A Senior Experiment on Thermal Conductivity

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In recognition of the need for physics experiments at the advanced undergraduate level, the American Association of Physics Teachers has recently sponsored the publication of the Taylor Manual (1959) to assist teachers in planning laboratory courses of this nature. Although designed prior to publication of the manual, it is felt that the following paper presents a good example of how one of the suggested experiments can be expanded to include experimental and computational techniques from both the fields of physics and mathematics.

The problem is to determine the thermal diffusivity (defined as the thermal conductivity divided by the density times specific heat) of brass by the periodic heat flow method. It should also be emphasized that the experiment has research implications since refined applications of the techniques involved have been used to determine the thermal conductivity of metals over a wide range of temperature (Sidles and Danielson, 1954) the thermal diffusivity of liquids (Hurt *et al.*, 1958) and most recently, the specific heat of germanium (Abeles *et al.*, 1959).

Figure 1 is a block design of the apparatus used. The potentiometer arrangement was used to read thermocouple voltages in order to determine the temperature. The reference junction of the thermocouple was immersed in an ice-water bath. The double-pole-double-throw switch was used for ease in switching thermocouples. The 2 v. source furnished the power to the potentiometer; the 4 v. source furnished power for the heater. The heater switch provided a square-wave heating current of 1/6 cycle per minute frequency, by being alternately opened and closed every three minutes. Not shown is a glass tube that surrounded the sample and reduced random air currents.

Figure 2 indicates the dimensions of the sample. It consisted of a quarter-inch brass rod about 120 cm. in length. A hole for the heating element was drilled in one end of the sample to a depth of 2 cm. The electric heating coil was wound around a form, inserted in the hole in the end of the sample and held in place by Saureisen cement. The heater had a resistance of 17 ohms which gave a heater power of 0.93 watts with the 4 v. source. The first thermocouple hole was drilled 1 cm. from the end of the heater hole and the second thermocouple hole was drilled 6 cm. beyond the first. These holes were 0.0225 inches in diameter. Iron-constantan thermocouples were used; the wires for the sample junctions were wound together and soldered in the thermocouple holes.

In order to indicate the measurements needed to obtain the thermal diffusivity an expression will be derived for it. Consider the Fourier heat conduction equation for three dimensions (equation 1). If it is assumed that the isothermal surfaces in the sample are parallel planes perpendicular to its length chosen as the x-direction, and that Newton's law of cooling holds for the sample's lateral heat losses, ν, equation (1) becomes equation (2).

$$\partial T/\partial t = k(\partial^2 T/\partial x^2 + \partial^2 T/\partial y^2 + \partial^2 T/\partial z^2) \tag{1}$$

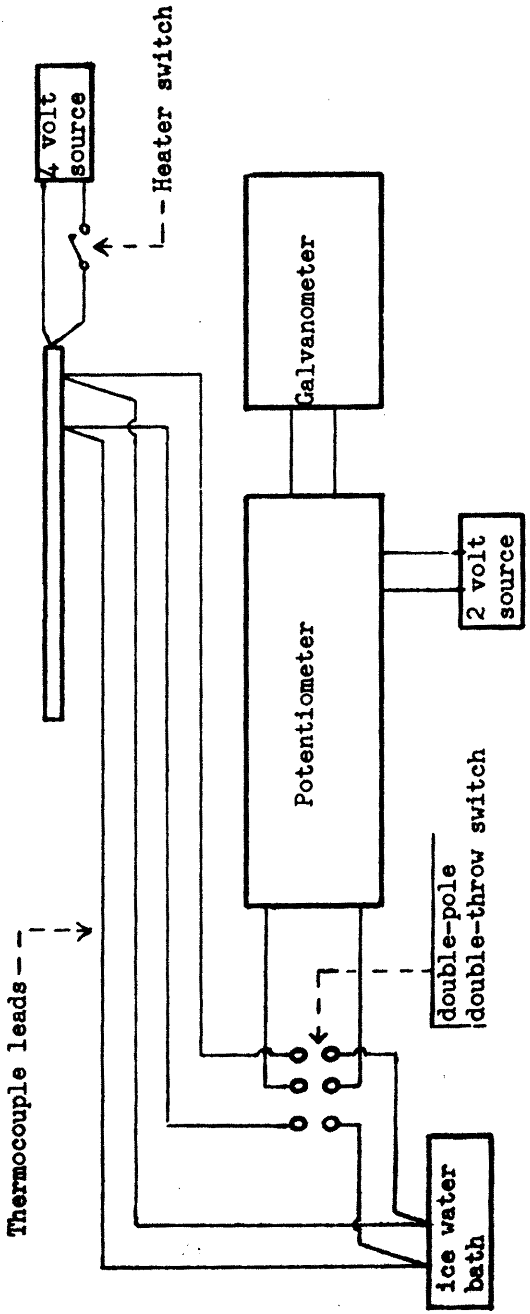

FIGURE 1. BLOCK DIAGRAM OF APPARATUS

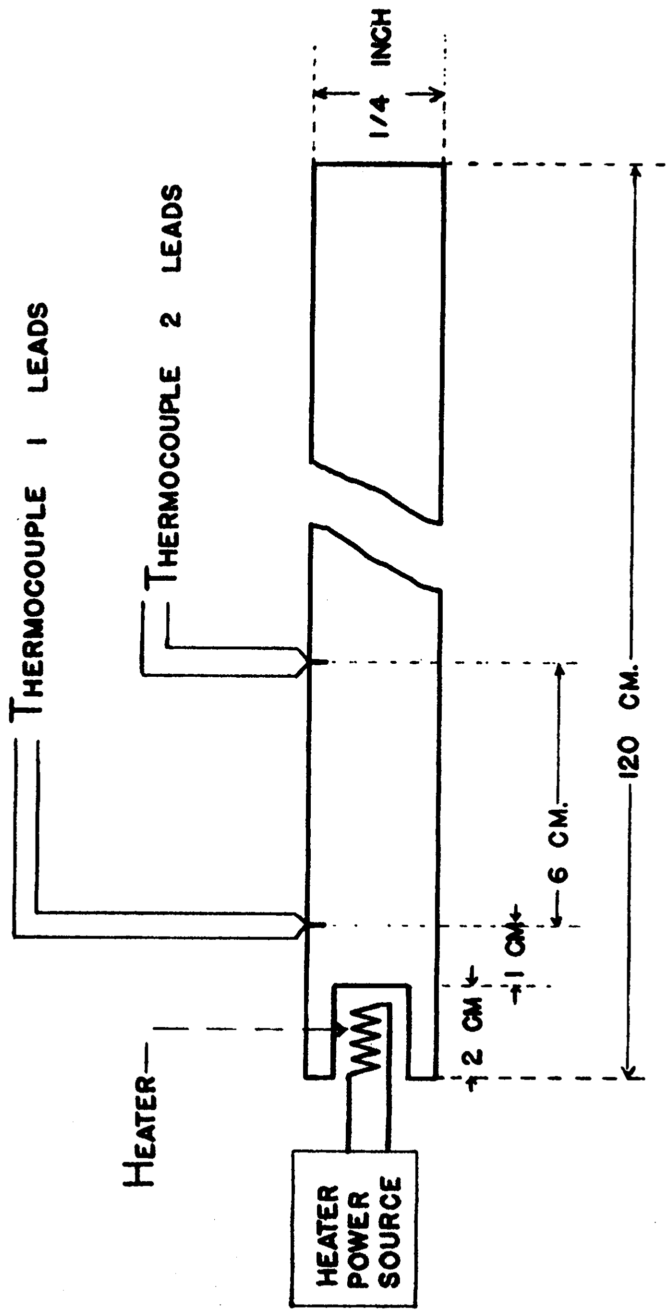

FIGURE 2.

FIGURE 2. SCHEMATIC OF SAMPLE ROD AND HEATER

$$\partial T/\partial t = k\partial^2 T/\partial X^2 - uT \quad (2)$$

The square wave heating current gave rise to a periodic, but not sinusoidal, temperature variation. However, if the temperature variation is assumed to be sinusoidal at the heater, this facilitates solution of equation (2), and a correction to the values obtained by this assumption can be made later. The boundary conditions are:

$$X = \text{origin};\ T = T_0 \sin wt$$
$$X = \text{infinity};\ T = 0$$

This gives a damped oscillatory solution

$$T = T_0 \exp(-ax) \sin(wt + Bx) \quad (3)$$

with a and B given by the expressions

$$a^2 - B^2 = u/k$$
$$2aB = -w/k \quad (4)$$

These lead to a value for the damping constant

$$a = [u/2k + (u^2 + w^2)^{1/2}/2k]^{1/2} \quad (5)$$

which indicates that the damping increases with an increase in frequency of the impressed temperature variation.

The temperature variations at two given positions X_1 and X_2 in the rod are

$$T_1 = T_0 \exp(-ax_1) \sin(wt + BX_1) = A_1 \sin(wt + p_1)$$
$$T_2 = T_0 \exp(-ax_2) \sin(wt + BX_2) = A_2 \sin(wt + p_2)$$

Observation of the amplitude ratio, A_1/A_2, and the phase difference, $(P_1 - P_2)$, of the temperature waves at the two points leads to values a and B

$$a = \ln(A_1/A_2)/(X_2 - X_1)$$
$$B = (p_1 - p_2)/(X_1 - X_2) \quad (6)$$

which can be combined on the basis of equation (4) to yield a value for the diffusivity from which the lateral heat loss constant, u, has been eliminated.

$$k = (X_2 - X_1)^2\, w/2(p_1 - p_2) \ln(A_1/A_2) \quad (7)$$

Thus, to measure k, the specimen need only be formed into a long thin rod, a sinusoidal heater fastened to one end, and the amplitude ratio and phase difference of the temperature variations at two locations a known distance apart measured. In the experiment that was performed, the temperature variation was not large so that the thermocouple voltage was approximately linear with temperature; hence it was possible to plot only thermocouple voltages *vs.* time to obtain the amplitude ratios and phase differences needed.

Measurements were begun after four or five heating cycles had been imposed upon the sample so that average temperatures had reached reasonably stationary values. Starting at this time, each thermocouple was monitored in a semi-continuous fashion with a Type K potentiometer. Specific readings were taken at thirty second intervals. It was not necessary to monitor both thermocouples simultaneously since the same time base was used throughout and a set of twelve or more readings on one could be followed by a similar number on the other with the time axes then being shifted to coincide.

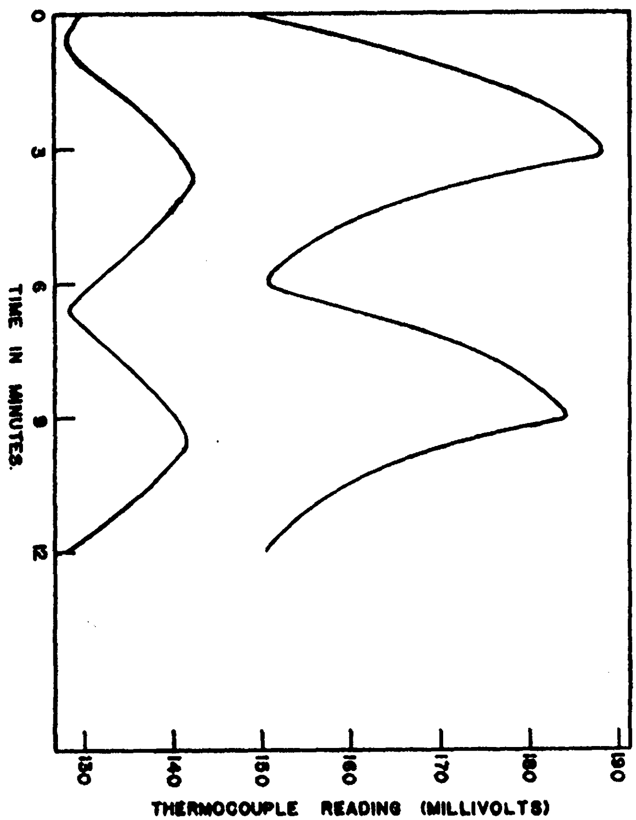

FIGURE 3. THERMOCOUPLE VARIATION WITH TIME

Figure 3 shows the thermocouple voltage *vs.* time variation for both thermocouples in the experiment performed. As mentioned previously, the temperature variation of the heater was not sinusoidal but was furnished by a square-wave heating current and thus the results are not true sine curves. The fact that the configuration at X_2 is much more like a sine curve than that at X_1 can be explained by expressing the temperature variation in the Fourier Series form as a sum of terms which vary harmonically in multiples of the fundamental frequency w. According to equation (5), higher frequencies suffer greater damping and it is to be expected that the thermocouple at X_2 will see a temperature variation more like the first harmonic term in the Fourier Series, i.e. a sinusoidal variation at the fundamental frequency.

Using the experimental curves of Figure 3 to compute k would lead to large error since the temperature variation at the heater was not sinusoidal. A better value can be obtained by expressing the variations in Fourier Series form, obtaining the equations of the first harmonic terms at the two locations, and measuring the amplitude ratio and phase difference of the two curves. Expressed as a Fourier Series, the thermocouple voltage at a fixed position as a function of time has the form

$$y = A_0 + \sum_{n=1}^{\infty} A_n \sin nx + \sum_{n=1}^{\infty} B_n \cos nx \qquad (x = wt) \tag{8}$$

Thus the equation of the first harmonic is

$$y_1 = A_1 \sin X + B_1 \cos X = C_1 \sin (X + S)$$

The constants C_1 and S are related to A_1 and B_1 by

$$C^2_1 = A^2_1 + B^2_1 \ ; \ B_1/A_1 = \tan^{-1} S$$

If the value of y is measured on the waveform at frequent intervals (say every 15 degrees), the sum of these values multiplied by $\sin x$ and divided by the number of intervals gives approximately the average height of $y \sin x$ curve. Hence

$$\langle y \sin x \rangle = 1/2\pi \int_0^{2\pi} y \sin x \, dx \cong 1/24 (y_{15} \sin 15^\circ + y_{30} \sin 30^\circ + \ldots + y_{360} \sin 360^\circ) \tag{9}$$

Also

$$\langle y \sin x \rangle = 1/2 \int_0^{2\pi} y \sin x \, dx/\pi = A_1/2 \tag{10}$$

and the Fourier coefficient, A_1, is obtained to a good approximation as

$$A_1 = 1/12 \ (y_{15} \sin 15^\circ + y_{30} \sin 30^\circ + \ldots + y_{360} \sin 360^\circ) \tag{11}$$

In similar fashion

$$B_1 = 1/12 \ (y_{15} \cos 15^\circ + y_{30} \cos 30^\circ + \ldots + y_{360} \cos 360^\circ) \tag{12}$$

Using these equations the expression for the first harmonic term can be obtained.

Figure 4 is a plot of the final result which was used to obtain values for the amplitude ratio and phase difference. For the brass sample used in the experiment the thermal diffusivity k was found to be 0.318 cm.²/sec., a very acceptable value according to handbook sources (Chemical Rubber Publishing Co., 1954)

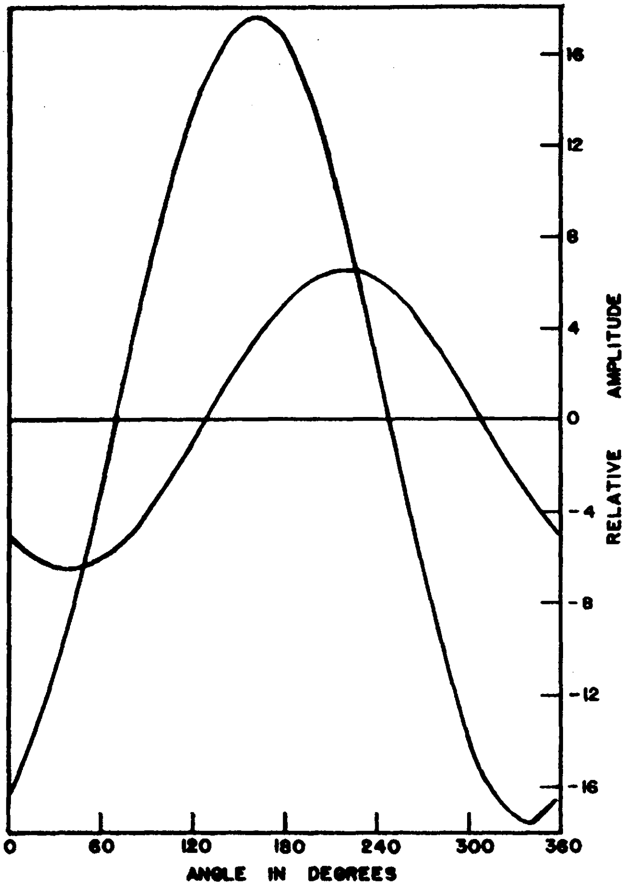

FIGURE 4. FIRST HARMONIC VARIATION WITH TIME

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