A Medieval Treatise on Graphing

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According to a logical-physical tradition present in certain medieval scholarly circles and inherited from Greek antiquity, a physical object could be regarded as possessing various "forms" and qualities. Some of these qualities might be natural to the object, such as the wetness of water, and some might be "accidental," as color in colored water or hotness in a brick.

An example used frequently by medieval logicians was the distribution of hotness in a body. A body equally hot throughout was an object throughout which the form heat was evenly distributed, while a body hot at one end and cold at the other was an object throughout which the form was unevenly distributed. As the medieval scholars recognized, the number of possible variations in this distribution is limited only by one's imagination.

During the twelfth and thirteenth centuries the subject of qualitative change underwent such detailed and extensive logical analysis and discussion that it acquired a technical terminology all its own. As a consequence, one could refer to the subject in general by speaking of the "intension and remission of forms" or, even more briefly, of the "latitudes of forms." A given amount of hotness at one point in a body might be referred to as "a degree of hotness more intense" than a cooler hot spot elsewhere in the body, or the cooler of the two hotnesses might be described as "an increment of hotness more remiss" than the hotter of the two hotnesses.

Until about the middle of the fourteenth century, all of the discussions of the subject were purely verbal, purely rhetorical, and without reference to any concise formulistic or mathematically symbolic statements. But a Latin treatise of 1346 makes rather confused incidental reference to the possibility of employing helpful geometric diagrams to depict the distribution of a quality in a body.

In this tract an Italian scholastic, Giovanni di Casali, remarks:

> . . . let there be throughout a uniform hotness just as a parallelogram is formed between two equidistant lines such that any part you wish is equally wide with another; because the latitude of any part whatever of such a one [i.e., hotness] is measured in reference to the base of that parallelogram. And let there be throughout a uniformly difform hotness such that it is a triangle; and so that uniformly difformly hot body is terminated at zero degree in one extreme. (Clagett, 1959, p. 887.)

A far clearer description of the application of geometric figures to this subject was given about a quarter of a century later in a 93-chapter work written by a schoolman who thoroughly and competently understood the details of the new technique that Casali had barely touched upon. This writer was Nicole Oresme (who died as Bishop of Lisieux in 1382), and his work was entitled *De configurationibus qualitatum*. His opening words demonstrate how acutely he recognized the role that geometry could play in assisting logical analysis of the intension and remission of forms:

> Every measurable thing except numbers is imagined by means of a continuous quantity. Therefore it is necessary, for its measurement, that points, lines, and surfaces or their properties be imagined. In these [points, lines, surfaces], as the Philosopher [Aristotle] holds, measure or proportion is found immediately. In other things, however, it [i.e., measure] is recognized by analogy, for by analogy those things are referred by the intellect to points, lines and surfaces. Although indivisible points or lines do not actually exist, yet it is necessary to picture them mathematically for the measure of things and for comprehending their proportions. Therefore, every intension successively acquirable is to be

> imagined by a straight line perpendicularly erected upon some point of space or of the subject of that intensible thing, as an example. (Oresme, 1360? folio 160r.)

Oresme's long treatise does not appear to have been the prime vehicle for disseminating the new geometrical technique, for a variety of reasons. Suffice to say that the treatise to be found scattered all over Europe in handwritten copies of the fourteenth and fifteenth centuries and in printed editions of the fifteenth and sixteenth centuries is a short manual entitled *De latitudinibus formarum*. This brief tract, often ascribed erroneously to Oresme, appeared probably very shortly after Oresme's long treatise, perhaps in the 1360's. The earliest extant copy, unknown author, known, is one dated 1395.

The treatise is simple and elementary in its treatment of the twin subjects of the latitudes of forms and the application to these of certain geometric figures. It derives its essential ideas probably from the fuller, more cogent discussions of Oresme's long treatise. But if the short work does not appear to be original, it presented in its most popular form the forerunner of the technique of graphing used a century and a half later by Rene Descartes in his analytical geometry (Descartes, 1637).

The fourteenth-century work assumed that its readers were already familiar with the concepts and terminology of the latitudes of forms. Its opening paragraph describes what it accomplished:

> Because the latitudes of forms are varied in many ways, this multiplicity is discerned with difficulty unless reference is made to geometric figures. Accordingly, having laid out certain divisions of latitudes with their definitions, I shall apply the infinite kinds of these [latitudes] to the infinite kinds of figures; from these the subject will appear more clear. (Pseudo-Oresme, 1395, folio 4r).

While the tract was principally a verbal discussion, it used and cited appropriate geometric figures, and it developed its material in an orderly, systematic way, beginning first with definitions of various kinds of latitudes and various kinds of geometric plane figures and then proceeding with "suppositions" and "propositions" to which logical proofs were attached.

Excerpted statements from the Suppositions provide an example of the treatise's method of treatment of its subject:

> All things which have some proportion to one another participate in the nature of quantity. . . .
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> Everything that by an increment of degree exceeds another or is exceeded by another is to be imagined by the mode of quantity. . . .
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> Everything that exceeds or is exceeded according to extension alone of its parts is to be imagined in this discussion as having one dimension alone and so is to be imagined as a line or longitude; everything that exceeds or is exceeded according to extension *and* intension is to be imagined as having two dimensions and so is to be imagined as a longitude and latitude, or as a surface. . . .
>
> Extension of form is to be imagined by a straight line, intension by a plane figure rising upon the straight line. (Pseudo-Oresme, 14th century, folio 14r.)

The Propositions demonstrate how the technique is to be used to its fullest to depict the distribution of a quality. Space permits only the briefest reference:

> Every latitude. . . is to be imagined by a plane figure rising upon a straight line. . . .
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> Every uniformly difform latitude beginning from a certain degree and terminated at zero degree is to be imagined by a triangle beginning from a right angle and terminated at an *acute* angle. . . .

By the end of the treatise the writer has described constant acceleration and has also pointed out the value of the geometric technique for describing, in addition to distributed qualities, the behavior of bodies in motion. While the work makes no reference to actual experience or to actual physical phenomena and no mention of physical bodies in free fall, the graphing technique that it employs is the same technique that Galileo applies two and a half centuries later in establishing his famous laws of falling bodies that describe acceleration due to gravity. (Galileo, 1638, pp. 169-174; Galileo, 1952, pp. 173-177.) The historical connections so far brought to light by historians of science indicate that Galileo's and Descartes' applications of the technique of graphing described in *De latitudinibus formarum* represent the culmination of a scientific tradition that had its roots in the Middle Ages.

Literature Cited

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