Preliminary Solution for the Hypervelocity Impact of a Porous Stone Sphere on Solid Aluminum[1]

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An earlier, analytical solution for the hypervelocity impact of a porous plate on a solid aluminum slab is extended to the impact of a sphere on a slab in this paper. This solution shows the compaction of the sphere and the flow of the compressed materials of the sphere and of the target in the early stages of the impact. This problem originated during an investigation of the impact of micrometeoroids on space vehicles in orbit. The analytically predicted velocities of impact range from 30,000 to 240,000 ft/sec. Since micrometeoroids tend to be spherical in shape, this paper considers a porous sphere of rock with a mass of 10^{-9} g. Micrometeoroids are most commonly silicates, mainly enstatite or olivine, with some composed of iron-nickel alloys (Donn, 1964). Micrometeoroids are usually not solid, but consist of a fluffy agglomerate. A typical, but not an average, density could be one, i.e., the same as that of water.

Papers on hypervelocity impact have already been presented to the Academy by members of this group (Lake, 1962; Sodek, 1963; Hardage, 1966) and this paper is a continuation of this series. Lake considered the shock propagation in aluminum from a hemispherical source. Sodex extended the solution to a solid aluminum sphere impacting on an aluminum slab but did not follow the position of the interface. Hardage considered a porous thin plate impacting on an aluminum slab and followed the position of the interface. All solutions made the usual assumption of inviscid flow of the highly compressed material. In this paper, a theoretical investigation of the early stages of the impact of a porous sphere of rock on a solid, aluminum slab is reported. The position of the interface is followed, as well as that of the shock front. There are several refinements in this solution which are expected to permit the solution to be continued at a later time to show the propagation of the shock from the fluid region into the plastic region where the shock front separates into two shock fronts and to follow this complicated shock into the elastic material of the target.

Initial Conditions

The initial and boundary conditions for the problem are summarized. A sphere is approaching normal to a semi-infinite slab at a velocity V and is assumed to touch at zero time; i.e. $t = 0$. The axis of symmetry, o-z, is determined by the point of contact, P, and lies along the axis of approach as indicated in Figure 1. Since the system has radial symmetry, a solution is required for only a thin, wedge-shaped piece of pie, with the point on the axis of symmetry. This three-dimensional wedge may be represented by a sketch in two dimensions that extend up from the axis of symmetry in the half-plane as far as may be required to show the effect of the impact. No flow may occur across the faces of this wedge, but material may flow out the back.

The origin of the spherical coordinate system is placed on the axis o-z, at a distance of three radii of the sphere above the point of contact. A two-dimensional mesh is formed by drawing lines of constant angle and lines of constant radius as indicated in Figure 2. The center of each cell in this mesh is given by two coordinates, L(r) and M(r,θ). The radial

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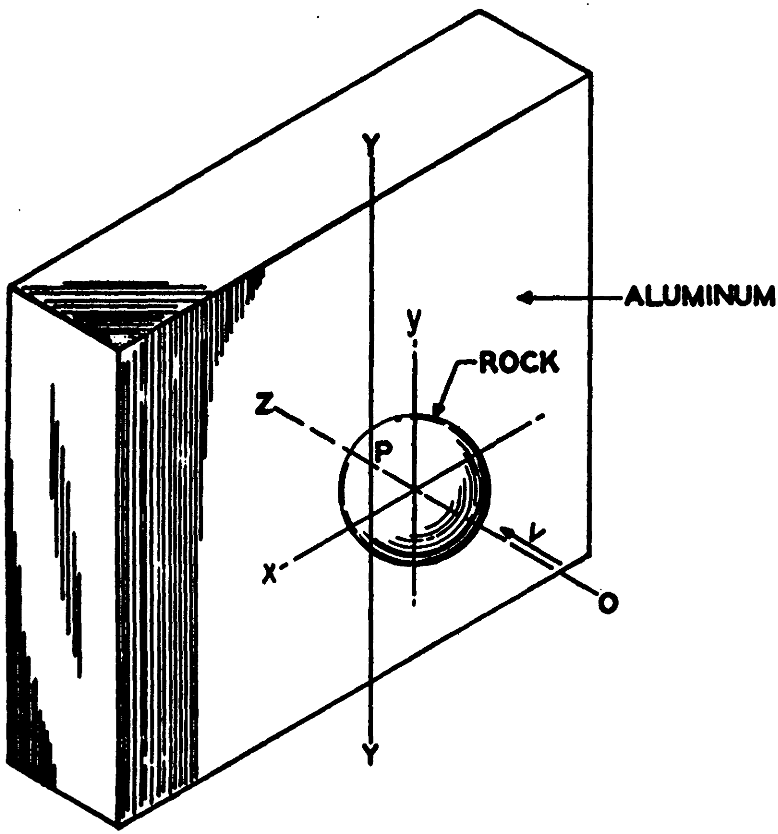

Figure 1. Coordinate system for porous rock impacting on "semi-infinite" slab of aluminum.

length is represented by L, and the angular position by M. The direction of increase of these quantities is indicated.

Hydrodynamic Equations for Viscid Flow

The equations for conservation of mass, momentum and energy are the fundamental equations of flow. The derivations of these equations are found in many books (Bird et al., 1960; Amer. Inst. Phys. Handbook, 1963). In vector form and with Eulerian coordinates, the three equations of flow with the viscosity are:

Conservation of mass, or the continuity equation

$$d\rho/dt - \rho\nabla\cdot V = 0 \qquad (1)$$

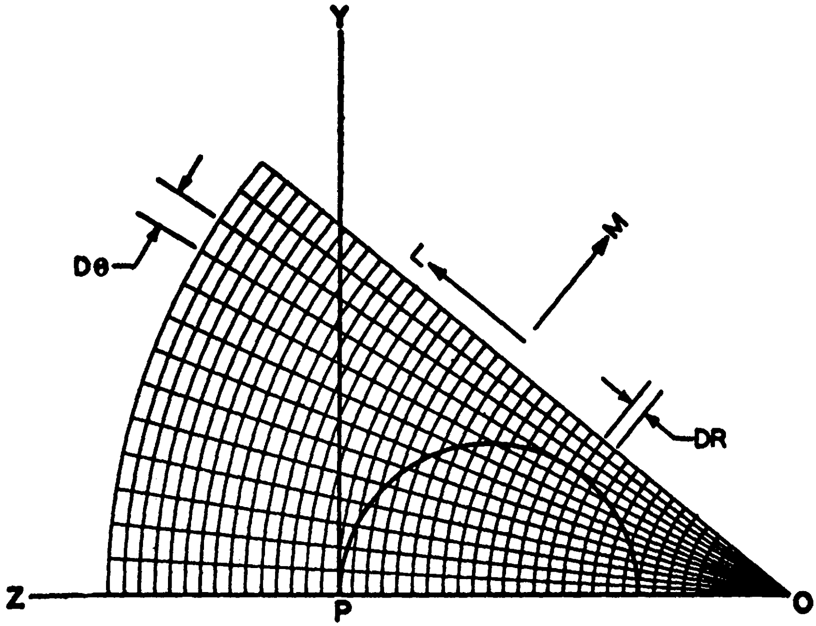

Figure 2. Polar coordinates for section of sphere on right entering aluminum slab on left.

Conservation of momentum, or the equation of motion

$$\rho\, dV/dt - \nabla \cdot p - \nabla \cdot S_{ij} = 0 \quad (2)$$

Conservation of energy, or the energy equation

$$\rho\, dE/dt - \nabla \cdot (pV) - \nabla \cdot (S_{ij} V_i) = 0 \quad (3)$$

The form for the energy equation was derived on the assumption of adiabatic energy flow. By using this equation for everything except the initial compression up the Hugoniot curve, any subsequent compression and/or expansion will remain on the same adiabat. The preceding equations are valid for both fluid and plastic flow. At the high pressures in the immediate vicinity of the impact, the flow is considered to be inviscid, or without viscosity. The equations for plastic flow are assumed to apply below a more or less arbitrarily selected pressure.

In the preceding equations, $V(v_i, v_j, v_k)$ is the velocity of the material in functional notation, ρ is the density, p is the pressure, E is the total energy and S_{ij} is the stress tensor. This tensor is zero for inviscid flow but, for plastic flow, it is very important and has the following form:

$$S_{ij} = (\eta_B - 2\eta/3) d_{KK} \delta_{ij} + 2\eta d_{ij} \quad (4)$$

where η_B is the bulk viscosity, and η is the shear viscosity. It is generally assumed that η_B is zero. The term δ_{ij} is the Kroneker delta. The strain rate components d_{ij} are defined by the relation

$$d_{ij} = 0.5[\partial v_j/\partial x_i) + (\partial v_i/\partial x_j)] \qquad (5)$$

If the material is elastic and not strained above the elastic limit,

$$S_{ij} = \lambda e_{KK}\delta_{ij} + 2\mu e_{ij} \qquad (6)$$

where μ is the modulus of rigidity and λ is the Lame's lambda. The terms e_{ij} are the strain components

$$e_{ij} = 0.5[(\partial r_j/\partial x_i) + (\partial r_i/\partial x_j)] \qquad (7)$$

where r_i and r_j are the components of the displacement vector for the material, which results from elastic deformation only. The displacement of a body through space at a constant velocity does not affect the magnitudes of these components.

For a plastic solid, the viscosity is a decreasing function of the strain rate tensor. The following relation was proposed for the viscosity of a plastic solid (Eirich, 1956).

$$\eta = \eta_0 + S_0/|S| \qquad (8)$$

The terms in this relation are the viscosity, η_0; the yield value in shear, S_0; and the value of the strain rate tensor $|S|$, which was defined in Equation 4. This model was used to describe plastic flow in hypervelocity impact (Riney, 1963), and it is employed in the plastic region for this study. In the literature, this model for the viscosity is designated as Bingham plastic flow.

The preceding terms for η_0 and S_0 are not constants, but are functions of the pressure and temperature. For a plastic solid it is known that the following inequalities are required: $\partial\eta_0/\partial p > 0$; $\partial S_0/p > 0$; $\partial\eta_0/\partial T < 0$; $\partial S_0/\partial T < 0$.

With these conditions, a first approximation for these two functions may have the following form.

$$\begin{aligned} \eta_0 &= a_1 + a_2 p + a_3/e \\ S_0 &= b_1 + b_2 p + b_3/e \end{aligned} \qquad (9)$$

where e is the internal energy and the a's and b's are constants which are selected by trial and error in order to obtain the normal values for η_0 and S_0 at standard temperature and pressure. Since e is proportional to the temperature, the inequalities for temperature are satisfied. The preceding approximations are employed for the calculations in this paper.

EQUATIONS OF STATE

The three conservation relations, Equations 1, 2 and 3, are not sufficient to solve for the shock and flow since there are four variables, **V**, p, ρ and E, in these relations. The fourth relation between these variables is usually taken as an empirical, or semi-empirical, equation of state. For aluminum, an equation of state was proposed by Tillotson (1962) which has been widely used and is used for this work. This equation has the form

$$P = (a + b/[E/E_0 m^2) + 1])\rho E + A\mu + B\mu^2 \qquad (10)$$

where P is the pressure in megabars, E is the specific internal energy, ρ is the density under compression, m is the ratio ρ/ρ_0 where ρ_0 is the normal density, and $\mu = m - 1$. The terms E_0, A, B, a, and b are constants.

A more complicated equation of state is required for the impacting sphere of porous rock. At the high pressures developed during the impact, inviscid fluid flow is generally assumed. This was assumed for aluminum in all calculations familiar to our group except some by Riney (1963). Wagner et al. (1964) have shown that the constants in the unmodified Tillotson equation of state cannot be adjusted to give a close fit to the Hugoniot curve that is found for porous stone by experiment. An equation of state for porous materials proposed by Soviet scientists provides a good fit with the experimental data (Kormer et al., 1962; Al'tshuler et al., 1962). McCloskey (1964) has modified and extended the Russian work. Wagner et al. (1964) have added some corrections to McCloskey's work. The proposed equation of state for porous rock assumes that P and E may each be written as the sum of three functions.

$$P(m,T) = P_c\ (m) + P_n\ (m,T) + P_e\ (m,T) \tag{11}$$

$$E(m,T) = E_c\ (m) + E_n\ (m,T) + E_e\ (m,T) \tag{12}$$

In these relations, T is the temperature. The three functions are distinguished by subscripts. The subscript c indicates compression with interactions of the atomic lattice at 0 K; the subscript n denotes the contribution from the thermal vibrations of the lattice ions; and the subscript e represents the contribution from thermally excited electrons. To illustrate the relatations in equation 11, the pressure is plotted in Figure 3 as a function of m for three isotherms.

Computing Procedures

The computer is programmed to solve the preceding equations with the boundary conditions (some have not been mentioned) to obtain values for ρ, u, w, E and P. The velocities u and w are the R and θ components of the velocity vector $\mathbf{V}$. The computer solution to obtain these variables as functions of space and time proceeds by the following steps.

1. Convert Equations 1, 2 and 3 into a finite difference form. Use central differencing for space derivatives, and forward differencing for time derivatives.
2. Label the center of each cell in Figure 2 with the initial values of ρ, u, w, E and P at time $t = 0$.
3. Solve the finite difference equations from step 1 for the values of ρ, u, w and E at time $t = t + \Delta t$.
4. Solve Equations 10 and 11 for the values of pressure at the center of each cell in the mesh at time $t + \Delta t$.
5. Replace each cell value of ρ, u, w, E and P with the new values at $t = t + \Delta t$.
6. Step time by an amount Δt.
7. Repeat steps 3 through 6 until a final time is reached, i.e., $t = t_{final}$

Results

The fluid flow solution agrees with and extends previous results from this laboratory. As an illustration of this work, a solution is shown at an instant during the formation of the crater in Figure 4. The relative positions of the compressed stone, the shock front and ejecta from the target are illustrated. The pressure has not yet decreased sufficiently to show the coupling to the plastic region. The initial conditions for the solution in the sketch were for a stone sphere of 50% porosity and mass 10^{-9} g which impacted on a thick aluminum slab at 36 km/sec.

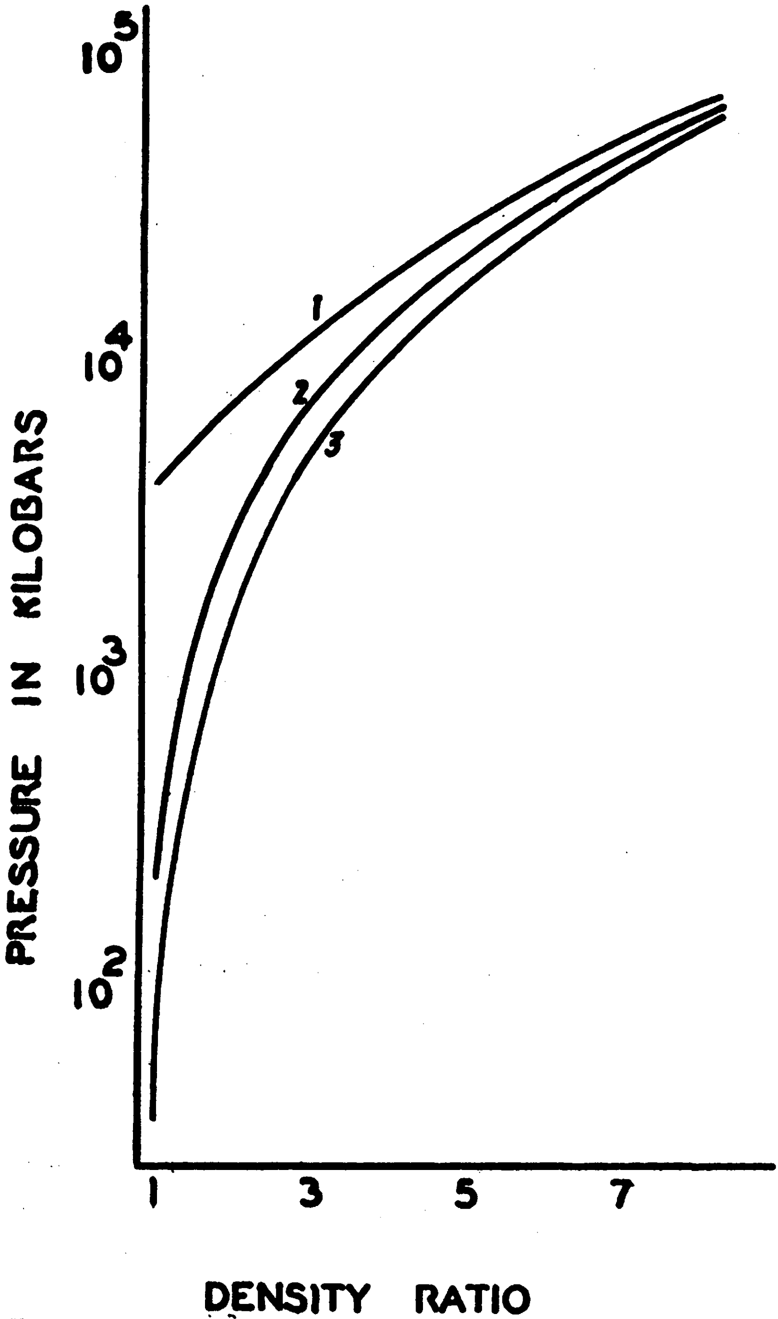

Figure 3. Isotherms at temperatures T_1, T_2 and T_3 for Equation 11.

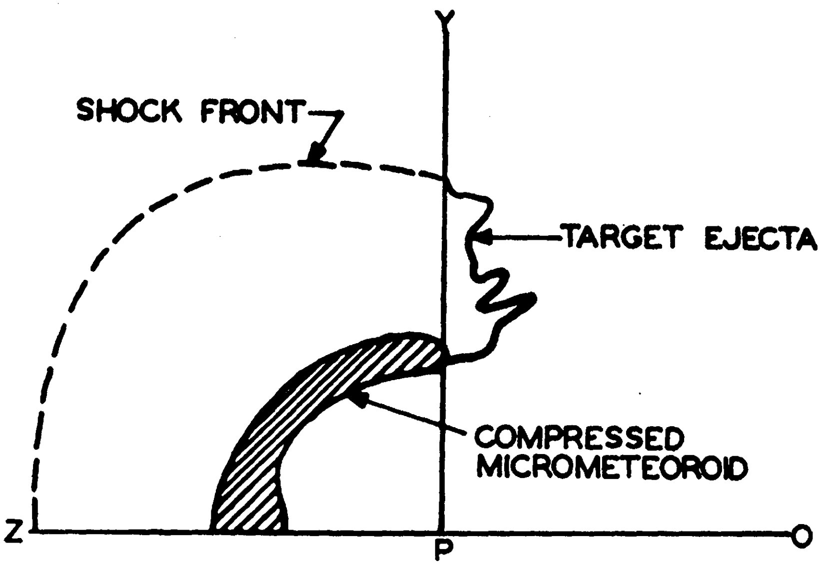

Figure 4. Section of crater that was calculated for one instant during the crater formation.

Computer solutions were obtained for a wide range of initial conditions. These include different initial velocities, two diameters of the impacting sphere and a range of porosities. Solutions were extended to shock coupling from the fluid to the plastic region and then to the elastic regions of the target.

Details of these further solutions will be published elsewhere.

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