AN AGGREGATE AGRICULTURAL PRODUCTION FUNCTION FOR THE U.S.A. IN 1965-1969[1]

James W. Richardson

Department of Agricultural Economics, Oklahoma State University, Stillwater, Oklahoma

An aggregate agricultural production function, *i.e.*, a mathematical description of farm output as a function of various inputs, including pesticides, fertilizer, cropland, machinery and labor, is presented.

A current production function, specifically one including pesticide as an input, is needed now more than ever before because of the environmental questions concerning pesticide use. The objective of this study is to develop such an aggregate agricultural production function for the U.S.A.

METHODS

An aggregate production function is estimated here for the 1965-1969 time period. The Cobb-Douglas functional form is used because of its previous use in aggregate economic studies (1, 2), its ease of estimating parameters, and its provision for diminishing factor returns and constant elasticity of factor substitution which are reasonably consistent with reality. Cobb-Douglas function parameters can be estimated by factor share, a methodology utilized by Tyner and Tweeten in studying optimum resource allocation for the U.S. agriculture (1).

Factor share for an input (X_i) is the ratio of total expenditures for the input and the total value of the output. Given competitive equilibrium conditions ($\partial Y/\partial X_i = (P_{xi}/P_y)$), then the right hand side of Equation 1, by definition of the factor share F_i for input X_i, is equal to:

$$F_i = \frac{\partial Y}{\partial X_i} \cdot \frac{X_i}{Y} = \frac{P_{xi}}{P_y} \cdot \frac{X_i}{Y} \quad \text{(Eq. 1)}$$

where

P_{xi} is price of factor X_i,

P_y is price of output Y,

and the left hand side of Equation 1 is, by definition, the elasticity of production for factor X_i. Since the elasticity of production is the exponential coefficient in a Cobb-Douglas function, the functions coefficients can be derived from aggregate expenditure data.

The assumption of equilibrium implicit in use of factor shares to estimate elasticities of production in Equation 1 is especially violated for pesticides, labor, and fertilizer. The adjusted factor share (1) F_t^* which is used as the elasticity of production is found by multiplying F_t by the ratio of the input's marginal revenue to marginal cost as in Equation 2:

$$F_i^* = F_i \frac{\partial Y}{\partial X_i} \frac{P_y}{P_{xi}} \cdot \quad \text{(Eq. 2)}$$

RESULTS AND DISCUSSION

Table 1 presents the estimated elasticities of production for the factors in Equation 3, with $F_i^* = b_i$:

$$Y = AX_1^{b_1} X_2^{b_2} \ldots X_9^{b_9} \cdot \quad \text{(Eq. 3)}$$

The input categories are similar to those used by Tyner and Tweeten (1) with the inclusion of pesticides and the exclusion of real estate taxes. Real estate taxes are a factor in determining the price of real estate and are, therefore, implicit in the real estate input. The output variable is adjusted farm output, the cash receipts (in dollars) for sales and adjusted for interfarm transfers and government payments for land diversion. The intercept A in equation 3 is 13.9885.

The elasticity of production for pesticides (b_1) is 0.0473, *i.e.*, a 1% decrease in the use of pesticide decreases farm output 0.047%. A reduction in pesticide use by 10% causes a 4.7% decrease in farm output or about a $145 million decrease in total farm output. The elasticity of production for fertilizer is 0.1266, *i.e.*, 10% increase in fertilizer use increases farm output by 12.6%, or about $386 million.

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TABLE 1. *Estimated elasticities of production for an aggregate agriculture production function for 1965-1969.*

Input	Production function[a]
Pesticides (lbs active ingredients)	.0473
Fertilizer and lime (tons purchased)	.1266
Livestock, feed, and seed ($)[b]	.1322
Labor on farms (man-hrs)	.1651
Machinery ($)[c]	.1238
Real estate ($)[d]	.1957
Machinery operating expenses ($)[e]	.1195
Miscellaneous current operating expenses ($)[f]	.0611
Crop and livestock inventory ($)[g]	.0528
Sum of production elasticities	1.0241

[a] Estimated for production function specified in Equation 3.
[b] Purchased from non-farm section; inter-farm transfers were excluded.
[c] Charges for annual investment (interest and depreciation) on machinery.
[d] Charges for annual investment in agriculture (acres cropland). Value adjusted for farm programs being capitalized into land values.
[e] Charges for fuel, repairs, and operations.
[f] Expenses do not include charge for pesticides.
[g] Charges for interest on inventory.

How do the parameters of Equation 3 compare to those estimated by other researchers? Table 2 compares the elasticities of production for Equation 3 to those estimated by Headley (2), for his is the most recent (1963) aggregate agricultural production function which included pesticide as an input.

TABLE 2. *Comparison of production elasticities of inputs estimated by two different studies.*

Input	Richardson 1965-1969	Headley(2) 1963
Pesticides	.0473	.0406
Fertilizer and lime	.1266	.1663
Livestock, feed and seed	.1322	—
Labor on farms	.1651	.1517
Machinery	.1238	.3178
Real estate	.1957	.1845
Machinery operating expense	.1195	—
Miscellaneous operating expense	.0611	—
Crop and livestock inventory	.0528	—

The elasticity of production for pesticide is almost identical for the two studies. The slight difference is most likely due to the increased use of pesticides in 1965-1969 relative to 1963. The elasticity of production for fertilizer in this study is lower than that reported by Headley (Table 2). However, another source (3) suggests that the factor share (elasticity of production) for fertilizer was slightly larger than 0.10, which lends support to this study's estimate of 0.1266.

The estimated elasticities for farm labor and real estate are almost identical for the two studies, with only slight differences due to changes in factor costs and levels of use between 1963 and 1965-1969.

Machinery's elasticity of production as estimated by Headley appears to be large compared to this study's estimate. The reason for the difference is that Headley includes machinery repairs, direct costs of operation, and the annual investment in machinery to compute the elasticity for machinery. In this study, the sum of elasticities of production for machinery, machinery operating expenses, and miscellaneous operating expenses, equals a value of 0.344, approximately equal to Headley's estimate of 0.3178 for machinery. Tyner and Tweeten (1) suggest that Headley's estimate is high and should be dealt with accordingly. Headley did not estimate the elasticities of production for livestock, feed, and seed or for crop and livestock expenses separately but included them in a lump sum referred to as "other."

The aggregate agricultural production function presented here is useful to researchers because it is for a recent time period and from it can be estimated the marginal rate of substitution between inputs, the optimal input use, and the impact of pesticides or fertilizer restrictions on output. There are other policy uses for this function, but it should be used with the same caution given all aggregate functions.

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REFERENCES

1. F. H. TYNER and L. G. TWEETEN, *J. Farm Econ.* 47: 1462-1467 (1965).
2. J. C. HEADLEY, *Amer. J. Agric. Econ.* 50: 13-23 (1968).
3. L. G. TWEETEN, *Foundations of Farm Policy*, University of Nebraska Press, Lincoln, 1970, p. 289.