The Prediction of the Flow-Oscillation Threshold in Heated Ducts

BILL J. WALKER[1] and DARREL G. HARDEN[2]

University of Oklahoma, Norman

INTRODUCTION

The onset of combined pressure and flow oscillations in fluid flow systems with heat addition are of importance to designers in various areas. Among these areas are:

(a) Regenerative heating of a rocket nozzle

(b) Supercritical conventional and nuclear power plants

(c) Seawater desalination plants

(d) Cool-down of helium cryopanels

A convenient experimental apparatus for use in studying combined pressure and flow oscillations is a natural-circulation loop. Figure 1 shows a schema of a natural-circulation loop. The fluid is heated on one side and cooled on the other. Because of the buoyancy forces a natural circulation occurs and in this case the flow is in a counterclockwise direction.

---

[1]Graduate Student, School of Aerospace and Mechanical Engineering.

[2]Assistant Professor, School of Aerospace and Mechanical Engineering.

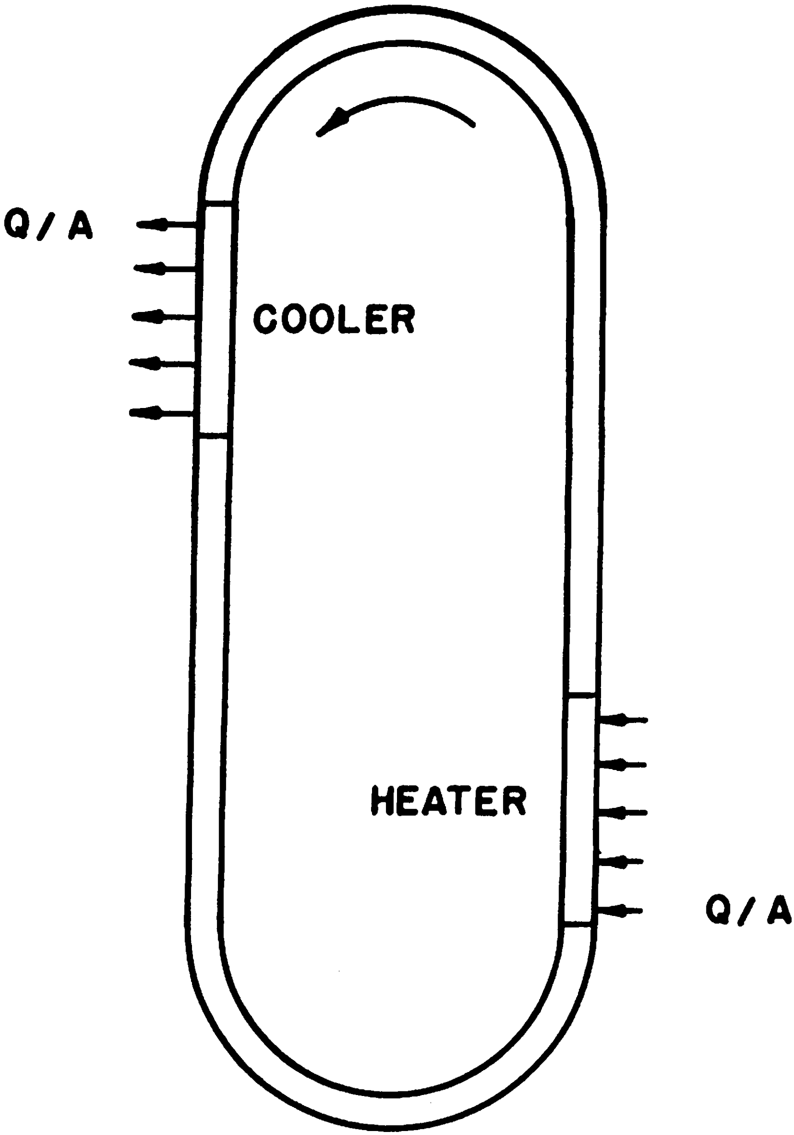

Figure 1. Natural-circulation loop schema

Figure 2 shows how the loop typically reacts to step changes in power to the heater section. As the power is increased to the first level the mass velocity overshoots a certain level and then follows a damped oscillation to that level. The same pattern occurs for the second power increase. However, at some level of power a further increase results in a sustained oscillation. Unless the operating conditions are changed, this oscillation would continue indefinitely. This is the type oscillation that occurs in the natural-circulation loop. The prediction of the threshold for this type of oscillation is outlined in this paper.

MODEL

Figure 3 shows how the natural-circulation loop was idealized in the model. The loop was taken as three sections:

(a) Upstream adiabatic — L_0

(b) Heater section — L_e

(c) Downstream adiabatic — L_1

There is a constant volumetric heat flux, W, to the fluid in the heater section. The driving pressure ΔP is a function of $U_0(T)$, the inlet velocity at the heated section, and its time derivative $dU_0(T)/dT$

Figure 4 shows the idealized equation of state for the fluid. The nonlinear change of density R with enthalpy H is postulated as being the driving mechanism for the oscillations and is the only mechanism introduced into the model. This is called "density effect" introduced analytically by Boure (1965), and is identical with the $(\rho h)_{max}$ theory of Harden (1963) (Walker, 1967).

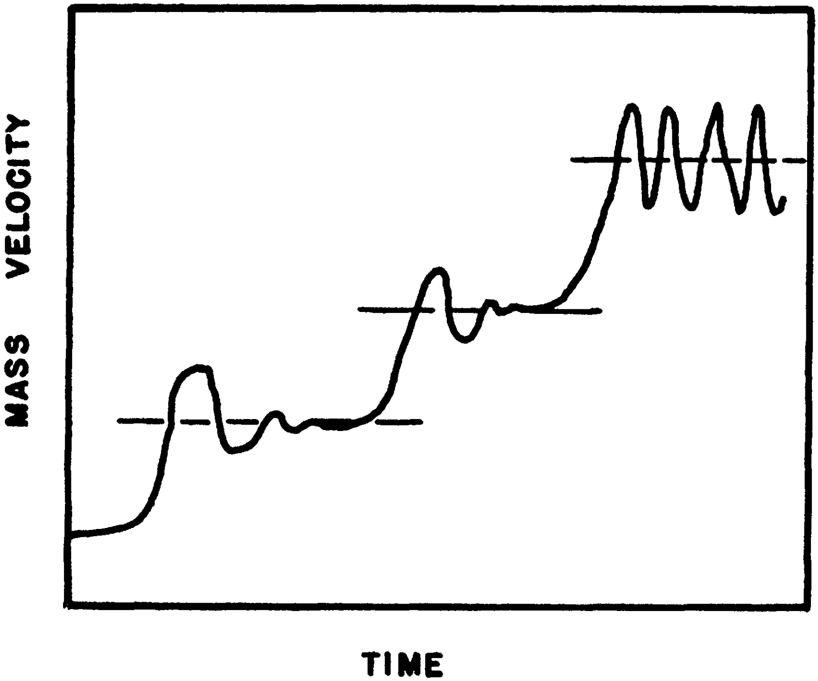

Figure 2. Flow as a function of time

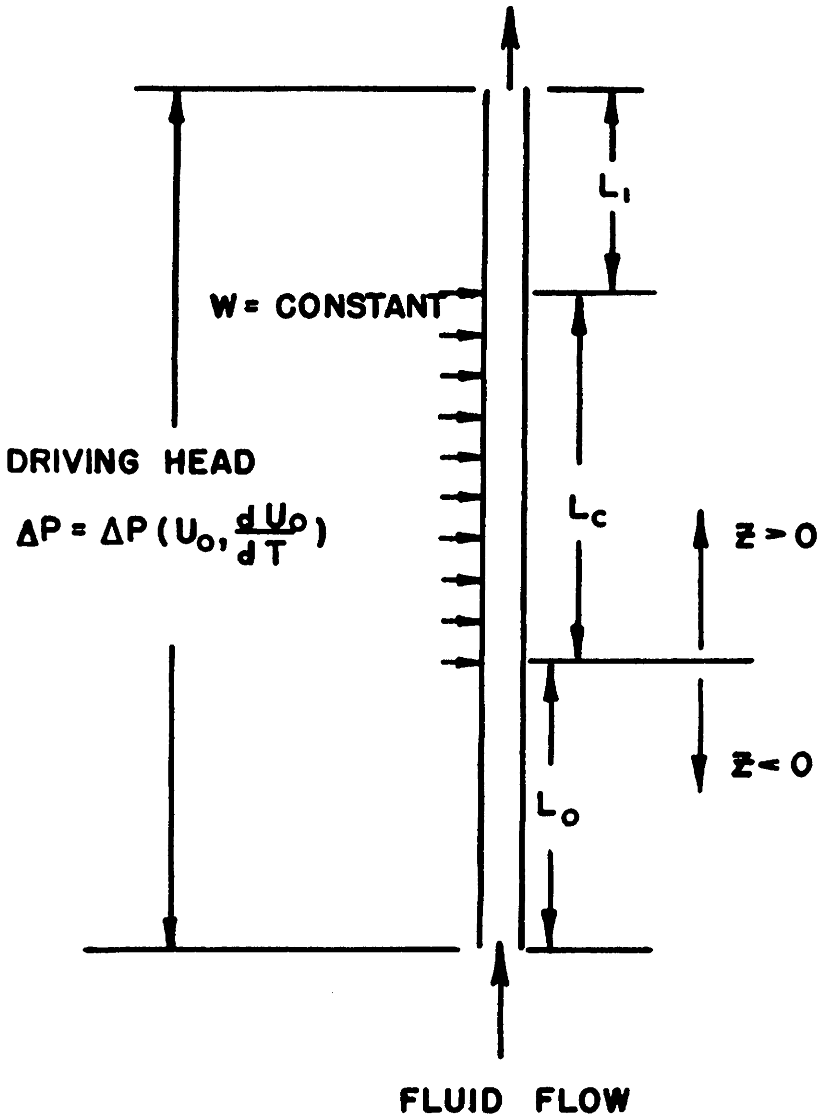

Figure 3. Idealized schema of the natural-circulation loop

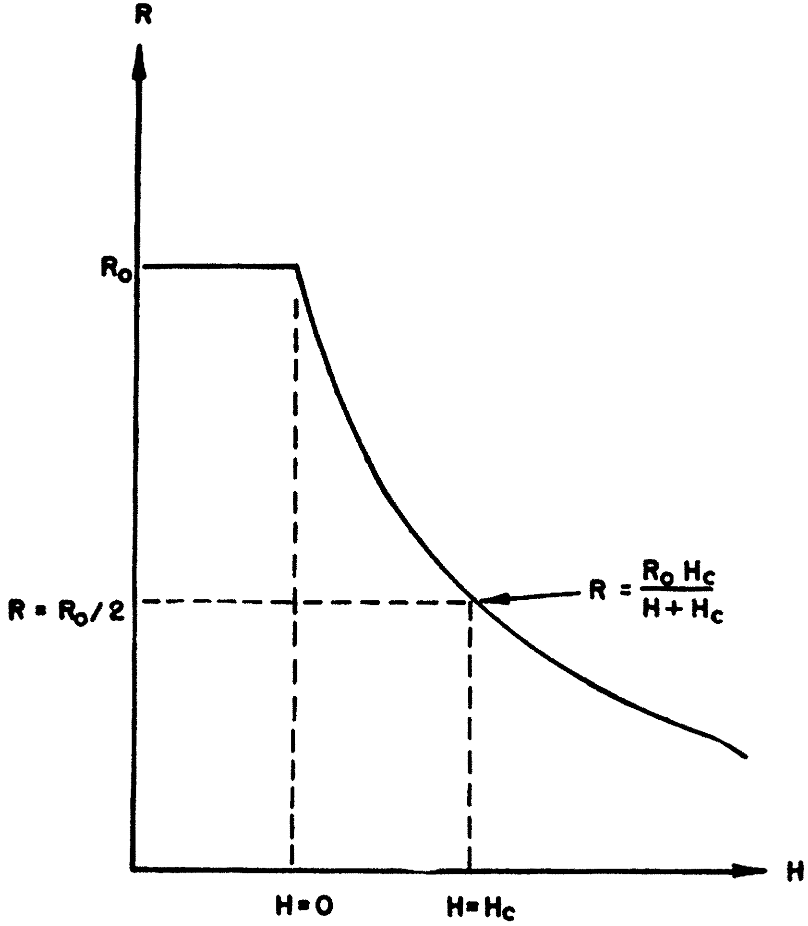

Figure 4. Model equation of state

Considering these idealizations, the mathematical model consists of the simultaneous solution of:

(a) Continuity equation

(b) Momentum equation

(c) Energy equation

(d) Equation of state

Making the appropriate assumptions, these equations are:

Continuity

$$(\partial R/\partial T) + [\partial (RU)/\partial Z] = 0 \tag{1}$$

Energy

$$R\ (\partial H/\partial T) + RU\ (\partial H/\partial Z) = W \tag{2}$$

Momentum (3)

$$(\partial P/\partial Z) + R\ (\partial U/\partial T) + RU\ (\partial U/\partial Z) + RG + (FRU^2/2D) = 0$$

State

$$R = R_0 \qquad H \leq 0$$

$$R = (R_0H_c/H + H_c)\ H \geq 0 \tag{4}$$

If (1) and (2) are combined, it follows that

$$\partial U/\partial Z = W\ [d(l/R)/dH] \tag{5}$$

The left hand side of (5) has units of time, therefore we define

$$l/Ⓗ = W\ [d(l/R)/dH] \tag{6}$$

The parameter Ⓗ for $H \geq 0$ is found from (4) and (6) and we define

$$Ⓗ_c \equiv H_cR_0/W \tag{7}$$

The number of parameters in the model was reduced by making equations (1) - (4) nondimensional. They were nondimensionalized by utilizing the following parameters and their characteristic values:

(a) Length — L_c

(b) Density — R_0

(c) Time — Ⓗ$_c$

The nondimensional continuity and energy equations were written for each of the three loop sections. These equations were combined with the equation of state and integrated over each section. Lagrangian coordinates were utilized in this integration in order to express the parameters as functions of nondimensional time. In performing this integration, a new parameter s was introduced where

$$s = H_0/H_c \tag{8}$$

and is called nondimensional subcooling since $(-H_0)$ is the enthalpy as the fluid enters the heater section.

The resulting equations were then introduced into the nondimensional momentum equations for each section. These equations were combined and linearized and the expression for the entrance velocity was found to be

$$u_0(t) = u_{\infty} + v_0\, e^{ct} \tag{9}$$

The method of small perturbations was then applied and an "equation in c" was formulated from the relations above.

c was then taken as

$$c = r + i\omega \tag{10}$$

and the resulting equations were investigated for the case of a pure oscillatory solution (neither damped nor amplified) for which $r \equiv 0$.

This led to a hypersurface (Σ) in the 6-dimensional space of the model parameters, i.e.,

$$(\Sigma) = \Sigma\ (l_o, l_1, s, u_\infty, g, f)$$

which corresponds to a pure oscillatory solution.

Finally, the steady state condition

$$\partial(\Delta p)/\partial u_\infty > 0 \qquad (11)$$

was utilized to eliminate the portion of the surface (Σ) where operation is impossible.

Results

Figure 5 shows how the experimental instability points compared with the instability threshold for the loop using Freon-114 as the heat transfer fluid operating at constant pressure. Figure 6 shows the instability threshold with subcooling, s, as a parameter. The experimental points shown were taken from tests with Freon-12 and Freon-114 as the heat transfer fluids at various values of subcooling. These figures show that it is possible to predict a flow instability threshold and obtain good agreement with experiment and a theory which includes first-order effects.

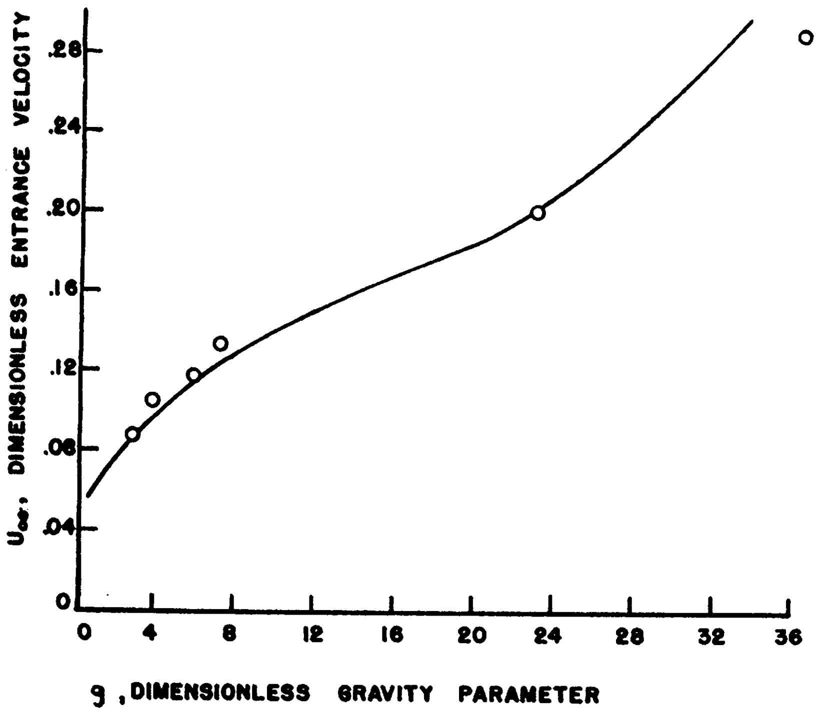

Figure 5. Results with Freon-114 at 495 psia

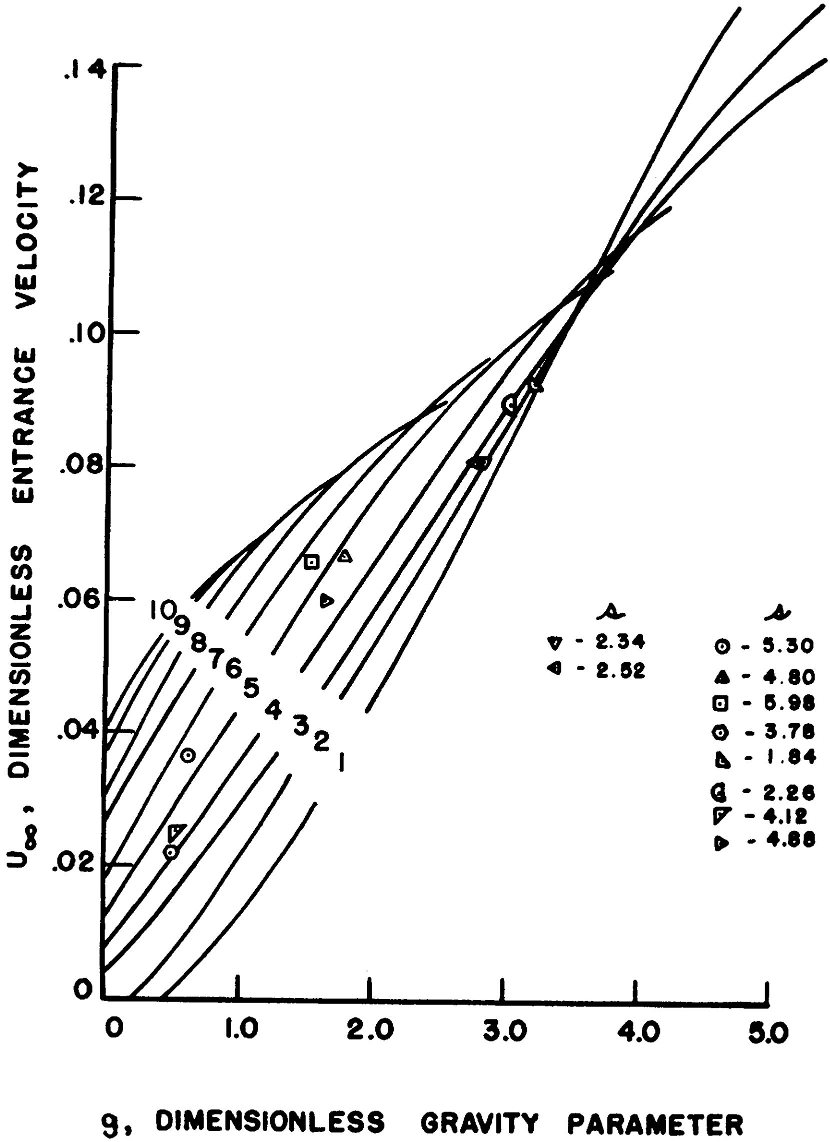

Figure 6. Results with Freon-12 and Freon-114 at various subcooling

NOMENCLATURE

Dimensional Quantities	Nondimensional Equivalent
L — Length	l
U — Velocity	u
W — Volumetric heat flux	w
ρh — Energy density	—
R — Density	ρ
T — Time	t
Z — Axial coordinate	z
P — Pressure	p
H — Enthalpy	h
G — Gravitational acceleration	g
F — Friction factor	f
D — Hydraulic diameter	—

Other Nondimensional Quantities

Ⓗ — Time
s — Subcooling
v — Transient velocity component
c — Complex constant
r — Real constant

REFERENCES

1. Harden, D. G. 1963. Transient behavior of a natural-circulation loop operating near the thermodynamic critical point. Argonne Nat. Lab. Rep. ANL-6710.

2. Walker, B. J. 1967. *Flow Instability Thresholds in a Natural-Circulation Loop,* Ph.D. Diss. Univ. Oklahoma (to be published).

3. Boure, J. 1965. *Contribution a l'étude théorique des oscillations dans les canaux chauffants a ébullition.* Thèse Docteur - Ingenieur - Centre d'Etudes Nucleaires, et Université, de Grenoble (France).