

## PRELIMINARY AND FINAL DESIGN OF A DIRECT-CURRENT MAGNETIZING COIL

NICHOLAS M. OBOUKHOFF, Oklahoma A. and M. College, Stillwater

A direct-current magnetizing coil is a substantial part or element of most electrical machines and apparatus. It is exemplified as mounted on an electromagnet in Figs. 1 to 4 showing openings of rectangular, circular, and other forms. Usually the conditions or requirements which should be complied with or fulfilled by a designer are as follows.

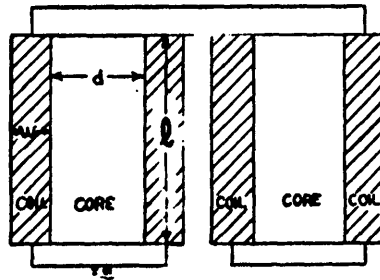


FIG. 1

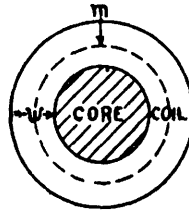


FIG. 2

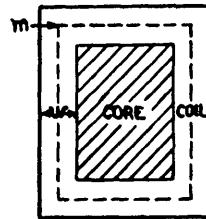


FIG. 3



FIG. 4

1. A terminal voltage  $V$ ; it may be determined by line voltage or otherwise.
2. A fixed number of ampere-turns ( $ni$ ) where  $n$  is the number of winding turns and  $i$  the current in the winding.
3. A surface temperature rise  $T_f$ ; in other words, an outside rise—a difference between the surface temperature of the coil and the ambient temperature.
4. Also, very often, a specific size and shape of coil opening.

5. Sometimes special requirements as to length and/or width of the winding space; we call them simply the length and width, or thickness, of the coil.

6.  $V$  is given as prescribed by the current source; ( $ni$ ) and also the size and shape of the coil opening are usually predetermined on the basis of the magnetic-circuit specification and performance of the electrical machine or apparatus; the temperature rise  $T_r$  is mostly chosen within limits depending on the ambient temperature and *restrictions imposed by insulating materials at the hottest spot inside the coil*, or, in other words, an appropriate temperature  $T_h$  there is assumed.

There are given: number of ampere-turns ( $ni$ ); outside temperature rise  $T_r$ ; terminal voltage  $V$ ; size and form of coil opening,  $p$  being the length of its perimeter.

There are sought: size of conductor, i. e., its cross-section area  $q$ ; area of winding space indicated by  $l$  and  $w$ , its length and width respectively, which are at the same time the length and thickness of the coil; number of ampere-turns  $n$ .

The derivations are brought about by using auxiliary formulas, as follows.

The electrical losses or generated heat in watts:

$$W = i^2R = V^2/R \dots\dots\dots(1)$$

The length of the mean turn:

$$m = p + kw \dots\dots\dots(2)$$

where the value of  $k$  depends upon the form of the opening. For openings of circular form (Fig. 2), of rectangular shape with perfectly rounded corners, and of the type represented in Fig. 4,  $k = \pi$ ; for the rectangular opening with square corners (Fig. 3)  $k = 4$ ; for the rectangular opening having corners not perfectly rounded  $\pi < k < 4$ —the first plausible assumption in this case would be averagely to make  $k = 3.57$ .

The cooling surface:

$$A = 2mw + pl + (p + 2kw)l = 2(w + l)(p + kw) = 2m(w + l) \dots\dots(3)$$

The outside temperature rise:

$$T_r = W / cA \dots\dots\dots(4)$$

where  $c$  is the cooling coefficient,  $A$  the area of the cooling surface, and  $W$  given by (1).

The total winding resistance:

$$R = \rho n(p + kw) / q \dots\dots\dots(5)$$

where  $\rho$  designates resistivity at the average temperature of the coil wire.

The number of winding turns:

$$n = slw / q \dots\dots\dots(6)$$

where  $s$  is the space factor which is computable or available from graphs if  $q$  and the mode of insulation of the conductors are known or determinable.

The current in the coil winding:

$$i = Vq / \rho n(p + kw) \dots\dots\dots(7)$$

Further derivations are as follows.<sup>1</sup>

Because of (7)

$$(ni) = Vq / \rho(p + kw) \dots\dots\dots I.$$

Referring to (4)

$$W = cT_r A \dots\dots\dots (8).$$

Again, because of (1),

$$i^2 R = cT_r A \dots\dots\dots (9).$$

Referring to (2) and (5)

$$R = qnm/q \dots\dots\dots (10).$$

Then from (9) and (10)

$$i^2 R = i^2 nq m/q = i(ni) q m/q = cT_r A \dots\dots\dots (11).$$

Again, denoting current density by  $\Delta$ , it obtains that

$$i = q\Delta, \Delta = (ni)/swl, \text{ and } i = q(ni)/swl \dots\dots\dots (12).$$

Substituting (12) for  $i$  and (3) for  $A$  we derive from (11) the fundamental equation:

$$q(ni)^2/swl = 2cT_r(w+l) \dots\dots\dots \text{II.}$$

Then

$$q(ni)^2/2wscT_r = lw+l^2 \dots\dots\dots (13).$$

Let

$$Q_w = q(ni)^2/2wscT_r \dots\dots\dots \text{III.}$$

Now (13) transforms to the quadratic equation:

$$l^2 + wl - Q_w = 0 \dots\dots\dots \text{IV.}$$

From this it follows that

$$l = -w/2 + \sqrt{w^2/4 + Q_w} \dots\dots\dots \text{V.}$$

Equation V gives  $l$  in terms of the assigned or known data,  $w$  being among them.

Likewise from equation II we get finally (if  $l$  is known or assigned):

$$w = -1/2 + \sqrt{l^2/4 + Q_l} \dots\dots\dots \text{VI}$$

where

$$Q_l = q(ni)^2/2lscT_r \dots\dots\dots \text{VII.}$$

Both equations III and VII are practically important although only auxiliary.

Expressions for  $Q_w$  and  $Q_l$  can be given other forms. Let us substitute for  $(ni)$  its value in equation I, then finally

$$Q_w = qV^2/2wsR_m(p+kw)cT_r \dots\dots\dots \text{III}_a$$

$$Q_l = qV^2/2lsR_m(p+kw)cT_r \dots\dots\dots \text{VII}_a$$

where  $R_m$  stands for resistance of a mean turn at average temperature of the coil.

Likewise substituting from (4), it obtains that

$$cT_r = 1/A_1 \text{ and } A_1 = 1/cT_r \dots\dots\dots (14)$$

where  $A_1$  stands for  $A/W$  and denotes area of the cooling surface per 1 watt dissipated; formulas III and VII, also III<sub>a</sub> and VII<sub>a</sub>, transform to

$$Q_w = qA_1(ni)^2/2ws = qA_1V^2/wsR_m(p+kw) \dots\dots\dots \text{III}_b$$

$$Q_l = A_1(ni)^2/2ls = qA_1V^2/2lsR_m(p+kw) \dots\dots\dots \text{VII}_b.$$

<sup>1</sup> Roman numerals are used to indicate practically important formulas or fundamental equations.

Raising equation (6) to the rank of practical importance we write:

$$n = \pi w l / q \dots\dots\dots \text{VIII.}$$

There are now three fundamental equations, namely, I, V (or VI), and VIII to determine four basic magnitudes pertaining to the coil, i. e.,  $q$ ,  $w$ ,  $l$ , and  $n$ . This is practically advantageous because it permits a design to be adapted to other requirements arising from structural and economical considerations.

*Suggestions and remarks.* There are cases in which  $w$  must or can be estimated on the basis of some conditions or considerations; for example, the field coils of a machine may call for this estimation. With the shape and size of the opening known or assigned and the average temperature  $T_a$  of the winding reasonably assumed or estimated, equation I makes it possible to compute  $q$  for an assigned number of ampere-turns; then if necessary it is rounded to a wire-table figure. A more accurate value for  $w$  should then be computed from equation I substituting there the above table value. Now by using equations III (or III<sub>b</sub>) and V,  $l$  becomes computable.

Denoting ambient temperature by  $T$ , outside and inside temperature rises by  $T_r$  and  $T_i$  respectively, and hottest spot and average temperature by  $T_h$  and  $T_a$  respectively, it obtains that

$$T_h = T + T_r + T_i \dots\dots\dots (15)$$

and also with sufficient accuracy that

$$T_a = T + T_r + T_i / 2 \dots\dots\dots (16).$$

$T_i$  usually varies from 5° to 30° C as ampere-turns change within the range 500 to 18,000. For a continual load in the absence of exceptional conditions it is suggested that 95° C  $\leq T_h < 100^\circ$  C. It is understood that if a load is intermittent or space limitations are stringent higher values of  $T_h$  are allowable, but in the latter case this would of necessity shorten the life of the coil. The cooling coefficient  $c$  usually varies from 0.005 to 0.010 depending on the nature of the ambient medium, conditions of ventilation, etc. Substituting values for  $T$ ,  $T_h$ , and  $T_i$  in equations (15) and (16) one finds  $T_r$  and  $T_a$  which in combination with chosen  $c$  and  $w$  will set the system of fundamental equations and their auxiliaries to work.  $T_a$  can usually be chosen beforehand within the range 85° to 95° C if there are no exceptional conditions such, for instance, as drastic limitations of winding space.

Another approach to the development of a design is to begin with a reasonable assumption of  $A_1$  instead of the temperature and cooling coefficient; then equations III<sub>b</sub> and VII<sub>b</sub> should be used. Usually  $A_1$  varies within the range 2 to 3.3, being about 2.65 for average cases. Also there are situations which require that  $l$  should be estimated first and then  $w$  determined from equations VI and VII. Since  $q$  is not yet known in this case, the space factor  $s$  should be estimated and checked afterwards.

The half-empirical formula

$$T_i = 5 \sqrt{(\pi i) / 500} \dots\dots\dots (17)$$

for an estimation of  $T_i$  in degrees C at the beginning of work on a design is offered as helpful and adequately accurate.