"Science a Vehicle in Mathematics, or Mathematical Methods in Science"

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The title of my paper states two sides of a big subject, "Science a Vehicle in Mathematics, or Mathematical Methods in Science." I must ask my listeners to think first of my being a high school teacher of mathematics, then they will appreciate my statement of the subject. Think of my use of the word 'mathematics', which uses science as a vehicle, not as pure abstract mathematics, but mathematics that must be implemented, must be useful, must be motivated, and finally must be sought after by people at a unique time in their lives.

Science itself is so vigorous and surrounds us so intimately at every level of experience that the use of its interesting applications to mathematics comes automatically. There is no other body of knowledge so fertile and applicable. If the teaching of mathematics had stayed nearer the growing edge of science through the last hundred years we would not have found our curriculum so full of dead matter, so crowded with obsolete illustrations and techniques. Morris Kline, professor of Mathematics at N.Y.U., in discussing the present ferment of the high school mathematics curriculum said, "The trouble with the present curriculum lies rather in the way in which we approach the material we teach. Mathematics is commonly presented as a series of techniques, processes, and theorems. There is no motivation for the material; the subject is isolated from all other bodies of knowledge and human interests; and knowledge acquired seems useless except perhaps to prepare students to pursue more of this unattractive material."

If this is the difficulty the remedies are obvious. First of all we must take the time and the trouble to arouse interest in mathematics. In view of the intimate connection between mathematics and the physical sciences most of the problems will come from this area. Of course mathematics must not be submerged in a host of applications. To *formulate* a problem mathematically one must introduce mathematical concepts. To *solve* the problem one must learn and apply some techniques. Moreover, the mathematical methods of solution must be taught and distinguished from methods used in other sciences and fields of thought. Obviously mathematical methods are different and possess their own effectiveness else mathematics would not be the powerful subject it now is. Hence the nature, concepts, and role of mathematics would be the primary object of attention despite the use of physical and other problems to motivate the subject and to demonstrate the value of mathematics. Hence there should be no question that one is teaching mathematics and not physics or some other science.

It is important in talking about problems for motivation to distinguish the trivial from the fundamental. Some problems are neither basic nor broad enough in appeal; they may serve for vocational training but not for liberal education. On the other hand, problems involving straight line and projectile motion, astronomy, light, sound, electricity, and the elements of chemistry and utilizing broad physical principles such as the Newtonian laws of motion or the law of gravitation are not only basic but involve familiar phenomena. Moreover, these physical topics do not require that much science be taught; the subject matter is partly familiar and intuitive. Furthermore, the applications should even enlighten students about their world.

The various branches of mathematics were created to help man understand the physical world and, in more recent years, to harness the processes of nature in behalf of man's necessities, convenience, comfort and health. The great mathematicians and the great scientists were the same people. Recall that the three greatest mathematicians of all time, by common consent, were Archimedes, Newton and Gauss. Recall also that the first two of these were also the greatest scientists of their eras and even Gauss made great contributions to astronomy, electricity and magnetism, map-making, systems of insurance and statistical studies of errors of measurement.

The use of physical problems to arouse interest, to give meaning to the mathematics by enabling the student to see physically what his formulas and theorems are saying, and to show the accomplishments of mathematics is good teaching. But there is a deeper reason for presenting mathematics in conjunction with other branches of knowledge, while of course emphasizing the mathematics; knowledge is a whole and mathematics is part of that whole.

I have thought a good deal about the approach to the mathematics curriculum which has just been sketched. I have even had the opportunity to teach physics and second year algebra in a physics laboratory. I have been granted time to try to teach both subjects to the same students in a laboratory atmosphere and I am convinced that mathematics, as a fine efficient machine, needs the fuel and the spark furnished by science to make it dynamic and useful to most high school students. True, a few students will respond to the sheer intellectual challenge of the subject. You may say, "Can a student learn mathematics and science simultaneously"? The amount of science presented would not be very much, just enough to be able to present the desired application. I will gladly take time to teach a little science if in doing so the mathematics will be made interesting and attractive.

Now some specific applications of science as a vehicle in mathematics. I shall not use much time showing how science is a trusty vehicle in arithmetic because my work is with other branches of mathematics. However, an interesting topic in arithmetic is the concept of the base of a number system, which can be taught in any mathematics course in high school. It adds excitement to otherwise boring material to learn how to change numbers from the base ten to base two and to know that this is the base used in electronic computing machines which are symbols of this scientific age. While in the business of changing to base two, base five, eight, twelve or any number makes an interesting way to teach understanding of numbers.

The branch of mathematics which can be most completely correlated with physics is algebra, because algebra is the language of the scientist. Some of the most profound statements ever made have been written in algebraic equations. Consequently, these inspired statements which captured the imagination of authors such as Newton, Archimedes and Einstein can not fail to excite students if presented in the proper way.

Problems involving speed of a moving object, freely falling objects, the effect of gravity, projectiles, objects put into orbit are all intensely interesting. Why would we ever need a problem so unimaginative as one on the age of John and Mary when they are surrounded by radio waves which travel at the speed of light or planes that exceed the speed of sound? The concept of weightlessness, which is before us in the news so often can implement some wonderful practice of quantitative thinking. Students who have not understood how satellites maintain their speed and stay in orbit or why they move into their orbits when projected from the earth grasp the idea when explained from Newton's thinking and conclusions. Sending a projectile out horizontally from the top of a mountain with greater and greater velocity would cause it to fall to the earth farther and farther away, so Newton reasoned. He concluded that if a projectile were shot out with enough speed it might begin to circle the earth. This experimenting and thinking led Newton to formulate the law of attraction between any two masses. Later when he had derived Kepler's laws of planetary motion he eliminated any doubts of the validity of the law of gravity and the established laws of motion. If we assume that the planetary paths are circular, then with just a little algebra we can see that the third Keplerian law does follow from the basic laws of motion and gravitation. Students literally submerge themselves in problems on Echo, Vanguard and more recent satellites. The experience of our astronauts should influence the ideals and aspirations of our teen agers.

Some other simple concepts from physics such as balances, fulcrums, levers, and moments of force are the bases of many interesting problems. The inclined plane contains ideas basic to other investigations such as projectile motion. Problems from this field along with those of the pendulum and the action of a spring can produce hours of interesting application. If the teacher can arrange to have the apparatus to demonstrate these problems he will have no trouble inspiring the students to solve exercises on the subject.

Another popular interest of the American people makes a pertinent scientific application for mathematics. I refer to the present interest in boating, skiing, surfboard riding and last the hobby of skin diving. The paraphernalia and gear that go with these activities have to be understood to be operated and therefore their users can appreciate the laws of water pressure, Pascal's law and motion in media with different densities.

The sextant and other reflecting mathematical instruments involve some very good applications of the law of the angle of incidence and reflection and simple geometric principles. The sextant can be introduced in geometry after the study of the axioms, triangles, and parallel and perpendicular lines. Basic physical laws of light are effectively used, combined with geometric applications regarding three or four fundamental axioms. For example, "if two angles have their sides respectively perpendicular . . .", "an exterior angle of a triangle . . .", "the sum of the angles about a point . . .", "if two lines are parallel . . ." and others. Students enjoy "The Sky and the Sailor", a very readable, and humorously written book for the math-science studies at the high school level by H. A. Calahan.

When studying tangents and secants to a circle it creates an interesting situation to let the circle be the earth and the point from which the tangent and secant are drawn be a pilot in a plane. The distance from the plane to the earth is the external segment of the secant and the tangent from the point is the pilot's line of sight over the horizon. Then it becomes interesting to speculate upon what area of the earth the pilot can see. Then letting the altitude of the pilot be the variable, how high would he have to be in order to see, say, one fourth of the earth's surface.

Of course there are many little trivial mechanical applications such as gears, fly wheels operated by bands, and hydraulic pressure devices apropos to the teaching of ratio and proportion and many other sections of geometry, but they are not broad enough to challenge thinking.

The laws of vibrating strings, or reeds, and the fact that all sound is the result of vibration or energy is another point where the thinking of physics and mathematics merge. With the use of an oscilloscope one can demonstrate the sine curve, varying its amplitude and period in numerous ways by producing sounds of different pitch and intensity. This piece of equipment should be a must in every trigonometry classroom, because it has the capacity to tie together an abstraction such as the graph of a sine curve with an ordinary gadget used in an electrical shop where radios are repaired. An interesting article appeared in the "School Science and Mathematics" in November, 1960 under the heading, "Using the Cathode-Ray Oscilloscope in the High School Trigonometry Classroom". Matthew H. Bruce, the author, says that he uses the oscilloscope to demonstrate the results of combining trigonometric functions. I can verify all that Mr. Bruce says about the value of demonstrations with the tuning fork, alternating current, a microphone and speaking, an organ pipe, and a bass fiddle. One of the most profitable and pleasant uses of mathematics is that of music. The important numerical ratios underlying musical harmony were discovered by the Euclideans. In fact, the Greeks, ever since the Pythagoreans, always spoke of arithmetic and geometry in connection with music and astronomy. Then trigonometry was created solely as a tool for the study of astronomy.

I am sure that you scientists have been waiting for me to discuss the second half of my title, "Mathematical Methods in Science". I have touched that side only incidentally because it would be a complete study in itself. My intention was that the scientist would be applying what I have been saying about my field to his field only in reverse. It was through an effort to study and understand the physical world that the major branches of mathematics were created. It is worth noting, incidentally, that the current great emphasis on mathematics is also due to the need for mathematics in science.