BINARY POPULATION GROWTH PROCESSES

Richard G. Fowler
Department of Physics, University of Oklahoma, Norman, Oklahoma 73019

The mathematical basis of the previously demonstrated empirical binary theory of human population growth is explored, and applied to certain special situations in which changes occurred historically or are still occurring. For application to predictions the practical limitations on the growth constants are also considered. A new answer is given to the question of how many hominoids have ever been born.

INTRODUCTION

It has been shown in a previous paper (1) by empirical analysis that the measures and estimates of world population obey an equation of increase that depends on the square of the extant population rather than the first power. The latter would be expected if growth were exponential (or equivalently geometric or doubling), as it is commonly supposed to be. The differential equation for the revised growth behavior takes the form

$$dN/dt = \alpha N^2 \qquad (1)$$

where N is population and α is the growth constant. It was further suggested in the previous paper that there is a logical basis for this equation in terms of general social support for the survival of the individual.

The differential equation has as its solution the reciprocal relation

$$\alpha N = (t\infty - t)^{-1} \qquad (2)$$

where t ∞ is the indicated date at which N is tending toward infinity (singularity). Values of α and t ∞ from the empirical studies are given in Table 1. These relations have applied with almost incredible accuracy since +1600 (Period II), and with a single change of constants fit the data as accurately as the data base is consistent with itself back into the mists of antiquity (Period I).

Equation 1 bears a superficial resemblance to the logistic equation, but it is not even conceptually similar. In the logistic equation α is negative, and N^2 is regarded as a mere correction on the first-power exponential growth term. It will later be shown that the binary equation is a description of a natural equilibrium state in population change, but when generalized to cases in which equilibrium is temporarily disturbed, as by a mighty plague, it may even briefly take the logistic form. It is the purpose of this paper, on the assumption of the validity of the empirical law developed in the previous paper, to discuss these questions of non-equilibrium in detail, with applications to both global transient phenomena and local situations.

Mathematical Considerations

World population changes in size because of a disparity between birth and death rates. Local populations change because of migrations also, which as far as population change is concerned merely alter the birth and death rates into influx and efflux rates. The success of Eq. 1 in modelling growth suggests immediately that we are actually dealing with a more general equation of the form

$$dN/dt = [B(N,t) - D(N,t)] N \qquad (3)$$

where B (N,t) and D (N,t) are birth (or influx) and death (or efflux) rate functions of population and time. If we expend both functions as a series in powers of N, we can express the equation as

$$dN/dt = (\beta_1 N + \beta_2 N^2 + \beta_3 N^3 ..) - (\delta_1 N + \delta_2 N^2 + \delta_3 N^3 ..) \qquad (4)$$

Now inquiry can be directed to the implications of the various terms, and their relative importance. The problem of the meaning of the terms has previously been partially addressed, but will be considered in greater depth here. Each of the coefficients of the powers of N may be an evanescent function of the time during the epoch following sporadic events such as floods, famines, plagues, and earthquakes.

TABLE 1. *Fitting constants for population data.*

Epoch	α	$t\infty$
+1978 to +1600	5.08×10^{-12}	2024.6
+1600 to —8000	2.10×10^{-12}	2591.5

On the basis of previously presented evidence for the empirical success of Eqs. 1 and 2, we can conclude that for averages of a few generations at a time during Period I, and almost from moment to moment during Period II, the following relations between the coefficients have held:

$$\beta_2 > \delta_2$$
$$\beta_1 - \delta_1 << (\beta_2 - \delta_2)\ N \tag{6}$$
$$\beta_2 - \delta_2 >> (\beta_n - \delta_n)\ N^{n-1} \qquad n > 2$$

Exactly how much these coefficients of the other powers are dominated by the second-degree coefficients can only be found by solving the differential equation with them present as correction terms, and then determining how small they must be at the extremes of the ranges of application to prevent them from modifying the result within the experimental error of the points. We now proceed to do this.

Let us first consider the effect of a linear term on the basic binary equation. If we set $\beta_2 - \delta_2 = \alpha$, and $\beta_1 - \delta_1 = \lambda$, then the solution is:

$$1/N = (\alpha/\lambda)\ [(1 + \lambda/\alpha\ N_0)\ \exp\ (-\lambda(t-t_0)) - 1] \tag{7}$$

where $N = N_0$ at $t = t_0$ Three distinct styles of behavior occur. (1) N declines exponentially to zero if λ is negative provided also that either α is negative, or α is positive *and* $\alpha\ N_0$ is less than the absolute value of λ. (2) N becomes infinite at time $t = t \infty$ if $\alpha > 0$ and either $\lambda > 0$ or $\alpha\ N_0 > |\lambda|$. (3) N approaches a stable saturation value equal to $\lambda/|\alpha|$ if $\lambda > 0$ and $\alpha < 0$. Case 1 describes the extermination of a species endangered by external attack and with either insufficient or negative intraspecies support. Case 3 is the logistic behavior in which an exponentially multiplying species is limited by the negative effects of its own numbers. Case 2, however, is the one which seems to describe our million years of human proliferation, and is the one we must investigate to answer the question posed at the beginning of the exercise about the historic size of the coefficient of the linear term.

The average deviation from the binary law for all data points lying between +1600 and +1978 is 4.3%. From Eq. 7 we can discern that the value of $[\lambda]$ which could not have been detected within this degree of data fluctuation is 1.5×10^{-6}. In Period I, - 8000 to +1600, the average deviation is 12%. The largest steady value of λ which could not have been detected over this range is 3×10^{-6}. Considering that the spontaneous death rate is roughly the reciprocal of life expectancy, so that δ_1 has run generally at about .02, then β_1 has been equal to δ_1 to better than .01%. Exponential growth has thus not been a significant part of world population change for the past ten thousand years at least, and it is the author's opinion that it has not been present for a million years. This will be made plausible when Fig. 1 is discussed.
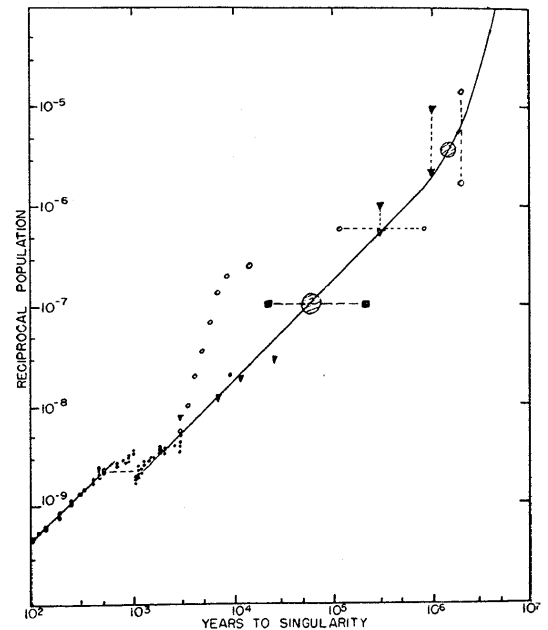

FIGURE 1. Reciprocal population estimate logarithms plotted against the logarithms of the time between the date and the date of population singularity. Modern and ancient data trend toward different singularities accounting for the offset at $t = 1000$. Solid curve from $t = 1000$ back to $t = 5.5$ million is calculated from Eq. 11.

The recent population estimates of McEvedy and Jones (2) for the period 0 to -8000 deviate systematically from those of all other writers, each of whose numbers center very well around the binary law's best fit curve. We can also use Eq. 7 to

determine what value of λ is needed to follow the trend they propose. It is 6×10^{-4}, not an impossible value, although much higher than applied above, and much higher than could have been present even in Australopithecene times. It is therefore probably unreasonable and if continued unabated backward in time, would have removed man from the earth by -30000. Thus McEvedy and Jones were forced to cut off exponential growth in the year -7000 and connect the prevailing population with that of remote antiquity (ca. -10^{-6}) by assuming population stagnation. The same problem faced Carniero and Hilse (3), and they proposed this same way out. But in many ways it is more difficult to imagine a stationary population over such a long period than to accept the possibility of the gradual increase described by the binary theory with occasional excursions up or down.

It will be proposed below that corrections to Eq. 5 above the second degree in N probably do not occur singly. Nevertheless it is desirable to conduct the same investigation as above using third-degree corrections. The exact solution for a combination of second and third powers of N has many of the same properties as before. Using λ as the coefficient of the correction term again, the solution is $\hspace{3cm}$ (8)

$$(\lambda/\alpha)\ [\ln(1 + \alpha/\lambda N) - \ln\ (1 + \alpha/\lambda N_0)] + 1/N_0 - 1/N = \alpha(t - t_0)$$

At $t \infty = t_0 + 1/\alpha N_0 - (\lambda/\alpha^2)\ ln\ (1 + \alpha/\lambda N_0)$, N becomes singular if $\lambda > 0$, or if λ is negative and $|\alpha\ \lambda N_0| < 1$. If $\lambda < 0$ and $|\alpha/\lambda N_0| > 1$, the population will level off at a constant value of $N = \alpha/|\lambda|$. Within this general framework, it can be said that if a third-degree term exists, it will always dominate the second-degree behavior eventually, just as the second inevitably supercedes the first. Within the present uncertainty of $\pm 4.3\%$, λ could be no larger than 5.5×10^{-23} without having made itself felt by now. And at this value, the third-degree term could not become dominant until $N = 8.8 \times 10^{10}$, an impossible population. We can therefore dismiss the possibility of a third-order correction term having ever played or now playing any role *by itself* in population growth.

Instructive as it is to consider these expansions of the B and D functions, and the effect of correction terms, the onset of terms like these does not adequately represent the trends of either the global population data at epochs when large deviations from the binary law are apparent, nor the data from special localities where transitions are even now in progress. Four such striking epochs can be identified, (a) the transition from Australopithecene times, (b) the Black Death, (c) the ca. +1600 growth acceleration, (d) the present population explosion. An example of a locality in which transition is in progress is the United States of America, which over the past 50 years has been responding to the closing frontier.

Basically transitions seem to be of two types: those which depend purely on population numbers, and those which are spurred by some external agency (in which I propose to include the occurrence of novel and compelling ideas). Thus they are centered around some epoch, but the calendar date of that epoch is of course unimportant, only the lapse of a certain interval of the order of a generation.

One might *a priori* expect that peoples could make sudden, irrational, and frequent
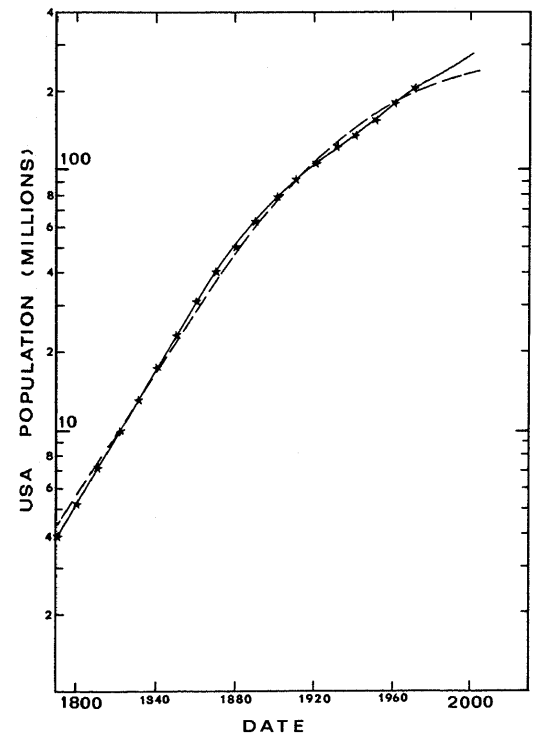

FIGURE 2. Population growth theory for the United States. Stars are census data. Solid curve is derived from Eq. 12. Dashed curve is best fit to the logistic theory.

changes in their growth patterns and therefore in the growth constants. Wars are often cited in this respect. The facts seem to be that war has affected only a tiny minority of the population at any one time and has had little to do with the growth in the number of females, which is the fundamental basis of population growth, so global growth has generally gone inexorably forward. Plotting the known data and best estimates on a graph which allows us to display the entire range of history (Fig. 1) reveals that except for small excursions like the one caused by the Black Death, or the great growth acceleration which took place at the end of the Dark Ages, there may have been no significant departures from binary growth following an initial zeroing of the linear growth terms probably at about the advent of *Homo erectus,* but perhaps even during the dominance of *Homo habilis.*

Under the common illusion that it is only recently that we have had close worldwide communication, one might wonder how the factors producing any relatively sudden changes observed could have propagated so quickly, and also how the great apparent general stability was maintained. In this connection it is interesting to speculate on the magnitude of the time constant for the propagation of information in primitive society. If we assume that information such as the existence of tools and living techniques is transferable whenever people came within seeing awareness of each other, a time constant can be derived from analogy with molecular collision theory, appropriately adjusted to two dimensions as

$$t = 1/wvn \qquad (9)$$

where w is the average diameter of an individual's territory (or spatial span of awareness), v is the average speed of agitation of members of the species, and n is the areal density of population. Insertion of crude estimates of $w = 0.1$ km, $v = 1$ km/day, and $n = N/10$ people/km^2, gives $t = 10^6/N$ years. Thus, if geographic boundaries are ignored, the time it took an idea to spread over the earth can have been less than a generation for over a million years, and a rapidly decreasing fraction of the average lifetime since the emergence of *Homo sapiens.* Since the time for mere propagation of observation has been so short for such a long time, in all probability the time constant is more often that for social change, involving belief structures, which popular wisdom over the ages has set at about a generation, that period during which impressionable youth can displace their idea-resistant seniors.

Accordingly we may surmise that the first class of transitions, those depending on the attainment of some critical value of population number, such as the initial zeroing of the linear growth terms, might take the form

$$B(N) = \beta'N + \beta''\exp[-f(N)]$$
$$D(N) = \delta'N + \delta''\exp[-g(N)] \qquad (10)$$

where $f(N)$ and $g(N)$ are two monotonically increasing functions of N which approach zero as N approaches zero, but which otherwise need not be of the same form. However if they are not the same, then one will always disappear before the other.

Exponentials such as these cannot be expanded prior to the integration of the differential equation except over extremely limited ranges of the variables. At the outset one can only speculate on the nature of the functions $f(N)$ and $g(N)$, so it is proper to assume that they take the simplest form possible in approximating the social pressures of the times. For example in the transition from African man to world-ranging man between -5×16^6 and -1×16^6, the critical population N at which a perception of the need for change set in among the naked hunter-gatherers must have been determined by the areal carrying capacity of the thermally suitable zone. This would have placed a limit of about 10^5 on man in that zone. The simplest function which contains the empirical elements is $\beta'- \delta' = \alpha$ coupled with $\beta''\exp[-f(N)] – \delta''\exp[-g(N)] = \lambda\exp[-N/N]$. Social pressures demanded a reduction in the birth rate or an increase in the death rate of fertile females, and since the latter seems to have been largely unthinkable in all cultures, the former must have been invoked either by ritualistic methods of birth control (tabus), or female infanticide, which is the mathematical equivalent to a birthrate reduction. When the above expression is fitted to the requirements that

it should conform to the binary trend with $\alpha = 2.099 \times 10^{-12}$ by about the year -5×10^5, and beginning with approximately one hominoid in about -5.5×16^6, then an excellent transition is achieved with

$$dN/dt = \alpha N + \lambda \exp(-N/N_c) \qquad (11)$$

where $\lambda = 4.71 \times 10^{-6}$ excess births per year, and $N_c = 8.93 \times 10^4$. It is remarkable that the usable range of the constants is not large, suggesting that we may be dealing in numbers which have more significance than might have been at first supposed, considering the speculative nature of the approach. The result is shown by the solid line leading back from the year +1000 in Fig. 1, which is calculated from the above numbers. That the transition from Malthusian to binary growth occurred precisely at the epoch when *Homo erectus* was replacing *Homo habilis* and Australopithecus strongly suggests that the former had techniques for population control, while the latter did not, and coincides with the observation of Martin (4) that at about -2×10^6 years *Homo erectus* must have been the first hominoid that needed to supply long-term care for infants since large postnatal brain growth first appeared in that species.

As a second example we turn to the fairly accurate census data of the United States. Examination of Fig. 2 will show that the growth was at first exactly exponential, as could be expected with any open frontier, and then underwent a decline. Since the colonial immigrants originated in a culture which had long conformed to the mores underlying the binary growth law, we may suppose that this element was latent in their growth rate, but was momentarily submerged by the opportunity for free growth which the expanding frontier permitted. Moreover, the numbers of immigrants at any one time has been geared to world population, not U.S.A. population. The situation was, in fact, precisely the one envisaged by Malthus. However, as in the development of early man, social pressures acted to reduce this exponential component until the world-controlled rate became the dominant element again. Subsequently a new phenomenon has set in which will be discussed later. The very short span in which all these changes have taken place can be traced to the already large numbers of the immigrants coupled with the relatively small domain available for them to exploit. The logical form in, which to express these processes is

$$dU/dt = [\lambda \exp<-f(U)> + \alpha N] \, U \qquad (12)$$

where U is the U.S.A. population. That the decline in the exponential component of the growth rate is a function of the national population rather than the world population follows from the fact that it is the local population which causes any pressures tending to limit Malthusian growth. The binary law component *is* proportional to world population N because in the limit we must be able to obtain the global binary law by summing over the growth rates in *i* disparate localities each of population N_i as follows

$$\sum_i (dN_i/dt) = \sum_i (\alpha N) N_i \qquad (13)$$

whence

$$d/dt \sum_i N_i = \alpha N \sum_i N_i \qquad (14)$$

and whence $dN/dt = \alpha N^2$ since $\sum_i N_i = N$.

Moreover we can deduce from the expanion of exp $[-f(U)]$ in a power series as

$$\exp[-f(U)] = (1 - aU + bU^2 \ldots) \qquad (15)$$

that α must be zero and $b < 0$, or the function would represent a steady diminution of the simple belief that "more is better" which has been inherent in the binary process at least in modern times. Since this belief was certainly widespread in 1790, we will assume $f(U) = [(U - U_0)/U_c]^2$ as being the simplest expression which meets the basic requirements. Extensive study of several other models has also shown that this fits the data best. The theoretical curve calculated from Eq. 12 with this choice of f (U) where $\lambda = .02551$, $U_0 = 1790$ population, $U_c = 7.384 \, 10^7$, and $\alpha = 5.079 \times 10^{-12}$ is shown as the solid line in Fig. 2, and fits the data within $\pm 1.41\%$. For an expression with only two adjustable constants this is excellent agreement. Modifications such as making the decline in exponentiality depend on world population double the imprecision of the fit. The fitted value of λ is in the appropriate range for pure exponential growth, as will be discussed below.

The logistic curve (given as the dashed line in Fig. 2), which was once proposed for modelling the declining growth rate of the U.S.A. poulation, has fallen into disfavor since it fits only over limited ranges and requires adjustments of the constants. Its general fit to the data is ±4.88%. It derives from a truncated expansion of the exponential in Eq. 12 and a replacing of N by U. Even the unexpanded parent equation does not fit nearly as well (±4%) as does the integral of Eq. 12.

The work on this section was completed before the 1980 U.S. Census data were known, and it was gratifying that the number predicted (223 million) should have been closer to the actual census than the Bureau's own prediction. However, both in the United States and in the world, the growth rate demanded by the binary law is rapidly approaching a value greater than the combination of female fertility with the maximum possibilities of mortality postponement. This problem has been addressed before (1), but will be studied in more detail in a following section. It is probable that the binary law can make a greater contribution as a yardstick to the past than to the future.

A possible third example of a transition process is the turbulent period from +1000 to +1600 which was marked by a definite acceleration of the growth rate after +1600 over that prevailing before +1000. The change was very sharp and took place in much less than 600 years, but was obscured by the large oscillation created by the great plagues. The author approaches this particular example with some reluctance since it is easy to overstretch a simplistic model like the one proposed in Eq. 1, whose great virtue is that it offers a long range of reasonable approximation, rather than a detailed description of all data points, many of which may themselves be questionable. Nevertheless it is an interesting exercise to formulate a transition function for as complex a change as the one which took place in medieval times, and then to examine its possible meaning, if any. In so doing, the first thing of note is that the acceleration in the growth rate by which α changed from 2.1×10^{-12} to 5.08×10^{-12} may have actually occurred around +1000, been lost in the plague oscillation, and then emerged into clear view only *after* +1600. In fact one might be led to conclude that it could have been the abnormal new growth bred of increasing worldwide urbanization with its accompanying unsanitary conditions which promoted the plagues. Since all these changes seem to have been imposed by some critical instant rather than critical population, they might be modelled by an expression such as (16)

$$dN/dt = \left[\alpha_1 + \alpha_2 \tanh(t - t_\infty)/t_c\right]\left[1 - k \exp(|t - t_1|/t_p)\right] N^2$$

where the first factor describes the acceleration beginning at about the year +900, and acting over a response time of the order of a generation (50 years fits well), while the second factor accounts for the plague oscillation with its world maximum at $t_1 = +1330$, its span of intensity $t_p = 75$ years, and its amplitude factor $k = 2$. The fit between this expression and the data is shown in Fig. 3 to be excellent, and can be made almost perfect if the reasonable change of t_p to 50 years is made for the period after 1330, implying that the recovery was more rapid than the onset, and if k is increased to 20 in +1500 to account for the resurgence of mortality in Europe. The final establishment of the modern growth rate may be connected with the successful general application of official public health measures which began largely with the fire of London.
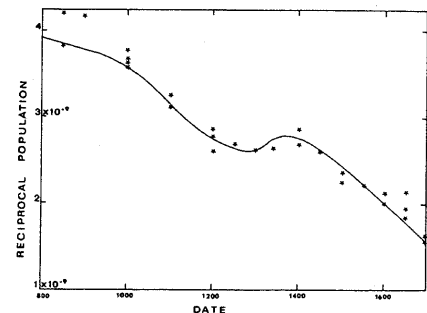

FIGURE 3. The world population changes during the Renaissance and the great plagues modelled by semi-empirical Eq. 16.

Birth and Death Rate Limitations

It is of some interest to compare the magnitudes of known and probable birth and death rates over history with the binary term αN. To begin the investigation we will use U.S.A. statistics. In the case of local populations, as already noted, immigrations and emigrations behave like births
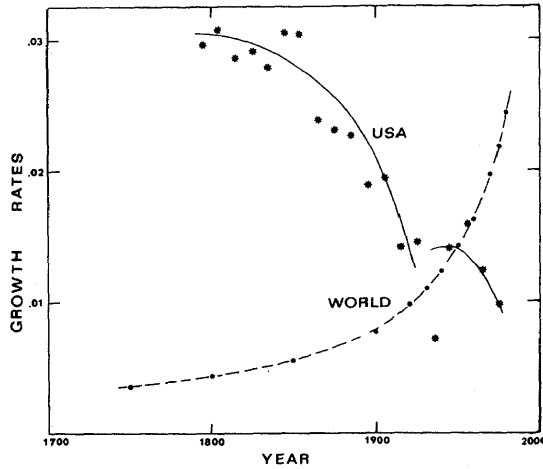
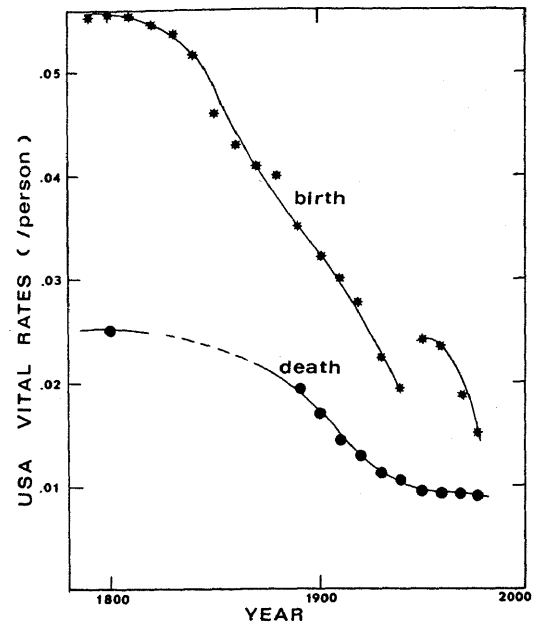FIGURE 4. Logarithmic growth rates for the U.S.A. and the world in recent times.

FIGURE 5. Birth and death rates for the U.S.A. Used in establishing a relation between death rate and mean lifetimes.

TABLE 2. *Relation of average life* $<L>$ *to death rate* δ.

Date	U.S.A.			World		
	δ	$<L>$	$\delta<L>$	δ	$<L>$	$\delta<L>$
1975	.0089	72.5	.645	.0101	72.5	.73
1970	.0095	70.9	.674	.0106	71	.75
1960	.0095	69.7	.664	.0112	70	.78
1950	.0096	68.2	.653	.0120	69	.83
1930	.0113	59.7	.673	.0129	67	.86
1900	.0172	46.0	.790	.0144	61	.88
1850	.0198	42.0	.830	.0161	58	.92

and deaths. The death rate often seems more difficult to assess than the birth rate, but since death is a spontaneous process it will very nearly be given by the reciprocal of the average life span when that has occasionally been estimated and recorded. In Table 2 the product of these two quantities is listed for the U.S.A. and the world over the periods for which data are available. This product would be unity if the hypothesis were precisely true, and it is seen to range from about 0.7 in recent years to around 0.9 in the early 19th century. We may then conclude that in default of firm data on death rates to use in studying the population growth of early agrarian societies, 90% of the reciprocal of average lifespan, if known, is a fair approximation to the death rate.

In Fig. 4 two curves are shown. Curve I is the actual logarithmic growth rate, $(dU/dt)/U$, for the U.S.A., census by census. Curve II is the same quantity for the world, smoothed by calculation from αN according to the binary hypothesis. We note that the world rate surpassed the U.S.A. rate in 1963 and that an immediate and rapid decline set in the U.S.A. logarithmic growth rate. It seems likely to be more than a coincidence that this date is very close to the first widespread expressions of concern in the U.S.A. over the impending population crisis. Had the U.S.A.

rate not declined, the world rate would not have passed it until 1980. This change in mores has not extended to the rest of the world yet. The U.S.A., it is the inescapable conclusion, is doing more to control its growth even including immigration than the world as a whole is doing for its simple birthrate.

In Fig. 5 birthrates and deathrates are plotted for the U.S.A. and in Fig. 6 the same rates are plotted for the world. For the U.S.A. the rates are plotted against the year, and have been steadily decreasing from a very large value of ≈ 0.06 births per year per human in 1790 to a value less than the world rate today. Because of this the U.S.A. growth rate would have declined rapidly also had it not been for immigration and a large concomitant decrease in the death rate. The world birth rate is largely inferred from a calculation of $\alpha N + 0.9/<L>$. For the world the rates have been plotted against population instead of data, and show clearly the linear trends of the form $(a + bN)$ that would be expected. It is noteworthy that the turning points in these curves are all at recognizable junctures in human history. A gap has been left in the curves between 1300 and 1500. To fill it in, Fig. 3 can be consulted.

The ultimate limit of socially fostered growth is the fertility of females, the unattainable maximum of which is about 1.3 child per mother per year. Empirically the maximum is nearer 0.7. When this is combined with the fact that the modern fraction of females aged 16 to 40 is about 0.1 of the total population, of whom about 10% remain spinster, the ultimate value at β is not much more than .06 (see the above U.S.A. value in 1790). This will not be reached however before the year 2018, so that no biological limit stands in the way of present runaway growth before the turn of the century.

Population Prediction

The inferences of the last section can be used to make refined guesses about world population for the near future, barring the worldwide catastrophe that unabated current practices will certainly bring on. Since socially fostered growth can continue beyond +2000, and shows no sign of change yet, the predictions for +1990 and +2000 are 5.7 and 8.0 billion. For the U.S.A., on the other hand, where growth seems to have returned to Malthusian behavior currently because negative social pressure has balanced positive pressures, the predictions for 1990 and 2000 are 246 and 278 million.
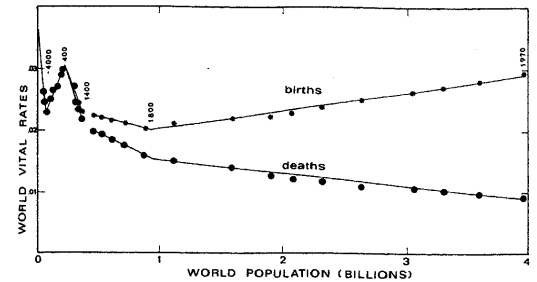

FIGURE 6. Birth and death rates for the world. Modern data are extended to ancient times by use of the empirical reciprocal relation between death rate and mean lifetimes. Birth rates are then obtained by adding growth rates to death rates.

How Many People Have Ever Lived?

This classical question was answered most recently by Keyfitz (5) who found 69 billion prior to 1966 and by J. F. Wellmeyer [as reported by Cook (6)], who estimated 77 billion to 1962. Both calculations were based on exponential growth. The theory of socially fostered growth allows a new estimate which is larger than either of the above. We divide the calculation into seven periods over which the birthrate and reciprocal population show linear trends according to Figs. 1 and 6.

TABLE 3. *Births by historic periods (billions).*

Period	Births (binary)	Births (Wellmeyer)	Births (Keyfitz)
pre-600,000	24.6	—	
—600,000 to —6000	60.0	12.0	
—6000 to +1650	25.2	42.0	69.0
+1650 to +1980	9.88	23.0 (to 1962)	

The calculation over any period is made by the following expression

$$\text{births} = \int \beta N \, dt = \int \beta N \, dn/F(N, t) \qquad (17)$$

where $F(N,t) = (dN/dt)$ is given by any of the appropriate expressions derived from Eq. 3 such as Eq. 1.

The results grouped according to the periods chosen by Wellmeyer are given in Table 3. Although the total is much the same, the distribution by periods is not. The differences lie in the lower estimates of birth rate used here, and in the recognition of the vast length of the prehistoric period. Thus, the total births in the long African period are indicated to have been 24.6 billion, while those of world-ranging man since then total 95.1 billion up to 1980.

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