Digital Computer Solution for the Propagation of a Spherical Shock Wave in Aluminum[1]

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INTRODUCTION AND STATEMENT OF PROBLEM

This paper presents an analytical study and computer solution for the propagation of a hemispherical shock wave into a semi-infinite block of aluminum. With this statement, the solution to be presented is accurate and the paper is properly titled. The origin of the problem is from micrometeoroid impact on space vehicles which explains the reason for employing the particular numerical values that follow.

Micrometeoroids are arbitrarily defined as particles with a mass of less than about 10^{-4} grams and with velocities in the range from 30,000 to 240,000 feet per second (Collins & Kinard, 1960). There is a lower limit on the mass of these particles since the pressure from sunlight will push very small particles outside of the orbit of the earth. For numerical calculations, a typical micrometeoroid is assumed to have a mass of 10^{-9} grams and a velocity of 118,000 feet per second. According to Whipple, some micrometeoroids have very low densities and are presumably aggregates of material from the tails of comets. Other micrometeoroids are of normal stone with a density of about 3.9 and a few are of an iron-nickel eutectic with a density of 8.0. The following calculation is not applicable for the very low density micrometeoroids.

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Experiments are reported in the literature in which simulated micrometeoroids are made to impinge at different velocities on a target. From these experiments (Chartres, 1960), a typical curve of depth of penetration as a function of the velocity of impact is reproduced in Fig. 1. Typical shapes of cavities from the impacts in three different velocity ranges are illustrated in this figure. It is evident that there are at least two different mechanisms of impact with an overlapping region between them. The penetration at low velocities, which is actually in the velocity range of the so-called high-velocity rifle bullet, is characterized by shear about the entering bullet. In the highest velocity range, the craters are nearly hemispherical in shape and evidence of shear is not so apparent. The transition between low- and high-velocity craters appears to develop when the initial velocity of the micrometeoroid first exceeds the velocity of sound in the material of the target.

The physical mechanisms during the initial phases of impact are not yet well understood; and consequently, the true boundary conditions cannot be specified for an accurate analytical solution. The problem is solved, however, with conditions that are probably incorrect in detail in order to obtain information that will assist in evaluating the relative importance of possible mechanisms and to gain experience for a more exact solution when the boundary conditions are better known. This approach was employed in the successful study of atomic blasts. In particular, information is obtained on the order of magnitude of the pressure developed, the duration of shock propagation and the volume of the region through which the shock is propagated. The analytical solution was obtained on an IBM 650 computer and the limited capacity of this machine required further simplification of the boundary conditions in order for the solution to be obtained in a reasonable time. The basic assumptions for the initial boundary conditions are that the impacting micrometeoroid is spherical, that it is incompressible and that it produces a hemispherical shock wave that propagates away from the point of impact as a hydrodynamic wave in a non-viscous fluid.

CONDITIONS AND EQUATIONS FOR SHOCK PROPAGATION IN ALUMINUM

The pressure drop across the shock wave is initially many hundreds of times the elastic limit of the aluminum. Under these conditions, Bjork (1959) and others have assumed that the shock wave is propagated as a hydrodynamic shock through a non-viscous liquid and that the usual hydrodynamic equations of flow apply. The hydrodynamic equations may be expressed in the Eulerian or the Lagrangian form. The Eulerian equations are with respect to space coordinates that are fixed. The Lagrangian equations describe the motion in terms of the paths of motion of the shocked material. When the motion is symmetrical, such as a hemispherical shock wave, experience has shown that the Lagrangian coordinates are generally the simplest to solve. An identity may be used to convert either the differential equations or the solution from one form of coordinates to the other.

The hydrodynamic equations for the propagation of a spherical shock wave have the form, in Lagrangian coordinates, which are presented in Fig. 2. The basis of each equation is indicated. These equations and an equation of state are necessary and sufficient to describe the motion of the material at all points in the medium except across the shock front. These equations do not contain terms for viscous, or other, losses.

Bethe (1942) has shown that the propagation of a shock through any medium can be solved with the hydrodynamic equations of motion provided the equation of state is known from theory or experiment for the

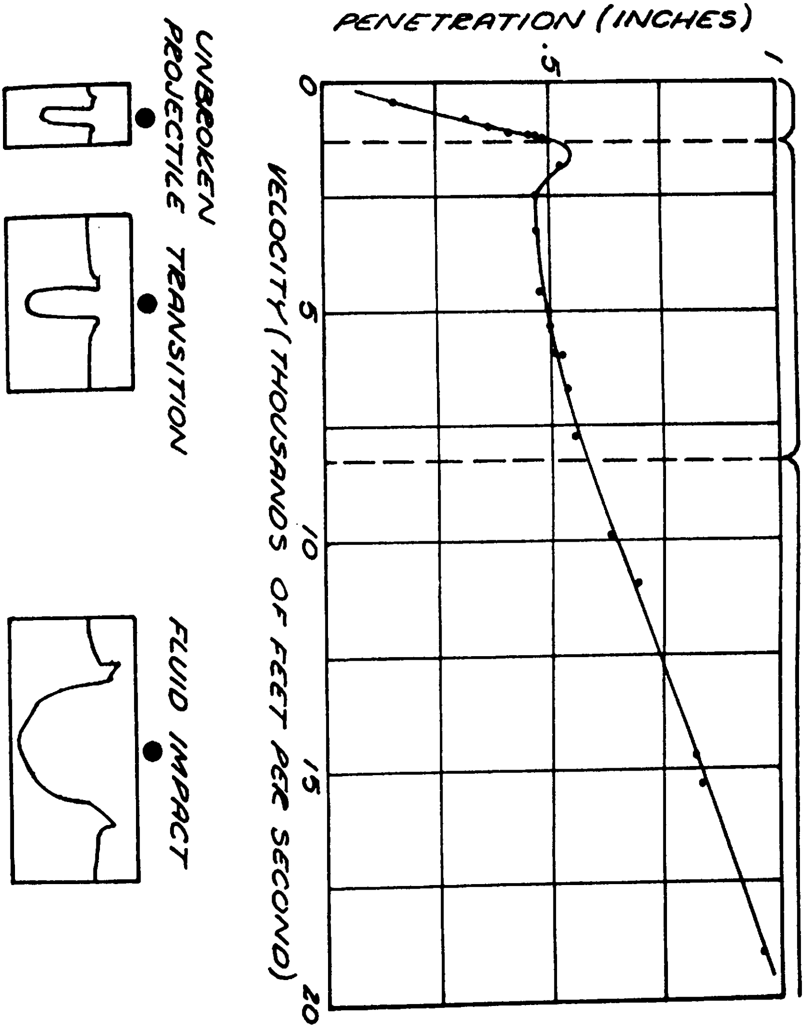

FIGURE 1. EFFECT OF VELOCITY ON PENETRATION

medium over the entire pressure range of the shock. There are two additional conditions which Bethe included and which are satisfied in this problem. With the equation of state and the hydrodynamic equation of flow, it is also necessary to specify that the entropy increases across the shock front. These two equations are presented in Fig. 3. The last equation in this figure may be derived from the preceding equations and is the Rankine-Hugoniot relation. This equation relates the internal energy, pressure and specific volume at the crest of the shock wave to similar values for the medium which the shock has not yet reached. The pressure-volume relation of a material during compression by a shock may be expressed by the Rankine-Hugoniot function, $P_H = f(V_H, E)$. This curve is not known along the entire pressure range. As in the procedure employed by Bjork (1960), the curve may be interpolated between the curve for pressures up to one megabar, determined experimentally by Fyfe *et al.* (1961), and the theoretical curves given by the Thomas-Fermi statistics (Courant and Friedrichs, 1948) for a plasma at pressures a little above twenty megabars. The values of P_H and V_H on this curve are the values in the Grunisen quation of state that also appears in Fig. 3. The shape of the interpolated, Hugoniot curve is indicated by the heavy line in Fig. 4. This interpolated curve is fit by an equation for use in the calculations. The lighter lines, which are extrapolated by dotted lines are typical adiabatic curves along which the calculations show that the material expands after it has been compressed by the shock.

NUMERICAL SOLUTION FOR PROPAGATION OF THE SHOCK

Two methods (Fyfe *et al.*, 1961) are recognized for obtaining a solution of these simultaneous equations with a digital computer. One method takes advantage of features of hyberbolic differential equations and uses

Conservation of Mass

$$\frac{1}{\rho} = \frac{1}{\rho}\left(\frac{R(r,t)}{r}\right)^2 \frac{\partial R}{\partial r} \qquad (1)$$

Conservation of Momentum

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\left(\frac{R(r,t)}{r}\right)^2 \frac{\partial P}{\partial r} \qquad (2)$$

Conservation of Energy

$$\frac{\partial E}{\partial t} = \frac{P}{\rho^2}\frac{\partial \rho}{\partial t} \qquad (3)$$

The velocity in Lagrangian coordinates

$$u = \frac{\partial R(r,t)}{\partial t} \qquad (4)$$

μ velocity
P pressure
ρ density
ρ_o reference density
r Eulerian coordinate
R Lagrangian coordinate

Fig. 2. HYDRODYNAMIC EQUATIONS FOR SYMMETRICAL FLOW IN SPHERICAL COORDINATES

the method of characteristics. A representative description of this method appears in Courant and Friedrichs (1948) and was the first method developed. The second and probably better known method is that of von Neumann and Richtmyer (1950) in which the equations are differenced, and integration across the shock is accomplished by introducing a pseudo-viscosity term which causes the shock front to change from a line of discontinuity to a narrow zone of large gradients. For a practical spacing of the net points, the pseudo-viscosity term is not closely related to the true viscosity. Either of these methods could be used and their complexities are comparable for a simple problem of the type being described here. The treatment of the shock propagation through a multiple layer medium is probably a little more direct by the von Neumann and Richtmyer method. Since our problem requires a study of shock propagation through a multilayer medium, this method was used.

The dissipative term for the solution of the problem of propagation of a spherical shock wave is the one that was suggested by von Neumann and Richtmyer in their original work. The dissipative term is taken as Q, which has the form that is given by Equation 8 in Fig. 5. The dissipative term is added to the pressure, P, in the preceding equations. This requires that Equations 2 and 3 be modified to the form in Equations 9 and 10, respectively.

Equation of State in Gruneisen Form

$$P - P_H = \gamma\rho\,(E - E_H) \tag{5}$$

Condition across Shock Front

$$S\,(\text{entropy}) \geq 0 \tag{6}$$

Rankine-Hugoniot Relation (Derived)

$$(E_H - E_o) = \tfrac{1}{2}(P_H - P_o)(V_o - V_H) \tag{7}$$

P, V and E are pressure, volume, and internal energy
subscript 0 is in front of shock
subscript H is peak, or minimum values in the shock front.
γ is the Gruneisen ratio and changes slowly with the density, only.

Fig. 3. ADDITIONAL EQUATIONS FOR SOLUTION OF SHOCK PROPAGATION

To obtain the most general solution to the problem, the preceding equations are converted to non-dimensional coordinates by a substitution that was proposed by Brode. The resulting equations were converted to difference equations by standard methods and prepared for solution on the computer.

Initial Boundary Conditions and Solutions

The difference equations that are obtained give a solution by the usual iterative method that is employed with digital computers. To apply these equations, the initial boundary conditions must be specified. As already mentioned, these boundary conditions are selected to require a solution in the form of a symmetrical hemispherical shock wave. This is necessary in order to obtain a solution in a reasonable time with an IBM 650 computer.

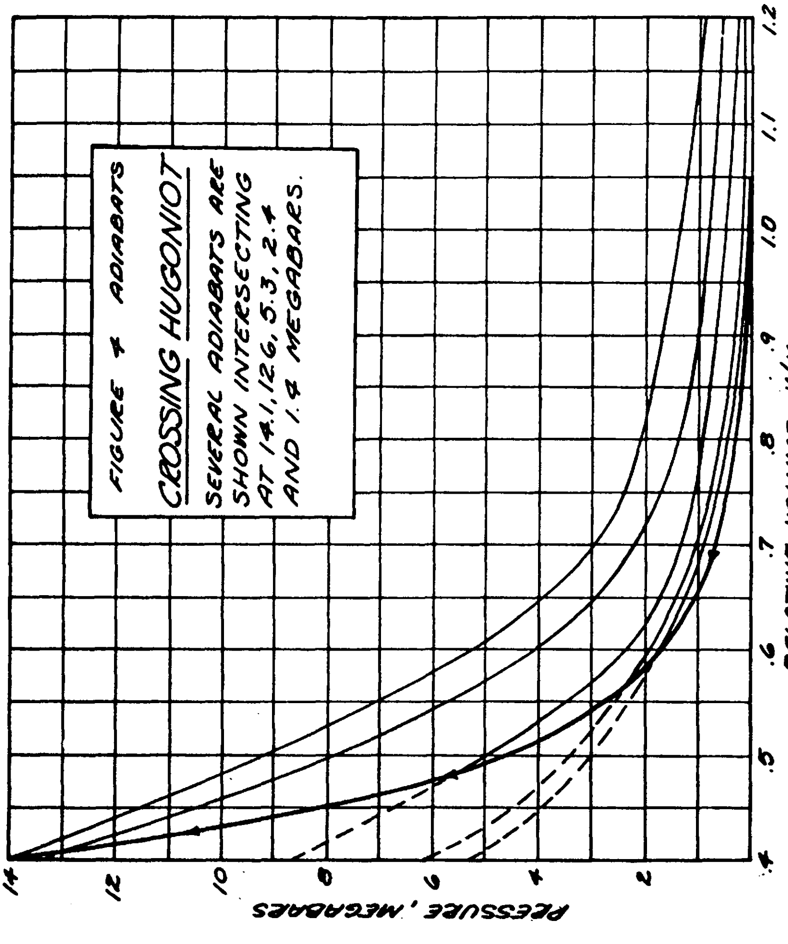

FIGURE 4 ADIABATS CROSSING HUGONIOT

SEVERAL ADIABATS ARE SHOWN INTERSECTING AT 14.1, 12.6, 5.3, 2.4 AND 1.4 MEGABARS.

Dissipative term which is only significant for large pressure gradients

$$Q = A^2 \rho \frac{\partial u}{\partial x} \left| \frac{\partial u}{\partial x} \right| \tag{8}$$

where A^2 is a constant that is determined by trial and error.

Conservation of Momentum

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho_0} \left(\frac{R(r,t)}{r} \right)^2 \frac{\partial (P+Q)}{\partial r} \tag{9}$$

Conservation of Energy

$$\frac{\partial E}{\partial t} = \frac{(P+Q)}{\rho^2} \frac{\partial \rho}{\partial t} \tag{10}$$

Fig. 5. INTRODUCTION OF VISCOUS DAMPING TERM FOR INTEGRATION ACROSS SHOCK

Within the limits imposed by the computer, the physical conditions are approximated. The incident micrometeoroid impacts at a velocity far in excess of the velocity of sound in the target material and the micrometeoroid is assumed to be spherical in shape and incompressible. At the time that the micrometeoroid has penetrated half-way into the target material, the physical situation is indicated in Fig. 6. The material of the target is assumed to be compressed between the surface of the micrometeoroid and the dotted line which indicates the position of the shock front. The pressure is 14.1 megabars, or a little over 14 million atmospheres for the typical micrometeoroid of 10^{-9} grams impacting with an initial velocity of 118,000 feet per second. Using the equations of flow and the Hugoniot-Rankine relation, but omitting the conservation of mass, the radius of the shock front is 13 per cent greater than the radius of the incompressible micrometeoroid.

From the initial boundary conditions, the shock wave propagates and decreases in amplitude as the energy of the shock increases in area with increase in radius of the shock front. The pressure distribution during the propagation is indicated by the series of shock fronts that are indicated in the lower part of Fig. 6. Since the solution was obtained with non-dimensional coordinates, the solution may be readily modified for other sizes and speeds of impact. An illustration of the flexibility of the solution is indicated in Fig. 7. This figure shows successive shock fronts for the impact of a simulated meteoroid with a mass of one milligram and a velocity at impact of 15,600 feet per second.

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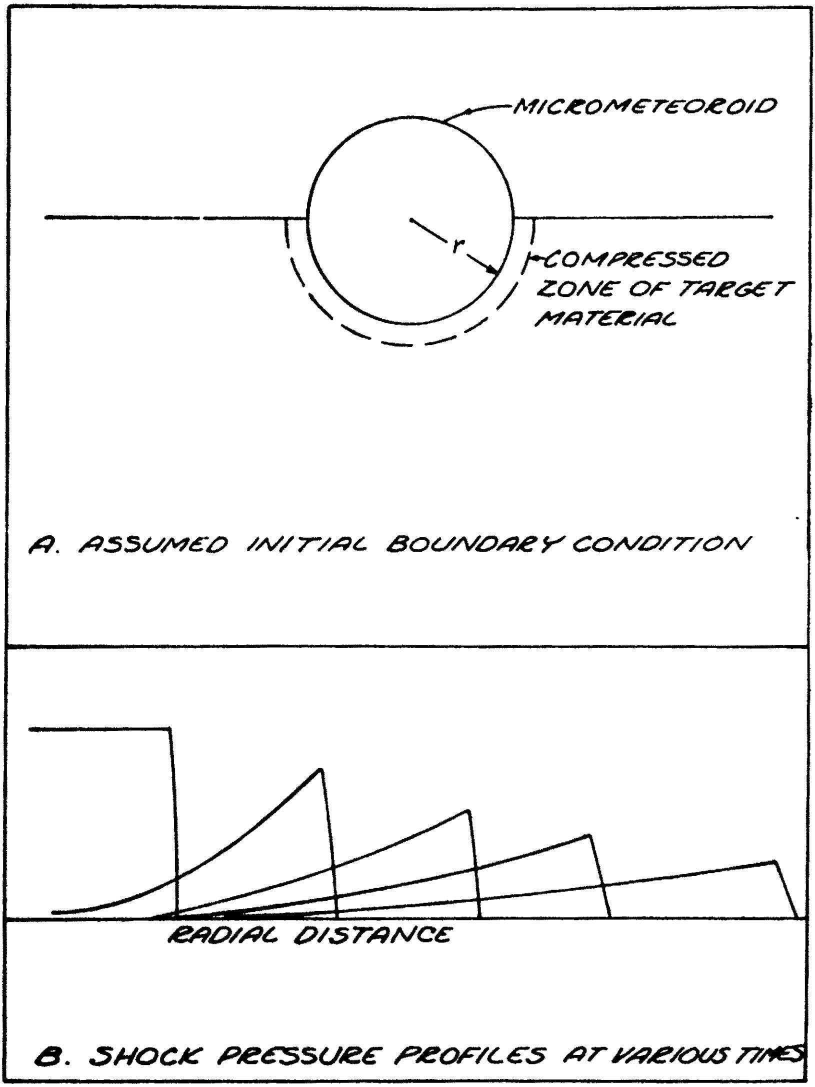

FIGURE 6. INITIAL BOUNDARY CONDITION AND SHOCK PROPAGATION

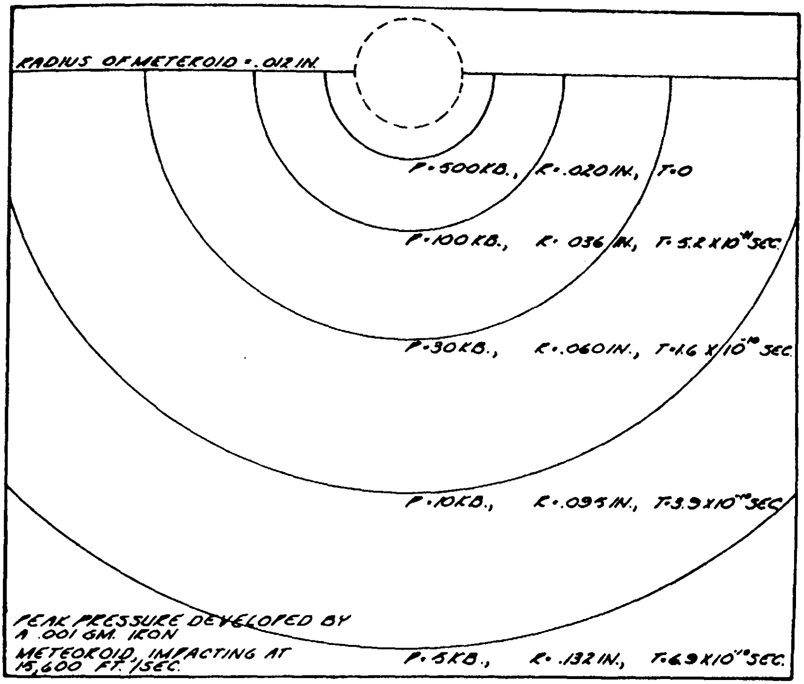

FIGURE 7. IMPACT OF METEOROID

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