

SPACE-DEPENDENT FAST REACTOR STABILITY

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A fast reactor may be defined as a reactor in which the neutron energy distribution differs as little as possible from the energy distribution of the neutrons emitted in fission. The system contains no moderators, or, in some special cases, a minimum amount of moderators; the fast neutrons are thus being slowed down much less than they are in other reactors. The bulk of fissions occurs at an energy region high enough where η , or the average number of neutrons produced per neutron absorbed in the fuel, is relatively large. For this reason, a fast reactor is capable of producing more neutrons than it actually consumes, thus, providing economic advantages. It should not be surprising that, in the near future, fast reactors would become a major source of nuclear power.

One major factor in the stability of a fast reactor is the reactivity, k , and the manner in which it varies as a result of changes in reactor operating conditions, such as the temperature of reactor core materials or the insertion of a control rod. Reactivity determines whether the neutron population within the system increases, decreases, or remains at a constant level. It is a function of the amount and type of materials that are present in the various regions of the reactor and the dimensions of these regions.

In a fast reactor, the temperature of the reactor core materials plays a decisive role in the variation of reactivity, more so than in a thermal reactor. Materials expand as they are being heated, and there are then fewer atoms per unit volume present. Neutrons then travel a longer distance between collisions and have a greater chance of escaping without causing fission. In a thermal reactor where neutrons travel slower, thermal expansion of core materials is responsible for the negative temperature feedback which, in turn, provides an inherent criterion

of stability for the thermal reactor. In a fast reactor, however, there are other effects resulting from the change of temperature even in the absence of thermal expansion; one such major effect which gives rise to an undesirable positive feedback is the so-called "Doppler effect."

The probability of a neutron causing fission or being captured by a nucleus is dependent on relative velocity. At certain velocities interaction probabilities are much higher than on the average. These maxima in probabilities are called resonances. If the fuel nuclei were at rest, then upon plotting capture or fission interaction probability versus neutron speed, resonances would show up as tall but narrow peaks. However, nuclei are not at rest, but vibrate due to thermal agitation. Thermal agitation decreases the height and increases the width of resonance peaks. This is the so-called Doppler effect of broadening of resonances. In a thermal reactor where neutrons travel at thermal speed, such Doppler effect is not observed. In a fast reactor which is highly enriched in U-235, there is very likely positive Doppler coefficient of reactivity. The basic problem in the stability of a fast reactor is to determine whether the negative temperature feedback due to fuel expansion responds fast enough to dominate over the positive Doppler effect and maintain an inherent criterion of stability in the event of a large perturbation of neutron flux.

Several recent papers (1-3) have dealt with stability of a fast reactor. However, the model chosen for the analytic studies has been based on a one energy group, one point neutron kinetics model. That one energy group representation is not valid in fast reactor kinetics studies is obvious from the above discussions, especially when the Doppler broadening effect is being considered. That the one-point kinetics model is inadequate if the reactor has a spatial di-

mension several times the migration length of the neutron has amply been demonstrated (4). With the trend of building larger power reactors due to economic considerations, spatial effects in the temporal behavior of neutron kinetics become increasingly important. It is the purpose of this paper to study space-dependent stability of a fast reactor with the Doppler coefficient dependent on the energy groups.

A standard notion which has been increasingly adopted in automatic control systems is to define stability in the sense of Liapunov, i.e., a system is said to be stable if for any disturbances the solutions will remain close to the original solution for all future time. Liapunov's definition of stability has adequately been covered elsewhere (5), and will not be reproduced here; however, it has several distinct advantages. It is a general definition and does not depend on the linearity or nonlinearity of the system. Moreover, it takes into account the actual operating conditions of dynamic systems by studying the stability in reference to the neighborhood of the given initial conditions.

The reactor model considered here consists of a two-region reactor with coupling between the two regions. Each fuel region is subcritical when considered separately, but coupling between the two regions allows the system to become critical. If the reactivity of one region is disturbed, the neutron density of the other will be disturbed only through its coupling to the first. Such a coupled-core reactor model will provide an additional safety measure in a fast reactor where controls by means of conventional control rods are not sufficient since the absorption cross sections become much smaller at high energies.

Consider the multi-group, two-point coupled neutron kinetics equations neglecting delayed neutrons:

$$\begin{aligned} \dot{x}_1 &= \frac{\epsilon_1}{\xi} x_1 + \frac{\epsilon_2}{\xi} x_2(t-\theta) \\ \dot{x}_2 &= \frac{\epsilon_2}{\xi} x_2 + \frac{\epsilon_1}{\xi} x_1(t-\theta) \\ \dot{z}_1 &= bx_1 - \omega z_1 \\ \dot{z}_2 &= bx_2 - \omega z_2 \end{aligned} \quad \text{Eq. 1}$$

$$\begin{aligned} \text{where } x(t) &= \frac{P(t) - P_0}{P_0} \\ z(t) &= \frac{T(t) - T_0}{T_0} \\ \omega &= \frac{HwC_c}{MC_r(H + wC_c)} \end{aligned} \quad \text{Eq. 2}$$

MC_r is mass multiplied by the specific heat of the core, H is the effective heat transfer coefficient between core and coolant, w is the mass flow rate, C_c is the specific heat of coolant, ω is called the inverse of the characteristic heat exchanger time constant; b is a constant denoting the inverse of MC_r ; $P(t)$ is a G-element column matrix of the power, which is proportional to neutron densities, G being the number of energy groups used to replace the continuous energy dimension, the subscripts 1 and 2 refer to the first and second reactor regions, respectively; $T(t)$ is a G-element column matrix of the temperature; ϵ is the effective reactivity for the respective regions; ξ is the over-all prompt-neutron lifetime in the system; θ is the transit time of neutrons between regions; ξ is the coupling coefficient. The transit time of neutrons between regions is usually a very small number in a fast reactor where neutrons travel relatively fast; for this reason the time-delay effect will not be considered in this paper. The total reactivity referred to prompt critical, $\epsilon(t)$, is a superposition of the ingoing pulse-generating reactivity, $\epsilon_0(t)$, and the feedback terms, ϵ_{Elas} and ϵ_{Dopp} :

$$\epsilon(t) = \epsilon_0(t) + \epsilon_{Elas}(t) + \epsilon_{Dopp}(t) \quad \text{Eq. 3}$$

The input function $\epsilon_0(t)$ is approximated by a rectangular function¹:

$$\epsilon_0(t) = \begin{cases} -u & \text{if } t < 0 \text{ or } t > t_0 \\ \lambda & \text{if } 0 < t < t_0 \end{cases} \quad \text{Eq. 4}$$

By modifying Randles' expression for the thermoclastic feedback reactively to the multi-group case:

$$\epsilon_{Elas}(t) = 4acn \max_{z \in G} \int_0^t J \left(\frac{\partial \epsilon_i}{L} \right) z(t-t') dt' \quad \text{Eq. 5}$$

where the kernel $J(\theta)$ is a periodic triangular function:

$$J(\theta) = \begin{cases} 0 & \text{for } 0 \leq \theta < 1 \\ 2 - \theta & \text{for } 1 \leq \theta < 3 \\ 0 - 4 & \text{for } 3 \leq \theta < 5 \\ 6 - \theta & \text{for } 5 \leq \theta < 7 \\ 0 - 8 & \text{for } 7 \leq \theta < 9 \dots \text{etc.} \end{cases} \quad \text{Eq. 6}$$

α is the coefficient of linear thermal expansion of the fuel; c is the speed of axial compression waves in fuel slugs; L is the half-length of fuel slugs; η is the axial fuel expansion coefficient of reactivity.

In order to describe the Doppler effect, a function $D_i(x)$ is defined such that if a segment of thickness dx at position x on the i -th fuel pellet is raised in temperature by a unit amount then the reactivity of the whole system changes by an amount $D_i(x)dx$. From this definition, it follows that the Doppler feedback reactivity is given by:

$$\epsilon_{\text{Dopp}}(t) = \max_{z \in G} \{ \gamma z(t) \} \quad \text{Eq. 7}$$

where λ is the Doppler coefficient of reactivity for the individual reactor region and is defined as follows: L

$$\gamma = 2 \int_0^L \int_0^L D_i(x) dx \quad \text{Eq. 8}$$

It is then readily seen that equations 1 are nonlinear because the thermoelastic and Doppler feedback reactivity terms as defined in equations 5 and 7 all depend on temperatures. Randles obtains the stability criteria by simply assuming both the thermoelastic and Doppler feedback reactivity terms go to zero. A different approach will be used in this paper. It is believed that the thermoelastic and Doppler feedback reactivity terms, after being linearized, will affect the stability of the fast reactor together with the ingoing pulse-generating reactivity and the coupling coefficients between the two reactor regions.

Rewriting equations 1 in the matrix form:

$$\dot{x}(t) = A x(t) \quad \text{Eq. 9}$$

where

$$x(t) = \begin{bmatrix} x_1 \\ x_2 \\ x_1 \\ x_2 \end{bmatrix}, \quad A = \begin{bmatrix} \epsilon_{10}/t & \epsilon_{21}/t & 0 & 0 \\ \epsilon_{12}/t & \epsilon_{20}/t & 0 & 0 \\ 0 & 0 & -\omega & 0 \\ 0 & b & 0 & -\omega \end{bmatrix}$$

$$\epsilon_{10} = \epsilon_{10} + \epsilon_{\text{Elas}} + \epsilon_{\text{Dopp}}$$

$$\epsilon_{\text{Elas}} = \frac{4\alpha c^2 \eta}{L} \max_{z \in G} (z_0) \left(\frac{z}{\lambda}\right)^3 \left(1 + \frac{z}{\lambda}\right) \exp(\lambda t / L) \quad \text{Eq. 10}$$

$$\epsilon_{\text{Dopp}} = \max_{z \in G} (\gamma z_0)^{1/2} \left(1 + \frac{z}{\lambda}\right) \exp(\lambda t / L) \quad \text{Eq. 11}$$

For the system described in equation 9, a suitable Liapunov function⁶ is:

$$V(X) = X^T P X \quad \text{Eq. 12}$$

The matrix P is a symmetric positive definite matrix satisfying the equation:

$$A^T P + P A = -I \quad \text{Eq. 13}$$

where A^T is the transpose of A , and I is the unit matrix. If a matrix P that will satisfy this condition can be found, then the system described by equation 9 is asymptotically stable. This requirement is both necessary and sufficient. This matrix equation leads to the ten simultaneous equations with ten unknowns. The positive definiteness of the matrix $P = (p_{ij})$ requires that the determinant of P_{11} must be greater than zero. A careful calculation yields the following three interesting criteria of stability for the fast reactor:

$$\epsilon_{10} > \omega t \quad \text{Eq. 14}$$

$$\omega^2 + b^2 = 0$$

$$\epsilon_{10}^2 \left[\frac{1}{4t^2} (1 + \epsilon_{10}^2 \omega^2 + \epsilon_{10}^2 \omega^2 + \epsilon_{10}^2 \omega^2) - \epsilon_{10}^2 \omega^2 (1 + \epsilon_{10}^2 \omega^2) \right] + \epsilon_{10} \epsilon_{20} + \omega^2 t^2 \left(1 + \frac{\epsilon_{10}}{20} \right)$$

where ϵ_{10} is given by equations 10 and 11. The last of the three criteria of stability is probably the most interesting one: it gives an upper limit of the coupling coefficient between the two coupled cores in terms of the input reactivity, thermoelastic and Doppler feedback reactivities and the constants ω and average neutron lifetime

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