
Plasma Temperatures

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INTRODUCTION

It has been pointed out by Coffman (1966) in a recent paper that there is a cooling effect on plasmas due to ionization when plasmas are produced in a confined environment. Such plasmas are commonly produced by electrically exploding wires which are submerged in a liquid. We wish to show the theoretical basis of this cooling effect from classical theory and to show an important feature regarding the use of the Boltzmann distribution function in the calculation of temperatures in plasmas. To illustrate the application of the theory and its correlation with experimental data, an analysis is included of some recent data obtained by Sherk (1964), who exploded wires under water and measured their brightness temperatures; and we present a correlation of the theory with some data given by Martin (1960), who exploded tungsten wire under water and made an analysis of the temperature of the resulting channel.

THEORY

It is important to distinguish between the brightness temperature T_b , which is defined as the temperature of a blackbody having the same spectral radiance at the wave length in question, and the kinetic temperature T , which is the temperature defined by equating the average energy per particle imparted to the system to $3kT/2$, wherein k represents the Boltzmann constant. It is the kinetic temperature that must be used in the Boltzmann distribution function as will be seen in the succeeding development.

We assume that the energy per atom imparted to the system is uniformly distributed among the particles; that is, equipartition of energy is

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assumed. That the energy per atom is proportional to the number of atoms in a metal is understandable if one looks at the excitation means as being the sudden application of an electric field. In this case the conduction electrons receive virtually all of the energy imparted by the source, and this energy is redistributed subsequently among the atoms of the system. In the case where the system is a wire, such an event is easy to demonstrate in the laboratory. If the wire is placed under water or in an oil bath, then the explosion of the wire is confined. Consequently, it may be assumed that the energy is truly distributed equally among all the free particles.

If V_1, V_2, \dots represent the first, second, etc., ionization potentials of the element of which the wire is composed, then one may relate the kinetic temperature and brightness temperature by the following equation: Kinetic energy of N atoms = Kinetic energy of ions and electrons plus ionization energy
or

$$3kTN/2 = 3kT_b N/2 \left\{ 1 + \sum_{j=1}^{(\infty)} \exp \left[- \left(e/kT \right) \left(\sum_{i=1}^j v_i \right) \right] \right\} + Ne \sum_{j=1}^{\infty} v_j \exp \left[- \left(e/kT \right) \left(\sum_{i=1}^j v_i \right) \right] \quad (1)$$

The left-hand side of Equation (1) is just the sum of the kinetic energies of N atoms, while the first term on the right represents the kinetic energy of N ions. The first sum on the right with its coefficient represents the kinetic energy contributed by the electrons due to first, second and higher order ionizations. The second sum on the right of Equation (1) with its coefficient accounts for the energy required for the ionization process. In every term on the right, a Boltzmann distribution is assumed.

As the number of atoms N is a common factor, in Equation (1) it follows that the per atom equation may be written in the form

$$3kT/2 = 3kT_b/2 \left\{ 1 + \sum_{j=1}^{\infty} \exp \left[- \left(e/kT \right) \left(\sum_{i=1}^j v_i \right) \right] \right\} + e \sum_{j=1}^{\infty} v_j \exp \left[- \left(e/kT \right) \left(\sum_{i=1}^{\infty} v_i \right) \right] \quad (2)$$

The reduction of Equation (2) to a simple form is possible if one realizes that the exponential terms occurring under the summation signs are very nearly either one or zero. If a sum which forms one of the exponents in Equation (2) is near zero, then the exponential may be approximated by the number one; similarly a very large exponent allows one to approximate the corresponding exponential term by the number zero. Therefore, for a plasma consisting of a singly ionized gas, Equation (2) may be written in the simplified form

$$3kT/2 = (3kT_b/2) + eV_1, \quad (3)$$

where $\exp (-eV_1/kT)$ has been set equal to unity and it has been assumed that $eV_1 \gg kT$ when $i > 1$.

After n ionizations, one may suppose $eV_{\dots} \gg kT$. Then it follows that the exponentials of Equation (2) may be replaced by zero for $j > n$. In addition, if

$$e \sum_{j=1}^n V_j \ll kT,$$

the remaining exponentials may be replaced by unity. Then Equation (2) reduces to the form

$$3kT/2 = (3kT_b/2)(1 + n) + e \sum_{j=1}^n V_j. \quad (4)$$

This is a simplified form of Equation (2) to describe the relationship of the brightness temperature to the ionization potentials, assuming a given atomic kinetic temperature to be determined by the input energy.

This simplified form points out the linearity between the sum of the ionization potentials and the brightness temperature. If the kinetic temperature is not high enough for second ionizations to become important, only the first ionization potential of the element will occur in the sum and Equation (4) reduces to Equation (3). This is expected to be the answer to a question posed by Sherk (1964) in regard to his empirical analysis of the data he collected on brightness temperatures of the elements aluminum, copper, tungsten and platinum. However, it can be seen that Sherk selected his data for constructing his linear relationship between the brightness temperature and the ionization potential in such a way that the slope of his graph is -5330 degrees per electron volt while our theory requires that the slope of this graph be -3860 degrees per electron volt. As we shall see, this discrepancy is easily resolved in favor of our theory by a reconsideration of the data of Sherk.

In the graphical presentation of his data Sherk excluded certain of his measured values. His basis for the exclusion is that the brightness temperature obtained at a wavelength of 5017 Å was a better approximation to the true temperature than were the brightness temperatures which were obtained at the wavelengths of 4000 Å and 6800 Å.

We wish to reconsider all of Sherk's published data and have presented from his data in Table I the average temperature in electron volts per particle together with its standard deviation for each of the elements aluminum, copper, tungsten and platinum. It is noted that the smallest standard deviation associated with Sherk's measured data is for the element platinum. Therefore, we take the measurement of the brightness temperature for platinum together with its ionization potential and calculate the average energy per particle imparted to the wire by Sherk's apparatus, obtaining a value of 13.80 eV. Setting $3kT_b/2e = 2.46$ eV, and $V_1 = 8.88$ eV into Equation (3), one gets $3kT/2e = 13.80$ eV. Of course, this $3kT/2e$ is the amount of energy per atom imparted by Sherk's apparatus for his particular platinum wires. Assuming that other wires had a like amount of energy per atom imparted to them in the explosion by Sherk's apparatus, we calculate the brightness temperature for each of the elements on which Sherk gave experimental data. The results of these calculations are indicated in Table I also; and it can be observed that in every case the calculated value of the brightness temperature is within the standard deviation obtained from Sherk's data. It should be noted that the brightness temperature is not directly given in Table I but is implied by the factor $3kT/2e$ so that the number appearing there has the physical meaning of the energy per particle measured in electron volts after one accounts for the cooling effect of ionization. In his paper Sherk observed the linearity between the ionization potential and the measured brightness temperatures of the elements in question and suggested that the explanation of the relationship was not completely clear. We believe the explanation given in this paper gives us an understanding of the linearity which was observed by Sherk.

TABLE I. REVISED BRIGHTNESS TEMPERATURES FROM DATA BY SHERK AND PRESENT THEORY.

Element	Experimental ($3kT_b/2e$)	Standard Deviation	Calculated ($3kT_b/2e$)
Al	4.12	0.62	3.92
Cu	3.25	0.32	3.06
W	2.92	0.35	2.93
Pt	2.46	0.24	2.46

We further calculate for the use of Sherk's apparatus or one of similar construction the predicted brightness temperatures for a number of metals by the use of Equation (4), and the results of these calculations are given in Table II together with the ionization potentials which were

TABLE II. CALCULATED BRIGHTNESS TEMPERATURES FROM PRESENT THEORY FOR AN APPARATUS DESIGNED FOR $3kT/2e = 13.80$ eV. AS SHERK USED WIRES OF CONSTANT VOLUME, HIS APPARATUS AND DATA CORRELATE WELL WITH THE DATA GIVEN IN THIS TABLE.

Element	$3kT_b/2e$
Al	3.92
Cu	3.06
W	2.93
Pt	2.46
Li	4.22
Na	4.34
Zn	3.44
Pb	3.21
Ag	3.23
Ni	3.06
Au	2.31
Hg	1.70

used in the calculation. It may be observed from Table II that sodium constitutes the best wire for a given specific energy input if the goal is to obtain a hot plasma. Lithium is not far behind, and aluminum serves very well as a source of hot plasma. Perhaps it is interesting to note that mercury gives the poorest yield of all in this respect. These predicted brightness temperatures, which are presented again in the form of remaining kinetic energy after the cooling effects of ionization have been considered, may be used as a further test of the theory when experimental data is available.

Of course, we realize that the apparatus of Sherk may not be available to every experimenter. Therefore, we present the above analysis and predictions for Sherk's apparatus more as a form of analysis than as a set of predictions which are of fundamental importance. However, it is obvious that any apparatus which imparts 13.80 eV per particle to a wire exploded under water would do for checking the predictions given in Table II. Furthermore, should one wish to experiment with some other

apparatus, it is only necessary to make an initial determination of the amount of energy imparted per atom to a wire before the analysis can be done and the predictions made which would again serve as a test for the theory. We are convinced that the success in explaining the data of Sherk is important in support of the theory.

Now we turn to a more complicated problem where it is not possible to ignore second ionizations, and we begin to see what steps are necessary when the postulates required for reducing Equation (2) to Equation (3) are no longer valid.

As a second application of this theory, let us turn to the work reported by Martin (1960). Martin used the explosion of a tungsten wire under water in an experimental study of a high-energy density, high-pressure arc plasma. While Martin was interested primarily in the energy transmitted to the channel containing ionized oxygen and hydrogen from the water surrounding the wire, he did record some useful data from his work for the application of the present theory. Let us take his data as recorded and make the necessary calculations to examine the relationship between brightness temperature and the atomic kinetic temperature. Martin emphasized the complete ionization of the initiating tungsten wire and included in his calculations the kinetic temperature of 200,000 K. With the known volume of tungsten initiating wire and the simultaneous Saha equations, Martin calculated the number densities of tungsten atoms in various stages of ionization. The results indicated that the populations of the singly and doubly ionized levels of the tungsten were about equal. If we use the first ionization potential (Moore, 1958) of tungsten as 7.98 eV and the value of 17.7 eV for the second ionization potential (Finkelburg and Humback, 1955) of tungsten, we can calculate from the Boltzmann factor the atomic kinetic temperature at which the population levels of the first and second ionization states are equal. Writing

$$\exp [-(e/kT) (V_1 + V_2)] = \frac{1}{2} = \exp(-0.693) \quad (5)$$

one gets

$$kT/e = (V_1 + V_2)/0.693 = 37.1 \text{ eV} \quad (6)$$

Using the above results together with the simplified theory expressed by Equation (4), one obtains a calculated brightness temperature of 48,000 K. Martin measures a maximum value of about 30,000 K, but when one uses the more exact form of the theory and substitutes one-half for the proper exponential factor to account for the tungsten atoms, which are once ionized, being 50 per cent doubly ionized, one gets a calculated brightness temperature of 30,400 K. This points up the fact that the resulting plasma temperature must be truly such that the conditions $eV_i \gg kT \ll eV_{i-1}$ for each i must be fulfilled before the simplified theory expressed by Equation (4) can be applied. However, there is good evidence in the correlation of the data of Martin and of Sherk that the Boltzmann theory can be used to gain insight into the relationship between the energy per atom applied to a confined system and the resulting brightness temperatures. Now let us turn to a precaution in the application of the Boltzmann theory.

We note in various places in the literature, and especially in the comments of Martin (1960), that the use of the Saha equations requires appropriately reduced ionization potentials to be used. Appropriately reduced ionization potentials are derived from a concept of excluded volumes formulated by Urey and Fermi, who are referenced by Martin. If one uses the Boltzmann distribution function together with measured ionization potentials and properly notes that the temperature to be used in the Boltzmann factor is the atomic kinetic temperature (and not the brightness temperature), then we believe consistent results with experiment are obtained from the Boltzmann theory. Fowler (1955) notes the importance

of applying the Boltzmann distribution function to systems which are statistically all alike. In practice, this means the Boltzmann statistics should be applied to systems constituting atoms and not to systems constituting atoms and ions and electrons. The use of the atomic kinetic temperature together with the measured ionization potentials in the Boltzmann theory makes it unnecessary to reduce the measured ionization potentials in the calculation of the population densities of the various atomic states. It is easy to see that if one uses the brightness temperatures in the Boltzmann factor, then it is necessary to reduce the potentials to obtain the right population densities of the various atomic states. Therefore, we caution that Boltzmann statistics may be used with precision only when the kinetic atomic temperatures are known together with the measured ionization potentials. Further, it is essential to consider the detailed population when kT is near the value of eV , for some ionization state. On the other hand, usual calculations may be made simply and easily giving the relationship of the brightness temperature to the kinetic temperature from Equation (3) if the above precautions are observed.

CONCLUSIONS

From the above theory and applications to two specific sets of data, we see that there is a strong theoretical connection via the Boltzmann statistics of the brightness temperature to the kinetic atomic temperature. We see that this connection may be simplified under the experimental conditions described by Sherk (1964) to a linear relationship between the ionization energy and the brightness temperature. In this theory, we limit such a linear relationship to the conditions that only a first ionization has occurred in the formation of the plasma and that the plasma has been confined until equipartition of energy results. A theoretical examination of the temporal behavior of electrons and nuclei in a wire during an explosion has been given by Coffman (1964) and supplies an estimate that a confinement time of less than a nanosecond is required. Data which was obtained by Martin (1960) was taken under a circumstance in which second ionizations were present in sufficient quantities that one could not use Equations (3) and (4). Therefore, we made more detailed calculations in consideration of this data including the terms involving the exponential factor for the second ionization potential, as this factor could not be replaced by either one or zero. When this was done, we found good agreement between the observed brightness temperatures which were obtained by Martin and the calculated brightness temperatures which were obtained from this theory.

The resulting expressions of this theory are necessary in the interpretation of data obtained from stellar processes. In these cases the brightness temperature constitutes the observable quantity and determines the rate at which nuclear reactions might be expected to occur, while the total energy of the system is directly dependent on the atomic kinetic temperature.

It is a brightness temperature that is usually measured in the laboratory when a plasma is generated by exploding wires. Much difficulty has been expressed (Chace and Moore, 1959, 1962, 1964) in correlating the brightness temperatures with other data, and we believe that part of this difficulty has been due to the failure to recognize the connection between the brightness temperature and the kinetic temperature as we have expressed it in the theory.

Therefore, we suggest an extension of the concepts of change of phase to the concepts of changes in the state of ionization of a gas. We wish to add to the changes of phase (Maninger, 1964) from solid to liquid and liquid to vapor the successive ionization processes; and we wish to make the parallel between the way the temperature behaves in change of phase to the way the brightness temperature behaves during ionization. Thus, a confined system may have energy added to it to the point of beginning

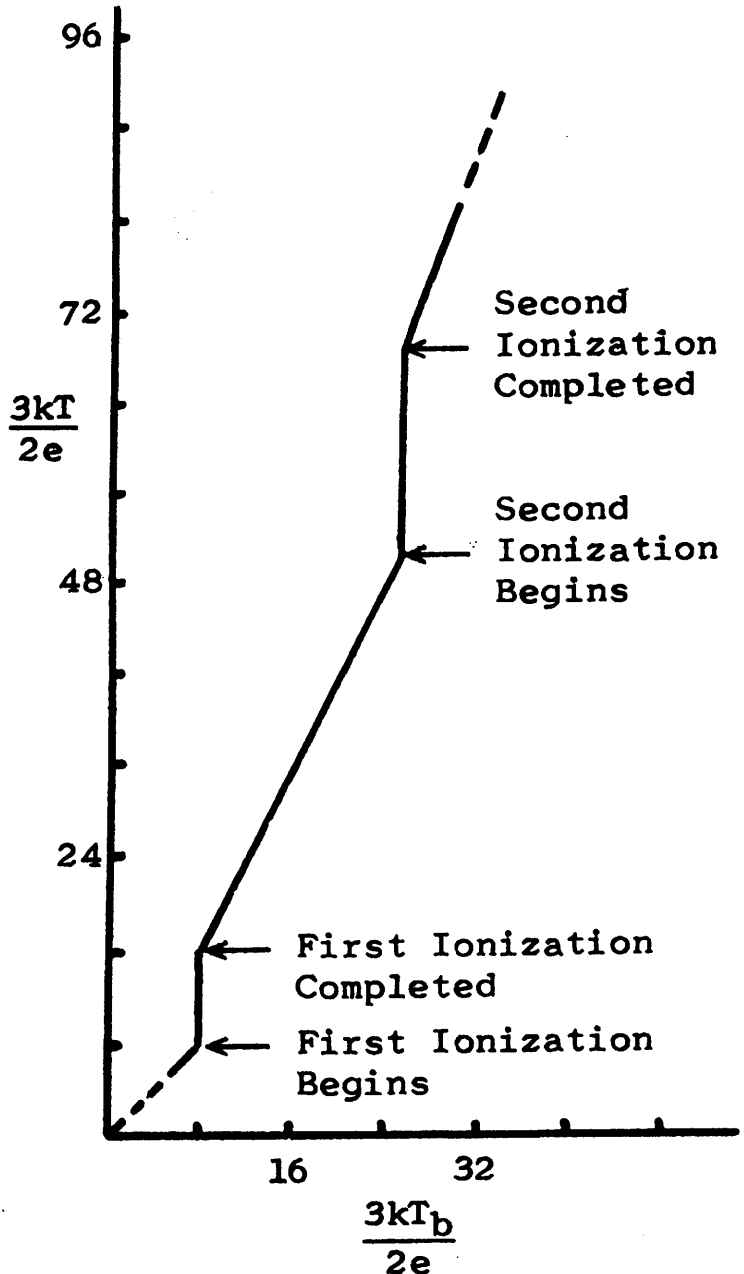


Figure 1. Theoretical relationship of brightness temperature T_b to atom kinetic temperature T under the assumptions of the present theory.

the first ionization, and its brightness temperature will remain constant, as it does during time required for a change of phase, until the first ionization has been completed; whereupon the brightness temperature will rise again until the system reaches an adequate kinetic temperature for second ionizations to become significant, and such a process repeats for each successive ionization. A graph (which shows the assumption) has been prepared and includes the first two ionization potentials of tungsten; and it may be seen in Figure 1 where the kinetic temperature is graphed as a function of the brightness temperature. Also, it may be seen from Figure 1 that the parallel concept to change of phase is a reasonable approximation to the brightness temperature changes resulting from ionization. It must be borne in mind that we have assumed the system to be confined and the temperature changes to have taken place in a time short enough that radiation losses are negligible. Under these assumptions, we believe the experimental data to which we have referred support the concepts introduced by the theory.

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