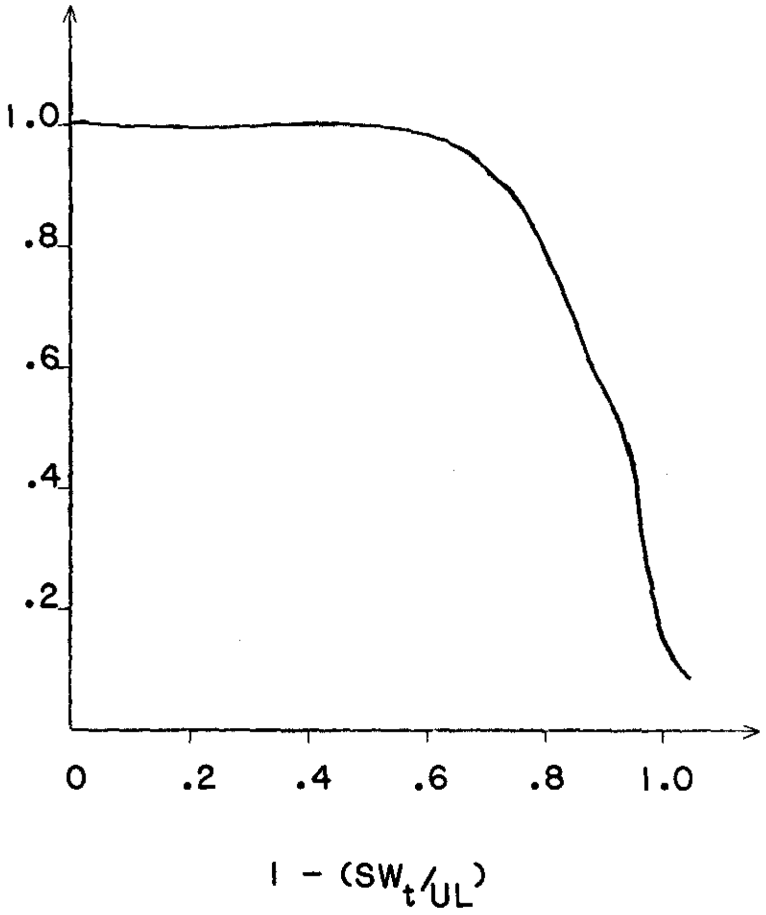

Figure 4. The effect of the Crop Efficiency Parameter $[\varepsilon_2]$ Corresponding to Available Soil Water on Net Photosynthesis.

248.80, 287.00, 357.50, 336.50, 340.80, 272.30, 209.30, and 116.00 $cm^2$, respectively (Arkin).

The planting date for grain sorghum is June 15 (Stone, Griffen, Ott). Plant population and row spacing are different for the dryland and irrigation scenarios. Dryland planting is wider spaced and has fewer plants per acre than irrigated production. For dryland production, the row spacing is 40 inches (101.6 cm) with a plant population per acre of 24,700 plants (61,000 plants/hectare). For irrigated production, the row spacing is 28 inches (70.0 cm) with a plant population of 100,000 per acre (247,000 plants/hectare). Daily climatological data are also required by the dynamic grain sorghum plant model and climate data are supplied to the model from the Environment Component. Table 1 lists the input data required for the grain sorghum plant growth model.

## ENVIRONMENT COMPONENT

Input data for the grain sorghum model include daily precipitation, maximum and minimum temperature, and solar radiation. These data are used to trace the daily development of the plant. Climatic data for May through October are required by the grain sorghum plant growth model. Twenty-three consecutive years of climatic data (1953 to 1975) for Dodge City, Kansas were obtained and are used in this analysis because solar radiation data are not available from a weather station in the Oklahoma Panhandle. Dodge City, located in Southwestern Kansas near the Oklahoma Panhandle, has weather conditions very similar to those in the study area.

Table 1. Input Data Required for the Grain Sorghum Plant Growth Model

| Data Required | Data Value |
|---|---|
| I. Plant Data | |
| A. Leaf number | 17 |
| B. Maximum individual leaf area ($cm^2$) | |
| Leaf 1 | 0.88 |
| Leaf 2 | 2.30 |
| Leaf 3 | 7.60 |
| Leaf 4 | 12.30 |
| Leaf 5 | 22.80 |
| Leaf 6 | 42.50 |
| Leaf 7 | 69.50 |
| Leaf 8 | 113.00 |
| Leaf 9 | 170.80 |
| Leaf 10 | 248.80 |
| Leaf 11 | 287.00 |
| Leaf 12 | 357.50 |
| Leaf 13 | 336.50 |
| Leaf 14 | 340.80 |
| Leaf 15 | 272.30 |
| Leaf 16 | 209.30 |
| Leaf 17 | 116.00 |
| II. Planting Data | |
| A. Planting date | June 15 |
| B. Plant population (plants/acre) | |
| 1. Dryland | 24,700 |
| 2. Irrigated | 100,000 |
| C. Row width (inches) | |
| 1. Dryland | 40 |
| 2. Irrigated | 28 |
| III. Climatic Data (Daily values from planting until maturity) | |
| A. Maximum daily temperature (°C) | a/ |
| B. Minimum daily temperature (°C) | a/ |
| C. Solar raditation (ly/day) | a/ |

Table 1. (Continued)

| Data Required | Data Value |
|---|---|
| D. Rainfall (cm/day) | a/ |
| IV. Soil Data | |
| A. Available water holding capacity (inches) | 7.63 |
| B. Initial available water content (inches) | b/ |
| V. Location Data | |
| A. Latitude (degrees) | 37° |

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a/ Daily values for study location from weather station.

b/ Value for each individual study period calculated from referenced study by Mapp et al.

### Rainfall Data

In the Oklahoma Panhandle, rainfall patterns fluctuate widely with abundant rainfall in some months and below average precipitation levels in other months. Rainfall during the grain filling stage of grain sorghum development is very critical. Sufficient rainfall results in plentiful yields while deficit precipitation levels cause sorghum yields to be reduced. Daily rainfall data for the 23-year services were used as input to the grain sorghum plant growth model.

### Temperature Data

Temperature is another environmental factor that may cause plant stress. The temperature stress coefficient ($\varepsilon_1$) used in the model may cause net photosynthesis to be less than potential photosynthesis and reduce plant growth. Maximum and minimum daily temperatures are used to derive an average daily temperature value for the plant growth model.

### Solar Radiation Data

Daily solar radiation is used by the grain sorghum plant growth model to derive photosynthesis for the plant. Cloud cover and other atmospheric conditions may cause net radiation to be less than potential radiation. A cloudy or rainy day, therefore, has lower radiation levels than a day with clear skies. Solar radiation is calculated in langley per day, where a langley is defined as a one calorie per square centimeter.

## Beginning Soil Moisture

The initial soil moisture level is an input required for the grain sorghum plant growth model. Field capacity of the Richfild clay loam soil is 16.32 inches of water and the permanent wilting point is a soil moisture level of 8.69 inches. The maximum extractable soil moisture level is 7.63 inches (16.32-8.69) and the minimum extractable soil moisture level is 0.0 inches (8.69-8.69). Beginning soil moisture level is derived from an equation by Mapp et al. (1975):

$$(3)\quad SW_b = 8.69 + 0.22\, R_{ma} + 2.33 R_{lwa}$$

where:

$SW_b$ is beginning soil moisture, $R_{ma}$ is rainfall during April, and $R_{lwa}$ is rainfall during the last week in April

## INPUT AND OUTPUT PRICES, PRODUCTION COSTS, REVENUES, AND NET RETURNS COMPONENTS

In the Input and Output Prices Component, enterprise budget costs are derived for use in the Production Costs Component. The grain sorghum price ($3.98 per hundredweight) is also transmitted to the Revenues Component. Enterprise budgets for dryland and irrigated grain sorghum were developed using the OSU Crop Budget Generator. Harvesting costs were varied depending on the final yield generated by the grain sorghum plant growth model. Fixed and variable irrigation costs were computed by the OSU Irrigation Cost Generator. Variable irrigation costs are a function of the gross quantity of groundwater pumped. Variable cost per acre inch for irrigation are multiplied by acre inches pumped during the season to determine total variable irrigation costs.

Production in hundredweights per acre is derived by the grain sorghum plant growth model for dryland and various irrigation scenarios. Revenues are derived by multiplying the simulated yield by the grain sorghum price ($3.98 per hundredweight). Net returns are calculated by subtracting the costs of production derived in the Production Costs Component for the particular grain sorghum enterprise from the gross revenues determined in the Revenues Component. Each irrigation strategy is replicated 23 times, based on the 23 years of weather data available, and results are evaluated based on the 23 net returns derived for each strategy.

The net returns series generated are used in several types of analyses. In the stochastic dominance analysis, twenty-three replications of each irrigation strategy are used to generate beta distributions of net returns. Stochastically dominate irrigation practices can be identified based on the relationships among the beta distributions. In the optimal control analysis, feedback loops between the Net Returns and Irrigation Decisions Components are used to derive the quantity and timing of irrigation water which maximizes net returns to the producer. These results specify the quantity of irrigation water to be applied at various stages of grain sorghum plant development to maximize producer net returns.

## VERIFICATION AND VALIDATION OF THE MODEL

Verification and validation are essential components in evaluating the performance of the simulation model. Verification involves establishing that the computer program is executing as intended without errors. Validation involves determining that the simulation model is a reasonable representation of reality in terms of the system being studied. Validation needs to be performed at all levels including data inputs, model elements, subsystems, and interface points.

Verification of input data of the grain sorghum plant parameters was accomplished through conversations with faculty member of the Department of Agronomy at Oklahoma State University and staff members at the Blackland Conservation Research Center at Bushland, Texas. Daily climatic data for the grain sorghum model, including maximum and minimum temperature, precipitation, radiation, and beginning soil water levels, were verified by examining printouts of the data. Validation of the grain sorghum plant growth model is accomplished by simulating dryland and contemporary irrigation practices and comparing yields from both production scenarios to actual yields achieved in the area. The generated yields were evaluated by experts in the area and compared to yields reported in a recent area publication (Gray).

### Dryland Production

Table 2 presents the results of the dryland grain sorghum plant growth simulations. For the dryland scenario row space for the plants is greater than for irrigated production and consequently the number of plants per dryland acre are less than under irrigation. In two of the 23 years simulated, grain sorghum does not produce a stand and zero yields are recorded. The average yield for the 23 year period is 14.80 hundredweights/acre or 26.4 bushels/acre. The 26.4 bushel/acre

Table 2. Annual Yields, Revenues, Costs and Net Returns for Dryland Grain Sorghum Production.

| Replication | Field Yield (CWT/AC) | Revenue ($/AC) | Costs ($/AC) | Net Return ($/AC) |
|---|---|---|---|---|
| 1 | 3.83 | 15.24 | 32.33 | -17.09 |
| 2 | 7.56 | 30.09 | 33.45 | -3.36 |
| 3 | 10.59 | 42.15 | 34.36 | 7.79 |
| 4 | 0.00 | 0.00 | 31.18 | -31.18 |
| 5 | 12.66 | 50.39 | 34.98 | 15.41 |
| 6 | 16.66 | 66.31 | 36.18 | 30.13 |
| 7 | 4.78 | 19.02 | 32.61 | -13.59 |
| 8 | 7.38 | 29.37 | 33.39 | -4.02 |
| 9 | 20.22 | 80.48 | 37.25 | 43.23 |
| 10 | 30.28 | 120.51 | 40.26 | 80.25 |
| 11 | 5.13 | 20.42 | 32/72 | -12.30 |
| 12 | 25.39 | 101.05 | 38.80 | 62.25 |
| 13 | 17.78 | 70.76 | 36.51 | 34.25 |
| 14 | 22.00 | 87.56 | 37.78 | 49.78 |
| 15 | 19.19 | 76.38 | 36.94 | 39.44 |
| 16 | 28.38 | 108.97 | 39.69 | 69.28 |
| 17 | 15.56 | 61.93 | 35.85 | 26.08 |
| 18 | 0.00 | 0.00 | 31.18 | -31.18 |
| 19 | 7.19 | 28.62 | 33.34 | -4.72 |
| 20 | 26.92 | 107.14 | 39.26 | 67.88 |
| 21 | 16.56 | 65.91 | 36.15 | 29.76 |
| 22 | 27.20 | 108.26 | 39.34 | 68.92 |
| 23 | 15.12 | 60.18 | 35.72 | 24.46 |

average compares favorably to the estimated average production for Richfield clay loam soil, 0 to 1 percent slope, of 22.0 bushels per acre (Gray). A reasonably high level of management ability is assumed for the farm situation being simulated. For the years in which a stand is achieved, the yields range from a minimum of 3.83 cwt/acre in 1953 to maximum of 30.28 cwt/acre in 1972. From Table 2 both the level and variability in yields between years are characteristic of dryland yields in the Oklahoma Panhandle. Returns per acre to dryland producers are highly variable due to yield fluctuations, as is reflected in Table 2. Based on conversations with agronomic and farm management experts in the area, the model was judged to be performing satisfactorily for dryland grain sorghum.

## Contemporary Irrigation Practices

To validate the model under irrigated conditions, the contemporary practice of applying 24 acre inches of water per acre is simulated. Producers following contemporary practices typically irrigate on a rather regular schedule. To simulate these practices, irrigations are assumed initiated on specified Calendar dates, and the net quantities of water applied are used by the grain sorghum plant growth model. One preplant and five post plant irrigations are initiated on the dates indicated in Table 3. Over the course of the irrigation season, a total of 24.0 inches per acre is pumped yielding a net application to the grain sorghum plant of 16.0 acre inches per acre.

Planting of grain sorghum is assumed to occur on June 15 with the May 25 irrigation being preplant to insure sufficient soil moisture at

Table 3. The Scheduling and Rates of a 24 Inch Application for Grain Sorghum

| Date | Calendar Day | Gross Application (inches) | Net Application (inches) |
|---|---|---|---|
| May 25 | 145 | 6.0 | 4.0 |
| June 22 | 173 | 3.6 | 2.4 |
| July 6 | 187 | 3.6 | 2.4 |
| July 20 | 201 | 3.6 | 2.4 |
| August 3 | 215 | 3.6 | 2.4 |
| August 17 | 229 | 3.6 | 2.4 |
| Total | | 24.0 | 16.0 |

planting time. The next scheduled irrigation is on July 6 and subsequent irrigations are scheduled every two weeks with a constant 3.60 inches assumed applied each period. Yields, revenues, costs and returns are presented in Table 4. The average yield for this scenario is 59.20 cwt/acre or 105.7 bushels/acre. The simulated yields compare favorably to irrigated production for grain sorghum on Richfield clay loam soils of 0 to 1 percent slope (115 bushels/acre) and 1 to 3 percent slope (105 bushels/acre) (Gray).

The maximum production for the 24 acre inch scenario is 72.32 cwt/acre with minimum production of 50.11 cwt/acre. Returns to producers average $78.86 per acre, and are more stable than returns under dryland production. Variations in annual yields, even with intensive irrigation, are due to other climatalogical factors, such as variations in temperature and solar radiation.

The results of the contemporary irrigation scenario compare favorably to those reported by Gray. Results were also shown to agronomic experts at Okahoma State University and the Blackland Conservation Research Center at Temple, Texas and were judged to be very reasonable in terms of yield level and variability.

Having verified and validated the grain sorghum plant growth model, the next phase of the research was to analyze water conservation strategies that allow the producer to reduce water use while maintaining crop yields and net returns. In subsequent sections of this report, evaluations are made of irrigation schedules according to the level of available soil water and stage of grain sorghum plant development. Stochastic efficiency analysis is used to identify risk efficient irrigation strategies and optimal control procedures are

Table 4. Simulated Grain Sorghum Yields, Revenues and Returns From Constant 24 Inch Irrigation Water Application Using 1953-75 Climatic Data

| Replications | Field Yield (CWT/AC) | Revenue ($/AC) | Return ($/AC) |
|---|---|---|---|
| 1 | 61.30 | 243.97 | 87.22 |
| 2 | 56.78 | 225.98 | 69.23 |
| 3 | 63.33 | 252.05 | 95.30 |
| 4 | 62.54 | 248.91 | 92.16 |
| 5 | 62.28 | 247.87 | 91.12 |
| 6 | 65.90 | 262.28 | 105.53 |
| 7 | 67.26 | 267.69 | 110.94 |
| 8 | 66.09 | 263.04 | 106.29 |
| 9 | 55.75 | 221.88 | 65.13 |
| 10 | 56.32 | 224.15 | 67.40 |
| 11 | 56.99 | 226.82 | 70.07 |
| 12 | 53.00 | 210.94 | 54.19 |
| 13 | 56.14 | 223.44 | 66.69 |
| 14 | 52.30 | 208.15 | 51.40 |
| 15 | 51.71 | 205.81 | 49.06 |
| 16 | 53.38 | 212.45 | 55.70 |
| 17 | 50.70 | 201.79 | 45.04 |
| 18 | 65.13 | 259.22 | 102.47 |
| 19 | 72.32 | 287.83 | 131.08 |
| 20 | 50.11 | 199.44 | 42.69 |
| 21 | 63.52 | 252.81 | 96.06 |
| 22 | 52.49 | 208.91 | 52.16 |
| 23 | 66.24 | 263.64 | 106.89 |
| AVG. | 59.20 | 235.61 | 78.86 |

applied to determine the optimal amount and timing of water for grain sorghum production.

## ANALYSIS AND RESULTS

The firm level irrigated grain sorghum simulator permits investigation of the effects of different irrigation strategies on grain sorghum yields, water use, and producer net returns. Proposed irrigation scenarios which initiate applications based on soil water level and stage of plant growth are investigated to determine their value as alternatives to contemporary intensive irrigation practices. Stochastic dominance analysis and optimal control theory are used to derive efficient and optimal irrigation schedules.

### WATER STRESS AND IRRIGATION SCHEDULING

Water stress in field crops leads to changes in the plant, including a decrease in nutrient uptake (Marias and Wiersma), a decrease in leaf area with corresponding decrease in plant size, an increase in the rate of leaf senescence, a decrease in length of the growing period, and a decrease in yield (Daugherty).

The effects of water stress on crop yield depend upon both the timing of the stress and the portion of the plant that gives economic yield. Water stress is most noticeable when the yielding portion of the plant is undergoing rapid growth. Thus, crops whose yields comprise the bulk of the above ground portion of the plant, such as tobacco, pastures, and silage, are more susceptible to water stress. With fruit and vegetables that are sold on a fresh weight basis, soil water stress needs to be avoided until harvest (Begg and Turner).

However, grains which are harvested dry are less affected by water shortage at physiological maturity (Salter and Goode). Irrigation scheduling based on crop water needs rather than on a regular time sequence, such as the contemporary irrigation practice, may result in less water application, little or no loss in yield, and the possibility of an increase in net returns.

Many irrigation scheduling approaches have been investigated. Visual appraisal of the condition of the crop is one method of determining when to initiate an irrigation application. With grain sorghum, a distinctive change in the crop appearance, such as leaf curl, signifies water stress. However, visual crop appraisal of the plant as a means of initiating an irrigation may be inaccurate, as the crop could already be experiencing water stress.

Irrigation scheduling based on measured soil parameters has centered on soil water content and soil water tension. Soil water content can be accurately determined by gravimetric sampling or a neutron soil water meter. The neutron probe can be calibrated and is designed to continually monitor soil water content. However, radiation hazards and high costs restrict the use of the neutron probe.

Soil water tension (or soil water potential) has been measured by tensiometers (Rose). A tensiometer can be placed in the root zone with irrigations initiated at certain predetermined soil water tensions. However, Jensen (1975) reported that the price of a tensiometer service in California was approximately $8 per acre in 1970, which suggests that this method may be too costly for widespread adoption.

In this model, rather than measure soil moisture, meteorological data are used to predict soil moisture content. The grain sorghum model uses this prediction to determine plant stress and to initiate irrigations.

### No Stress Irrigation Schedule

An alternative to the contemporary practice of applying 24 inches per acre regardless of climatic conditions is to attempt to schedule irrigation applications in accordance with the needs of the plant. One approach to monitoring plant needs is measuring extractable soil water and initiating irrigation applications when extractable soil water falls below some critical level at which plant stress occurs. For this scenario, the critical extractable soil water level is defined as an extractable soil water ratio at or below 45 percent.[3] This approach would use a combination of visual and meterological data to insure against severe soil water stress. When the extractable soil moisture ratio is at or below 45 percent, an irrigation application initiated. It requires fifteen days to apply the 3.0 inch application on a 155 acre field under surface irrigation. The extractable soil moisture ratio is then ignored until the fifteen day application period is completed. The significance of the no delay or no stress scenario is that irrigations are initiated during all growth

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[3] The 45 percent value is termed the critical extractable soil moisture ratio based on conversations with Dr. Joe T. Ritchie, Soil Scientist, Blackland Soil and Water Research Laboratory, USDA, SEA/AR, Temple, Texas. At the 45 percent level, the leaves of the grain sorghum plant begin to curl, a phenomenon that may be observed by the grain sorghum producer.

stages; that is, an irrigation is initiated regardless of the particular growth stage of the grain sorghum plant. In later scenarios, the significance of delaying or withholding irrigation applications when the grain sorghum plant is in a particular growth stage is evaluated.

Yields, water applied, costs and returns for the no stress scenario are presented in Table 5. The largest return is $140.54 per acre while the smallest return is $53.82 per acre with a mean return of $93.98 per acre for the 23 replications. Yield per acre for the no stress scenario varies from a high of 72.30 cwt/acre to a low of 50.07 cwt/acre with an average yield of 52.84 cwt/acre. Water pumped for this scenario ranges from a high of 22.5 inches to a low of 4.5 inches with an average quantity of groundwater pumped being 14.09 inches. In comparing the no stress scenario with contemporary irrigation practices, a 24-inch application results in an average yield of 59.20 cwt/acre while the no stress scenario yields 52.84 cwt/acre. Under the no stress scenario, an average of only 14 acre-inches per acre of groundwater is pumped, about a 10 inch saving in water application, with yields approximately equal to those obtained under contemporary practices. Returns to the producer from better water management using the no stress scenario average about $15 per acre greater than returns under contemporary practices.

## Irrigation Scheduling in Accordance With Growth Stages

Producers who are monitoring soil water are also able to observe the stage of grain sorghum development and could irrigate in accordance with growth stages. This scenario evaluates the premise

Table 5. Yields, Water Use, Revenues, Irrigation Costs and Net Returns for the No Stress Irrigation Scenario

| Replication | Yield | Water Applied | Revenue | Irrigation Costs | Returns |
|---|---|---|---|---|---|
| | (CWT/AC.) | (INCHES) | ($/AC.) | ($/AC.) | ($/AC.) |
| 1 | 60.97 | 22.50 | 242.66 | 35.78 | 88.30 |
| 2 | 56.68 | 18.00 | 225.59 | 20.62 | 78.38 |
| 3 | 63.11 | 13.50 | 251.18 | 21.47 | 111.12 |
| 4 | 62.20 | 22.50 | 247.56 | 35.78 | 93.19 |
| 5 | 62.05 | 13.50 | 246.96 | 21.47 | 106.90 |
| 6 | 65.80 | 9.00 | 261.88 | 14.31 | 128.98 |
| 7 | 67.04 | 18.00 | 266.82 | 28.62 | 119.61 |
| 8 | 65.85 | 18.00 | 262.08 | 28.62 | 114.87 |
| 9 | 55.73 | 9.00 | 221.81 | 14.31 | 88.91 |
| 10 | 56.28 | 9.00 | 223.99 | 14.31 | 91.09 |
| 11 | 56.66 | 18.00 | 225.51 | 28.62 | 78.30 |
| 12 | 52.93 | 9.00 | 210.66 | 14.31 | 77.76 |
| 13 | 55.87 | 13.50 | 222.36 | 21.47 | 82.31 |
| 14 | 52.28 | 13.50 | 208.07 | 21.47 | 68.02 |
| 15 | 51.53 | 9.00 | 205.09 | 14.31 | 72.19 |
| 16 | 53.30 | 9.00 | 212.13 | 14.31 | 79.23 |
| 17 | 50.51 | 18.00 | 201.03 | 28.62 | 53.82 |
| 18 | 64.95 | 22.50 | 258.50 | 35.78 | 104.14 |
| 19 | 72.30 | 18.00 | 287.75 | 28.62 | 140.54 |
| 20 | 50.07 | 4.50 | 199.28 | 7.16 | 73.53 |
| 21 | 63.28 | 13.50 | 251.85 | 21.47 | 111.80 |
| 22 | 52.47 | 9.00 | 208.83 | 14.31 | 75.93 |
| 23 | 66.00 | 13.50 | 262.68 | 21.47 | 122.62 |

that a plant can be stressed in a particular stage of growth as long as the value of the water saved is greater than the value of the yield reduction which occurs. Thus, it may be economic to stress the plant during early stages of development as long as intensive irrigations occur during later critical stages of development. Greater returns to the producer would be expected from irrigating by growth stages than result from the 24-inch scenario if per acre yields do not decline significantly. Under this scenario, the plant receives a 3-inch application at preplant if the soil moisture ratio is 45 percent or less. Irrigations for plant growth stages 1, 2, 3 and 4 are initiated at the 45 percent soil moisture ratio unless a particular plant growth stage or stages are to be stressed. For example, by eliminating an irrigation in Stage 1 while initiating irrigations at other stages of growth, the effects on yields and returns from plant growth stress during stage 1 can be estimated. Similar runs are made in which irrigations are restricted in each stage and combinations of stages. One purpose of this scenario is to determine the most critical plant growth stage in terms of irrigation water requirements. Comparisons of net returns, yields and water use are summarized in Tables 6, 7 and 8, respectively.

From Table 7, the mean yield of withholding water during stage 1 of grain sorghum plant growth is quite similar to the no stress scenario which suggests that water stress during stage 1 of plant growth has little impact on final yield. In comparing mean yields of different scenarios, the no stress and no irrigation in stage 1 scenarios are quite similar, and both scenarios generate higher mean net returns (Table 6) than the contemporary practice of applying two

Table 6. Comparison of Net Returns by Different Growth Stage Irrigation Scenarios

| Irrigated Scenario | Mean | Variance | Coef. of Var. | Largest | Smallest |
|---|---|---|---|---|---|
| Contemporary | 78.86 | 613.77 | 0.31 | 131.08 | 42.69 |
| No Delay | 93.98 | 472.50 | 0.23 | 140.54 | 53.82 |
| No Irr. Stage 1 | 94.23 | 471.11 | 0.23 | 140.31 | 53.44 |
| No Irr. Stage 2 | 92.76 | 484.24 | 0.24 | 140.53 | 44.79 |
| No Irr. Stage 3 | 53.33 | 444.01 | 0.23 | 140.53 | 52.67 |
| No Irr. Stage 4 | 91.29 | 368.54 | 0.21 | 128.98 | 53.84 |
| No Irr. Stage 1&2 | 92.45 | 524.43 | 0.25 | 146.43 | 44.79 |
| No Irr. Stage 1&3 | 93.54 | 454.39 | 0.23 | 140.31 | 52.67 |
| No Irr. Stage 1&4 | 91.39 | 371.23 | 0.21 | 128.98 | 53.84 |
| No Irr. Stage 2&3 | 88.12 | 502.88 | 0.25 | 140.53 | 42.67 |
| No Irr. Stage 2&4 | 75.74 | 979.26 | 0.41 | 128.98 | 0.68 |
| No Irr. Stage 3&4 | 52.32 | 1629.64 | 0.77 | 105.25 | -22.08 |
| No Irr. Stage 1,2,&3 | 84.86 | 478.92 | 0.26 | 135.21 | 42.67 |
| No Irr. Stage 2,3,&4 | 25.60 | 3574.55 | 2.34 | 105.11 | -84.19 |
| No Irr. Stage 1,2,3,&4 | 11.43 | 4131.72 | 5.62 | 105.11 | -84.19 |

Table 7. Comparison of Yields by Different Growth Stage Irrigation Scenarios

| Irrigation Scenario | Mean | Variance | Coef. of Var. | Largest | Smallest |
|---|---|---|---|---|---|
| Contemporary | 59.20 | 38.75 | 0.11 | 72.32 | 50.11 |
| No Delay | 59.04 | 36.31 | 0.10 | 72.30 | 50.07 |
| No Irr. Stage 1 | 59.28 | 38.84 | 0.10 | 72.24 | 50.07 |
| No Irr. Stage 2 | 58.57 | 38.41 | 0.11 | 72.30 | 48.24 |
| No Irr. Stage 3 | 58.64 | 35.98 | 0.10 | 72.30 | 49.96 |
| No Irr. Stage 4 | 57.19 | 23.79 | 0.09 | 65.80 | 50.07 |
| No Irr. Stage 1&2 | 58.18 | 38.08 | 0.11 | 71.98 | 48.24 |
| No Irr. Stage 1&3 | 58.62 | 36.52 | 0.10 | 72.24 | 49.96 |
| No Irr. Stage 2&3 | 56.78 | 37.12 | 0.11 | 72.30 | 45.91 |
| No Irr. Stage 2&4 | 52.74 | 61.25 | 0.15 | 65.80 | 35.36 |
| No Irr. Stage 3&4 | 46.23 | 96.84 | 0.21 | 59.84 | 28.09 |
| No Irr. Stage 1,2,&3 | 55.57 | 32.13 | 0.10 | 69.16 | 45.91 |
| No Irr. Stage 2,3,&4 | 38.57 | 218.21 | 0.38 | 58.03 | 11.03 |
| No Irr. Satge 1,2,3,&4 | 34.55 | 257.20 | 0.46 | 58.00 | 11.03 |

Table 8. Comparison of Water Use by Different Growth Stage Irrigation Scenearios

| Irrigation Scenario | Mean | Variance | Coef. of Var. | Largest | Smallest |
|---|---|---|---|---|---|
| Contemporary | 24.00 | 0.07 | 0.00 | 24.00 | 24.00 |
| No Delay | 14.09 | 25.19 | 0.36 | 22.50 | 4.50 |
| No Irr. Stage 1 | 13.89 | 22.74 | 0.34 | 22.50 | 4.50 |
| No Irr. Stage 2 | 13.70 | 23.73 | 0.36 | 22.50 | 4.50 |
| No Irr. Stage 3 | 13.50 | 28.17 | 0.39 | 22.50 | 4.50 |
| No Irr. Stage 4 | 11.15 | 15.62 | 0.35 | 18.00 | 4.50 |
| No Irr. Stage 1&2 | 12.91 | 21.67 | 0.36 | 22.50 | 4.50 |
| No Irr. Stage 1&3 | 13.30 | 25.49 | 0.38 | 22.50 | 4.50 |
| No Irr. Stage 1&4 | 10.96 | 13.78 | 0.34 | 18.00 | 4.50 |
| No Irr. Stage 2&3 | 12.13 | 21.90 | 0.39 | 22.50 | 4.50 |
| No Irr. Stage 2&4 | 9.78 | 9.95 | 0.32 | 18.00 | 4.50 |
| No Irr. Stage 3&4 | 8.22 | 13.47 | 0.45 | 13.50 | 0.00 |
| No Irr. Stage 1,2,&3 | 11.15 | 12.10 | 0.31 | 18.00 | 4.50 |
| No Irr. Stage 2,3,&4 | 5.87 | 9.57 | 0.53 | 13.50 | 0.00 |
| No Irr. Stage 1,2,3,&4 | 4.70 | 6.12 | 0.53 | 9.00 | 0.00 |

acre-feet per acre. Only three of the proposed scenarios have mean returns less than returns from contemporary practices. In each of these three irrigation schedules, water was withheld in both stage 3 and 4 of plant growth which are crucial for the development of the economic yield.

In evaluating the combined growth stage irrigation schedules, results suggest that withholding irrigation in stage 1 and 2 is an interesting option. The mean level of net return for this scenario is $92.45 per acre with a high of $146.43 and a low of $44.79 per acre. The mean net return for the no delay scenario is $93.98 per acre. Both schedules exceed the mean net return of $78.86 per acre for the contemporary irrigation practice. The mean level of net returns for other combined growth stage irrigation scenarios (such as stage 2 and 3 or stage 3 and 4) are lower than for the no stress schedule because growth stress is occurring in the stages when economic yield is developing.

For the combined irrigation scenario of omitting applications in growth stages 1 and 2, the average yield is 58.18 cwt/acre as compared to 59.04 cwt/acre under the no stress scenario and 59.20 cwt/acre under the contemporary schedule. Substantial differences exist in water use. The irrigation scenario of not applying water during stage 1 and 2 requires an average of 12.91 inches per acre as compared to 14.09 inches per acre for the no stress scenario and 24 inches per acre for the contemporary scenario.

In comparing the scenarios, returns average around $90 per acre for seven of the proposed irrigation schedules. Scenarios which withhold irrigation water during the growth stage 4 have lower returns and yields. In three irrigation scenarios where stage 4 irrigation water is withheld, negative returns to the producer for some of the replications are realized.

### Irrigation Scheduling By Critical Soil Moisture Ratios and Days Until Stress

The surface irrigation system utilized in this analysis has the capacity to irrigate 155 acres of cropland with a 3-inch application in 15 days. If irrigations are initiated at lower and lower levels of extractable soil water, more and more days of plant stress occur. For this set of irrigation scenarios, various critical extractable soil moisture ratios are used. The critical extractable soil moisture ratio is the result of the current level of extractable soil moisture divided by the upper level of extractable soil moisture. The critical extractable soil moisture ratios used in this analysis are 45 percent, 30 percent, 20 percent, and 0 percent. An irrigation application is initiated if the number of days required for the extractable soil moisture ratio to decline to the specified level is 15 days or less.

An alternative strategy is investigated to permit the producer to delay an irrigation if the plant is in a particular growth stage. Also, for this scenario a producer may skip up to two successive growth stages. When the plant is not in a designated growth stage, irrigations are initiated when 15 days or less are required to reach the critical extractable soil moisture ratio. A final irrigation scenario investigated involves the use of different critical extractable soil moisture ratios at different stages of plant growth.

In all of these scenarios, all preplant irrigations are based on the 45 percent extractable soil moisture ratio.

### Forty-five Percent Ratio

Tables 9 through 11 show the results of the 45 percent critical soil moisture ratio scenario. Under this scenario the largest mean return occurs for the irrigation schedule which eliminates irrigations during growth stages 1 and 2. Returns under the no irrigation in stage 1 and 2 schedule average $88.58 per acre with mean yields of 59.22 cwt/acre. Mean yields for the 45 percent ratio scenario (Table 10) vary from a high of 59.22 cwt/acre to a low of 55.17 cwt/acre. Water pumped varies for the 45 percent ratio scenario (Table 11) from a maximum of 22.11 inches to a minimum of 14.28 inches per acre. The irrigation schedule of withholding water during stage 1 and 2 of plant growth seems favorable when compared to contemporary irrigation procedures of applying 24 acre inches. Average returns under the strategy withholding irrigation in stage 1 and 2 (Table 9) are approximately $10.00 per acre greater than contemporary practices with a water savings of water of approximately 7 inches per acre.

### Thirty Percent Ratio

Tables 12 through 14 show the result of the 30 percent critical soil moisture ratio scenario. Mean net returns for the different scenarios average around $90 per acre with no irrigation in stage 4 having the highest mean net returns. However, the no irrigation in stage 4 schedule also has the largest variance in net returns when compared to the other schedules. Irrigations are scheduled 15 days in

Table 9. Comparison of Net Returns by Different Growth Stage Irrigation Scenarios for Critical Extractable Soil Moisture Ratio of Forty-Five Percent

| Irrigation Scenario | Mean | Variance | Coef. of Var. | Largest | Smallest |
|---|---|---|---|---|---|
| No Delay | 81.96 | 466.43 | 0.25 | 134.45 | 40.21 |
| No Irr. Stage 1 | 84.41 | 474.33 | 0.26 | 134.40 | 47.23 |
| No Irr. Stage 2 | 89.60 | 456.09 | 0.25 | 134.43 | 46.55 |
| No Irr. Stage 3 | 83.24 | 467.47 | 0.26 | 134.41 | 47.29 |
| No Irr. Stage 4 | 88.42 | 532.93 | 0.26 | 139.97 | 47.37 |
| No Irr. Stage 1&2 | 86.58 | 454.21 | 0.24 | 134.40 | 44.79 |
| No Irr. Stage 2&3 | 87.16 | 440.76 | 0.24 | 134.41 | 49.21 |
| No Irr. Stage 3&4 | 78.26 | 746.19 | 0.39 | 129.22 | 19.13 |

Table 10. Comparison of Yields by Different Growth Stage Irrigation Scenarios for Critical Soil Moisture Ratio of Forty-Five Percent

| Irrigation Scenario | Mean | Variance | Coef. of Var. | Largest | Smallest |
|---|---|---|---|---|---|
| No Delay | 59.22 | 39.17 | 0.11 | 72.59 | 50.15 |
| No Irr. Stage 1 | 59.21 | 39.14 | 0.11 | 72.55 | 50.10 |
| No Irr. Stage 2 | 59.15 | 39.20 | 0.11 | 72.55 | 50.10 |
| No Irr. Stage 3 | 59.15 | 38.64 | 0.11 | 72.56 | 50.10 |
| No Irr. Stage 4 | 58.89 | 36.18 | 0.10 | 72.15 | 50.10 |
| No Irr. Stage 1&2 | 58.70 | 38.61 | 0.11 | 72.55 | 48.24 |
| No Irr. Stage 2&3 | 58.65 | 36.46 | 0.10 | 72.56 | 49.35 |
| No Irr. Stage 3&4 | 55.17 | 51.08 | 0.10 | 67.66 | 37.21 |

Table 11. Comparison of Water Use by Different Growth Stage Irrigation Scenarios for Critical Extractable Soil Moisture Ratio of Forty-Five Percent

| Irrigation Scenario | Mean | Variance | Coef. of Var. | Largest | Smallest |
|---|---|---|---|---|---|
| No Delay | 22.11 | 26.26 | 0.23 | 31.50 | 9.00 |
| No Irr. Stage 1 | 20.54 | 22.59 | 0.23 | 27.00 | 9.00 |
| No Irr. Stage 2 | 19.76 | 33.00 | 0.29 | 31.50 | 4.50 |
| No Irr. Stage 3 | 21.13 | 23.64 | 0.23 | 31.50 | 9.00 |
| No Irr. Stage 4 | 17.22 | 11.71 | 0.20 | 22.50 | 9.00 |
| No Irr. Stage 1&2 | 16.63 | 21.90 | 0.28 | 22.50 | 4.50 |
| No Irr. Stage 2&3 | 17.41 | 21.67 | 0.27 | 27.00 | 4.50 |
| No Irr. Stage 3&4 | 14.28 | 11.71 | 0.24 | 22.50 | 9.00 |

Table 12. Comparison of Net Returns by Different Growth Stage Irrigation Scenarios for Critical Extractable Soil Moisture Ratio of Thirty Percent

| Irrigation Scenario | Mean | Variance | Coef. of Var. | Largest | Smallest |
|---|---|---|---|---|---|
| No Delay | 87.53 | 478.99 | 0.25 | 134.35 | 47.30 |
| No Irr. Stage 1 | 87.81 | 466.56 | 0.25 | 134.39 | 47.23 |
| No Irr. Stage 2 | 89.21 | 466.61 | 0.24 | 141.48 | 46.89 |
| No Irr. Stage 3 | 88.96 | 456.50 | 0.24 | 134.35 | 47.32 |
| No Irr. Stage 4 | 91.56 | 497.93 | 0.24 | 124.22 | 47.30 |
| No Irr. Stage 1&2 | 89.79 | 465.70 | 0.24 | 141.48 | 44.79 |
| No Irr. Stage 2&3 | 89.40 | 427.31 | 0.23 | 141.21 | 50.31 |
| No Irr. Stage 3&4 | 89.06 | 411.95 | 0.24 | 129.22 | 51.75 |

Table 13. Comparison of Yields by Different Growth Stage Irrigation Scenarios for Critical Extractable Soil Moisture Ratio of Thirty Percent

| Irrigation Scenario | Mean | Variance | Coef. of Var. | Largest | Smallest |
|---|---|---|---|---|---|
| No Delay | 39.21 | 39.12 | 0.11 | 72.54 | 50.10 |
| No Irr. Stage 1 | 39.21 | 39.10 | 0.11 | 72.54 | 50.10 |
| No Irr. Stage 2 | 59.17 | 39.00 | 0.11 | 72.53 | 50.10 |
| No Irr. Stage 3 | 59.18 | 39.01 | 0.11 | 72.54 | 50.05 |
| No Irr. Stage 4 | 58.90 | 33.73 | 0.10 | 67.66 | 50.10 |
| No Irr. Stage 1&2 | 56.69 | 38.60 | 0.11 | 72.53 | 48.24 |
| No Irr. Stage 2&3 | 58.43 | 35.97 | 0.10 | 72.47 | 49.63 |
| No Irr. Stage 3&4 | 56.09 | 28.54 | 0.10 | 67.46 | 47.20 |

Table 14. Comparison of Water Use by Different Growth Stage Irrigation Scenarios for Critical Extractable Soil Moisture Ratio of Thirty Percent

| Irrigation Scenario | Mean | Variance | Coef. of Var. | Largest | Smallest |
|---|---|---|---|---|---|
| No Delay | 10.59 | 21.67 | 0.25 | 27.00 | 9.00 |
| No Irr. Stage 1 | 18.39 | 22.74 | 0.26 | 27.00 | 9.00 |
| No Irr. Stage 2 | 17.41 | 25.19 | 0.29 | 27.00 | 4.50 |
| No Irr. Stage 3 | 17.61 | 25.19 | 0.29 | 27.00 | 9.00 |
| No Irr. Stage 4 | 15.26 | 17.15 | 0.27 | 22.50 | 9.00 |
| No Irr. Stage 1&2 | 15.85 | 19.14 | 0.28 | 22.50 | 4.50 |
| No Irr. Stage 2&3 | 15.46 | 20.82 | 0.30 | 22.50 | 4.50 |
| No Irr. Stage 3&4 | 12.33 | 14.47 | 0.31 | 18.00 | 4.50 |

advance of the attainment of the critical soil moisture level. Therefore, an irrigation commencing in the latter days of stage 3 is completed in stage 4 of plant growth. Mean returns for the no irrigation in stage 4 schedule are higher than the other single stage irrigation elimination schedules, but the variance of returns is higher, mean yields are lower, and corresponding water use is lower.

Mean yields (Table 13) for the 30 percent ratio scenario range from 59.21 cwt/acre to 56.09 cwt/acre with the average quantity of water pumped ranging from 18.59 inches to 12.33 inches (Table 14). For the no irrigation in stage 1 and 2 schedule, the average quantity of water pumped is 15.85 inches and net returns average $89.79 per acre. Net returns are higher and water use lower for the no irrigation in stage 1 and 2 schedule than for the contemporary irrigation practice.

### Twenty Percent Ratio

Tables 15 and 17 show the results of the 20 percent critical soil moisture ratio scenario. Mean returns for six of the proposed scenarios average around $92.00 per acre. There is greater variation in the net returns for each irrigation schedule because stress on the plant is greater at the lower ratio level. Mean yields for the twenty percent ratio scenario (Table 16) range from 59.16 cwt/acre to 54.30 cwt/acre, and the average quantity of water pumped for this scenario (Table 17) ranges from 15.26 inches to 10.57 inches. For the schedule eliminating irrigations in stage 1 and 2, returns average $91.22 per acre. The average level of returns is lower than for the 30 percent scenario indicating growth stress is more pronounced when the critical

Table 15. Comparison of Net Returns by Different Growth Stage Irrigation Scenarios for Critical Extractable Soil Moisture Ratio of Twenty Percent

| Irrigation Scenario | Mean | Variance | Coef. of Var. | Largest | Smallest |
|---|---|---|---|---|---|
| No Delay | 92.61 | 508.06 | 0.26 | 141.18 | 47.30 |
| No Irr. Stage 1 | 92.99 | 508.64 | 0.24 | 141.18 | 47.23 |
| No Irr. Stage 2 | 92.03 | 496.72 | 0.24 | 141.00 | 47.23 |
| No Irr. Stage 3 | 92.31 | 500.08 | 0.24 | 141.10 | 47.30 |
| No Irr. Stage 4 | 92.94 | 393.41 | 0.21 | 129.22 | 53.96 |
| No Irr. Stage 1&2 | 91.22 | 495.07 | 0.24 | 141.00 | 46.79 |
| No Irr. Stage 2&3 | 88.78 | 421.56 | 0.23 | 141.08 | 52.67 |
| No Irr. Stage 3&4 | 80.71 | 572.68 | 0.30 | 129.22 | 5.50 |

Table 16. Comparison of Yields by Different Growth Stage Irrigation Scenarios for Critical Extractable Soil Moisture Ratio of Twenty Percent

| Irrigation Scenario | Mean | Variance | Coef. of Var. | Largest | Smallest |
|---|---|---|---|---|---|
| No Delay | 59.16 | 39.02 | 0.11 | 72.40 | 68.10 |
| No Irr. Stage 1 | 59.16 | 39.04 | 0.11 | 72.46 | 50.10 |
| No Irr. Stage 2 | 59.02 | 37.93 | 0.10 | 72.43 | 50.12 |
| No Irr. Stage 3 | 59.09 | 38.46 | 0.11 | 72.46 | 49.95 |
| No Irr. Stage 4 | 58.31 | 28.19 | 0.09 | 67.66 | 50.19 |
| No Irr. Stage 1&2 | 58.64 | 38.59 | 0.11 | 72.43 | 68.24 |
| No Irr. Stage 2&3 | 57.65 | 31.59 | 0.10 | 72.49 | 49.96 |
| No Irr. Stage 3&4 | 54.30 | 37.68 | 0.11 | 67.66 | 34.79 |

Table 17. Comparison of Water Use by Different Growth Stage Irrigation Scenarios for Critical Extractable Soil Moisture Ratio of Twenty Percent

| Irrigation Scenario | Mean | Variance | Coef. of Var. | Largest | Smallest |
|---|---|---|---|---|---|
| No Delay | 15.24 | 22.43 | 0.31 | 22.32 | 4.50 |
| No Irr. Stage 1 | 15.26 | 22.43 | 0.31 | 22.50 | 4.50 |
| No Irr. Stage 2 | 15.26 | 22.43 | 0.31 | 22.50 | 4.50 |
| No Irr. Stage 3 | 15.24 | 22.43 | 0.31 | 22.50 | 4.50 |
| No Irr. Stage 4 | 12.91 | 18.14 | 0.33 | 22.50 | 4.50 |
| No Irr. Stage 1&2 | 14.87 | 21.90 | 0.31 | 22.50 | 4.50 |
| No Irr. Stage 2&3 | 13.89 | 19.22 | 0.32 | 22.50 | 4.50 |
| No Irr. Stage 3&4 | 10.57 | 16.92 | 0.39 | 18.00 | 0.00 |

soil moisture ratio is permitted to decline to 20 percent prior to initiation of an irrigation.

### Zero Percent Ratio

Tables 18 through 20 show the results of the 0.0 percent critical soil moisture ratio. The largest mean net return is for either the no delay or no irrigation in stage 1 schedule with a value of $93.48 per acre. Average net returns for the zero percent scenario (Table 18) range from $93.48 per acre to $55.04 per acre. The range of the net returns for the zero percent scenario indicates the effects of water stress because of the low critical ratio used to initiate irrigations. Mean yields for the zero percent ratio scenario (Table 19) range from 58.82 cwt/acre to 47.07 cwt/acre with the average quantity of water pumped for this scenario ranging from 13.11 inches to 8.61 inches (Table 20). Less water is pumped for these sets of irrigation schedules because of the low critical extractable soil moisture ratio. However, a consequence of the low critical extractable soil moisture ratio is lower yields which translates into lower returns for this scenario.

### Combination of Critical Ratios for Growth Stages

Under this scenario, different critical soil moisture ratios for different growth stages are investigated. Under this scenario, growth stages 1 and 2 have the same critical ratio and growth stages 3 and 4 have the same ratio. Critical ratios used in this scenario are 0 percent, 20 percent, 30 percent, and 45 percent. Tables 21 through 23 list the different critical extractable soil moisture ratios for this

Table 18. Comparison of Net Returns by Different Growth Stage Irrigation Scenarios for Critical Extractable Soil Moisture Ratio of Zero Percent

| Irrigation Scenario | Mean | Variance | Coef. of Var. | Largest | Smallest |
|---|---|---|---|---|---|
| No Delay | 93.48 | 419.71 | 0.22 | 142.82 | 53.76 |
| No Irr. Stage 1 | 93.48 | 419.71 | 0.22 | 142.82 | 53/74 |
| No Irr. Stage 2 | 91.88 | 459.33 | 0.23 | 142.82 | 44.79 |
| No Irr. Stage 3 | 93.47 | 420.93 | 0.22 | 142.82 | 52.64 |
| No Irr. Stage 4 | 86.11 | 235.08 | 0.18 | 117.45 | 53.75 |
| No Irr. Stage 1&2 | 91.88 | 459.33 | 0.23 | 142.82 | 44.79 |
| No Irr. Stage 2&3 | 87.97 | 461.99 | 0.24 | 142.82 | 42.67 |
| No Irr. Stage 3&4 | 55.04 | 1496.11 | 0.67 | 100.36 | -21.94 |

Table 19. Comparison of Yields by Different Growth Stage Irrigation Scenarios for Critical Extractable Soil Moisture Ratio of Zero Percent

| Irrigation Scenario | Mean | Variance | Coef. of Var. | Largest | Smallest |
|---|---|---|---|---|---|
| No Delay | 58.52 | 34.14 | 0.10 | 71.07 | 50.04 |
| No Irr. Stage 1 | 58.52 | 34.14 | 0.10 | 71.07 | 50.05 |
| No Irr. Stage 2 | 58.04 | 35.25 | 0.10 | 71.07 | 48.24 |
| No Irr. Stage 3 | 58.36 | 33.73 | 0.10 | 71.07 | 49.99 |
| No.Irr. Stage 4 | 55.65 | 18.30 | 0.05 | 64.70 | 48.74 |
| No Irr. Stage 1&2 | 58.04 | 35.25 | 0.10 | 71.07 | 48.24 |
| No Irr. Stage 2&3 | 56.43 | 33.23 | 0.10 | 71.07 | 45.90 |
| No Irr. Stage 3&4 | 47.07 | 86.97 | 0.20 | 58.61 | 28.09 |

Table 20. Comparison of Water Used by Different Growth Stage Irrigation Scenarios for Critical Extractable Soil Moisture Ratio of Zero Percent

| Irrigation Scenario | Mean | Variance | Coef. of Var. | Largest | Smallest |
|---|---|---|---|---|---|
| No Delay | 13.11 | 24.50 | 0.38 | 22.50 | 4.50 |
| No Irr. Stage 1 | 13.11 | 24.50 | 0.38 | 22.50 | 4.50 |
| No Irr. Stage 2 | 12.91 | 26.95 | 0.40 | 22.50 | 4.50 |
| No Irr. Stage 3 | 12.72 | 24.04 | 0.39 | 22.50 | 4.50 |
| No Irr. Stage 4 | 10.57 | 16.02 | 0.39 | 18.00 | 4.50 |
| No Irr. Stage 1&2 | 12.91 | 26.05 | 0.40 | 22.50 | 4.50 |
| No Irr. Stage 2&3 | 11.35 | 17.38 | 0.37 | 18.00 | 4.50 |
| No Irr. Stage 3&4 | 8.61 | 12.17 | 0.41 | 13.50 | 0.00 |

analysis and the results derived. Also a preplant irrigation for all these schedules is initiated when soil moisture falls below the 45 percent critical extractable soil moisture ratio.

Mean returns for these schedules (Table 21) range from $93.02 for a critical ratio of 20 percent in stages 1 and 2 and a critical ratio of 0 percent in stages 3 and 4 to $85.35 per acre for a critical ratio of 30 percent in stages 1 and 2 and a critical ratio of 45 percent in stages 3 and 4. Mean yields for this scenario (Table 22) range from 59.16 cwt/acre for three of the twelve schedules investigated to a low of 58.86 cwt/acre for the schedule with a critical ratio of 20 percent in stages 1 and 2 and a critical ratio of 0 percent in stages 3 and 4. Average quantity of water pumped (Table 23) ranges from 19.96 inches for the schedule with a critical ratio of 30 percent in stages 1 and 2 and a critical ratio of 45 percent in stages 3 and 4 to 13.50 inches for the schedule with a critical ratio of 20 percent in stages 1 and 2 and a critical ratio of 0 percent in stages 3 and 4. Because the simulation model generates a distribution of returns for each scenario, it is possible to determine expected net returns and measure the variability of those returns. Some irrigation schedules permit the producer to achieve a higher expected return, but may also increase the variability of net returns. Other strategies would produce lower and more stable net returns. Which of these strategies a producer would prefer depends to some extent upon preferences for or aversion to income variability or tradeoffs the producer is willing to make between expected income and variability of income.

In the next section of this analysis, stochastic efficiency concepts are explained and stochastic dominance procedures are used to

Table 21. Comparison of Net Returns for Different Combinations of Critical Extractable Soil Moisture Ratios and Growth Stages

| Irrigation Scenario | Mean | Variance | Coef. of Var. | Largest | Smallest |
|---|---|---|---|---|---|
| S12-0%&S34-20% | 62.75 | 482.33 | 0.24 | 141.08 | 53.76 |
| S12-0%&S34-30% | 60.41 | 470.79 | 0.24 | 141.48 | 53.76 |
| S12-0%&S34-45% | 88.89 | 485.02 | 0.25 | 134.40 | 46.62 |
| S12-20%&S34-0% | 93.02 | 405.20 | 0.22 | 138.80 | 53.96 |
| S12-20%&S34-30% | 89.36 | 448.36 | 0.24 | 134.36 | 47.38 |
| S12-20%&S34-45% | 87.86 | 461.44 | 0.24 | 134.39 | 47.38 |
| S12-30%&S34-0% | 61.97 | 493.56 | 0.24 | 138.81 | 45.39 |
| S12-30%&S34-20% | 89.48 | 518.60 | 0.25 | 141.19 | 47.38 |
| S12-30%&S34-45% | 85.38 | 487.34 | 0.26 | 134.40 | 47.38 |
| S12-45%&S34-0% | 88.38 | 481.73 | 0.25 | 138.81 | 44.78 |
| S12-45%&S34-20% | 87.70 | 502.11 | 0.26 | 141.19 | 47.37 |
| S12-45%&S34-30% | 85.96 | 490.08 | 0.26 | 134.35 | 47.37 |

Table 22. Comparison of Yields for Different Combinations of Critical Extractable Soil Moisture Ratios and Growth Stages

| Irrigation Scenario | Mean | Variance | Coef. of Var. | Largest | Smallest |
|---|---|---|---|---|---|
| S12-0%&S34-20% | 59.12 | 38.91 | 0.11 | 72.43 | 50.10 |
| S12-0%&S34-30% | 59.16 | 38.95 | 0.11 | 72.53 | 50.10 |
| S12-0%&S34-45% | 59.16 | 38.96 | 0.11 | 72.55 | 50.10 |
| S12-20%&S34-0% | 58.56 | 34.62 | 0.10 | 71.86 | 50.05 |
| S12-20%&S34-30% | 59.20 | 39.12 | 0.11 | 72.54 | 50.10 |
| S12-20%&S34-45% | 59.22 | 39.15 | 0.11 | 72.55 | 50.10 |
| S12-30%&S34-0% | 58.69 | 36.75 | 0.10 | 71.87 | 50.05 |
| S12-30%&S34-20% | 59.16 | 38.96 | 0.11 | 72.46 | 50.10 |
| S12-30%&S34-45% | 59.22 | 39.17 | 0.11 | 72.55 | 50.11 |
| S12-45%&S34-0% | 58.57 | 35.92 | 0.10 | 71.87 | 50.04 |
| S12-45%&S34-20% | 59.18 | 39.02 | 0.11 | 72.46 | 50.10 |
| S12-45%&S34-30% | 59.21 | 35.13 | 0.11 | 72.54 | 50.10 |

Table 23. Comparison of Water Use for Different Combinations of Critical Extractable Soil Moisture Ratios and Growth Stages

| Irrigation Scenario | Mean | Variance | Coef. of Var. | Largest | Smallest |
|---|---|---|---|---|---|
| S12-0%&S34-20% | 15.07 | 20.44 | 0.30 | 22.50 | 4.50 |
| S12-0%&S34-30% | 16.63 | 20.14 | 0.27 | 27.00 | 4.50 |
| S12-0%&S34-45% | 17.61 | 22.74 | 0.27 | 27.00 | 4.50 |
| S12-20%&S34-0% | 13.50 | 24.65 | 0.37 | 22.50 | 4.50 |
| S12-20%&S34-30% | 17.41 | 26.95 | 0.30 | 27.00 | 4.50 |
| S12-20%&S34-45% | 18.39 | 26.26 | 0.25 | 27.00 | 4.50 |
| S12-30%&S34-0% | 14.48 | 24.58 | 0.34 | 22.50 | 4.50 |
| S12-30%&S34-20% | 17.22 | 20.52 | 0.26 | 27.00 | 4.50 |
| S12-30%&S34-45% | 19.96 | 20.82 | 0.23 | 27.00 | 9.00 |
| S12-45%&S34-0% | 16.43 | 20.44 | 0.28 | 27.00 | 9.00 |
| S12-45%&S34-20% | 18.39 | 20.98 | 0.23 | 27.00 | 9.00 |
| S12-45%&S34-30% | 19.57 | 20.44 | 0.23 | 27.00 | 9.00 |

identify risk efficient irrigation techniques. One of the underlying assumptions is that producers are risk averse. That is, they are willing to accept more income variability only if expected income is also expected to increase.

## STOCHASTIC EFFICIENCY ANALYSIS

The use of stochastic efficiency to order preferences was first proposed by Quirk and Saposnik and has been extended by Hadar and Russell, and Whitmore. Stochastic efficiency analysis has been used in statistics (Blackwell and Grischick), in inventory control (Karlin), and has had extensive use in portfolio and capital budgeting theory (Levy and Hanoch). Anderson and Anderson, Dillon, and Hardaker discuss the use of stochastic dominance to evaluate new technology in agriculture.

Anderson states:

> "It is believed that, whenever research is addressed to the development of new varieties and practices, etc., that are intended for adoption by "risk-averse" farmers, the principles of stochastic efficiency are pertinent and indeed offer an important method of filtering out inefficient technological packages (i.e. packages that would not be preferred and adopted by those averse to risk) so that they are not extended to the farming community."

Three ordering rules of stochastic efficiency are first-degree stochastic dominance (FSD), second-degree stochastic dominance (SSD), and third-degree stochastic dominance (TSD). It is assumed that the farm operator has preferences which are a function of a single uncertain quantity, x, which is net returns. The utility function relating operator preferences to x is encoded as U(x) and the ith derivative with respect to x is $U_i(x)$. If the producer is evaluating two alternative technologies, e.g. two irrigation schedules, the net returns series associated with the schedules are assumed to be continuous random variables, x, over the range of the net returns $a \leq x \leq b$, with the frequency distribution associated with the irrigation schedules given as f(x) and g(x). The rules of stochastic dominance will determine whether the irrigation schedule generating net return frequency distribution f(x) or g(x) would be preferred by the decision maker.

## FIRST-DEGREE STOCHASTIC EFFICIENCY (FSE)

First order stochastic dominance (FSD) rests on the behavioral assumption of Bernoulli's principle. In general if action $a_1$ is preferred to $a_2$ then the utility associated with $a_1$ is greater than the utility associated with $a_2$, or $U(a_1) > U(a_2)$. First degree stochastic dominance (FSD) assumes that if x is an unscaled consequence, such as net returns to the agricultural producer, the decision-maker always prefers more to less of x. This assumption implies that the utility function U(x) is monotonically increasing in range from a to b wherein the first derivative of the function is strictly positive, i.e. $U_1(x) > 0$. If one wished to evaluate irrigation schedules F and G, irrigation schedule F would dominate irrigation schedule G by first degree stochastic dominance if $F_1(R) \leq G_1(R)$ for all R in the range from a to b with at least one strong inequality (i.e., the < holds for at least value of R).

Graphically, this rule means that a first-degree stochastically dominant cumulative distribution function must be nowhere to the left of a dominated curve. In Figure 5, cummulative distribution function $F_1$ dominates $G_1$, but does not dominate $G_1$ according to first degree stochastic dominance. Distributions that are dominant are said to be stochastically efficient of the first degree (FSE). Selecting the best or single most preferred distribution from the efficient set requires knowing more about the decision-makers preferences than is assumed for FSD.

Anderson, Dillon and Hardaker indicate that, as an empirical matter, relatively few strategies can be eliminated by the FSD rule. Thus, it is important to have more restrictive concepts of efficiency so that a larger number of alternatives can be eliminated leaving a smaller efficient set.

## SECOND-DEGREE STOCHASTIC EFFICIENCY (SSE)

Second order stochastic dominance (SSD) provides rules to further define an efficient set. Second order stochastic dominance requires the added assumption is that successive amounts of x, say net returns, have diminishing value to the decision-maker or that the decision-maker is averse to risk. For SSD, the second derivative of the utility function must be negative, i.e. $U_1(x) > 0$ and $U_2(x) < 0$. This additional condition implies that the utility function over the range of [a,b] is monotonically increasing and concave downward.

The ordering rule can again be stated in terms of cumulative distribution functions. The distribution function $F_1$ in Figure 6 is said to dominate $G_1$ if it lies more to the right in terms of

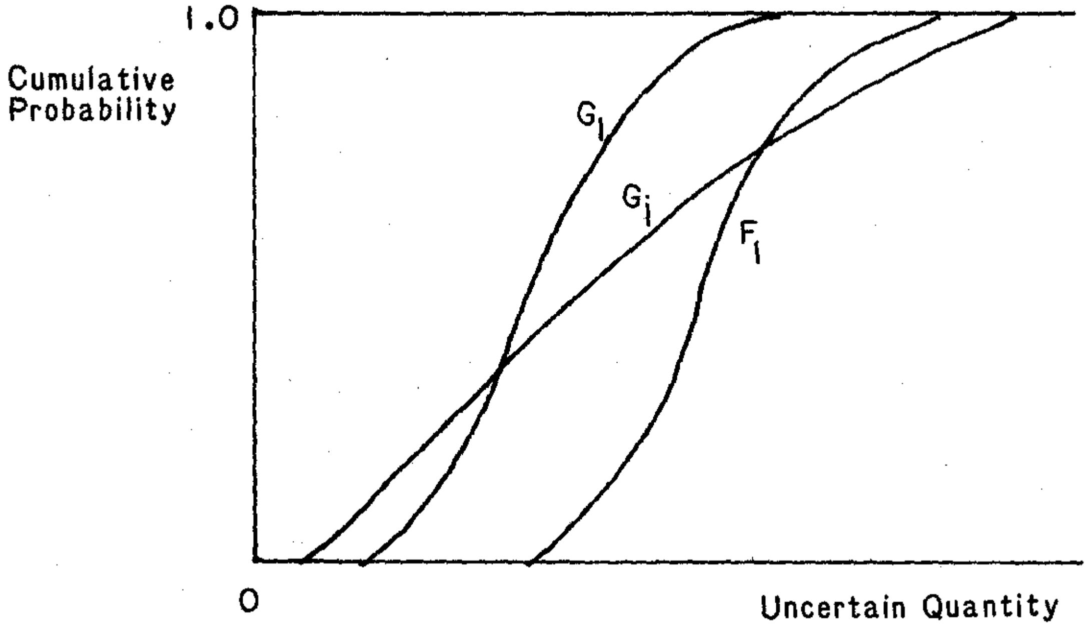

Figure 5. Illustration of First-Degree Stochastic Dominance

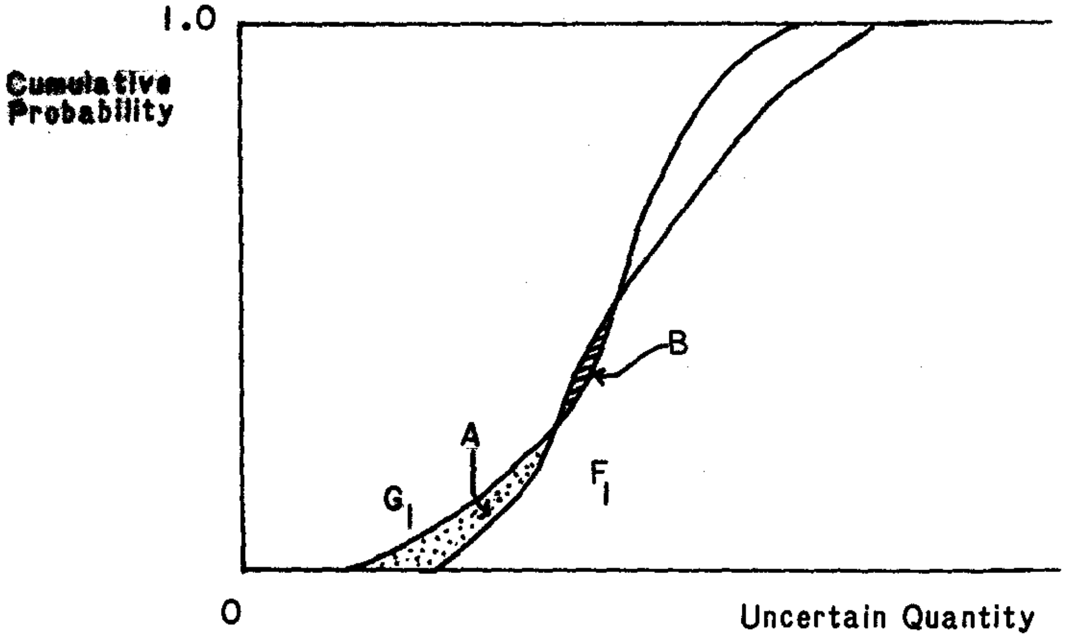

Figure 6. Illustration of Second-Degree Stochastic Dominance

difference in areas between $F_1$ and $G_1$. Since area A exceeds area B, CDF $F_1$ dominates $G_1$ by second-degree stochastic dominance. The dominated distribution would never be preferred by risk averse, utility-maximizing decision makers. Second degree stochastic efficiency is thought to be of considerable practical importance in defining efficient sets of technologies or strategies. Further narrowing of the efficient set requires more restrictive assumptions regarding decision maker preferences.

## THIRD-DEGREE STOCHASTIC EFFICIENCY (TSE)

Third degree stochastic dominance (TSD) requires the additional assumption that the third derivative of the utility function is strictly positive, i.e. $U_3(x) > 0$ with $U_1(x) > 0$ and $U_2(x) < 0$. This assumption implies that as people becomes wealthier they become decreasingly averse to risk. Also, TSD implies that the decision-maker prefers positive skewness in the distribution of returns to negative skewness. The ordering rule for TSD is that the distribution of net returns from irrigation schedule F dominates the distribution of net returns from irrigation schedule G by TSD if and only if $F_3(R) \leq G_3(R)$ for all R in the range of $[a,b]$ with at least one strict inequality, and $F_2(b) \leq G_2(b)$ when b is the upper range of returns, which is equivalent to $E_F[x] \geq E_G[x]$.

The third-degree stochastically efficient set cannot be any larger than the second-degree or first-degree sets, and Anderson, Dillon and Hardaker suggest that the SSE and TSE sets may be very similar. Thus, the TSE rule, which requires a more restrictive behavioral assumption regarding decision-maker preferences, may add little power to stochastic efficiency analysis. A stochastically dominant irrigation schedule is one that is more profitable on the average and also less prone to low outcomes under unfavorable conditions.

In this study, the filtering characteristic of stochastic dominance is used to ascertain which of the proposed irrigation schedules (and the associated net returns distributions) dominate the contemporary practice of applying 24 acre-inches of irrigation water. Returns to the contemporary irrigation schedule and the alternative schedules are derived through the firm level simulation model which utilizes the grain sorghum plant growth model.

For the irrigation scenario comparisons, continuous functions of returns from different irrigation schedules are derived. For continuous functions, a theoretical continuous distribution is assumed. The distribution most often assumed is the normal because it is symmetric and is easily defined by specifying the mean and variance. However, the normal distribution may assume values in the range from positive to negative infinity. Net returns from different irrigation schedules lie in some bounded range and are often skewed. Thus, a Beta distribution is used for this analysis as the distribution of net returns. Mean, variance, maximum, and minimum net return values for each irrigation schedule are required to derive Beta distributions of net returns. Stochastic dominance for these schedules is derived through a computer algorithm (SDOM) developed by Anderson and is listed in Anderson, Dillon, and Hardaker.

Stochastic dominance analysis is performed to compare alternative irrigation strategies to the contemporary practice of applying 24 acre-inches per acre. The general irrigation strategies evaluated include contemporary practices, initiating irrigations by growth stages with no stress allowed, and initiating irrigations based on stage of growth with plant stress allowed as irrigations are reduced.

## STOCHASTIC DOMINANCE FOR IRRIGATIONS INITIATED BY GROWTH STAGES

Net return data on the mean, variance, maximum, and minimum values for each irrigation scenario are used to derive individual cumulative Beta distributions. The SDOM computer algorithm is used to derive the set of stochastically dominant irrigation schedules. The contemporary irrigation schedule is the base portfolio while the proposed irrigation schedules are the challenger portfolios. If the challenger portfolios are stochastically dominant by first, second, or third degree stochastic dominance, the degree of stochastic dominance is assigned. If the base portfolio is stochastically dominant, the degree of stochastic dominance is assigned.

Table 24 shows the degree of stochastic dominance between contemporary irrigation procedures and the series of irrigation schedules based on growth stages. A major use of stochastic dominance procedures is to derive those technologies which may be incorporated by risk averse producers. From Table 24, six of the proposed irrigation schedules are stochastically dominant by the first degree over contemporary irrigation practices and two are second degree stochastically dominant. Two of the irrigation technologies are neither stochastically dominant nor dominated and four of the 14 irrigation technologies are found to be stochastically inefficient by

Table 24. Degree of Stochastic Dominance Between Contemporary Irrigation Practices and Proposed Irrigation Schedules Based on Growth Stages

| Irrigation Practice | Expected Net Returns[a] (dollars) | Probability of Expected Return as Great as Contemporary Practice (Percent) | Degree of Stochastic Dominance |
|---|---|---|---|
| Contemporary Practices | 86.89 | - | - |
| No Delay in Irrigation | 97.18 | 61.9 | FSD over Contemporary |
| No Irrigation In: Stage 1 | 97.07 | 61.8 | FSD over Contemporary |
| No Irrigation In: Stage 2 | 92.66 | 56.0 | FSD over Contemporary |
| No Irrigation In: Stage 3 | 96.60 | 61.0 | FSD over Contemporary |
| No Irrigation In: Stage 4 | 91.41 | 56.0 | SSD over Contemporary |
| No Irrigation In: Stage 1&2 | 95.61 | 58.6 | FSD over Contemporary |
| No Irrigation In: Stage 1&3 | 96.49 | 61.0 | FSD over Contemporary |
| No Irrigation In: Stage 1&4 | 91.41 | 56.0 | SSD over Contemporary |
| No Irrigation In: Stage 2&3 | 91.60 | 54.8 | No Dominant Strategy |
| No Irrigation In: Stage 2&4 | 64.82 | 32.8 | FSD by Contemporary |
| No Irrigation In: Stage 3&4 | 41.59 | 14.4 | FSD by Contemporary |
| No Irrigation In: Stage 1,2,&3 | 88.94 | 52.2 | No Dominant Strategy |
| No Irrigation In: Stage 2,3,&4 | 10.46 | 9.6 | FSD by Contemporary |
| No Irrigation In: Stage 1,2,3,&4 | 10.46 | 9.6 | FSD by Contemporary |

[a] A Beta distribution for net returns is assumed.

first degree stochastic dominance. All four of the stochastically dominated schedules involve a failure to irrigate grain sorghum in the fourth stage of growth. These results signify the importance of irrigation during the grain filling stage of plant development.

The eight strategies that are stochastically dominant over contemporary practices can be incorporated into producers production processes. They have returns higher than contemporary practices on the average and also have higher returns under unfavorable conditions.

In Table 25, net returns for all fifteen irrigation schedules are compared simultaneously in order to derive the stochastically efficient set of irrigation schedules among the fifteen candidates. Assuming a risk averse irrigation producer, there are three irrigation schedules among the fifteen investigated which exhibit first degree stochastic dominance. These three schedules are the no delay irrigation schedule, no irrigation in stage 1, and no irrigation in stage 1 and 2. Under the additional behavioral assumptions required for second and third degree stochastic efficiency, only two schedules remain in the efficient set. The two schedules which are stochastically efficient by second and third degree criteria are the no delay irrigation schedule and no irrigation in stage 1. Greater knowledge of the individual producer's utility functions is required to determine a preference between the no stress irrigation schedule and the no irrigation in stage 1 schedule. However, stochastic efficiency procedures have permitted derivation of two irrigation schedules from the fifteen presented which can be adopted by risk averse producers to increase net returns.

Table 25. First, Second and Third Order Stochastic Dominance of the Fifteen Proposed Irrigation Schedules Based on Growth Stage

| Stochastic Dominance Order |
|---|
| Efficient Prospects of the First Degree:<br>1) No Delay Scenario<br>2) No Irrigation in Stage 1<br>3) No Irrigation in Stage 1 & 2 |
| Efficient Prospects of the Second Degree:<br>1) No Delay Scenario<br>2) No Irrigation in Stage 1 |
| Efficient Prospects of the Third Degree:<br>1) No Delay Scenario<br>2) No Irrigation in Stage 1 |

## STOCHASTIC DOMINANCE FOR IRRIGATION BY CRITICAL SOIL MOISTURE RATIOS AND DAYS UNTIL STRESS

From Tables 9, 12, 15, and 18, the mean, variance, largest, and smallest values of net returns for each irrigation scenario are used to derive individual net returns beta cumulative distributions. By using the SDOM computer algorithm, stochastic dominance for these scenarios is derived. Stochastic dominance for each of the four different soil moisture ratios (45%, 30%, 20%, and 0%) to contemporary irrigation practices is derived to ascertain which proposed technologies may be adopted by risk averse producers.

Table 26 shows the degree of stochastic dominance between contemporary irrigation practices and schedules under the critical ratio of 45 percent. Results show that all but two of the proposed schedules dominate contemporary practices by the first degree. The no delay scenario and the contemporary practice do not dominate each other. The no delay scenario uses more water than the other proposed scenarios and, therefore, net returns are lower for the no delay strategy. Also, the contemporary practice dominates by the first degree the irrigation schedule of not irrigating in growth stage 3 and 4. By initiating irrigations by the 45 percent critical ratio, an irrigation commences earlier than for the other critical ratio scenarios. Therefore, an irrigation occurs in the early part of stage 2 and, when the plant is stressed again in the early part of stage 3, an irrigation is prohibited and the grain sorghum plant is stressed for an extended length of time.

Table 27 shows the degree of stochastic dominance between

Table 26. Degree of Stochastic Dominance Between Contemporary Irrigation Practices and Proposed Irrigation Schedules Based on Growth Stage and a Critical Extractable Soil Moisture Ratio of Forty-Five Percent

| Irrigation Practice | Expected Net Returns[a] (Dollars) | Probability of Expected Return as Great as Contemporary Practice (Percent) | Degree of Stochastic Dominance |
|---|---|---|---|
| Contemporary | 86.89 | - | - |
| No Delay | 87.19 | 61.9 | FSD over Contemporary |
| Stage 1 | 97.07 | 61.8 | FSD over Contemporary |
| Stage 2 | 92.66 | 56.0 | FSD over Contemporary |
| Stage 3 | 96.60 | 61.0 | FSD over Contemporary |
| Stage 1 & 2 | 91.41 | 56.0 | SSD over Contemporary |
| Stage 1 & 3 | 95.61 | 58.6 | FSD over Contemporary |
| Stage 1 & 4 | 91.41 | 56.0 | SSD over Contemporary |
| Stage 2 & 3 | 91.60 | 54.8 | No Dominant Strategy |
| Stage 2 & 4 | 64.82 | 32.8 | FSD by Contemporary |
| Stage 3 & 4 | 41.59 | 14.4 | FSD by Contemporary |
| Stage 1, 2, & 3 | 88.94 | 52.2 | No Dominant Strategy |
| Stage 2, 3 & 4 | 10.46 | 9.6 | FSD by Contemporary |
| Stage 1, 2, 3 & 4 | 10.46 | 9.6 | FSD by Contemporary |

[a] A Beta distribution for net returns is assumed.

contemporary irrigation procedures and schedules under the 30 percent critical ratio. From Table 27, all proposed irrigation schedules dominate the contemporary practice while the schedule of no irrigation in stage 3 and 4 dominates in the second degree. In comparing Tables 26 and 27, there is a reversal in stochastic dominance for the irrigation schedule of not irrigating in stage 3 and 4. With the 30 percent critical ratio, it takes longer to reach the critical 15-day irrigation initiation value because the critical ratio is lower than the 45 percent criterion. Therefore, the irrigations before entering stage 3 for the 30 percent ratio are likely to be later in stage 2 than the same irrigation for the 45 percent ratio. Thus, the plant is stressed less severely in stages 3 and 4 where no irrigations are allowed than for the 45 percent critical ratio scenario. In comparing the nature of stochastic dominance for the irrigation schedule of not irrigating in stage 3 and 4 for ratios of lower values (Tables 28 and 29) as compared to the 30 percent criterion, the 20 percent and zero percent critical ratios have a more severe effect on crop development than the 30 percent critical ratio. The effects of the soil water stress at these lower critical ratios are seen in the stochastic dominance results in Tables 28 and 29.

Table 28 shows the degree of stochastic dominance between contemporary irrigation procedures and schedules under the 20 percent critical ratio. All proposed irrigation schedules which do not include irrigations in stage 4 are stochastically dominant by the first degree. When this lower critical ratio is accompanied by not irrigating in stage 3 and 4, the contemporary practice of applying 24 inches of groundwater is stochastically dominant by the first degree.

Table 27. Degree of Stochastic Dominance Between Contemporary Irrigation Practices and Proposed Irrigation Schedules Based on Growth Stage and Critical Extractable Soil Moisture Ratio of Thirty Percent

| Irrigation Practice | Expected Net Returns[a] (Dollars) | Probability of Expected Return as Great as Contemporary Practice (Percent) | Degree of Stochastic Dominance |
|---|---|---|---|
| Contemporary | 86.89 | - | - |
| No Delay | 90.86 | 54.6 | FSD over Contemporary |
| Stage 1 | 90.79 | 54.5 | FSD over Contemporary |
| Stage 2 | 94.18 | 57.7 | FSD over Contemporary |
| Stage 3 | 90.83 | 54.5 | FSD over Contemporary |
| Stage 4 | 88.30 | 51.7 | SSD over Contemporary |
| Stage 1 & 2 | 93.13 | 56.5 | FSD over Contemporary |
| Stage 2 & 3 | 91.22 | 59.8 | FSD over Contemporary |
| Stage 3 & 4 | 90.49 | 54.7 | SSD over Contemporary |

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[a] A Beta distribution for net returns is assumed.

Table 28. Degree of Stochastic Dominance Between Contemporary Irrigation Practices and Proposed Irrigation Schedules Based on Growth Stage and a Critical Extractable Soil Moisture Ratio of Twenty Percent

| Irrigation Practice | Expected Net Returns[a/] (Dollars) | Probability of Expected Return as Great as Contemporary Practice (Percent) | Degree of Stochastic Dominance |
|---|---|---|---|
| Contemporary | 86.89 | - | - |
| No Delay | 94.28 | 57.9 | FSD over Contemporary |
| Stage 1 | 94.20 | 57.8 | FSD over Contemporary |
| Stage 2 | 94.15 | 57.7 | FSD over Contemporary |
| Stage 3 | 94.28 | 57.9 | FSD over Contemporary |
| Stage 4 | 91.59 | 56.2 | SSD over Contemporary |
| Stage 1 & 2 | 92.93 | 56.3 | FSD over Contemporary |
| Stage 2 & 3 | 96.87 | 61.3 | FSD over Contemporary |
| Stage 3 & 4 | 67.39 | 34.2 | FSD by Contemporary |

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[a/] A Beta distribution for net returns is assumed.

Table 29 shows the degree of stochastic dominance between the contemporary irrigation procedure and schedules under the zero percent critical ratio. When a very low critical soil moisture ratio is used and irrigations in particular growth stages are skipped, fewer of the proposed irrigations schedules dominate contemporary practices. By lowering the critical ratio, the time delay in applying irrigation water lengthens which increases the soil moisture stress to the grain sorghum plant. These interactions return to the producer and affect the degree of stochastic dominance for some of the proposed irrigation schedules.

## STOCHASTIC DOMINANCE FOR IRRIGATIONS BY COMBINATIONS OF CRITICAL SOIL MOISTURE RATIOS

Data in Table 21 on the mean, variance, largest, and smallest net return values for each irrigation schedule are used to derive individual schedule cummulative Beta distributions. Procedures as outlined in the previous section are used to derive stochastic dominance. Table 30 shows the degree of stochastic dominance between contemporary irrigation procedures and schedules using variable critical soil moisture ratios at different growth stages. The same critical soil moisture ratio is used for growth stages 1 and 2, and a similar ratio is applied for growth stages 3 and 4. Also under the different combination of critical soil moisture ratios scenario, irrigations were permitted in all growth stages for the grain sorghum plant. Therefore, these irrigation schedules are similar in procedure to the no delay schedules except that different soil moisture values

Table 29. Degree of Stochastic Dominance Between Contemporary Irrigation Practices and Proposed Irrigation Schedules Based on Growth Stage and a Critical Extractable Soil Moisture Ratio of Zero Percent

| Irrigation Practice | Expected Net Returns[a] (Dollars) | Probability of Expected Return as Great as Contemporary Practice (Percent) | Degree of Stochastic Dominance |
|---|---|---|---|
| Contemporary | 86.89 | - | - |
| No Delay | 98.29 | 62.8 | FSD over Contemporary |
| Stage 1 | 98.29 | 62.8 | FSD over Contemporary |
| Stage 2 | 93.80 | 57.1 | FSD over Contemporary |
| Stage 3 | 97.73 | 62.0 | FSD over Contemporary |
| Stage 4 | 85.60 | 48.0 | No Dominant Strategy |
| Stage 1 & 2 | 93.80 | 57.1 | FSD over Contemporary |
| Stage 2 & 3 | 92.75 | 55.9 | No Dominant Strategy |
| Stage 3 & 4 | 39.41 | 11.0 | FSD by Contemporary |

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[a] A Beta distribution for net returns is assumed.

are used at various stages of plant growth. Since the various no delay scenarios (Table 26 and 29) dominate by first degree the contemporary practice of applying irrigation water, it is not surprising that all of the schedules using different combinations of critical soil moisture ratios dominate the contemporary irrigation practice by the first degree.

Assuming a risk averse producer in the Oklahoma Panhandle, stochastic efficiency analysis shows that there are a number of alternative schedules superior to contemporary practices. Eliminating irrigations in stages 1 and 2 may be beneficial to the producer, but eliminating irrigations in stage 4 of plant growth may be unwise. Also, irrigation schedules using variable soil moisture ratios at different stages of plant growth prove to be stochastically dominant over the contemporary schedule. Those alternative irrigation schedules that are dominant over the contemporary irrigation practice also have greater mean net returns and higher net returns under unfavorable conditions.

## OPTIMAL CONTROL ANALYSIS

The objective of optimal control theory is to determine the control signals that will cause a process to satisfy the fiscal constraints and minimize or maximize some performance criterion (Kirk). The formulation of an optimal control problem requires a mathematical description of the process to be controlled, such as the equations which make up the grain sorghum plant growth model; a statement of the physical constraints, such as minimum and maximum supplies of groundwater; specification of control variables, such as a

Table 30. Degree of Stochastic Dominance Between Contemporary Irrigation Practice and Proposed Irrigation Schedules Based on Variable Critical Extractable Soil Moisture Ratios at Different Stages of Plant Growth

| Irrigation Practice | Expected Net Returns[a] (Dollars) | Probability of Expected Return as Great as Contemporary Practice (Percent) | Degree of Stochastic Dominance |
|---|---|---|---|
| Contemporary | 86.89 | - | - |
| Stage 1 & 2 at 0%<br>Stage 3 & 4 at 20% | 97.42 | 62.1 | FSD over Contemporary |
| Stage 1 & 2 at 0%<br>Stage 3 & 4 at 30% | 97.62 | 62.2 | FSD over Contemporary |
| Stage 1 & 2 at 0%<br>Stage 3 & 4 at 45% | 90.51 | 54.1 | FSD over Contemporary |
| Stage 1 & 2 at 20%<br>Stage 3 & 4 at 0% | 96.38 | 61.2 | FSD over Contemporary |
| Stage 1 & 2 at 20%<br>Stage 3 & 4 at 30% | 90.87 | 54.6 | FSD over Contemporary |
| Stage 1 & 2 at 20%<br>Stage 3 & 4 at 45% | 90.88 | 54.6 | FSD over Contemporary |
| Stage 1 & 2 at 30%<br>Stage 3 & 4 at 0% | 92.10 | 55.6 | FSD over Contemporary |
| Stage 1 & 2 at 30%<br>Stage 3 & 4 at 20% | 94.28 | 57.9 | FSD over Contemporary |
| Stage 1 & 2 at 30%<br>Stage 3 & 4 at 45% | 90.89 | 54.6 | FSD over Contemporary |
| Stage 1 & 2 at 45%<br>Stage 3 & 4 at 0% | 91.80 | 55.2 | FSD over Contemporary |
| Stage 1 & 2 at 45%<br>Stage 3 & 4 at 20% | 94.28 | 57.9 | FSD over Contemporary |
| Stage 1 & 2 at 45%<br>Stage 3 & 4 at 30% | 90.86 | 54.6 | FSD over Contemporary |

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[a] A Beta distribution for net returns is assumed.

scheduled irrigations; and specification of a performance criterion, such as net returns to the producer.

For this analysis, the amount and timing of irrigation water is the controlled input. The grain sorghum plant growth model determines the daily soil moisture level and growth stage of the plant. The producer controls water applications. Optimal control derives the amount and timing of irrigation water for grain sorghum over the course of the growing season that will maximize net returns to the producer.

The optimal allocation of groundwater is depicted as:

$$(4)\quad S[x(t),U(t),t] = \sum_{t=0}^{T-1} F[x(t),U(t),t] + F(X_T)$$

where S[x(t),U(t),t] is the objective function (the summation of total returns that are earned over the entire study period) subject to N constraints (the quantities of groundwater applied) and to any boundary conditions which may apply. The term F[x(t),U(t),t] is the intermediate function and shows dependence of the functional on the time paths of state variables x(t), control variables, U(t), and time within the relevant period.

The system is described by N first order difference equations which can be expressed as:

$$(5)\quad X_i(t+1) - X_i(t) = f_i[x(t),U(t)t] \quad i = 1,2,\ldots N$$

$$X(t_o) \text{ is given} \qquad t = 0,1,\ldots,T-1$$

where x(t) is an n-vector of variables which describe the state of the system (such as yields or returns) at period t and U(t) is an m-vector of variables to be controlled which, in this case, would include the quantity of groundwater pumped. The controller or producer determines the optimal levels of the input signal and the dynamic behavior of

$X(t_o)$ is given.

The values of the control variables U(t) are restricted by the following constraint:

(6) $gi[U(t)] \leq b_i(t) \quad i = 1,\ldots,m$

$$t = 0,\ldots,T-1$$

where $b_i$ is a constant, such as 3 inches of groundwater. Any control variable U(t) that satisfies the constraint is referred as an admissable or feasible control variable. The control problem becomes one of deriving the value of the control variable, U(t), through time such that the following system is solved.

(7) Maximize: $S[x(t), U(t),t] = \sum_{t=0}^{T-1} F[x(t),U(t),t] + F(x_T)$

Subject to:

(8) $X(t+1)-x(t) = f[x(t),U(t),t] \qquad i = 1,2,\ldots,m$

$g_i[U(t)] \leq b_i\ (t) \qquad t = 1,2,\ldots,T-1$

and where $x(t_o)$ is given.

In the optimal control scenario, the procedure used is to derive an input signal for the simulation model which optimizes the objective function through the growing season. Optimizing procedures for simulation models include response surface methodology and direct search techniques. Response surface methodology involves estimating first and second order differential equations to approximate the simulation response surface based on appropriate experimental design and replications of the simulation runs. Because the objective function in this study is not expressed in terms of the decision variables, optimization techniques which rely on derivatives cannot be applied directly to this problem (Pedgen). Direct search techniques, however, do not require derivative information. For this study, the

Box-Complex, a nonlinear programming direct search procedure, was modified and used to derive optimal irrigation schedules. Unlike the Hooke-Jeaves Pattern search, Rosenbrock's method of rotating coordinates, and the simplex method by Nelder and Mead, the modified Box-Complex can be used for problems which incorporate constraints. The original Box-Complex algorithm presented in Kuester and Mize is modified to incorporate all the routines of the unconstrained Nelder-Mead Flexible polyhedral search.

## IRRIGATION SCHEDULING BY OPTIMAL CONTROL

In this section, optimal control procedures are used to derive irrigation applications under three different scenarios. Under the first scenario, irrigations are initiated whenever the daily extractable soil moisture ratio is equal to or below 45 percent. The constraint for this scenario is that an irrigation application be no less than 1 and no more than 3 inches. For the second scenario, the optimal control is used to derive a critical soil moisture ratio for the entire growing season. For the critical soil moisture ratio scenario, the application of a 3 inch irrigation requiring 15 days for completion initiates an irrigation sequence. The third scenario uses optimal control theory to derive optimal soil moisture ratios for each stage of growth for the grain sorghum plant. The Modified Box-Complex is used to derive irrigation schedules that maximize net returns for 23 replications of each of the three proposed irrigation scenarios.

## Optimal Control for a 1 to 3 Inch Application

Optimal control is used to derive irrigation applications which are initiated when the soil moisture ratio is equal to or below the 45 percent level. Irrigation applications are constrained to range from 1 to 3 inches with the objective being the maximization of the following performance function:

(9) Maximize: $NR = GR - GSTI - CIG$

Subject to: $1 \leq IR_i \leq 3 \qquad i = 1,2,\ldots,n$

where, NR is net returns ($/acre), GR is gross returns which is the price received by producers for grain sorghum ($3.98/hundredweight) time the quantity of grain derived by the model (hundredweight/acre); CSTI is the cost of grain sorghum production less the variable cost of irrigation ($/acre), CIG is the total variable cost of irrigation water per acre inch times the number of acre-inches applied per acre. The quantity of water used in the plant growth model is a net irrigation figure; that is, the gross quantity of water pumped less quantities lost due to evaporation and distribution.

With unconstrained direct search models, significantly dissimilar starting points may be used to increase the likelihood of deriving a global maximum. The Modified Box-Complex uses a pseudo-random number sequence which permits derivation of different initial configurations from a single initial point and the solution can be considered a global maximum. For each of the 23 replications, three runs with different pseudo-random numbers are used to derive the optimal irrigation strategy.

Table 31 shows the results for the first optimal control scenario. Field yields range from a maximum of 72.11 cwt/acre to a

Table 31. Simulated Grain Sorghum Yields, Revenues, Costs and Net Returns for the Optimal Control Scenario to Determine a Single Season Critical Extractable Soil Moisture Ratio Using 1953-1975 Climatic Data

| Replications | Field Yield | Revenue | Water Pumped | Var. Irr. Cost | Total Cost | Net Return |
|---|---|---|---|---|---|---|
| | (CWT/AC) | ($/AC) | (AC. Inch) | ($/AC) | ($/AC) | ($/AC) |
| 1 | 60.98 | 242.70 | 13.56 | 21.56 | 140.15 | 102.55 |
| 2 | 56.38 | 224.39 | 12.11 | 19.25 | 137.84 | 86.55 |
| 3 | 62.87 | 250.22 | 13.85 | 22.02 | 140.61 | 109.61 |
| 4 | 62.28 | 247.87 | 15.32 | 24.36 | 142.95 | 104.93 |
| 5 | 61.84 | 246.12 | 7.89 | 12.69 | 131.28 | 114.84 |
| 6 | 65.43 | 260.41 | 9.30 | 14.79 | 133.38 | 127.03 |
| 7 | 66.96 | 266.50 | 14.35 | 22.82 | 141.41 | 125.09 |
| 8 | 65.72 | 261.57 | 13.54 | 21.53 | 140.12 | 121.45 |
| 9 | 55.56 | 221.13 | 5.49 | 8.73 | 127.32 | 93.81 |
| 10 | 56.00 | 222.88 | 4.81 | 7.65 | 126.24 | 96.64 |
| 11 | 56.67 | 225.55 | 11.80 | 18.76 | 137.35 | 88.19 |
| 12 | 52.84 | 210.30 | 3.97 | 6.31 | 124.90 | 85.40 |
| 13 | 55.87 | 222.36 | 8.08 | 12.85 | 131.44 | 90.93 |
| 14 | 52.17 | 207.64 | 8.37 | 13.31 | 131.90 | 75.74 |
| 15 | 51.56 | 205.21 | 4.30 | 6.84 | 125.43 | 79.78 |
| 16 | 53.25 | 211.98 | 8.10 | 12.88 | 131.47 | 80.74 |
| 17 | 50.48 | 200.91 | 12.45 | 19.80 | 138.39 | 62.52 |
| 18 | 64.89 | 258.26 | 14.97 | 23.80 | 142.39 | 115.87 |
| 19 | 72.11 | 287.00 | 10.78 | 17.14 | 135.73 | 151.27 |
| 20 | 50.09 | 199.36 | 1.61 | 2.56 | 121.15 | 78.21 |
| 21 | 63.15 | 251.34 | 11.67 | 18.56 | 137.15 | 114.19 |
| 22 | 52.46 | 208.79 | 7.25 | 11.53 | 130.12 | 78.67 |
| 23 | 66.07 | 262.96 | 13.34 | 21.21 | 139.80 | 123.16 |
| AVG. | 58.94 | 234.58 | 9.87 | 15.69 | 134.28 | 100.30 |

minimum of 50.09 cwt/acre with an average yield of 58.94 cwt/acre. Water pumped ranges from maximum of 15.32 inches to a minimum of 1.61 inches with an average groundwater pumped of 9.87 inches per acre. Net returns range from a maximum of $151.27 per acre to a minimum of $62.52 per acre with an average return for the 23 replications of $100.30 per acre. With the optimal control scenario, only two out of the 23 replications do not require a preplant irrigation because soil moisture is greater than the 45 percent ratio. However, when preplant irrigations are scheduled, an average of 2.07 inches is applied.

## Optimal Control for a Single Season

## Soil Moisture Ratio

Optimal control procedures are used to derive a soil moisture ratio for the entire season that maximizes the performance function of net returns. In this scenario, an irrigation is scheduled 15 days before the critical soil moisture ratio is reached. Examining the results of the single critical soil moisture ratio presented earlier in this chapter aids in determining the constraints of the mode. An understanding of the expected shape of the performance function aids in deriving starting points and constraints for the Modified Box-Complex, reducing the time required to derive a global maximum solution.

Table 32 shows the results of an optimal single season critical soil moisture ratio procedure. For the entire season an average critical soil moisture ratio derived is approximately 16 percent with a maximum of approximately 23 percent and minimum of 0.0 percent. Yields average 59.07 cwt/acre with a maximum yield of 71.97 cwt/acre

Table 32. Simulated Grain Sorghum Yields, Revenues, Costs, and Net Returns for the Optimal Control Scenario of Initiating an Irrigation Aplication at the Forty-Five Percent Soil Moisture Ratio Using 1953-1975 Climatic Data

| Replication | Yield | Revenue | Water Pumped | Var. Irr. Cost | Ratio | Returns |
|---|---|---|---|---|---|---|
| | (CWT/Acre) | ($/AC) | (Inches) | ($/AC) | | ($/AC) |
| 1 | 60.93 | 242.50 | 18.00 | 28.62 | 0.0318 | 95.29 |
| 2 | 60.74 | 285.83 | 18.00 | 28.62 | 0.1907 | 78.62 |
| 3 | 63.20 | 251.54 | 13.50 | 21.46 | 0.1567 | 111.48 |
| 4 | 62.51 | 248.79 | 22.50 | 35.77 | 0.1880 | 94.42 |
| 5 | 62.24 | 247.72 | 13.50 | 21.46 | 0.2299 | 107.66 |
| 6 | 65.83 | 262.00 | 13.50 | 21.46 | 0.2197 | 121.95 |
| 7 | 67.29 | 267.81 | 18.00 | 28.62 | 0.2272 | 120.60 |
| 8 | 65.29 | 259.85 | 13.50 | 21.46 | 0.0781 | 119.80 |
| 9 | 55.53 | 221.01 | 9.00 | 14.31 | 0.0000 | 88.11 |
| 10 | 56.33 | 224.19 | 9.00 | 14.31 | 0.2118 | 91.29 |
| 11 | 56.99 | 226.82 | 18.00 | 28.62 | 0.1957 | 79.61 |
| 12 | 52.95 | 210.74 | 9.00 | 14.31 | 0.2286 | 77.84 |
| 13 | 55.89 | 222.44 | 9.00 | 14.31 | 0.1947 | 89.54 |
| 14 | 52.12 | 207.44 | 9.00 | 14.31 | 0.0634 | 74.54 |
| 15 | 51.53 | 205.09 | 9.00 | 14.31 | 0.0000 | 72.19 |
| 16 | 53.31 | 212.17 | 9.00 | 14.31 | 0.1496 | 79.27 |
| 17 | 50.54 | 201.15 | 18.00 | 20.62 | 0.1253 | 53.94 |
| 18 | 65.09 | 259.06 | 22.50 | 35.77 | 0.2105 | 104.69 |
| 19 | 71.97 | 286.44 | 13.50 | 21.46 | 0.0634 | 146.69 |
| 20 | 50.10 | 199.40 | 4.50 | 7.16 | 0.2604 | 73.65 |
| 21 | 63.52 | 252.81 | 13.50 | 21.46 | 0.2542 | 112.75 |
| 22 | 52.48 | 208.87 | 9.00 | 14.31 | 0.1903 | 75.97 |
| 23 | 66.21 | 263.52 | 13.50 | 21.46 | 0.2084 | 123.46 |
| AVG. | 59.07 | 235.09 | 13.30 | 21.15 | 0.1599 | 95.35 |

and a minimum of 50.10 cwt/acre. Average yield for the contemporary schedule of 24 inches is 59.20 cwt/acre. Average quantity of water pumped by this schedule is 13.30 inches with a maximum of 22.50 inches and a minimum of 3.50 inches. With a water savings of approximately 10 inches by the optimal control single season ratio, and with no significant decrease in average yields when compared to contemporary practices, the net returns of the proposed schedule are greater than for contemporary practices. From Table 32 , average returns for the single season optimal control soil moisture ratio schedule are $95.35 per acre with a maximum return of $146.39 per acre and a minimum return of $53.94. The net return for this schedule are substantially above the $78.86 per acre average contemporary irrigation return and the optimal soil moisture ratio scenario saves approximately 10 inches of water per acre. Also this procedure suggests that a critical ratio of 16 percent on the average is adequate as an initiator of irrigation applications for the entire season.

## Optimal Control for a Multiple Season Critical Soil Moisture Ratio

For this scenario, optimal control theory is used to derive critical soil moisture ratios for each stage of grain sorghum plant growth that maximize net returns to the producer. Critical soil moisture ratios are derived for stage 1, stage 2, and stage 4 of plant growth. Net returns for the various combinations of soil moisture ratios at different stages of plant growth are examined to better understand the shape and response of the performance function surface.

Also, these results aid in determining the upper and lower constraints for the Modified Box-Complex to permit more efficient derivation of optimal critical soil moisture ratios.

Table 33 shows the results of an optimal multiple critical soil moisture ratio schedule. The average critical soil moisture ratios derived are 22.7 percent in stage 1, 19.7 percent in stage 2, 22.1 percent in stage 3, and 16.5 percent in stage 4. All of these figures average around the 20 percent level which has been suggested by agronomic experts as a serious soil moisture ratio in grain sorghum plant growth.

Average yields under this schedule are 69.06 cwt/acre with a maximum of 72.55 cwt/acre and a minimum of 50.11 cwt/acre. Average quantity of groundwater pumped is 13.50 inches with a maximum of 22.50 inches and a minimum of 4.50 inches. In comparing these results with the optimal single season critical soil moisture ratio, there are no appreciable differences in yields, quantity of water use, and returns. However, under the contemporary irrigation schedule, the average return for the 23 replications is $78.86 while the average return for the multiple critical soil moisture ratio is $95.02 per acre. This proposed schedule seems promising when compared to contemporary practices because returns are higher and water use on the average is 10 inches less. When evaluating the single season critical soil moisture ratio, an average ratio of 16 percent is derived for the entire season. However, under the multiple critical soil moisture ratio evaluation, ratios vary from 13.5 to 22.4, and average more than 16 percent.

Table 33. Simulated Grain Sorghum Yields, Water Pumped and Net Returns for the Optimal Control Scenario to Determine Critical Extractable Soil Moisture Ratio for Each Growth Stage Using 1953-1975 Climatic Data

| Replication | Yield | Water Pumped | Ratio Stage 1 | Ratio Stage 2 | Ratio Stage 3 | Ratio Stage 4 | Returns |
|---|---|---|---|---|---|---|---|
| | (CWT/AC) | | | | | | ($/AC) |
| 1 | 40.98 | 18.00 | 0.4046 | 0.1493 | 0.2273 | 0.0234 | 95.37 |
| 2 | 56.74 | 18.00 | 0.0847 | 0.3199 | 0.1979 | 0.1115 | 78.62 |
| 3 | 63.21 | 13.50 | 0.3807 | 0.1601 | 0.3977 | 0.3200 | 111.52 |
| 4 | 62.51 | 22.50 | 0.1603 | 0.1316 | 0.0848 | 0.1603 | 94.42 |
| 5 | 62.24 | 13.50 | 0.2537 | 0.1253 | 0.2322 | 0.2110 | 107.66 |
| 6 | 65.83 | 13.50 | 0.3636 | 0.1159 | 0.3138 | 0.1229 | 121.95 |
| 7 | 67.29 | 18.00 | 0.1080 | 0.2865 | 0.1697 | 0.2400 | 120.60 |
| 8 | 65.79 | 13.50 | 0.2187 | 0.1629 | 0.1783 | 0.0401 | 121.79 |
| 9 | 56.74 | 13.50 | 0.2360 | 0.3405 | 0.1384 | 0.1935 | 81.79 |
| 10 | 54.87 | 4.50 | 0.2500 | 0.2500 | 0.0000 | 0.0000 | 92 64 |
| 11 | 56.98 | 18.00 | 0.2860 | 0.1852 | 0.2650 | 0.1818 | 79.57 |
| 12 | 52.94 | 9.00 | 0.1847 | 0.3239 | 0.0638 | 0.2537 | 77.80 |
| 13 | 55.89 | 9.00 | 0.1907 | 0.0239 | 0.4298 | 0.1844 | 89.54 |
| 14 | 62.13 | 9.00 | 0.1559 | 0.2200 | 0.0479 | 0.0922 | 74.58 |
| 15 | 51.53 | 9.00 | 0.2271 | 0.0159 | 0.2957 | 0.1844 | 72.19 |
| 16 | 53.31 | 9.00 | 0.1431 | 0.1832 | 0.3537 | 0.0745 | 79.27 |
| 17 | 50.54 | 18.00 | 0.1836 | 0.2090 | 0.1832 | 0.0393 | 53.94 |
| 18 | 65.09 | 22.50 | 0.1748 | 0.1600 | 0.1646 | 0.2015 | 104.69 |
| 19 | 72.55 | 18.00 | 0.1846 | 0.1695 | 0.2886 | 0.3750 | 141.54 |
| 20 | 50.11 | 4.50 | 0.3695 | 0.0239 | 0.3277 | 0.2459 | 73.69 |
| 21 | 63.52 | 13.50 | 0.2133 | 0.2283 | 0.2234 | 0.2535 | 112.75 |
| 22 | 52.48 | 9.00 | 0.2371 | 0.2059 | 0.1194 | 0.2804 | 75.97 |
| 23 | 66.22 | 13.50 | 0.4119 | 0.1027 | 0.3740 | 0.1653 | 123.50 |
| AVG. | 69.06 | 13.50 | 0.2248 | 0.1780 | 0.1979 | 0.1719 | 95.00 |

# COMPARISON OF RESULTS BETWEEN OPTIMAL CONTROL, STOCHASTIC EFFICIENCY, CONTEMPORARY IRRIGATION PRACTICES, AND DRYLAND GRAIN SORGHUM PRODUCTION

Table 34 shows the average water use, yields, and net returns for the various methods used to derive production of grain sorghum in this analysis. The set of stochastically efficient irrigation schedules are those derived in Table 25 which apply only to irrigations by growth stages.

As expected dryland production of grain sorghum has the lowest average yields and net returns of all the production methods investigated. The production method with the largest average returns per acre is the optimal control scenario of applying between 1 to 3 inches of water. Also from Table 34, this irrigation schedule has the lowest average per acre yields of the irrigated production methods, requires the lowest quantity of water pumped, and has the higher per acre net returns.

In comparing the average returns of the stochastically efficient irrigation schedules to other irrigated production methods, the average returns are only slightly less than the returns derived under optimal control. Also these stochastically efficient production methods initiate irrigations on a constant 45 percent critical extractable soil moisture level and apply a constant 3 inches of groundwater. The optimal control scenario however requires monitoring to insure the proper quantity of groundwater or critical extractable

Table 34. Comparison of Average Water Use, Yields, and Net Returns for the Optimal Control Runs, the Stochatic Efficient Sets of Irrigation Schedules from Irrigation by Growth Stage, Contemporary Irrigation Practices, and Dryland Production.

| | Water Use (Inches) | Yield (cwt/ac) | Net Returns ($/ac) |
|---|---|---|---|
| Optimal Control: | | | |
| 1. The 1 to 3 Inch Applications | 9.87 | 58.94 | 100.30 |
| 2. Single Season Soil Moisture | 13.30 | 59.07 | 95.35 |
| 3. Multiple Season Soil Moisture | 13.50 | 59.06 | 95.02 |
| Stochastic Efficiency[a/] | | | |
| 1. No Delay Scenario | 14.09 | 59.20 | 93.98 |
| 2. No Irrigation in Stage 1 | 13.89 | 59.04 | 94.23 |
| Contemporary Irrigation Practice | 24.0 | 59.20 | 78.86 |
| Dryland Production | 0.0 | 14.80 | 23.11 |

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[a/] These stochastic efficient irrigation schedules are derived from Table and incorporates only irrigation by growth stage.

soil moisture ratio is used. The managerial costs of such surveillance of plant parameters are not included in the optimal control scenario. Thus, net returns for the optimal control procedures are slightly higher than they would be if those costs were considered. Also, the optimal control scenarios are developed based on the assumption that no delays would occur in applying water when needed. Thus, it is possible that optimal control irrigation models that exclude irrigations at specific stages of plant growth would result in somewhat higher net returns.

## AGGREGATE IMPLICATIONS

Analysis of irrigation scheduling for this study has centered on the water savings for the individual 155 acre farm. However, incorporation of this technology in the Oklahoma Panhandle has important implications for reducing quantities of groundwater pumped and the decline of the static water table for the entire region. In 1979, irrigated grain sorghum acreage totaled 119,500 acres in Oklahoma Panhandle (Thompson, Mapp, Slogget). Under the assumption that all irrigated grain sorghum acreage in the region receives 2 acre-feet per acre as would be applied using contemporary practices the total quantity of groundwater applied in the region would be 239,000 acre-feet.

The decline in static water level can be derived from the equation:

$$(10) \quad s = \frac{w \cdot (1-R)}{cs \cdot a}$$

where:

d is the decline in static water level in feet, w is the volume of water pumped,

R is the recirculation coefficient[5/], R = 0.2

cs is the coefficient of storage[6/], cs = 0.1, and

a is the surface land area.

On the average, only about one acre in five is irrigated in the Oklahoma Panhandle. The Ogallala aquifer lies under all of the acres and contributes irrigation water to irrigated operators in the region. Thus, in equation (10) the land area is usually about five times as great as the number of acres irrigated. Thus, by dividing the 119,500acres of grain sorghum irrigated by .20, the surface area for equation (10) is 597,500 acres. The static water level declines by 3.20 feet from the contemporary practice of pumping 24 acre inches of groundwater.

If the no delay irrigation schedule were to be adopted, the average per acre quantity of groundwater pumped would be 14 acre inches or 1-1/6 acre feet. The quantity of water necessary to irrigate the 1979 irrigated grain sorghum acreage in Oklahoma Panhandle would be 139,147 acre-feet or an aggregate savings of 99,853 acre-feet of groundwater. The decline in the static water level wouldbe reduced from 3.20 feet under contemporary practices to 2.33 feet under the no delay irrigation schedule.This 27 percent reduction

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[5] The recirculation coefficient is defined as the percentage of water applied that percolates back through to the water table (Hart, Hoffman, Goemaat).

[6] This implies that the volume of water the aquifer releases by gravity is only 10 percent of the volume of the saturated material (Hart, Hoffman, Goematt).

in the decline in the water table would lengthen the life of the groundwater supply. Similar aggregate groundwater saving result for other proposed irrigation schedules relative to the contemporary practice.

The results indicate substantial potential for irrigated producers in the Oklahoma Panhandle to reduce water use, energy use, and the decline in the static water level, maintain crop yields, and increase net returns through irrigation scheduling based on soil water levels. Some additional equipment and management costs would be required to improve the timing of water applications and these costs are ignored in the analysis. However soil water levels can be monitored using soil tensiometers, neutron probes or other devices. Some of these work as well in clay soils. Tensiometers also require maintenance to insure proper operation, and reading tensiometers may require some training. The cost of monitoring soil moisture is expected to be less than the difference in net returns of contemporary and proposed irrigation schedules. Linking soil moisture monitoring devices to a microcomputer may greatly reduce the labor required to monitor soil moisture and could facilitate automatic initiation of irrigation applications.

The grain sorghum plant growth model has an important subroutine not used in this analysis. The model has the capability of updating computations at any point in the growing season with actual values based on field observations. If the model indicates that for a specific Calendar day the grain sorghum plant contains eight leaves and field observation reveals that the plants only have four leaves, the feedback routine is initiated to correct the model to reflect four leaves. This feedback routine will be important as the grain sorghum plant growth model is developed into a field model which can be used to schedule irrigations.

## SUMMARY AND CONCLUSIONS

Irrigated production has increased significantly in the Oklahoma Panhandle over the past three decades, with irrigated acreage increasing from 11,500 acres to more than 400,000 acres between 1950 to 1981 (Schwab). The Ogallala aquifer which is the primary source of irrigation water in the regions contains a finite quantity of water because of its isolation from major sources of recharge. Water withdrawls by irrigated producers continue to exceed recharge. Continued overdraft of the aquifer results in a declining water table and will lead to eventual economic exhaustion of the water resources. Increases in the price of natural gas and other fossil fuels further reduce the economic life of the aquifer.

With declining groundwater levels, interest in developing irrigation practices which reduce water and energy use while maintaining current levels of net returns to agricultural producers has increased. The major objective of this study is to derive irrigations strategies that reduce water and energy use in the production of grain sorghum in the Oklahoma Panhandle while maintaining the level of net returns.

Agronomic research on the relationships between soil moisture and plant growth suggests that the timing of soil moisture stress in relation to the stage of plant growth is more significant than the total seasonal soil moisture deficiency. A firm level decision model is used to evaluate the effects of alternative irrigation strategies on yields, water use, and net returns. This closed system simulation model uses information feedback loops between principle components of the firm level model to initiate irrigations. A major component of the firm level model is the dynamic grain sorghum plant growth model developed by Arkin, Vanderlip, and Ritchie. The firm level model simulates different irrigation schedules and the results are compared to the contemporary practice of applying 24 acre inches per acre.

The contemporary irrigation schedule is simulated by a series of time events. A preplant irrigation of 6 inches is applied on May 25 and five postplant irrigations of 3.6 inches are applied every two weeks commencing on June 22. In addition to the contemporary practice, an number of alternative irrigation schedules are investigated. A "no stress" irrigation schedule is analyzed which initiates irrigations whenever the critical extractable soil moisture ratio reaches 45 percent, a level at which leaf curl may be observed by irrigators in the field. Another set of irrigation schedules uses the 45 percent critical soil moisture level to initiate irrigations, but also allows the producer not to irrigate if the plant is in a specific state or stages of growth where water stress is not critical.

The evaluation of proposed irrigation schedules relative to the contemporary irrigation practice is performed using stochastic efficiency procedures. Stochastic efficiency assumes that the producer is risk averse and that, based on the producers risk preferences, a set of risk efficient irrigation strategies can be identified. A stochastic efficient irrigation strategy is not only more profitable on the average but is less prone to low outcomes under less favorable conditions.

For the stochastic efficiency analysis, a Beta distribution of net returns is assumed. The Beta distribution is more desireable than a normal distribution because the Beta distribution restricts net returns to some bounded range while the normal distribution allows net returns to range from positive to negative infinity.

The stochastic efficiency analysis reveals that most of the irrigation schedules which include irrigation in stage 4 of plant growth are stochastically dominant over contemporary practices. Most of the dominant irrigation schedules are first degree stochastically dominant over contemporary irrigation practices. Those schedules which do not include irrigation in stage 4 of plant growth have lower yields because the economic yield is developing in that growth stage.

Stochastic efficiency does not derive optimal irrigation schedules. Thus, optimal control is used to derive the quantity of groundwater use through time which maximize returns to the producer, given constraints (such as that the producer irrigates between 1 and 3 inches each time an application is initiated).

The Modified Box-Complex algorithm is used to derive optimal solutions for the simulation model. Under the optimal control scenario, an irrigation application is initiated whenever the daily extractable soil moisture ratio is 45 percent or below. The constraint for this scenario is that the producer will irrigate between 1 to 3 inches for each application while maximizing net returns. The optimal control scenario is run for each of the 23 replications in order to derive the quantity of groundwater for the entire irrigation season that maximizes the net returns function. The average yield for the 23 replications is 58.94 cwt per acre with average net returns to the producer being $100.30 per acre. The average quantity of groundwater applied is 9.87 inches, which represents nearly a 14 inch savings in irrigation water compared to contemporary practices of applying 24 acre inches per acre.

The simulation modeling of different irrigation schedules is focused on a single 155 acre grain sorghum field. However, incorporation of irrigation scheduling technology in the Oklahoma Panhandle could significantly reduce total groundwater pumping and reduce the rate of decline in the static water table. In 1979, total irrigated grain sorghum acreage in the Oklahoma Panhandle was 119,500 acres (Thompson, Mapp, and Sloggett). Assuming that 2 acre-feet or 24 inches of water are applied to the 119,500 acres, a total of 239,000 acre-feet are pumped. With 239,000 acre-feet pumped the corresponding decline in static water level is 3.20 feet. If, however, the no stress irrigation schedule is used to apply an average of 14 inches or 1 1/16 acre-feet of groundwater per acre, the total quantity of groundwater applied is 139,147 feet, thus savings 99,853 acre-feet of groundwater. Also the decline in the water table is reduced to 2.33 feet. Similar aggregate groundwater savings result if other proposed irrigation schedules are adopted in the Oklahoma Panhandle.

# PUBLICATIONS RESULTING FROM PROJECT

## Ph.D. Dissertation

Harris, Thomas R. Analysis of Irrigation Scheduling for Grain Sorghum in the Oklahoma Panhandle. Unpublished Ph.D. Dissertation, Oklahoma State University, May, 1981.

## Publications

Harris, Thomas R. and Harry P. Mapp, Jr. "A Control Theory Approach to Optimal Irrigation Scheduling in the Oklahoma Panhandle." Professional Paper No. P-791 of the Oklahoma Agricultural Experiment Station. Presented at the Southern Agricultural Economics Association Meetings, Hot Springs, Arkansas, February 1980.

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Several additional reports and articles are being prepared. The contribution of OWRT will be noted and copies of the publications furnished as they become available.

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