CONSEQUENCES OF A SIX-COMPONENT QUARK IN SU(3) SPACE

D. S. Carlstone

Department of Physics, Central State College, Edmond, Oklahoma 73034

An SU(6) model, which should not be confused with the union of spin and unitary spin, is constructed based on a picture of the quark belonging to the six-dimensional representation of SU(3). The spin is then combined with the SU(6) to form SU(12)j. It is shown that, in this case, the spin-1/2 octet and spin-3/2 decuplet can both be accommodated in the 220-dimensional representation, which is completely antisymmetric in three indices. The resulting predictions for mass and magnetic moments are discussed.

The success of the quark model in the description of "elementary" particles is well known. For example, when the quark spin is taken into account the model predicts that the $(1/2)^+$ baryon octet and $(3/2)^+$ decuplet are united in the 56 – dimensional representation of $SU(6)_j$. However, this presents a puzzle, since the $\underset{\sim}{56}$ is a completely symmetric representation of $SU(6)_j$. According to Fermi statistics, one would expect that three bound quarks which form the baryon are in an antisymmetric state. It has been suggested (1) that this can be understood if quarks obey parastatistics. It is the purpose of this work to explore another possibility.

We consider the possibility that the fundamental quark belongs to the six-dimensional representation of SU(3). Such a scheme implies that there is a symmetry higher than SU(3), namely SU(6). This SU(6) is not to be confused with the union of SU(3) and spin, which has been denoted $SU(6)_j$. It is shown that when the spin is added to $SU(6)_j$ to form $SU(12)_j$ the octet and decuplet can be accommodated in a representation which is completely antisymmetric in three indices. There are, however, many new particle states predicted by this model. This may not automatically be a disadvantage of the model since there is evidence for the existence of resonances which cannot be accommodated in the usual $SU(6)_j$ models.

It should be noted that there is evidence for the existence of quarks (2). A quark with charge 2/3 may have been seen. A quark with this charge does occur in the SU(6) scheme. As of this writing, it is by no means certain that the existence of the quark will be confirmed. In the model investigated here a given particle state is a rather complex trilinear combination of quark states. The quark model loses much of its simplicity in this case. Therefore, the quark of the SU(6) model is regarded as a purely mathematical object. As such it is useful to refer to the quark, and such language will be used whenever convenient.

SU(6) and SU(12)j

The construction of the SU(6) algebra is based on a generalization of the Elliott model of SU(3) (3). The procedure here is more closely related to an SU(8) algebra which has been considered previously (4). In particular, the 6-dimensional representation of SU(3) is to be embedded in the 6-dimensional representation of SU(6). Let the correspondence be

$$q_1 \sim |1\,1\;2/3>$$

$$q_2 \sim |1\,{-1}\;2/3>$$

$$q_3 \sim |1\,0\;2/3>$$

$$q_4 \sim |1/2\,{-1/2}\,{-1/3}>$$

$$q_5 \sim |1/2\;1/2\,{-1/3}>$$

$$q_6 \sim |0\,0\,{-4/3}>,$$

where q_i transforms as the 6-dimensional representation of SU(6) and the quantum numbers are $|TT_3Y>$. The embedding will be represented by

$$\{6\} = [6],$$

where the bracket (brace) refers to the dimensionality of the SU(3) [SU(6)] representation.

The adjoint or 35-dimensional representation of SU(6) decomposes with respect to SU(3) according to {35} = [8] + [27]. Therefore, from the {35} a set of matrices can be selected which transforms the [6] as in SU(3). The operators correspond, of course, to the generators of SU(3). In terms of SU (6) indices they transform as

$$T_3 = A^2_2 - A^1_1 + 1/2(A^4_4 - A^5_5)$$

$$(T_-)^\dagger = T_+ = -A^4_5 - \sqrt{2}(A^2_3 + A^3_1)$$

$$(K_-)^\dagger = K_+ = -A^4_1 - \sqrt{2}(A^6_3 + A^5_1)$$

$$(L_-)^\dagger = L_+ = -A^5_1 - \sqrt{2}(A^6_4 + A^4_9)$$

$$Y = 1/3\,[-2(A^1_1 + A^2_2 + A^3_1) + (A^4_4 + A^5_5) + 4A^6_6]$$

T is the isotopic spin operator, Y is the hypercharge operator, and the K and L are the remaining operators of the SU(3) algebra.

Thus, SU(3) has been embedded in SU(6). The SU(3) generators have been identified, and the additional operators in the 35 transform as a [27] with respect to the SU(3) generators. If the operators are expressed in spherical tensor form, the commutation relations of the SU(6) algebra can be given in terms of SU(3) Clebsch-Gordon coefficients and reduced matrix elements, as in the SU(8) model mentioned above. However, such details will not be given here.

If this structure has physical meaning, then it is convenient to identify the mathematical quark with the fundamental, or six-dimensional, representation of SU(6). This object would be expected to possess baryon number 1/3. Therefore, it is assumed that mesons consist of quark-antiquark pairs. Thus mesons belong to a {1} or a {35} . Since there is no evidence as yet for a [27] of pseudoscalar mesons, they will not be given further consideration.

Of primary concern here are the baryon states which are assumed to be trilinear combination of quarks.

Of interest are those representations that occur in the product

{6} × {6} × {6}= {20} + {56} + {70} + {70}

The SU(3) content of these representations is given by

{20} = {10} + {10*}

{56} = {1} + {27} + {28}

{70} = {8} + {27} + {35}

If the [6*] had been chosen as the quark, each of the SU(3) multiplets would be replaced by its conjugate.

It is noted that only one octet and one decuplet occur in this product. At this point the main difference between SU(3) and SU(6), as far as the octet and decuplet are concerned, is that the decuplet belongs to a completely antisymmetric representation. The octet belongs to the {70} which has mixed symmetry. Since {35} occurs twice in the product {70*} x {70}, there are two independent ways to couple this product with a {35} to form an invariant. Thus, the D/F ratio is not determined by SU(6).

When spin is united with SU(6) to form $SU(12)_j$, the quark will belong to the 12 of SU(12) . The SU(6)⊗ SU(2) structure of 12 is denoted {6,2}. We are again interested in states occuring in the product 12 × 12 × 12 = 220 + 364 + 572 + 572. The SU(6) × SU(2) content of each of these is

220 = {70,2} + {20,4}

364 = {56,4} + {70,2}

572 = {20,4} + {70,2} + {20,2}

The explicit transformation properties of each component of the $SU(12)_j$ representations are obtained by combining the proper spin combination with the proper quark combination and imposing the required overall symmetry. The quark and spin classification of the [8,2] part of the 220 is given in Table 1, where α(B) is the spin-up (spin-down) state.

The most interesting observation here is that the spin —1/2 octet in the {70} and the spin —3/2 decuplet belong together in the 220, which is completely antisymmetric in three indices. It was, of course, initially suggested (5) that the baryon octet should be assigned to the 20-dimensional representation of $SU(6)_j$ which is also completely antisymmetric in three indices, in order to satisfy Fermi statistics. As it turns out, the 56-dimensional representation, which contains the spin —1/2 octet and spin —3/2 decuplet, provides the preferred description (6,7). In this model, both features are incorporated in the same representation. The price that must be paid for this "convenience" is that now there are many other particles that must be found.

Since this model does allow one to include the $(1/2)+$ octet and $(3/2)+$ decuplet in a completely antisymmetric representation, it is well to test further consequences. In the next section, the mass relations will be discussed, and the possibility that certain resonances belong to higher dimensional representations of SU(3) in the 220 will be explored.

MASS RELATIONS

There is no problem in reproducing the SU(3) mass sum rules in either SU(6) or $SU(12)_j$, if the mass operator is properly chosen. However, since the symmetry properties of the 220 of $SU(12)_j$ are different than those of the $\underline{56}$ of $SU(6)_j$, it is not expected that the mass sum rules relating different SU(3) multiplets will be the same in these two cases.

Although the algebra was constructed on the basis of the quark belonging to the [6], it could also belong to the [6*]. In fact, the [6*] might be preferred, since the [6*] and [3] have the same triality. With the [6*] choice, the highest isospin multiplets occur with negative hypercharge.

The quark model provides a very convenient method for calculating the mass formulae. We assume a mass operator of the form

$$M = m_\mu + aV_1 + bV_2 + cV_3$$

Here m_μ is the central mass of each SU(3) multiplet, and the the V_i are two body potentials

$$(V_1)_{ij} = 1/2\,(y_i + y_j)$$
$$(V_2)_{ij} = (y_i + y_j)\,(\sigma_i\cdot\sigma_j)$$
$$(V_3)_{ij} = \sigma_i\cdot\sigma_j$$

with y being the hypercharge operator. In practice the V_3 term may be included with m_μ. With a [6] assignment for the quark, the mass sum rules relating octet and decuplet masses are not in good agreement with experiment. However, for a [6*] quark assignment, these sum rules are

$$(N - \Xi) + 5/14(\Sigma - \Lambda) = (N^* - \Xi^*)$$
$$(\Xi - \Lambda) - 3/7(\Sigma - \Lambda) = (\Xi^* - \Sigma^*)$$

where the particle symbol stands for its mass. Although the first of these is not quite as good as the equivalent $SU(6)_j$ result (8), the second is somewhat better. If, for example, N, Σ, Λ, and Σ^* masses are used to fit the parameters, the predictions for N^*, Ξ^*, and Ω masses are about as good as SU(6).

Next, we attempt to fit the new resonances into this scheme. The following resonances (whether confirmed or not) are considered: N(1466) 1/2+, N(1751) 1/2+, Δ (1934) 1/2+, Δ (1688) 3/2+, N(1863) 3/2+, $Z_0(1863)$?, $Z_1(1910)$?, $Z_1(2190)$?, $Z_1(2280)$?. It is, of course, impossible to fit all of these into the 220, whether the [6] or the [6*] quark is assumed. The [6*] assignment permits more flexibility in the resonance assignments. This assignment does predict a $Y = +3$ (denoted ZZ) particle in the [35].

Various possibilities for the assignments of these resonances have been considered for the [6*] case. A few details should be mentioned. If the N(1863) is assigned to the [10*], this predicts a $Z_0(1671)$. With the $\Delta(1934)$ assigned to the [27], there would be a $Z_1(1766)$ and an N(1946). And, if $Z_1(2280)$ belongs to the [35*], the $Y=3$ resonance should be ZZ(2022). This model therefore fails to provide any successful predictions.

MAGNETIC MOMENTS

$SU(6)_j$ has proved to be so successful in predicting the ratio $su(\bullet)_j = -3/2$, that any alternative to $SU(6)_j$ must also provide a reasonable prediction for this ratio. Again, since the symmetry properties of the baryon octet are different in $SU(12)_j$, it is not expected that this result will be obtained.

Following the $SU(6)_j$ quark model, it is assumed that the magnetic moment is proportion to its charge. In this case, the quark charges are 4/3, 1/3, and $-2/3$. From the particle classifications in Table 1, it follows that

$$\mu(p) = \mu(\Sigma^+) = 17/15\,\mu_0$$
$$\mu(n) = \mu(\Xi^0) = -2\mu_0(\Lambda) = 2\mu_0(\Sigma^0) = 14/15\,\mu^0 \quad \text{Eq. 1}$$
$$\mu(\Sigma^-) = \mu(\Xi^-) = 1/5\,\mu^0$$

where μ_0 is the quark magnetic moment. From this,

$$Q = 1/3[2(A^2_2 + A^4_4 + A^6_6) - (A^3_3 + A^5_5) - 4A^1_1]$$

which is not nearly so impressive as the $SU(6)_j$ result. However, this value is considerably better that the $-1/2$ which is predicted if the octet belongs to the $\underline{20}$ of $SU(6)_j$. The fact that $SU(12)_j$ does not give a completely unreasonable prediction is not too discouraging.

The same results, of course, may be obtained without reference to the quark model. In terms of a coupling scheme, it may be assumed that the magnetic moment operator transforms as the charge

$$Q = 1/3\,[2(A_2^2 + A_4^4 + A_6^6) - (A_3^3 + A_5^5) - 4A_1^1]$$

In tensor form, the $\{70\}$ transforms as $N_{\alpha\beta,\gamma} = -N_{\beta\alpha,\gamma}$ with $N_{\beta\gamma,\alpha} + N_{\alpha,\beta} = 0$. The two independent couplings of $\{70^*\} \times \{70\}$ to $\{35\}$ are chosen to be

$$R_\nu^\mu = N^{-\alpha\beta,\mu} N_{\alpha\beta,\nu}$$

and

$$S_\nu^\mu = N^{-\alpha\mu,\beta} N_{\alpha\nu,\beta}$$

Therefore, with R_ν^μ and S_ν^μ transforming as the charged operator, the magnetic moments for the octet are

$$\mu(p) = (1/15)\,r - (8/15)\,s$$
$$\mu(n) = (-7/15)\,r + (7/30)\,s \qquad \text{Eq. 2}$$
$$\mu(\Xi^-) = (2/5)\,r + (3/10)\,s$$

plus the other relations which are consistent with the SU(3) predictions.

Again following $SU(6)_j$, the next step is to construct the baryon current in $SU(12)_j$ in order to obtain the S/R ratio. The $\{70,2\}$ part of the $\{220\}$ tensor transforms as

$$T_{\alpha i\beta j\gamma k} \sim X_{(ij)k} N_{\alpha\beta,\gamma} + X_{(jk)i} N_{\beta\gamma,\alpha} + X_{(ki)j} N_{\gamma\alpha,\beta}$$

where

$$X_{(ij)k} = 1/\sqrt{6}\,(\varepsilon_{ik}X_j + \varepsilon_{jk}X_i) \qquad X_1 = \alpha,\quad X_2 = \beta$$

The $\{70^*,2\} \times \{70,2\}$ part of the (traceless) baryon current is of the form

$$J_{\nu n}^{\mu k} \sim \bar{T}^{\alpha i\beta j\mu k} T_{\alpha i\beta j\nu n} \quad - \text{trace}$$

Omitting overall multiplicative constants, it is found that

$$J_{\nu n}^{\mu k} \sim (XX_3)_n^k\,(2S_\nu^\mu - R_\nu^\mu) + \delta_n^k(XX_1)\,(R_\nu^\mu + 2S_\nu^\mu - 1/2\,\delta_\nu^\mu\langle \bar{N}N\rangle)$$

Therefore, for the $\{35,3\}$ current, the S/R ratio is -2. With $s = -2r$ in equation 2, the relations of equation 1 are obtained. It is also noted that the $\{35,1\}$ cur-

$$|p\rangle \sim \sqrt{2/15}\{-1/2[q_3(\alpha)q_3(\beta)q_5(\alpha)] + 1/3(2[q_1(\alpha)q_2(\beta)q_5(\alpha)] - [q_1(\beta)q_2(\alpha)q_5(\alpha)] - [q_1(\alpha)q_2(\alpha)q_5(\beta)]) + \sqrt{2}/6(2[q_1(\alpha)q_3(\alpha)q_4(\beta)] - [q_1(\alpha)q_3(\beta)q_4(\alpha)] - [q_1(\beta)q_3(\alpha)q_4(\alpha)])\}$$

$$|\Sigma^+\rangle \sim \sqrt{2/15}\{-1/2[q_3(\alpha)q_5(\alpha)q_5(\beta)] - 1/3(2[q_1(\alpha)q_3(\alpha)q_6(\beta)] - [q_1(\alpha)q_3(\beta)q_6(\alpha)] - [q_1(\beta)q_3(\alpha)q_6(\alpha)]) - \sqrt{2}/6(2[q_1(\alpha)q_4(\beta)q_5(\alpha)] - [q_1(\beta)q_4(\alpha)q_5(\alpha)] - [q_1(\alpha)q_4(\alpha)q_5(\beta)])\}$$

$$|\Lambda\rangle \sim 1/\sqrt{45}\{1/2([q_3(\alpha)q_4(\beta)q_5(\alpha)] - [q_3(\alpha)q_4(\alpha)q_5(\beta)]) + \sqrt{2}([q_1(\alpha)q_2(\beta)q_6(\alpha)] - [q_1(\beta)q_2(\alpha)q_6(\alpha)]) + \sqrt{2}/2([q_1(\alpha)q_4(\alpha)q_4(\beta)] + [q_2(\alpha)q_5(\alpha)q_5(\beta)] + 2[q_3(\alpha)q_3(\beta)q_6(\alpha)])\}$$

$$|\Xi^-\rangle \sim \sqrt{2/15}\{1/2[q_4(\alpha)q_4(\beta)q_5(\alpha)] - 1/3(2[q_2(\beta)q_5(\alpha)q_6(\alpha)] - [q_2(\alpha)q_5(\beta)q_6(\alpha)] - [q_2(\alpha)q_5(\alpha)q_6(\beta)]) - \sqrt{2}/6(2[q_3(\beta)q_4(\alpha)q_6(\alpha)] - [q_3(\alpha)q_4(\beta)q_6(\alpha)] - [q_3(\alpha)q_4(\alpha)q_6(\beta)])\}$$

TABLE 1. *Classification of the octet in terms of quarks. The bracket implies complete antisymmetrization.*

rent corresponds to an F-type coupling, as expected.

Although the calculations in this section are based on the [6] quark assignment, the results are same for the [6*] assignment.

CONCLUSIONS

The point of view has been taken that this model may be of interest since it is possible to include the spin $-1/2$ octet and spin $-3/2$ decuplet in a completely antisymmetric representation of $SU(12)_j$. Immediate consequences of this assignment have been discussed. It is interesting to note that, with a [6*] assignment for the quark, the mass sum rules relating these two multiplets do provide a reasonable alternative to the $SU(6)_j$ model. However, in spite of this observation, the model does not provide any new predictions which can be immediately confirmed. For example, the attempt to describe the particle spectrum according to $SU(12)_j$ has not been successful. Of course, it is possible that the predicted particles will be found at higher energies. If it is assumed that the higher SU(3) multiplets exist at an arbitrarily high energy, it is also possible that the algebras of SU(6) and $SU(12)_j$ may be useful.

The failure of the model to correctly predict the $\mu(p)/\mu(n)$ ratio may be sufficient to discount the model. If so, then this work tends to reemphasize that success of the $SU(6)_j$ model is not understood. This may indirectly tend to imply support for the paraquark model.

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