Use of Mathematical Models in Water Quality Management

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The basic problem in water quality management is to measure maximum beneficial use of the resource. This necessitates assessment in terms of an appropriate criterion, say economic efficiency or welfare efficiency, and is also concerned with the ordering of human activities in geonomic (banal) and economic spaces. The use of macro- and microanalytical models representing inputs and outputs by sets of mathematical relationships is of considerable assistance in understanding the interactions (sensitivity analyses) and alternate policies of future operation (system analyses).

The author has utilized both macro- and micromodels. The essential difference is one of fineness. The macromodel deals with an entire basin as an entity and requires disaggregation techniques to create fineness. It deals with averages, and functions as a planning policy model and operates at a reasonable safe level through aggregation. The micromodel on the other hand deals with discrete elements and in essence synthesizes the whole from the parts. It is subject to the accuracy of the elements and the danger of double counting. Also, mathematically, to get solutions requires considerable variable reduction.

The models being currently used for water quality management are mostly micromodels and use solids or dissolved oxygen criteria. Water Quality Criteria, or better still damage functions, are essential to model evaluation. Damage functions are very difficult to develop, and at present, the water quality criteria are used in lieu of the more definitive values. The water quality standard implies a level above which there is damage, and below which there is no damage.

The required water quality standards (RQS) are decision level criteria, consisting of 49 discrete values for each of four categorical uses; namely, municipal, industrial, agricultural and recreational. The 49 discrete values can be grouped into six general groups depicting stream responses. The stream responses can be formulated as translations from upstream AQS values by category to effluent standards, by input/output response equations. Six categories of response equations are recognized; biodegradable (L), nutritional (N), persistent (P), sediments (S), thermal (T) and bacterial (B). Two of these, namely bacterial and sediments, can be considered constraints. This results in a four by four stream response matrix.

Thus, the model must be able to respond to "instream-standards" for a specified use—municipal, industrial, agricultural, and/or recreational. The decision variable will be the required treatment level to produce an effluent standard, so treatment in terms of the stream responses criteria and cost schedule must be developed.

Many types of models, mostly deterministic, have been proposed. All require a basin approach, and one actually uses the name "Basin Firm". This implies a need for a realignment of institutional constraints. This will be difficult, but the increasing magnitude of the water resources problem will force it into being. Models can be based on continuous or discrete functions. Waste treatment processes are by discrete increments, so the author has used the discrete route. The discrete route can be used with one plant being pivotal to provide a continuous spectrum.

These ideas can, I believe, be best conveyed by resorting to a greatly simplified example. Assume a basin with four discrete waste discharges (A,B,C,D) and one point of intake (I), the river distance between discharges being depicted as length divided by velocity or time (t) (Fig. 1).

Further assume that specific waste loading and volumes will be as follows:

Source	Location	Flow	BOD	L	N	T	P
Industrial	A	0.01	5000	50	2.5	Nil	Cd = 2.0
Municipal	B	1.0	200	200	10	Nil	Nil
Ind./Municipal	C	2.0	250	500	23	= 20°	Cr = 0.9
Municipal	D	0.6	200	100	5	Nil	Nil
Total	$ABCD$			850	42.5	—	—

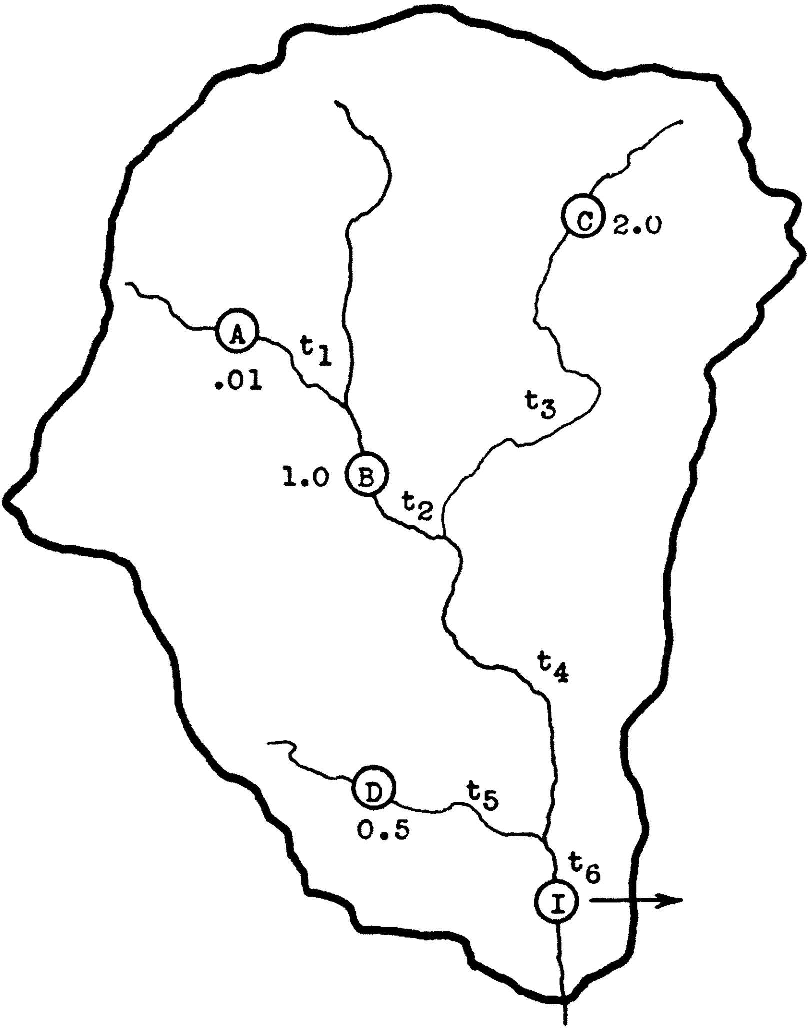

Fig. 1. River basin with four discrete waste discharges (A,B,C,D) and intake (I).

L, N, T, P represent biodegradable, nutritional, thermal, and persistent chemical loads, L & N as pounds per day of BOD and nitrogen, T as rise in degrees F, and P as ppm of a specified element.

To develop the model, it is necessary to formulate input-output relationships between RQS on one hand and stream responses on the other. For example, the biodegradable model would be as follows for a continuously applied load:

$$R = (S - RQS) \cdot Q \cdot C_L$$

where R is the permissible load that will maintain the RQS dissolved oxygen level, with a saturation level S, a stream flow Q, and stream characteristics C_L, which is a function of re-aeration, reach, velocity, number of successice re-uses, and degree of disaggregation required ($n \cdot (D/L) \cdot C_L\ k_2$).

Using this macrolevel model the following values of L_t were estimated:

Reach (identified by t)	Flow Q	Re-aeration or Permissible Load (R)
t_1	0.5	6.0
t_2	5.0	120
t_3	8.0	192
t_4	13.0	104
t_5	5.0	60
t_6	18.0	36
		Total = 518

Given the impressed biological load (850) and the regenerative capacity (518) the problem is to write an objective and constraint relationship in terms of possible decisions and subject it to an economic optimization. The decisions will be the level of treatment (e) required at A, B, C, and D. Using three discrete levels, primary, secondary, and tertiary with associated costs:

Treatment	Efficiency (e)		Residual Discharge (1-e)		Removal Cost (C)	
	L	N	L	N	C	Ce
Primary	0.3	(0.3)	0.7	(0.7)	15	5.0
Secondary	0.7	(0.5)	0.3	(0.5)	25	17.5
Tertiary	0.9	(0.7)	0.1	(0.3)	35	31.5

The minimum cost will be: Min $C = 50 \times Ce_a + 200 \times Ce_b + 500\ Ce_c + 100\ Ce_d$

subject to $50\ (1-e_a) + 200(1-e_b) + 500(1-e_c) + 100(1-e_d) \leq 518$

This problem can be put on a digital computer with instructions to try all possible combinations of e for A, B, C and D at three levels. It is of interest to note that it required only 50 minutes to program and 5 minutes to run at a total cost of $4.00.

The optimal solution is for primary treatment at A, C and D and secondary treatment at B. If secondary treatment had been required at each discharge, the basin would be charged in excess of 2.2 times (14,875/6759). Now, the nutritional load should also be examined; only the constraint is changed:

$$2.5\ (1-e_a) + 10\ (1-e_b) + 25\ (1-e_c) + 5\ (1-e_d) \leq 25.9$$

Using the 1-2-1-1 decision

$$2.8\ (0.7) + 10\ (0.5) + 25\ (0.7) + 5\ (0.7) = 27.75 > 25.9$$

Since this combination results in a value greater than 25.9, a new combination below 25.9 should be sought. Thus, treatment would be based on N rather than L. To meet this requirement ($\Sigma\ N_t\ (1-e_t) \leq 25.9$) we found the treatment combination 3-2-1-2 to give the minimum cost.

$$2.5(0.3) + 10(0.5) + 25(0.7) + 5(0.5) = 25.75 < 25.9$$

Cost = $9825.

Treatment (*e*) has no effect on *P* or *T* and these must be handled as requiring dilution.

Source	Dilution (Q/q)	*P*	*T*
A	180	0.01 = *RQS*	—
B	18	—	—
C	9	0.01 = *RQS*	*T* = 2° 10° allowable
D	30	—	—

Thus, all 81 possible combinations of treatment have been examined for four stream responses. Any new entrants, or changes in uses or criteria, can quickly be evaluated in a similar fashion.

Now that the optimal treatment levels have been assigned it remains to apportion the cost among the dischargers. For example, A must pay B the following:

$$\text{Cost to A} = L_a\ [(\text{Total Cost}/\Sigma\ L_i) = C_a]$$

$$\text{or } 50\ [(9325/850) - 5.0] = \$3000$$

The author has presented a discussion of water quality standards and criteria and an outline of a system analysis approach to implementation based on an economic efficiency criterion.