
Dividing a Line Externally and Hyperbola Construction

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To divide a line externally into a given ratio:

For example, if the segment from (2,3) to (5,7) is to be extended to four times its original length, what is the terminal point?

In the formula for internal division,

$$x = \frac{r_2x_1 + r_1x_2}{r_1 + r_2} \qquad y = \frac{r_2y_1 + r_1y_2,}{r_1 + r_2}$$

let (x_2,y_2) be the unknown, let (2,3) be (x_1,y_1) and let (5,7) be (x,y), so that:

$$5 = \frac{3(2) + 1x_2}{1 + 3} \qquad 7 = \frac{3(3) + 1y_2}{1 + 3}$$

$$20 = 6 + x_2 \qquad 28 = 9 + y_2$$

$$14 = x_2 \qquad 19 = y_2$$

Construction by ruler and compass:

Given: hyperbola and transverse axis, find foci and directrices:

The general equation of hyperbola is $x^2/a^2 - y^2/b^2 = 1$. By inspection, a point (5/4 a, 3/4 b) will lie on the curve. With the curve and transverse axis given, the vertices and distance a are readily available.

By a double bisection 1/4 a can be obtained, and a vertical line $x = 5/4$ a can be constructed. The point where this line crosses the curve will then be 3/4 b above the axis.

After trisecting the distance 3/4 b, the minor axis can be determined; then the location of foci and directrices are easily determined.