SECTION C, PHYSICAL SCIENCES

Digital Computer Method for Calculating Rocket Trajectories from Acceleration Data[1]

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In recent years, the Research Foundation of Oklahoma State University has been making calculations of rocket trajectories that involve the time variation of rocket acceleration, velocity, and position. These calculations have been based primarily on data obtained from longitudinal accelerometers mounted within the rocket payload. The acceleration information was transmitted to ground-based recording stations by way of radio telemetry. Past efforts in rocket trajectory analysis have been limited in accuracy by the design of the longitudinal accelerometers and the quality of the telemetry records available for use in extrapolating data. Recently a digital computer program has been developed for calculating trajectories of multistage rockets. This computer program which is applicable primarily to the IBM 650 computer has greater versatility than the desk calculator methods used in the past for trajectory calculations.

All of the accelerometers which have been used on the most recent rockets have been designed for presentation of both positive and negative acceleration information. This gives output data for both the thrust and drag periods of the atmospheric flight of the rockets in terms of g, the acceleration due to gravity. The output data from the accelerometers are not usable on the down-leg of the flight, because the rocket tumbles on reentering the atmosphere.

It is well known from the laws of mechanics that the rate-of-change of distance with time represents velocity information and the rate-of-change of velocity with time gives acceleration information. Conversely, if an acceleration-versus-time variation is integrated between time limits, velocity information may be obtained, and by integrating the velocity information between the same time limits, distance information may be obtained. This technique is utilized by numerical methods to obtain the velocity and position of a rocket when acceleration-time information is known. The area enclosed by an acceleration curve bounded by two time limits gives a value equivalent to velocity at the upper time boundary in units of g-seconds. To obtain the total velocity of the vehicle, one must add the new velocity to the velocity at the beginning of the period which is being integrated. A second integration will provide a change of distance and this distance, added to the value of the distance at the beginning of the period of integration, gives the cumulative distance for the vehicle.

When acceleration information, such as is obtained from rocket accelerometers, is being used for velocity and distance calculation, it is not practical to do mathematical integration using definite integrals because, in general, the mathematical curves are not expressed as continuous mathematical functions. For this reason it is necessary to integrate the area under the acceleration curves by numerical techniques. Different methods have been used in the Electronics Laboratory involving tedious tabulation of results computed by desk calculators. Methods used for numerical integration are the Trapezoidal rule and Simpson's one-third rule, both of which require the division of the time scale of the information into equal increments. The planimeter method for obtaining the area of

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acceleration curves has been used to some extent in the field for rough trajectory calculations, but this is less accurate than the numerical techniques mentioned.

To reduce the time and labor involved in computing a rocket trajectory from accelerometer information, a program has been written for use on an IBM 650 computer, one of which is available at Oklahoma State University. This digital computer trajectory program was written in SOAP (Symbolic Optimum Assembly Procedure) language. The computer program utilizes acceleration information obtained from telemetry records, which are accurate charts of data similar to that shown in Fig. 1, except with an expanded time scale. The computer program makes trajectory calculations of acceleration, velocity, and position in three orthogonal directions utilizing a preselected increment of time for the atmospheric portion of the rocket flight. This atmospheric portion includes all thrust and drag periods of the upleg flight. A different increment of time, usually larger than the first, is used for the velocity and position information following the atmospheric portion. The computer program utilizes both positive and negative values of acceleration for use in the thrust and drag regions.

The accelerometer information to be used in connection with the computer program is obtained from the telemetry records with the greatest accuracy presently possible. The accelerometers most recently used are of the segmented type. One type, in particular, has five segments which may be used to cover the entire anticipated range of acceleration, both negative and positive. The internal construction consists basically of five linear resistive elements spaced end-to-end in such a manner that a wiper will traverse the entire series of five segments between the two extreme *g* values. This design improves the resolution of the acceleration data from the telemetry records. In most cases an accurate gage voltage of five volts is applied across the accelerometer with the resistive elements in parallel so that each segment is supplied with the gage voltage. The output voltage from the wipers of the accelerometers varies over a range of zero-to-five volts per segment as a linear function of the acceleration. This output is supplied to the telemetry system for data transmission.

The IBM 650 computer trajectory program is written to utilize a series of acceleration values which are recorded as a function of time. Fig. 1 illustrates an acceleration curve which includes most of the time when the rocket is under the influence of atmospheric drag and shows the acceleration due to thrust of the first and second stage motors. It can be seen that the acceleration includes values of atmospheric drag following both the first and second stage thrust. The computer program considers the drag as negative acceleration.

The basic information which is supplied to the computer at the beginning of a trajectory analysis includes the rocket code number, the time increments to be used in making the calculations, the value of *g* at the launch site, trigonometric constants for the elevation angle of launch, and values of elapsed time at ignition and burnout for the various stages of the rocket. Constants may also be introduced to modify the data and to generate artificial drag periods, if this should become necessary. When wind data are available, they also are introduced prior to the acceleration data. The acceleration data are punched into input data-cards with seven values of acceleration per card, together with a code and card sequence number word. The output information which is obtained from the computer program are the rocket code number; the elapsed time; acceleration in the X, Y, Z, and tangential directions; velocity in the X, Y, Z, and tangential directions, position in the X, Y, and Z directions; the range of the rocket from the launch point; and the elevation and azimuth angle of the rocket measured as angles tangent to the rocket trajectory at the elapsed time when the calculation is made.

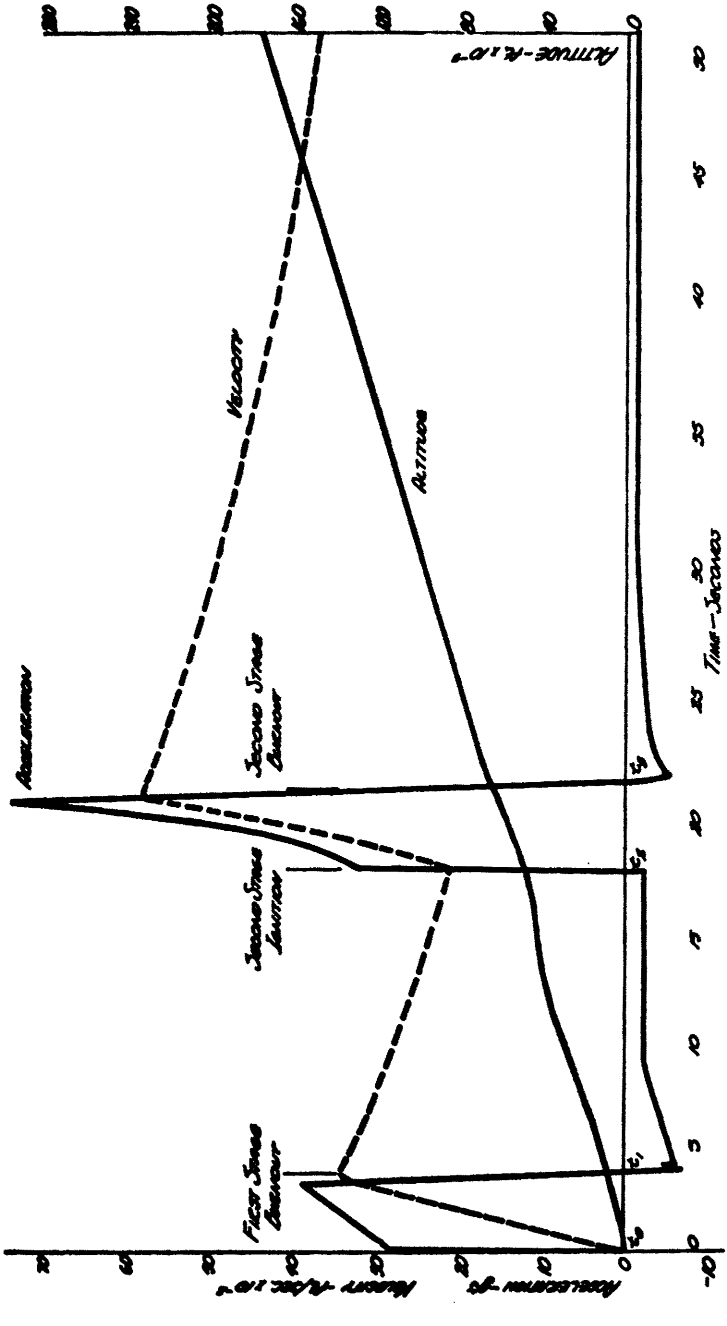

Fig. 1. Acceleration, velocity, and altitude curves for a two-stage rocket.

The IBM 650 computer program has been written to be as versatile as possible for use with various types of rockets with which Oklahoma State University, Research Foundation is associated. The program will accommodate any number of thrust and drag periods; however, it is designed specifically for rockets which have two periods of thrust and two periods of drag.

The computer program will smooth rough data by means of the "least squares curve fit" method as an option, using parabolic or cubic equation curve fitting, whichever is more desirable. The curve fit routine utilizes five adjacent acceleration data points for calculating a smooth curve, the mid-point of which gives the value of acceleration used in making the calculations. The program does not utilize smoothed data for the first and last two points in the series of acceleration data points since five points in sequence must be available for the curve fit routine.

The computer program will re-evaluate the gravitational attraction constant of the earth, g, as an inverse square function of the altitude of the rocket. This re-evaluation may be done with each incremental calculation of the altitude of the rocket and each new g value introduced into the subsequent calculation.

Elevation angles of launch less than 90° will introduce values of acceleration and velocity in the X or forward direction, as well as in the Z direction. The longitudinal acceleration values as derived from the accelerometer are separated into vector components for calculating trajectory information. This technique is illustrated in Fig. 2. The computer program will introduce the effects of winds as forces operating at any given altitude and considers the winds as occupying definite altitude strata. The winds have the effect of changing the elevation and azimuth angles of the rocket as measured at the instantaneous position of the rocket in its trajectory. These wind corrections of the elevation and azimuth angles are included in the vector components of acceleration and velocity illustrated in Fig. 2.

The computer program will generate artificial drag information for one or two periods such as may be needed for a two-stage rocket whose thrust periods are separated by a time interval. These artificial drag calculations may be used when accurate drag data are not available from telemetry records. They are simple exponential curves and originate from a predetermined initial amplitude of drag and a constant term for the exponent. This technique of obtaining drag information is not the ultimate in accuracy, but it may be useful in instances where the initial values of the drag curve are available but the remainder of the data is uncertain.

The calculation procedure for the IBM 650 computer program does not utilize the Simpson's one-third rule or the Trapezoidal rule used in desk calculator techniques, rather, it calculates velocity and position information by considering the accelerometer curve as a series of small rectangles as shown in Fig. 2. In other words, acceleration is assumed to be constant from one value of elapsed time to the next, the values being separated by the incremental value of time, Δt, mentioned earlier. As can be seen in Fig. 2, the area under a rising acceleration curve is somewhat greater than the true area depending upon the time increments used while the area under the falling portion of the curve is somewhat smaller than the true area. Thus the errors partially cancel.

Fig. 2 also illustrates acceleration vectors on an X, Y, Z set of orthogonal axes. The measured value of acceleration, a_m, as obtained from the telemetry records is the magnitude of the accelerometer reading in units of g, and a_x, a_y, a_z are the vecotor components of acceleration. It

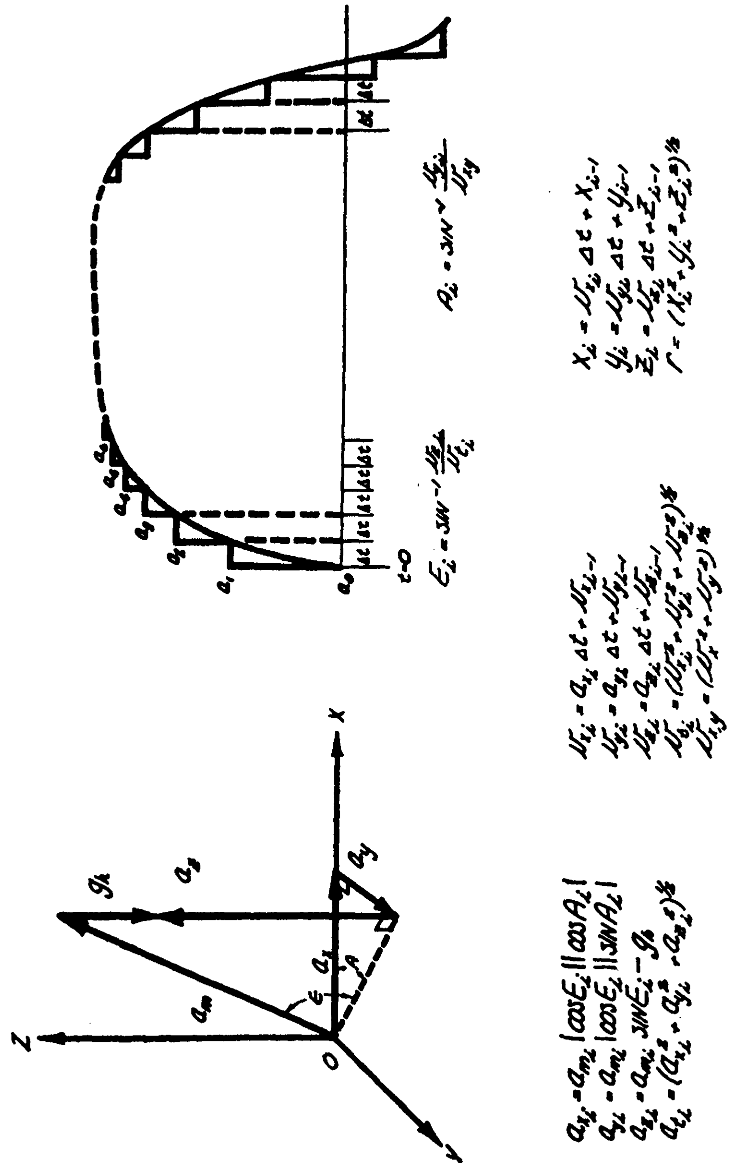

Fig. 2. Acceleration vector diagram and computation technique.

can be seen that the Z component is corrected by the value of *g* at the altitude at which the calculations are being made. The tangential acceleration, a_t, is derived from the three orthogonal components of acceleration as shown in the figure. The vector components of velocity, v_x, v_y, v_z, are obtained from the sum of the acceleration-time increment product and the velocity of the previous calculation. The tangential velocity and the x-y velocity, v_{xy}, are derived in a manner similar to that for the tangential acceleration. The X, Y, and Z positions are derived in a similar manner to that of the velocity using the product of the velocity and the time increment added to the position determined by the previous calculation. The range, *r*, is derived from the three orthogonal X, Y, and Z positions of the rocket. The elevation and azimuth angles, E and A, are determined from the inverse sines of two velocity quotients as indicated in Fig. 2.

When wind velocity corrections are considered, a direct wind velocity subtraction is made from v_x and v_y. The wind velocity is given a positive value if in the direction of the positive X and Y axes. The effects of wind on a rocket under thrust is such that the rocket veers toward the wind. When the component velocities are changed by subtraction of the wind components, the angles E and A are modified. This, in turn, results in a change of the acceleration components in the X, Y, and Z directions.

The computer program was designed to be as versatile as possible; consequently, many tests and branching operations are involved. The branching operations include "reading in" acceleration data cards, utilizing the least-squares-curve-fit routine, generating artificial drag data, re-establishing the curve fit at the beginning of the second thrust period, making calculations determined only by the acceleration of gravity, and arranging the readout format. The program is designed so that the results may be listed on the IBM 407 Accounting Machine or it may be punched on output data-cards.

The IBM trajectory program has already proven its usefulness on several rocket trajectories. In some instances, radar data were available for comparison with the computer results. When the accelerometer data were accurate, the radar and the calculated altitude results agreed within two per cent.