

DIMENSIONLESS PARAMETERS IN GROUNDWATER RECHARGE

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Field data on rainfall and well observations are mathematically analyzed and a dimensionless equation is presented for computing the net amount of discharge and recharge. Thus, the amount of available groundwater is more accurately predicted.

The accelerated use of groundwater during this decade to meet growing demands in rural, urban, and industrial areas has created a multitude of problems. Among these, recharge, either natural or artificial, is of prime significance. Natural recharge by rainfall is computed most often by the traditional coefficient method of $Q_r = C_r A$. Recent efforts to gain an understanding of hydrologic mechanisms through the dimensionless approaches of mathematical analysis have produced promising results (1, pp. 13-21; 2-6).

Similarly, the present study is an extension of these efforts in that it attempts by using field data to identify all the parameters controlling natural recharge, and establish interrelationships among these parameters.

METHOD OF INVESTIGATION

A formidable obstacle in a study of this nature is the unreliability of the available data, which have been usually obtained under uncontrolled conditions. The data from Frederick, Oklahoma, in this instance, were considered to be reliable. They pertained to wells located in unconfined terrace deposits and particularly provided information on static head readings and depth of wells, amount of pumpage, location of wells and rainfall intensities. The data reveal that the static level of the wells did not change during the first two or three months of the year. The little amount of pumpage must have been, in all probability, compensated for by the amount of rainfall during that month. Therefore, it was concluded that there was either no influent or effluent seepage, or the amount was small. Also, it was assumed that there were no observational errors. This assumption becomes meaningful in view of the

attempt to establish correlation of residuals for the equations presented later in this study.

Identification of parameters

The pertinent parameters to be used were identified and, with the assumptions made in the previous paragraph, the problem was reduced to a simple functional relationship of the form:

$$= f(A, t, R_0, h_0, h, h_w, K, Q) \quad \text{Eq. 1}$$

where

h_0 = piezometric head at time zero, ft.

h_w = piezometric head at time t , ft.

K = permeability, gal. per day per sq. ft.

Q = pumpage rate, cu. ft. per unit time

R_0 = radial distance corresponding to h_0 , ft.

Q_r = recharge rate, cu. ft. per unit time

i = rainfall intensity, in. per unit time

A = recharge area, acres

In order to reduce further the number of parameters in Equation 1 and best select those combinations which would characterize the problem in a dimensionless form, the following terms were introduced:

$$\underline{Y} = \frac{Q_r}{iA} \quad \text{Eq. 2}$$

$$\underline{X} = \frac{h_0}{h_w} \quad \text{Eq. 3}$$

$$\underline{Z} = \frac{Q}{KR_0^2} \quad \text{Eq. 4}$$

Thus, the problem became one of expressing Y as a function of X and Z , or $Y = f(X, Z)$.

INTERRELATIONSHIPS

To establish correlations among the various parameters tacitly indicated in the functional equation, multiple regression analysis was employed (7). Consequently, it was possible to put this equation into the form:

$$Y = B_0 + B_1 X_1 + B_2 X_2 \dots + B_n X_n \quad \text{Eq. 5}$$

The principle of least squares was used in order to find the unknown intercepts and coefficients of the regression equations (8, pp. 171-195).

The regression equations investigated herein were:

$$Y = B_0 + B_1 X + B_2 Z \quad \text{Eq. 6}$$

$$\ln(Y) = B_0 + B_1 \ln(X) + B_2 \ln(Z) \quad \text{Eq. 7}$$

$$1/\ln(Y) = B_0 + B_1 X + B_2 Z \quad \text{Eq. 8}$$

$$1/\ln(Y) = B_0 + B_1 \ln(Z) + B_2 \ln(Z) \quad \text{Eq. 9}$$

The numerical analysis of Equations 6,

7, 8, and 9 was executed by using the values of the independent variables given in Table 1. The values of Q_r , indicated in Table 1, were computed from the general equation $Q_r = CiA$, and as expected, for the same area the amount of recharge is the same as long as the rainfall intensity stays constant. Consequently, to this extent Q_r is a dependent variable. However, considered in light of the regression equation, Q_r assumes the same sense as the other variables. The contributory area of recharge for each well was calculated on the basis of the Theisen method (9). C was given as 11.5 percent (1, p. 22). Then, applying the IBM 1130 Computer Program ("Regre," 1130 Scientific Subroutine Package, IBM Manual No. H 20-0252) and single precision, the values of the dimensionless terms X , Y , and Z were computed and introduced into a subroutine program in order to obtain the regression coefficients, intercepts, R values, and F values for Equations 6, 7, 8, and 9.

DISCUSSION OF RESULTS

The pertinent mathematical characteristics for Equations 6, 7, 8, and 9 which have been obtained by making use of the method of least squares (7) are presented in Table 2.

TABLE 1. The values of the known parameters for the wells in the city of Frederick, Oklahoma ($K = 20,000$ gpd/sq. ft.).

Well	Well property area, sq. ft.	R_0	Data period, months	l	h_0	Parameters $\frac{h_w}{h_0}$	Q	Q_r
A	635,000	450	1	0.165	13.0	13.5	26,600	12,550
			6	1.050	13.0	13.5	362,000	80,000
			12	2.580	13.0	13.0	1,690,000	196,500
B	635,000	450	1	0.165	14.0	14.4	611,000	12,550
			6	1.050	14.0	12.0	1,548,000	80,000
			12	2.580	14.0	11.0	3,520,000	196,500
C	635,000	450	1	0.165	14.0	14.0	17,900	12,550
			6	1.050	14.0	13.2	483,000	80,000
			12	2.580	14.0	13.5	1,790,000	196,500
D	282,600	450	1	0.158	17.5	16.0	700,000	5,380
			6	1.050	20.0	17.5	293,000	47,000
			12	2.580	20.0	20.0	1,649,000	84,500
E	282,600	450	1	0.158	13.5	11.5	247,000	5,380
			6	1.050	16.0	11.5	303,000	47,000
			12	2.580	16.0	16.0	1,050,000	84,500

TABLE 2. Values for R , F , intercepts, and coefficients.

Equation number	R value	F value	B_0	B_1	B_2
6	0.623	3.802	0.055	0.068	-3.778
7	0.605	3.458	-2.592	0.494	-27.688
8	0.556	2.682	-2.252	0.576	-0.016
9	0.571	2.907	-0.439	-0.147	0.009

The equation with the highest coefficient of multiple correlation is selected for best fitting the data. In this instance, R is maximum at 0.623 for Equation 6. By substituting the values of B_0 , B_1 , B_2 , Y , X , and Z , Equation 6 assumes the form:

$$\frac{Q_r}{iA} = 0.055 + 0.068 \left(\frac{b_0}{h_w}\right) - 3.778 \left(\frac{Q}{KR^2_0}\right)$$

Eq. 10

The values of the F-test compared to 3.89, which is that for a 95 percent confidence interval and the same degrees of freedom as Equation 10, are not found to be significant (7).

Therefore, having obtained the different variables in Equation 10, the net recharge rate, Q_r , could be computed directly. Equation 10 could also be presented in a graphical form by plotting Q_r/iA as a function of A/KR^2_0 for constant values of b_0/h_w , as shown in Figure 1.

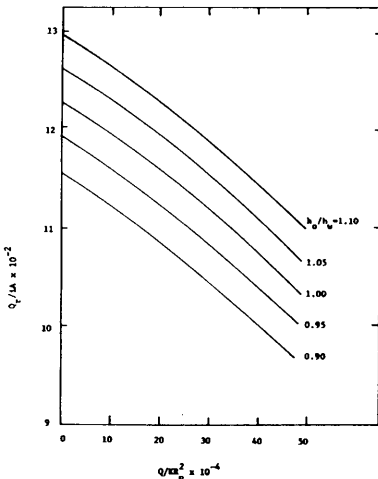


FIGURE 1. Graphical representation of Equation 10 for constant values of b_0/h_w .

Table 3 presents the residual values for Equation 10. These are the differences between the actual and estimated values of Q_r/iA for the three data periods and for each well as presented in Table 1 and they comply with the assumptions made for a fitted model (8, pp. 86-97). In Table 3 these sets of data are referred to as "Case

TABLE 3. Residual values for Equation 10.

Case no.	Y value	Y estimate	Residual
1	0.11978	0.12070	-0.00092
2	0.11998	0.11942	0.00055
3	0.11994	0.11687	0.00306
4	0.11978	0.11910	0.00067
5	0.11998	0.12681	-0.00683
6	0.11994	0.12853	-0.00859
7	0.11978	0.12327	-0.00349
8	0.11998	0.12563	-0.00565
9	0.12016	0.11902	0.00113
10	0.10290	0.12372	-0.02082
11	0.15839	0.13058	0.02780
12	0.12008	0.10923	0.01084
13	0.12049	0.13287	-0.01238
14	0.15839	0.14750	0.01088
15	0.12008	0.11430	0.00577

Numbers." A close examination of the residual values reveals the negligible amount of error associated with estimating the values of Q_r/iA using Equation 10. Along with the high coefficient of multiple correlation obtained for Equation 10, the study of the residuals gives further indications of success in computing the recharge rate by this equation.

It should be noted that Equation 10 takes into account the amount of pumpage Q ; consequently, it represents the net amount of recharge. A value of Q_r/iA less than zero will indicate that discharge is greater than recharge, which means that a condition of mining has developed. This equation could also be used to compute the total amount of recharge by setting Q equal to zero.

CONCLUSIONS

On the basis of the data available and the results obtained, the following are concluded:

1. Of the four different regression equations investigated, the one with the highest coefficient of multiple correlation has the form:

$$\frac{Q_r}{iA} = 0.055 + 0.068 \left(\frac{b_0}{h_w}\right) - 3.778 \left(\frac{Q}{KR^2_0}\right)$$

2. The mathematical model depicted by the equation represents the net amount of discharge or recharge; it also gives a truer picture of the availability of groundwater than the conventional coefficient method heretofore used.

3. Since the equation takes into account the amount of pumpage, the amount of recharge may be computed by setting Q equal to zero, if no pumping occurred.

4. A negative value of Q_r/iA indicates that discharge is greater than recharge, and it implies that a condition of "mining" has been established.

5. Although a predictive equation has been developed, it is not as comprehensive as desired. As more data become available, it would be possible to refine the mathematical model(s) so as to encompass any aquifer under any existing conditions.

ACKNOWLEDGMENTS

This study was funded by the Oklahoma Water Resources Research Institute (OWRR A-027-OKLA, OURI 1779). The data were provided by the City of Fredrick, Oklahoma.

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