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A DIMENSIONLESS PARAMETER STUDY OF GROUNDWATER RECHARGE

PHASE II

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ABSTRACT

The method of coefficients has been used to predict groundwater recharge for several years. A new approach was attempted using a "dimensionless parameter" concept to relate recharge to other known parameters, i.e., pumpage, permeability, rainfall, recharge area, etc. Data from a total of fifteen observation wells from two locations in Oklahoma and two locations in Kansas were used. The high-use municipal wells in southwestern Oklahoma show periodic "mining" which can be avoided if pumpage rates are modified. The wells in Kansas are located very far from other pumping locations thus rendering the recharge area excessivly large. Regression analysis was performed encompassing recharge periods of one month, six months, and twelve months. The resulting linear equations are multiterm, wherein positive coefficients imply no overuse while negative coefficients substantiate "water mining", and these equations predict groundwater recharge rates more accurately than heretofore.

ACKNOWLEDGMENTS

This report presents the results of a research study sponsored by the Oklahoma Water Resources Research Institute and the Oklahoma University Research Institute.

The data for this study were obtained from the Utility Superintendent of the City of Frederick, Oklahoma, the Soil and Water Conservation Division of the Agricultural Research Service, Chickasha, Oklahoma, and the U. S. Geological Survey office in Lawrence, Kansas. These offices were especially helpful and cooperative with respect to available data.

A special note of thanks goes to Mr. James Naney of Chickasha, Oklahoma and Dr. Don W. Goss of Bushland, Texas for their guidance and advice in obtaining these data and suggestions as to their applicability.

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CHAPTER I

INTRODUCTION

The water utilization today (1972) is being rapidly accelerated by industry, agriculture and municipalities in every sector of the world. Part of this demand is being met by the use of groundwater which may be available locally in sufficient amounts to justify development of wells and an attendant pipeline system to pump and distribute the water to the users.

Sometimes the replenishment of local groundwater resources falls below consumption rates, with the result that the water table is lowered. Furthermore, legal consequences may follow in that the need is created to establish use priorities of a continually diminishing supply. The losers in this water rights struggle must develop or find high-cost substitute sources of water if they are to remain in business.

In arid regions, such as the Great Plains area of the United States, most municipalities must depend upon the available groundwater with their true rate of consumption being controlled by the recharge rate of the supplying aquifer. Thus, groundwater recharge has become one of the most important problems facing specialists in the general area of water supply.

Groundwater supply is a function of reservoir size, recharge rate, and consumption requirements. In order to plan the economic and social development of an area dependent upon these resources, the reservoir size and the rate of recharge must be estimated accurately. The projected and actual growth and development of all municipal, commercial, industrial and agricultural facets of an area would provide consumptive amounts. These estimates, then, become an integral part of the design-analysis process and must be accurate to get optimum resource management.

The consumption rate of an underground reservoir is affected by pumping, percolation to another aquifer, effluent seepage, evaporation from areas near an air-water interface, and transpiration by plants whose roots are located in the aquifer. The latter two loss mechanisms may be the most important of those listed here and are probably the most difficult to estimate.

The coefficient method (1) is used by the Water Resources Board of Oklahoma to account for losses due to evaporation and transpiration. To calculate exact quantities of water losses due to evapotranspiration over a large area is difficult, if not virtually impossible, because the process requires exacting, precise data. The most difficult part of the analysis is probably obtaining data under controlled conditions over a large study area. This in itself is a problem of great magnitude.

Recharge of the aquifer is effected from percolation of stream water, soil water or water from other aquifers. Under some conditions, the replenishment of the groundwater supply may be accomplished by artificial recharge methods (2).

The coefficient method has been the most common procedure of computing recharge in the 1960's and 1970's. The recharge rate per unit time, Qr, is computed using the equation:

$$Q_r = CiA \qquad (1)$$

where C is a coefficient directly related to the infiltration and percolation of water through the soil layers, i is the intensity of rainfall in inches per hour, A is the area in acres through which infiltration occurs, and Q_r has units of cubic-feet per second.

Therefore, in order to arrive at a more accurate estimate of the recharge rate, the present study has been undertaken. It attempts to supplement the evaluation of Q_r by considerations of the water table fluctuations reflected in water levels in wells. Furthermore, it aims at deriving a recharge rate, Q_r, as a function of all the involved parameters in dimensionless form.

CHAPTER II

LITERATURE REVIEW

The history of hydrologic problems shows that most applications of rainfall-runoff relationships, recharge of groundwater aquifers, and evaporation of water from ponds and lakes are based on a quasi-analytical method because of the general world-wide acceptance of empirical equations and coefficient methods of solutions. Groundwater analyses have not been exempt from this approach. Thus, the expression $Q_r = CiA$ falls in the pattern of this simplistic approach. But by and in itself, does not constitute an accurate estimate of the groundwater available because it lacks the accountability function which relates water table fluctuations to area rainfall volumes. In addition, the expression tacitly indicates that the mathematical solution of a complex flow problem has been simplified by reducing its dimensionality. However, this oversimplification often introduces a large error which has to be adjusted subjectively by its user.

Since it is highly desirable to eliminate the problem of "water mining", the general category of groundwater recharge is a very necessary input to current water supply studies. Mining can cause irreversible physical changes and attendant damage to aquifers, thereby diminishing their recharge capabilities. This event has been legally paraphrased as "a depletion of an aquifer so that unreliable pumping and unavailable water are cause for frustration in subsequent usage" (3). Modern technology furnishes many of these mining and recharge solutions on a macroscopic scale which are based almost entirely on the Darcy equation, the Rational formula, and a simplified form of the Laplace equation. However, since constants are involved, their evaluation implies that a subjective decision has to be a priori. Consequently, most of the problems are solved without the help of a deterministic mathematical model.

Of all possible methods of solutions available, the concept of dimensionless parameters appears to be the one of easiest application to the problem. This method is listed as an explicit goal in a paper by Esmaili, et al. (4) which also includes a summary of the literature available on groundwater recharge. Also, a significant conclusion is reached and it is expressed in the statement "the dimensionless forms of the solutions make possible the application to any problem with similar boundary and initial conditions without any restriction on the value of the aquifer parameters." Finally, the paper states that the need for verification on this experiment had not been done but would need to be accomplished in the very near future. This line of thought seems to be in general agreement with many of the authors at the International Hydrological Conference held at Urbana, Illinois in August, 1969. This included George B. Maxey (5), A. Klute (6), W.C. Ackerman (7), W.C. Walton (8), Jacob Bear (9), J. Amorocho (10), H.N. Holtan and N.C. Lopez (11), R.K. Linsley (12), V. Yevjevich (13), and D.R. Dawdy (14).

Using this approach, it appears that such a mathematical model is applicable to the Southern Great Plains area including Oklahoma and contiguous areas. However, due to the scarcity of accurate field data, most of the previous studies have been inadequate to enable the formulation of a model with the desired degree of accuracy.

As a result of this study, the regression analysis method can be used for predicting groundwater recharge in much the same way that the Froude number, Grashef number, and Weber number have contributed to the understanding of heat transfer and fluid mechanics problems.

CHAPTER III

COLLECTION OF DATA

The data for this study were acquired from several different sources. Data for the five municipal wells belonging to the City of Frederick, Oklahoma, were taken from a report on this same subject matter by Bagdadipour, Harp, and Laguros (15). This was the only city among several contacted that had adequate information to permit the analyses set forth for the present study. Data from wells located near Anadarko and Chickasha in Oklahoma were furnished by the Department of Agriculture, Agricultural Research Service (ARS) in Chickasha, Oklahoma. The data for the two wells located in Kansas were obtained from the U.S. Geological Survey in Lawrence, Kansas.

All the data collected were considered accurate enough to be usable except for two of the four wells in the Kansas area where an unreasonably large recharge area and permeability were obtained. Due to the type of measurement methods and instrumentation used at these two locations, the values of these parameters were not dependable. Although the data were tried, the results obtained indicated that they were not comparable to the data obtained from the other wells. Therefore, they were not included in the final analysis.

The wells analyzed represent different types of usage. Those from the Frederick, Oklahoma area are city water supply wells characterized by a continuous but fluctuating use dependent on season and soil moisture availability. The ARS wells are in farm areas and are not used at all except for pumping tests and measurement of water-table fluctuations. Very little is known about the usage from the Kansas wells but the rates of pumpage suggest that they are probably used for municipal water supply.

The amount of pumpage for each of the wells is shown in Table 1 and their general locations are shown on the map in Figure 1. The data for the static head readings were obtained for each well. The plot for well "B" in the Frederick area is shown in Figure 2 in order to illustrate the seasonal and use-rate fluctuations typically encountered in this study.

The rainfall data obtained from the U.S. Weather Bureau, are given in monthly amounts shown in Table 2. The rainfall given in each case is for the year analyzed.

Other data required for the analysis including soil characteristics, depths of water tables, permeabilities, etc., are shown in Tables 3, 4, and 5.

Data from the High Plains of Texas were considered for use in this analysis. Upon the recommendation of Dr. Don W. Goss, Geologist for the U.S. Department of Agriculture, Bushland, Texas,* these data were deleted because of the lack on influence of precipitation upon the recharge characteristics. The tight soils and sparse rainfall in this region constitute a completely different type of recharge problem.

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* Personal interview, Dr. Don W. Goss, USDA, Southwestern Great Plains Research Center, Bushland, Texas 79012.

TABLE 1. WATER WITHDRAWAL RATES FOR ALL SAMPLE WELLS

Location	Well Identification	Data Period	Total Amount Pumped-Gallons
Frederick, Okla.	A	3-1 to 4-1	199,000
		1-1 to 6-30	2,707,000
		1-1 to 12-31	12,603,000
Frederick, Okla.	B	3-1 to 4-1	457,000
		1-1 to 6-30	1,565,000
		1-1 to 12-31	26,272,000
Frederick, Okla.	C	3-1 to 4-1	134,000
		1-1 to 6-30	3,612,000
		1-1 to 12-31	15,789,000
Frederick, Okla.	D	3-1 to 4-1	1,000
		1-1 to 6-30	2,194,000
		1-1 to 12-31	12,263,000
Frederick, Okla.	E	3-1 to 4-1	1,000
		1-1 to 6-30	2,265,000
		1-1 to 12-31	7,839,000
*Chickasha, Okla.	205	----	----
	213	----	----
	311	----	----
	312	----	----
	314	----	----
	507	----	----
	508	----	----
	509	----	----
Sharon Springs, Kansas	Sharon Springs	1-1 to 12-31	112,425,000
Burdett, Kansas	Burdett	1-1 to 12-31	66,750,000

* Observation wells, not used for water supply purposes.

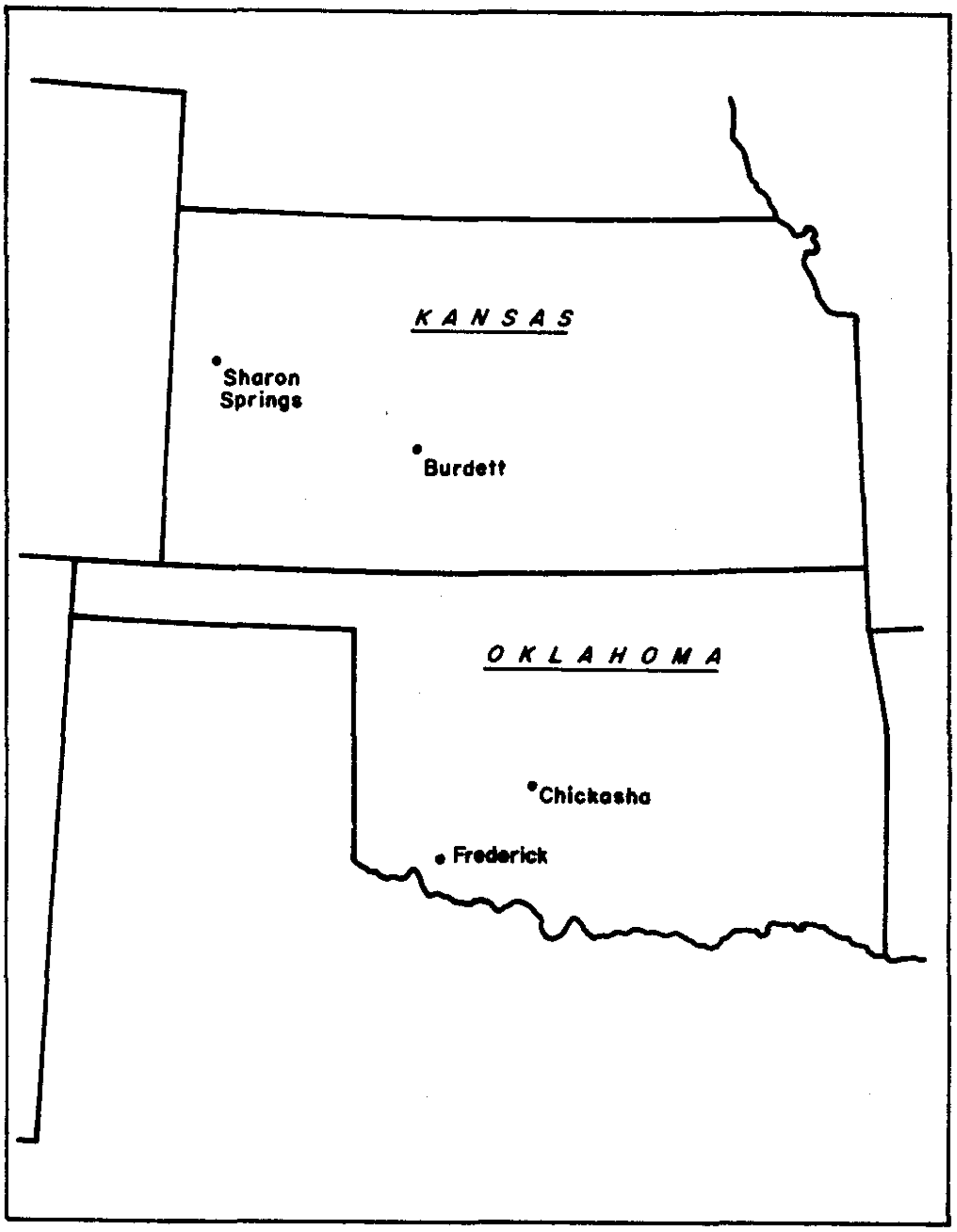

Fig.1.-General location map of observation wells

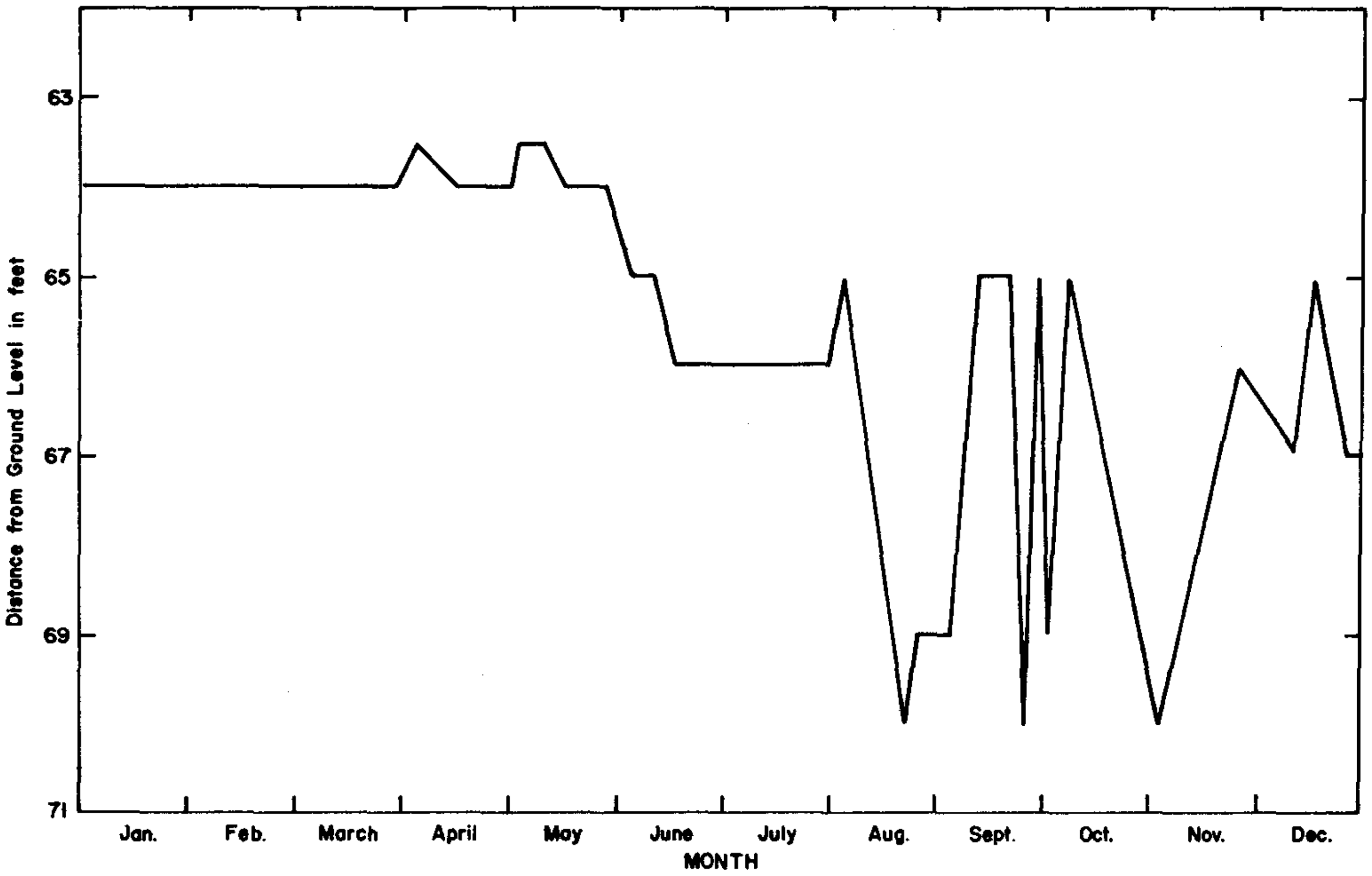

Fig. 2.-Example of typical static water levels for all test wells.

TABLE 2. MONTHLY AND ANNUAL RAINFALL, INCHES

Location	Well Identification	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Annual Total
Frederick, Okla.	A	0.22	2.46	1.98	0.91	4.37	2.62	1.90	3.95	8.43	3.12	0.32	0.69	30.97
Frederick, Okla.	B	0.22	2.46	1.98	0.91	4.37	2.62	1.90	3.95	8.43	3.12	0.32	0.69	30.97
Frederick, Okla.	C	0.22	2.46	1.98	0.91	4.37	2.62	1.90	3.95	8.43	3.12	0.32	0.69	30.97
Frederick, Okla.	D	0.22	2.46	1.98	0.91	4.37	2.62	1.90	3.95	8.43	3.12	0.32	0.69	30.97
Frederick, Okla.	E	0.22	2.46	1.98	0.91	4.37	2.62	1.90	3.95	8.43	3.12	0.32	0.69	30.97
Chickasha, Okla.	205	0.12	1.83	1.99	3.16	5.70	2.84	0.86	2.45	4.28	1.56	0.27	0.87	25.93
Chickasha, Okla.	213	0.12	1.83	1.99	3.16	5.70	2.84	0.86	2.45	4.28	1.56	0.27	0.87	25.93
Chickasha, Okla.	311	0.26	0.04	2.34	5.86	3.07	1.66	2.46	2.12	5.50	1.96	0.55	0.94	26.76
Chickasha, Okla.	312	0.26	0.04	2.34	5.86	3.07	1.66	2.46	2.12	5.50	1.96	0.55	0.94	26.76
Chickasha, Okla.	314	0.26	0.04	2.34	5.86	3.07	1.66	2.46	2.12	5.50	1.96	0.55	0.94	26.76
Chickasha, Okla.	507	0.16	0.07	2.38	5.30	4.67	1.47	3.28	0.66	6.85	2.53	0.44	0.95	28.76
Chickasha, Okla.	508	0.16	0.07	2.38	5.30	4.67	1.47	3.28	0.66	6.85	2.53	0.44	0.95	28.76
Chickasha, Okla.	509	0.16	0.07	2.38	5.30	4.67	1.47	3.28	0.66	6.85	2.53	0.44	0.95	28.76
Sharon Sp. Kansas	Sharon Springs	1.42	2.06	0.70	3.41	1.15	3.31	1.78	0.13	0.50	1.50	0.33	1.06	17.35
Burdett, Kansas	Burdett	1.67	0.72	1.84	1.68	3.44	6.09	1.80	2.10	0.57	5.06	trace	0.62	25.59

TABLE 3. THE KNOWN PARAMETERS OF ALL TEST WELLS FOR ONE MONTH DATA PERIODS

Location	Well Identification	Well Property		K	Parameters				
		Area (ft^2)	R_o(ft)	(gal/day/ft^2)	i (ft)	h_o (ft)	h_w (ft)	Q (ft^3/time)	Q_r (ft^3/time)
Frederick, Okla.	A	635,000	450	1630	0.165	13.0	13.5	26,000	12,550
	B	635,000	450	1530	1.165	14.0	14.4	611,000	12,550
	C	635,000	450	1570	0.165	14.0	14.0	17,900	12,550
	D	282,600	300	1375	0.158	17.5	16.0	700,000	5,380
	E	282,600	300	1913	0.158	13.5	11.5	274,000	5,380
Chickasha, Okla.	205	125,000	200	1548	0.180	66.66	66.56	---	2,225
	213	125,000	200	1548	0.180	26.07	26.22	---	2,225
	311	70,500	150	391	0.1858	42.08	42.14	---	1,375
	312	502,600	400	391	0.1858	60.75	60.70	---	9,800
	314	125,000	200	391	0.1858	5.81	5.74	---	2,435
	507	282,600	300	2315	0.1997	36.13	36.06	---	5,925
	508	282,600	300	2315	0.1997	33.33	33.28	---	5,925
	509	282,600	300	2315	0.1997	32.65	32.61	---	5,925

TABLE 4. THE KNOWN PARAMETERS OF ALL TEST WELLS FOR SIX MONTH DATA PERIODS

Location	Well Identification	Well Property		K	Parameters				
		Area (ft^2)	R_o (ft)	(gal/day/ft^2)	i (ft)	h_o (ft)	h_w (ft)	Q (ft^3/time)	Q_r (ft^3/time)
Frederick, Okla.	A	635,000	450	1630	1.050	13.0	13.5	362,000	80,000
	B	635,000	450	1530	1.050	14.0	12.0	1,548,000	80,000
	C	635,000	450	1570	1.050	14.0	13.2	483,000	80,000
	D	282,600	300	1375	1.050	20.0	17.5	293,000	47,000
	E	282,600	300	1913	1.050	13.5	11.5	303,000	47,000
Chickasha, Okla.	205	125,000	200	1548	1.081	66.76	66.66	---	14,175
	213	125,000	200	1548	1.081	26.04	30.60	---	14,175
	311	70,500	150	391	1.1025	41.97	41.70	---	8,160
	312	502,600	400	391	1.1025	60.64	60.04	---	58,100
	314	125,000	200	391	1.1025	6.09	6.41	---	14,450
	507	282,600	300	2315	1.1708	36.27	35.91	---	34,750
	508	282,600	300	2315	1.1708	33.39	33.11	---	34,750
	509	282,600	300	2315	1.1708	32.63	32.69	---	34,750

TABLE 5. THE KNOWN PARAMETERS OF ALL TEST WELLS FOR TWELVE MONTH DATA PERIODS

Location	Well Identification	Well Property		K (gal/day/ft^2)	Parameters				
		Area (ft2)	R_o (ft)		i (ft)	h_o (ft)	h_w (ft)	Q (ft^3/time)	Q_r (ft^3/time)
Frederick, Okla.	A	635,000	450	1630	2.580	13.0	13.0	1,690,000	196,500
	B	635,000	450	1530	2.580	14.0	11.0	3,520,000	196,500
	C	635,000	450	1570	2.580	14.0	13.5	1,790,000	196,500
	D	282,600	300	1375	2.580	20.0	20.0	1,649,000	84,500
	E	282,600	300	1913	2.580	16.0	16.0	1,050,000	84,500
Chickasha, Okla.	205	125,000	200	1548	2.161	66.76	67.16	----	28,350
	213	125,000	200	1548	2.161	26.04	27.35	----	28,350
	311	70,500	150	391	2.2300	41.97	41.80	----	16,500
	312	502,600	400	391	2.2300	60.64	60.57	----	117,650
	314	125,000	200	391	2.2300	6.09	5.10	----	29,250
	507	282,600	300	2315	2.3966	36.27	35.45	----	71,100
	508	282,600	300	2315	2.3966	33.39	32.90	----	71,100
	509	282,600	300	2315	2.3966	32.63	32.45	----	71,100
Sharon Spr. Kansas	Sharon Springs	13,854,000	2,100	845	1.4458	236.0	201.1	112,425,000	220,331,000
Burdett, Kansas	Burdett	2,010,000	800	2040	2.1325	61.0	46.5	66,750,000	47,150,000

CHAPTER IV

METHOD OF ANALYSIS

The previous analysis by Bagdadipour, Harp and Laguros (15), began by determining the relevant parameters considered in computing the recharge of producing aquifers. These parameters were then grouped into dimensionless terms such that all terms would be interrelated into a minimum number of "dimensionless parameters."

In this study, these parameters were reinvestigated and found to be sound. The final part of the study was to extend a multiple regression analysis so as to include the additional data and to find a prediction equation for the recharge capabilities of a well.

The involved parameters, shown in Figure 3, were:

Q = pumpage flow rate, cubic feet per unit of time.

Q_e= effluent flow rate, cubic feet per unit of time.

Q_i= influent flow rate, cubic feet per unit of time.

Q_r= recharge flow rate, cubic feet per unit of time.

A = the surface area contributing to recharge, square feet.

i = rainfall intensity, feet per unit of time.

h_o= piezometric head at beginning time, feet.

h_w= piezometric head at time "t", feet.

K = permeability in gallons per day per square feet

R_o= radial distance corresponding to h_o, feet

In the case of the wells under study, there is negligible seepage,

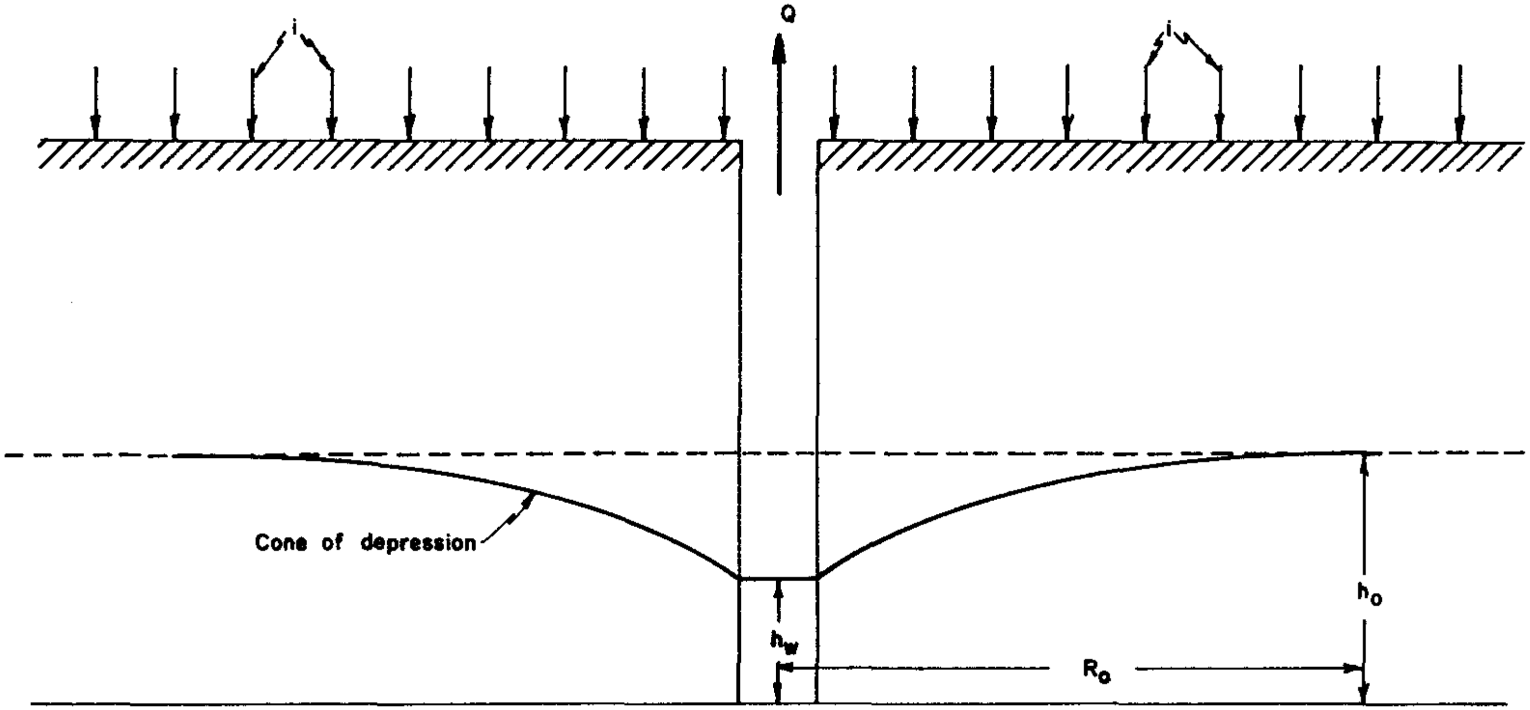

Fig.3.-Definition of variables for unsteady radial flow in an unconfined aquifer with a recharge rate "i" and a discharge rate "Q".

influent or effluent, indicated by the static head levels of the well. The piezometric head, hereafter referred to as head, remained relatively constant for periods when there was no pumpage but very small amounts of precipitation were observed; therefore, it was concluded that seepage, if any, was so small that it was negligible. Deleting these factors, then, the problem reduces to one of the functional form of:

$$Q_r = f\ (Q, A, i, h_o, h_w, K, R_o). \tag{2}$$

Using the Pi Theorem (16) to reduce these terms to dimensionless parameters, the following significant equations can be derived:

$$X = h_o/h_w \tag{3}$$

$$Y = Q_r/iA \tag{4}$$

$$Z = Q/KR_o{}^2. \tag{5}$$

Thus, the problem becomes one of relating Y, which contains the recharge parameter, as a function of X and/or Z. Using the multiple regression analysis technique the relationship assumes the form:

$$Y = B_o + B_1X_1 + B_2X_2 + \ldots + B_iX_i. \tag{6}$$

The use of a least-squares regression analysis is recognized as a statistical method that gives the best fit for a straight line equation while minimizing the residual sum of squares and variance of the parameter.

At this point, some consideration must be given to a nonlinear relationship between these parameters. There are two possible choices of analysis. First, an attempt can be made to make a direct nonlinear fit of the data; second, the data may be transformed in such a way that the relationships become linear or approximately linear. If the latter method is used, the regression analysis can still be applied. The transformation methods used are either logarithmic or reciprocal or both (17).

The residuals of these equations are shown on the IBM 360 computer printouts reproduced in Appendices II, III, and IV. These values list the difference between the actual data and the predicted values of Q_r/iA for each of the periods investigated and for each data point used. A close inspection of these values will show a small amount of error due to the prediction of values for the Q_r/iA variable using the equations previously indicated. A large deviation is shown in the Kansas data which is probably due to geomorphological differences of soil layers and measured soil characteristics. The measurements in the Chickasha and Frederick areas appear to be well instrumented and documented, while those of the Kansas area were not as extensive and complete as the other data.

While all the final equations take into consideration the amount of pumpage, Q, it is stated that not all holes had actual pumpage. If there was no pumpage, those terms of the equation associated with pumpage, (Q/KR_o^2) were equated to zero. By setting Q equal to zero in Equation 13, presented later in this report, an estimate of the recharge can be computed. This would indicate recharge or discharge only by the fluctuations of the water table levels.

In areas where there was pumpage, a negative value of Q_r/iA would show that "mining" was taking place. However, due to the limited length of data that are available, i.e., one to four years, these estimates indicate only the annual trends which may or may not concur with long-term (10 year or more) trends.

In the previous study (15), the four forms of equations investigated using regression analysis were:

$$Y = B_o + B_1X + B_2Z \tag{7}$$

$$\ln (Y) = B_o + B_1\ln (X) + B_2\ln (Z) \tag{8}$$

$$\ln (Y) = B_o + B_1X + B_2Z \tag{9}$$

$$1/\ln (Y) = B_o + B_1\ln (X) + B_2\ln (Z) \tag{10}$$

A commercial regression analysis program for a digital computer gives many statistical results including mean, variance, correlation, residuals, etc. The method employed herein to determine the "best-fit" was to select the analysis that gave the highest coefficient of multiple correlation, R^2.

If the ratio of the regression mean square to the residual mean square, or F-value, is significant, it is indicative that the regression coefficient take into account more of the variance in data than one would expect to be taken into account by chance alone in identical conditions and times. Using a five percent significance level, the F-test was employed to justify the use of each parameter in the regression equation. Following the method of the previous study, an equation was accepted as satisfactory when the F-ratio for the observed data were not greater than four times the selected percentage points of the F-distribution.

The residuals shown in Appendices II, III, and IV are the difference between the observed and the predicted values using the regression coefficients of the dependent variables. The foremost assumptions about the residual in a least-squares analysis, are that the data points are independent with constant variance and a mean equal to zero. Also, when an F-test is used, these points will give a normal distribution. A prediction equation or "model" is usually taken as being correct if all the above assumptions are satisfied (18).

While testing for correctness of the assumptions of constant variance and normality, no evidence was found to indicate anything to the contrary. The least-squares method is designed to give a sum of residuals equal to zero; therefore, no check is necessary on this point.

The values of the independent variables collected are shown in Table 3. The value of Q_r, the dependent variable used in the prediction equation, was calculated using the coefficient method of Equation 1. The C values used were 11.5 percent for the Frederick well, 10.5 percent for the Chickasha wells, and 11.0 percent for the Kansas wells. These values were characteristic of the infiltration coefficient for each soil type. The area used for each equation was found by the Thiesen weighting method (19).

Finally, by using a scientific subroutine for the IBM 360/50 computer the data were subjected to a regression analysis. Grouping the data in like time groups, separate runs were made of each group for each equation. Thus, for the one month period equation 10, for the six month period equation 8, and for the 12 month period equation 9 gave the best fit. The numerical forms of these equations are respectively, equations 11, 12, and 13 as presented below:

$$\frac{1}{\ln (Q_r/iA)} = -0.47862 + 0.00153 \ln (h_o/h_w) - 0.00093 \ln (Q/KR_o^2) \tag{11}$$

$$\ln (Q_r/iA) = -2.05622 + 0.58363 \ln (h_o/h_w) + 0.00453 \ln (Q/KR_o^2) \tag{12}$$

$$\ln (Q_r/iA) = -1.86867 - 0.56420 (h_o/h_w) + 107.44388 (Q/KR_o^2) \tag{13}$$

CHAPTER V

DISCUSSION OF RESULTS

The values obtained by using the least-squares approach are shown in Table 6.

The "best-fit" for the data was selected on the basis of the highest coefficient of multiple correlation. This value is the greatest, 0.96712, for the one month period using Equation 11. The F-test value required at a 95-percent confidence interval is 3.89. Since the data used give an F-value of 72.31235, it is concluded that Equation 11 is significant in predicting recharge rates applicable to the south-central Great Plains area on a monthly basis. However, caution should be used since these data are average monthly data. It was chosen because this is a period of near mean soil moisture for the problem area described.

Using the rationale for choice of "best-fit", the six-month data, i.e., the period from January 1 to June 30, yields Equation 12. The multiple correlation of 0.88674 and an F-value of 18.39871 indicate the utility value of this equation for predicting recharge rates for the first half of the calendar year.

The "best" equation to use would be one that would encompass all months, all seasons and all moisture conditions pertinent within a given length of time. For this type of application, the data listed in Table 5 as the 12 month or annual values use a one calendar year period of investi-

TABLE 6. VALUES FOR R, F, INTERCEPTS, AND COEFFICIENTS

Equation No.	Period (Months)	R Value	F Value	B_0	B_1	B_2
7	1	0.58301	2.57456	0.09511	0.01245	2.86722
	6	0.81092	9.60235	0.00189	0.10854	1.69876
	12	0.92210	34.07259	0.91024	-1.34104	259.17041
8	1	0.96196	61.98625	-2.08660	-0.00897	0.00447
	6	0.88674	18.39871	-2.05622	0.58363	0.00453
	12	0.67877	5.12615	-1.59420	8.70076	0.02126
9	1	0.57853	2.51536	-2.34301	0.11105	25.71475
	6	0.81592	9.95182	-3.02017	0.80919	17.84193
	12	0.92323	34.63809	-1.86867	-0.56420	107.44388
10	1	0.96712	72.31235	-0.47862	0.00153	-0.00093
	6	0.86931	15.46605	-0.48689	-0.14953	-0.00098
	12	0.64331	4.23636	-0.36790	1.66171	0.00283

gation and give a good estimate, as indicated by the R-value of 0.92323 and an F-value of 34.63809, when transformed to Equation 13. Figure 4 shows Equation 13 as a graphical solution for a constant (h_o/h_w) value.

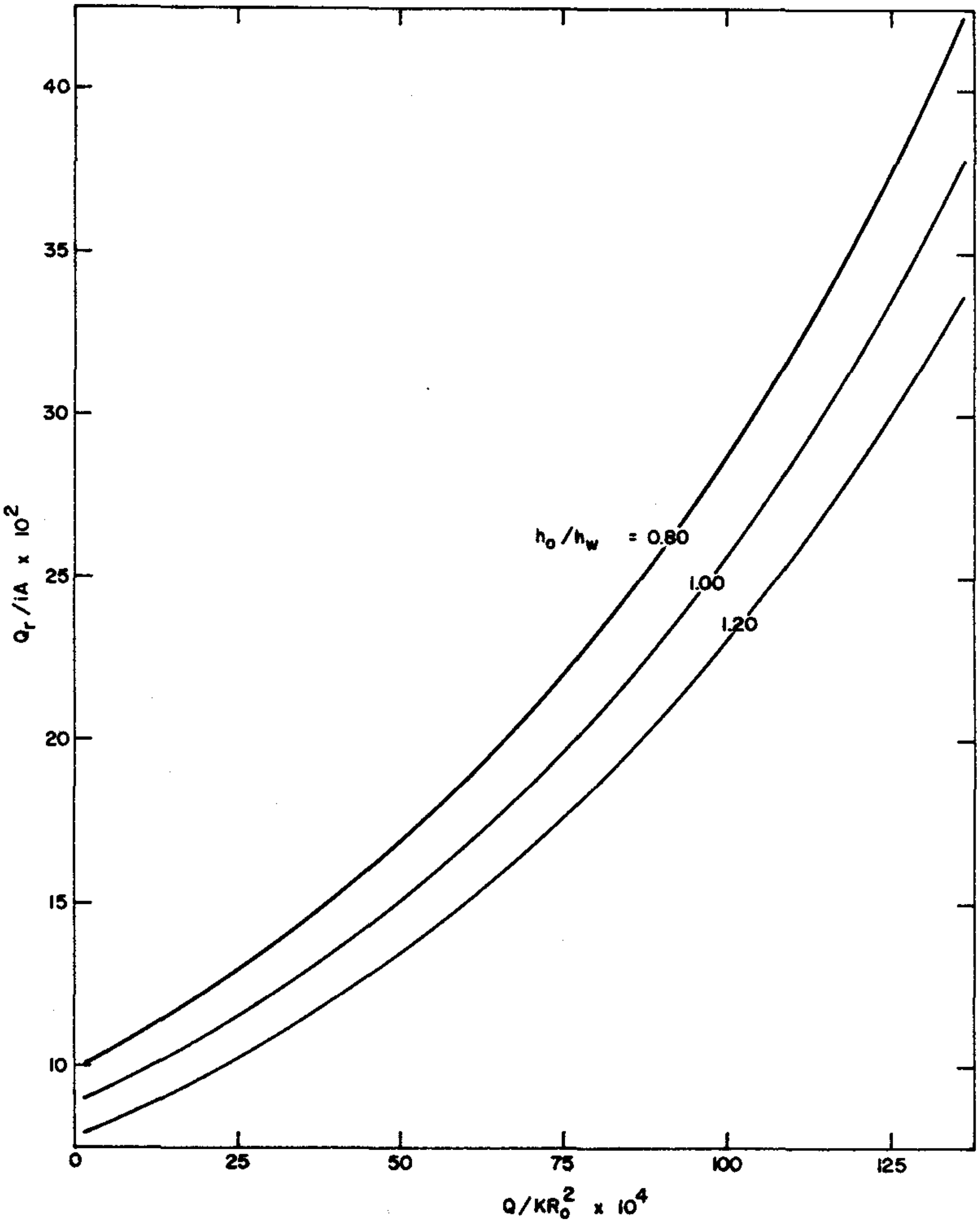

Fig.4.-A graphical solution of Equation 13 using constant values of h_o/h_w.

CHAPTER VI

SUMMARY AND CONCLUSIONS

Review of previous work indicated that the best method to compute groundwater recharge rates, as accurately as possible, was to take a two-fold approach. First, to consider recharge as a parameter in the total underground water budget thus expressing the recharge in terms of water table fluctuations which, when measured as water levels in wells, reflected water losses and/or withdrawals. Second, to derive a dimensionless parameter method for the actual prediction made.

The most pertinent problem of this study has been that of finding data that were both accurate and reliable enough to evaluate the models.

The parameters of each well were then grouped into dimensionless terms derived by the Pi Theorem method and were subjected to a multiple regression analysis using the variable Q_r/iA as the dependent term and the other terms, h_o/h_w and Q/KR_o^2 as independent variables.

The analysis of the available data produced the following significant results:

1. Of the equations studied, the ones with the highest correlation values are:

$$\frac{1}{\ln (Q_r/iA)} = -0.47862 + 0.00153 \ln (h_o/h_w) - 0.00093 \ln (Q/KR_o^2)$$

using one month values,

$$\ln (Q_r/iA) = -2.05622 + 0.58363 \ln (h_o/h_w) + 0.00453 \ln (Q/KR_o^2)$$

using six months values, and

$$\ln (Q_r/iA) = -1.86867 - 0.56420 (h_o/h_w) + 107.44388 (Q/KR_o^2)$$

using 12 month or annual values.

2. The mathematical equations given here help predict the net amount of discharge or recharge of a groundwater aquifer. They also indicate a truer quantity of groundwater available for use than the old coefficient method currently used.
3. If there is no pumpage, the recharge can be computed directly from Equation 13.
4. "Mining" can be confirmed when a negative value of Q_r/iA results from using the equations established herein.
5. Results from this study indicate that this method could easily be adapted for use in any area where the same contributing factors are significant, especially in the south central Great Plains areas.

Recommendations for further research include:

1. Better defined limits, therefore establishing the extent of applicability.
2. Adoption to local areas.
3. Investigation of errors in using this technique and how they can be minimized.

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APPENDIX I; SAMPLE REGRESSION PROGRAM FOR IBM 360

```
C
C     ...............................................................
C
C        SAMPLE MAIN PROGRAM FOR MULTIPLE REGRESSION - REGRE
C
C        PURPOSE
C           (1) READ THE PROBLEM PARAMETER CARD FOR A MULTIPLE REGRES-
C           SION, (2) READ SUBSET SELECTION CARDS, (3) CALL THE SUB-
C           ROUTINES TO CALCULATE MEANS, STANDARD DEVIATIONS, SIMPLE
C           AND MULTIPLE CORRELATION COEFFICIENTS, REGRESSION COEFFI-
C           CIENTS, T-VALUES, AND ANALYSIS OF VARIANCE FOR MULTIPLE
C           REGRESSION, AND (4) PRINT THE RESULTS.
C
C        REMARKS
C           THE NUMBER OF OBSERVATIONS, N, MUST BE GREATER THAN M+1,
C           WHERE M IS THE NUMBER OF VARIABLES.  IF SUBSET SELECTION
C           CARDS ARE NOT PRESENT, THE PROGRAM CAN NOT PERFORM MULTIPLE
C           REGRESSION.
C           AFTER RETURNING FROM SUBROUTINE MINV, THE VALUE OF DETER-
C           MINANT (DET) IS TESTED TO CHECK WHETHER THE CORRELATION
C           MATRIX IS SINGULAR.  IF DET IS COMPARED AGAINST A SMALL
C           CONSTANT, THIS TEST MAY ALSO BE USED TO CHECK NEAR-
C           SINGULARITY.
C
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C           CORRE  (WHICH, IN TURN, CALLS THE SUBROUTINE NAMED DATA)
C           ORDER
C           MINV
C           MULTR
C
C        METHOD
C           REFER TO B. OSTLE, 'STATISTICS IN RESEARCH', THE IOWA STATE
C           COLLEGE PRESS', 1954, CHAPTER 8.
C
C     ...............................................................
C
C     THE FOLLOWING  DIMENSIONS MUST BE GREATER THAN OR EQUAL TO THE
C     NUMBER OF VARIABLES, M..
C
      DIMENSION XBAR(40),STD(40),D(40),RY(40),ISAVE(40),B(40),
     1          SB(40),T(40),W(40)
C
C     THE FOLLOWING DIMENSION MUST BE GREATER THAN OR EQUAL TO THE
C     PRODUCT OF M*M..
C
      DIMENSION RX(1600)
C
C     THE FOLLOWING DIMENSION MUST BE GREATER THAN OR EQUAL TO
C     (K+1)*K/2..
C
      DIMENSION R(820)
C
C     THE FOLLOWING DIMENSION MUST BE GREATER THAN OR EQUAL TO 10..
C
      DIMENSION ANS(10)
C
C     ...............................................................
C
C        IF A DOUBLE PRECISION VERSION OF THIS ROUTINE IS DESIRED, THE
C        C IN COLUMN 1 SHOULD BE REMOVED FROM THE DOUBLE PRECISION
C        STATEMENT WHICH FOLLOWS.
C
C     DOUBLE PRECISION XBAR,STD,RX,R,D,B,T,RY,DET,SB,ANS,SUM
C
C        THE C MUST ALSO BE REMOVED FROM DOUBLE PRECISION STATEMENTS
C        APPEARING IN OTHER ROUTINES USED IN CONJUNCTION WITH THIS
C        ROUTINE.
C
C        ............................................................
C
    1 FORMAT(A4,A2,I5,2I2)
    2 FORMAT(25H1MULTIPLE REGRESSION.....,A4,A2//6X,14HSELECTION.....I2//
     1)
    3 FORMAT(9H0VARIABLE,5X,4HMEAN,6X,8HSTANDARD,6X,11HCORRELATION,4X,10
     1HREGRESSION,4X,10HSTD. ERROR,5X,8HCOMPUTED/6H    NO.,18X,9HDEVIATIO
     2N,7X,6HX VS Y,7X,11HCOEFFICIENT,3X,12HOF REG.COEF.,3X,7HT VALUE)
    4 FORMAT(1H ,I4,6F14.5)
    5 FORMAT(10H DEPENDENT)
    6 FORMAT(1H0/10H INTERCEPT,10X,F16.5//23H MULTIPLE CORRELATION  ,F13
     1.5//23H STD. ERROR OF ESTIMATE,F13.5//)
    7 FORMAT(1H0,21X,39HANALYSIS OF VARIANCE FOR THE REGRESSION//5X,19HS
     1OURCE OF VARIATION,7X,7HDEGREES,7X,6HSUM OF,10X,4HMEAN,12X,7HF VAL
     2UE/30X,10HOF FREEDOM,4X,7HSQUARES,9X,7HSQUARES)
    8 FORMAT(30H ATTRIBUTABLE TO REGRESSION    ,I6,3F16.5/30H DEVIATION F
     1ROM REGRESSION    ,I6,2F16.5)
    9 FORMAT(1H ,5X,5HTOTAL,19X,I6,F16.5)
   10 FORMAT(36I2)
   11 FORMAT(1H ,15X,18HTABLE OF RESIDUALS//9H CASE NO.,5X,7HY VALUE,5X,
     110HY ESTIMATE,6X,8HRESIDUAL)
   12 FORMAT(1H ,I6,F15.5,2F14.5)
   13 FORMAT(53H0NUMBER OF SELECTIONS NOT SPECIFIED.  JOB TERMINATED.)
   14 FORMAT(52H0THE MATRIX IS SINGULAR.  THIS SELECTION IS SKIPPED.)
C
C     ...............................................................
C
C     READ PROBLEM PARAMETER CARD
C
  100 READ (5,1) PR,PR1,N,M,NS
C        PR.........PROBLEM NUMBER (MAY BE ALPHAMERIC)
C        PR1........PROBLEM NUMBER (CONTINUED)
C        N..........NUMBER OF OBSERVATIONS
C        M..........NUMBER OF VARIABLES
C        NS.........NUMBER OF SELECTIONS
C
C     LOGICAL TAPE 13 IS USED AS INTERMEDIATE STORAGE TO HOLD INPUT
C     DATA.  THE INPUT DATA ARE WRITTEN ON LOGICAL TAPE 13 BY THE
C     SPECIAL INPUT SUBROUTINE NAMED DATA.  THE STORED DATA MAY BE USED
C     FOR RESIDUAL ANALYSIS.
C
      REWIND 13
C
      IO=0
      X=0.0
C
      CALL CORRE (N,M,IO,X,XBAR,STD,RX,R,D,B,T)
C
      REWIND 13
C
C     TEST NUMBER OF SELECTIONS
C
      IF(NS) 108, 108, 109
  108 WRITE (6,13)
      GO TO 300
C
  109 DO 200 I=1,NS
      WRITE (6,2) PR,PR1,I
C
C     READ SUBSET SELECTION CARD
C
      READ (5,10) NRESI,NDEP,K,(ISAVE(J),J=1,K)
C        NRESI.....OPTION CODE FOR TABLE OF RESIDUALS
C                  0  IF IT IS NOT DESIRED.
C                  1  IF IT IS DESIRED.
C        NDEP......DEPENDENT VARIABLE
C        K.........NUMBER OF INDEPENDENT VARIABLES INCLUDED
C        ISAVE.....A VECTOR CONTAINING THE INDEPENDENT VARIABLES
C                     INCLUDED
C
      CALL ORDER (M,R,NDEP,K,ISAVE,RX,RY)
C
      CALL MINV (RX,K,DET,B,T)
C
C     TEST SINGULARITY OF THE MATRIX INVERTED
C
      IF(DET) 112, 110, 112
  110 WRITE (6,14)
      GO TO 200
C
  112 CALL MULTR (N,K,XBAR,STD,D,RX,RY,ISAVE,B,SB,T,ANS)
C
C     PRINT MEANS, STANDARD DEVIATIONS, INTERCORRELATIONS BETWEEN
C     X AND Y, REGRESSION COEFFICIENTS, STANDARD DEVIATIONS OF
C     REGRESSION COEFFICIENTS, AND COMPUTED T-VALUES
C
      MM=K+1
      WRITE (6,3)
      DO 115 J=1,K
      L=ISAVE(J)
  115 WRITE (6,4) L,XBAR(L),STD(L),RY(J),B(J),SB(J),T(J)
      WRITE (6,5)
      L=ISAVE(MM)
      WRITE (6,4) L,XBAR(L),STD(L)
C
C     PRINT INTERCEPT, MULTIPLE CORRELATION COEFFICIENT, AND STANDARD
C     ERROR OF ESTIMATE
C
      WRITE (6,6) ANS(1),ANS(2),ANS(3)
C
C     PRINT ANALYSIS OF VARIANCE FOR THE REGRESSION
C
      WRITE (6,7)
      L=ANS(8)
      WRITE (6,8) K,ANS(4),ANS(6),ANS(10),L,ANS(7),ANS(9)
      L=N-1
      SUM=ANS(4)+ANS(7)
      WRITE (6,9) L,SUM
      IF(NRESI) 200, 200, 120
C
C     PRINT TABLE OF RESIDUALS
C
  120 WRITE (6,2) PR,PR1,I
      WRITE (6,11)
      MM=ISAVE(K+1)
      DO 140 II=1,N
      READ (13) (W(J),J=1,M)
      SUM=ANS(1)
      DO 130 J=1,K
      L=ISAVE(J)
  130 SUM=SUM+W(L)*B(J)
      RESI=W(MM)-SUM
  140 WRITE (6,12) II,W(MM),SUM,RESI
      REWIND 13
  200 CONTINUE
      GO TO 100
  300 CONTINUE
      END
```

APPENDIX II: DATA OUTPUT FOR ONE MONTH VALUES

```
VARIABLE      MEAN        STANDARD       CORRELATION     REGRESSION     STD. ERROR      COMPUTED
  NO.                     DEVIATION        X VS Y        COEFFICIENT    OF REG.COEF.    T VALUE
  1          0.01508       0.05259       -0.35977         0.00153        0.02676        0.05724
  2        -27.95786      16.94847       -0.96711        -0.00093        0.00008      -11.16293
DEPENDENT
  3         -0.45268       0.01621


INTERCEPT                   -0.47862

MULTIPLE CORRELATION         0.96712

STD. ERROR OF ESTIMATE       0.00452


                        ANALYSIS OF VARIANCE FOR THE REGRESSION

    SOURCE OF VARIATION         DEGREES       SUM OF           MEAN              F VALUE
                               OF FREEDOM     SQUARES          SQUARES
ATTRIBUTABLE TO REGRESSION         2          0.00295          0.00148         72.31235
DEVIATION FROM REGRESSION         10          0.00020          0.00002
    TOTAL                         12          0.00315



                        TABLE OF RESIDUALS

               CASE NO.     Y VALUE       Y ESTIMATE       RESIDUAL
                  1         -0.47123      -0.46994         -0.00129
                  2         -0.47123      -0.47289          0.00166
                  3         -0.47123      -0.46955         -0.00168
                  4         -0.47255      -0.47368          0.00113
                  5         -0.47255      -0.47240         -0.00015
                  6         -0.44367      -0.44079         -0.00288
                  7         -0.44367      -0.44080         -0.00287
                  8         -0.44367      -0.44080         -0.00287
                  9         -0.44364      -0.44080         -0.00284
                 10         -0.44359      -0.44080         -0.00279
                 11         -0.44340      -0.44078         -0.00262
                 12         -0.43220      -0.44080          0.00860
                 13         -0.43220      -0.44081          0.00861
```

APPENDIX III; DATA OUTPUT FOR SIX MONTH VALUES

```
VARIABLE      MEAN       STANDARD       CORRELATION    REGRESSION     STD. ERROR     COMPUTED
  NO.                    DEVIATION        X VS Y       COEFFICIENT    OF REG.COEF.   T VALUE
   1        0.03545       0.11938        0.79220        0.58363        0.24602       2.37225
   2      -27.49130      17.52397        0.81613        0.00453        0.00166       2.72554
DEPENDENT
   3       -2.16006       0.15200

INTERCEPT                   -2.05622

MULTIPLE CORRELATION         0.88674

STD. ERROR OF ESTIMATE       0.07697


                ANALYSIS OF VARIANCE FOR THE REGRESSION

     SOURCE OF VARIATION        DEGREES      SUM OF         MEAN            F VALUE
                               OF FREEDOM    SQUARES        SQUARES
ATTRIBUTABLE TO REGRESSION         2         0.21800        0.10900        18.39871
DEVIATION FROM REGRESSION         10         0.05924        0.00592
      TOTAL                       12         0.27725


                          TABLE OF RESIDUALS

             CASE NO.       Y VALUE       Y ESTIMATE       RESIDUAL
                 1         -2.12038       -2.10912        -0.01126
                 2         -2.12038       -1.99026        -0.13012
                 3         -2.12038       -2.05128        -0.06910
                 4         -1.84267       -2.00568         0.16301
                 5         -1.84267       -1.89221         0.04954
                 6         -2.25354       -2.23522        -0.01932
                 7         -2.25354       -2.23612        -0.01742
                 8         -2.25354       -2.24210        -0.01143
                 9         -2.25354       -2.23727        -0.01667
                10         -2.25520       -2.23524        -0.01996
                11         -2.25519       -2.27052         0.01573
                12         -2.25472       -2.24017        -0.01455
                13         -2.25472       -2.33521         0.08049
```

APPENDIX IV: DATA OUTPUT FOR ANNUAL VALUES

```
VARIABLE      MEAN       STANDARD        CORRELATION      REGRESSION     STD. ERROR       COMPUTED
  NO.                    DEVIATION         X VS Y         COEFFICIENT    OF REG.COEF.     T VALUE
   1        1.06561        0.11318        0.64345         -0.56420        2.29651        -0.24564
   2        0.00819        0.01444        0.92283        107.44388       18.00119         5.96871
DEPENDENT
   3       -1.59018        1.63178


INTERCEPT                    -1.86667

MULTIPLE CORRELATION          0.92323

STD. ERROR OF ESTIMATE        0.67724


                  ANALYSIS OF VARIANCE FOR THE REGRESSION

    SOURCE OF VARIATION        DEGREES       SUM OF           MEAN             F VALUE
                              OF FREEDOM     SQUARES          SQUARES
ATTRIBUTABLE TO REGRESSION        2          31.77388         15.88694         34.63809
DEVIATION FROM REGRESSION        12           5.50386          0.45865
     TOTAL                       14          37.27774



                           TABLE OF RESIDUALS

               CASE NO.      Y VALUE      Y ESTIMATE       RESIDUAL
                   1        -2.12075       -1.88276        -0.23799
                   2        -2.12075       -1.36619        -0.75456
                   3        -2.12075       -1.84886        -0.27189
                   4        -2.15507       -1.00172        -1.15335
                   5        -2.15507       -1.77854        -0.37653
                   6        -2.25399       -2.44593         0.19194
                   7        -2.25399       -2.44128         0.18729
                   8        -2.25399       -2.43600         0.18201
                   9        -2.25425       -2.43517         0.18092
                  10        -2.25408       -2.43352         0.17944
                  11        -2.25443       -2.54239         0.28796
                  12        -2.25425       -2.42951         0.17526
                  13        -2.25425       -2.40585         0.15160
                  14         2.39654        0.71069         1.68585
                  15         2.45633        2.88437        -0.42804
```