THE RELATION OF SOLAR RADIATION TO SUN-SPOT CYCLES

Clyde John Bollinger
University of Oklahoma

THE SUN-SPOT CYCLE

Sun-spots were discovered by Galileo with his newly made telescope in 1610 and their periodicity was announced by Schwabe in 1843. According to Abbot* sun-spots are great vortices in the solar atmosphere in which gases, estimated by Russell to have initial temperatures of 10,000-20,000 degrees centigrade rise to the surface of the sun.** Sun-spots occur in cycles. Upper line Fig. 1. The length of the interval from one year of greatest quiescence to the next averages eleven and one-eighth years, but has ranged from ten to thirteen years.

Each cycle is introduced by a sudden outburst of solar activity in about 30° north and south latitude of each solar hemisphere, followed by a gradual equatorward shift of the zones of greatest activity and final disappearance within a few degrees of the solar equator at about which time a new cycle begins in higher solar latitudes. The maximum number of sun-spots, it appears from graphs by Maunder,*** occurs when the centers of the zones of spottedness are near the fifteenth parallels of solar latitudes, at which time the mean daily spotted area occupies from 1,100 to 1,500 millionths of the visible solar hemisphere.

Hale's study of the magnetic fields and the direction of rotation of sun-spot whirls indicates that sun-spots exhibit a double cycle averaging twenty-two and one-fourth years in duration.+ Double cycles ended in 1889, 1913, and presumably in 1934. An examination of Fig. 1 indicates that the first and second phases of each double cycle have, since 1843, been of almost equal duration and that the number of sun-spots at the maxima of the first eleven year cycles has invariably been higher than the maxima of the second.++ In the four double cycles occurring between 1843 and 1934 the average number of sun-spots for the years of maximum spottedness in the first phase was 113.05 while that of the second was only 75.16. (Table I). The comparative flatness of the sun-spot curve of the second phase of the double cycle is sometimes accentuated by a failure of the two solar hemispheres to attain maximum spottedness at the same time. Thus in the second cycle of the double cycle of 1889-1913 the maximum of the southern hemisphere occurred in 1905 while that of the northern did not occur until 1907.†

RELATION OF SUN-SPOTS TO SOLAR RADIATION

The existence of solar radiation changes associated with sun-spots is well established as appears from the following statement by Clayton (1930). "Abbot and his associates have given evidence of the relation of solar radiation to solar contrast and groups of spots on the sun, Fowle has shown a relation to faculae, Bauer has shown a relation to terrestial magnetism, and Austin has found a marked parallelism between radio receptivity and changes in monthly values of solar radiation."††

*C. G. Abbott, The Sun and the Weather, p 259.
**Russel, Annual Report of Mount Wilson, 1921.
***E. W. Maunder, The Sun and Sunspots 1820-1920, Smithsonian Institution Miscellaneous Collections, Vol. LXXXVII(No. 2, Washington, 1933.
+G. E. Hales, Quoted by C. G. Abbot in Sun-spots and Weather, Smithsonian Miscellaneous Collections, Vol. LXXXVII No. 2, Washington, 1933.
++See also, Nicholson, S. B. and Losh, S. B., High and Low Maxima of Alternate Cycles, Contributions Mount Wilson Solar Observatory, No. 300, 1925, p 467
†See graph by Maunder. Loc. Cit.
††H. H. Cloyton, Smithsonian Collections, Vol. 82, No. 7, 1930, p 4.

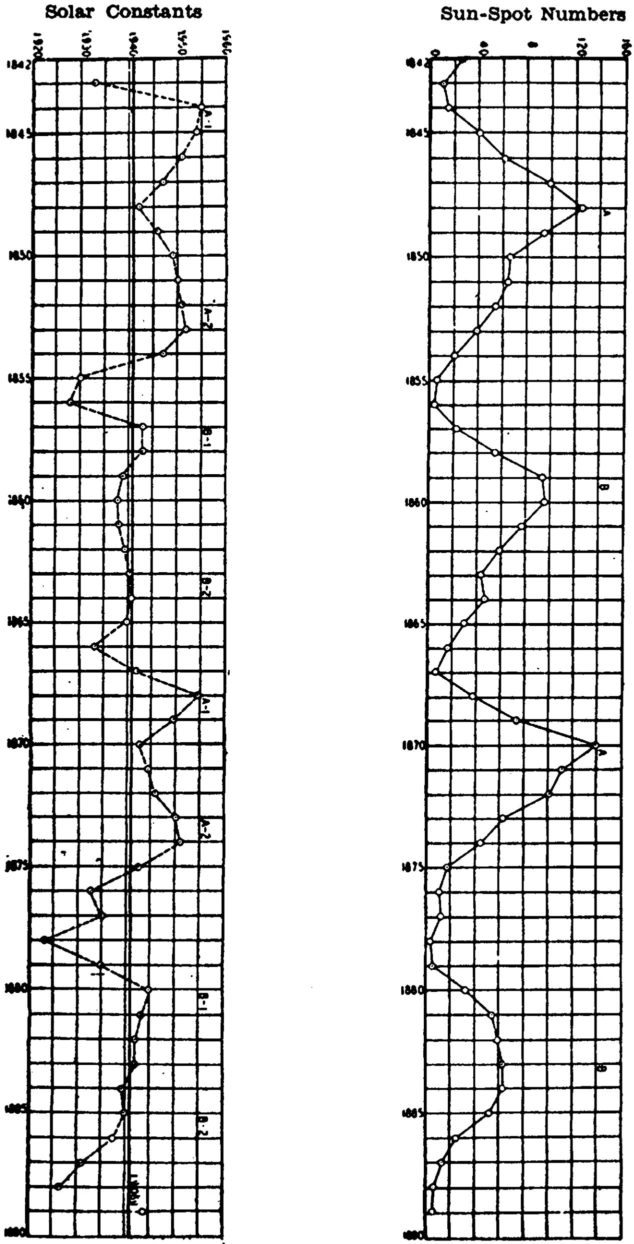


Fig. 1a. Sunspot Cycles and Hypothetical Solar Constants, 1842-1888

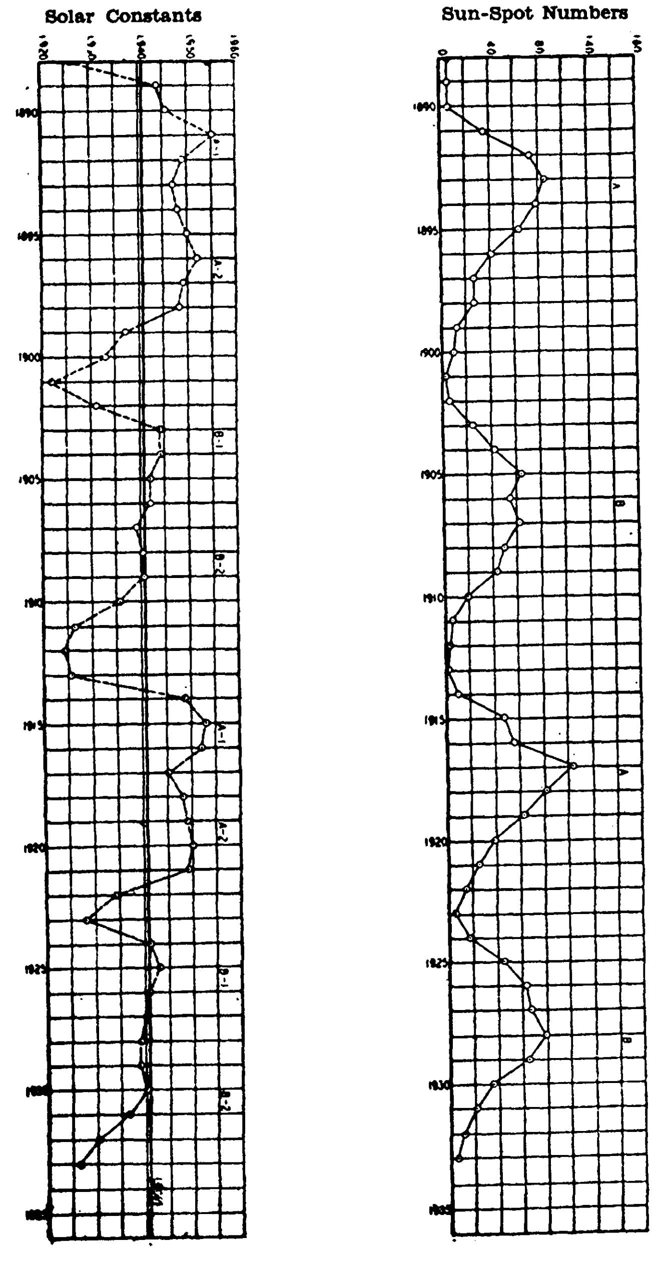


Fig. 1b. Sunspot Cycles and Hypothetical Solar Constants, 1888-1933

Shortness of the period of observation and unreliability of solar constant determinations, however, make it impossible, at present to draw more than tentative conclusions concerning the relation of solar radiation to sun-spot numbers. In addition to the possibility of instrumental errors there are uncertainties arising from the necessity of extrepolating pyroheliometric measurements to the top of the atmosphere. Monthly values considered fairly reliable, however, have been published by the Smithsonian Institution for the period 1918-1930. This study is based upon these values supplemented by the less dependable Mt. Wilson determinations for the summer months for the period 1908-1918. Since climatic and crop yield data exhibit considerable agreement with these values it appears that they are adequate for the purpose for which they are here employed.

TABLE I.

RELATIVE DURATION AND STRENGTH OF SUN-SPOT CYCLES, 1843-1934

(Smithsonian Miscellaneous Collection Vol. 79, and Monthly Weather Review)

Double Cycle	Duration Years	Average No. per Mo. on Years of Maxima First Phase	Second Phase
1843-1867	24	124.3	95.7
1867-1889	22	139.1	63.7
1889-1913	24	84.9	63.5
1913-1934	21	103.9	77.8
Average		113.05	75.16

An examination of the literature concerning sun-spots and solar radiation reveals a general agreement that the sun-spot binima are periods of relatively low solar radiation. According to Abbot* the solar constant is two or three per cent higher when sun-spots are most numerous than at the minimum. Kimball** states that "emitted solar radiation is strongest at the beginning of solar activity instead of at its maximum and analogously true of the minimum." Franz Bauer*** ,1932) has found evidence of a falling off of insolation of the sun-spot maximum. He has shown that hot solar faculae and sun-spot numbers are almost parallel, $(r = .89)$ and suggests that during intense solar activity matter is brought from the depths of the sun in quick succession and therefore the sun as a whole is hotter, but that hydrogen and calcium flocculi which change in quantity and intensity with sun-spots, increase the fraction of photospheric radiation which returns from the solar atmosphere to the photosphere. Dr. Angstrom found the Mt. Wilson solar constant values for the period 1915-1917 fairly represented by the formula $S = 1.903 + .011 \sqrt{n} - .006$, in which S is the solar constant and n the Wolf and

*R. H. Baker, Astronomy, An Introduction, p 300.

**H. H. Kimball, The Monthly Weather Review, Dec. 1932.

***Franz Bauer, Changes in the Solar Constant of Radiation, Monthly Weather Review, Dec. 1932, Wolfer sun-spot numbers.* Later, however, this relationship failed to continue which led to the expression of the "fear that no single formula will be found capable of expressing the relation of sun-spots to solar constants for more than a brief period."**

Quarterly Smithsonian solar constants for the period 1919-1930 when grouped according to quarterly sun-spot numbers for the same period reveal the following relationship:

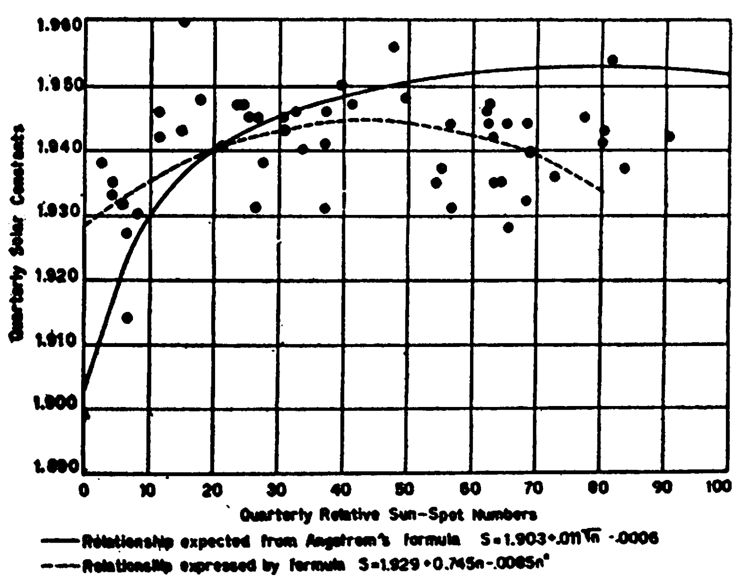
TABLE II.

Average Relation of Sun-Spots and Solar Constants Values 1919-1930
(Data Appendix Table I)

Number of Quarters	Sun-spot Range	Sun-spot Average	Solar Constant Average
6	0-10	5.2	1.9326
13	20-40	30.6	1.9438
21	Over 50	68.4	1.9400

The distribution is also shown in Fig. 2. Using the above averages as star points the relationship between quarterly sun-spots and solar constants

FIG. 2
Sun-Spots and Solar Radiation



for this period was found to be expressed in a general way by the formula: $S = 1.929 + 0.745\ n - .0085\ n^2$, in which S is the quarterly solar con-

*Annals of the Astrophysical Observatory, Vol. IV, 1922, p 215.
**Ibid. p 215.

stant and n the average number of sun-spots per month for each quarter. The parabola expressed by this formula may be compared with the heavy line in Fig. 2 showing the average distribution that would be expected from Angstrom's formula. Both lines confirm Abbot's statement that solar radiation is lowest on the sun-spot minimum. The data for 1919-1930 also confirms Kimball's statement that isolation is greatest at the beginning of the sun-spot cycle as well as Bauer's statement that there is a decline in radiation when sun-spots are most numerous. These data lead to the conclusion that each eleven year sun-spot cycle should be characterized by a minimum of solar radiation on the sun-spot minimum and that there should be a secondary minimum or radiation on the sun-spot maximum. Maximum insolation should occur early in the cycle and there should also be a secondary maximum of radiation following the secondary minimum.

In order to determine the relation between the time of maximum solar radiation and associated solar activity at time of few and also at times of many sun-spots, monthly solar constants were correlated with monthly sun-spot numbers of preceding and following months for two periods: (1) January 1921-August 1924 in which all months had less than forty sun-spots except March 1922 when sixty were reported and (2) January 1926-December 1928 in which all months had over fifty sun-spots. The results are shown in Table III.

TABLE III.

Relation of Monthly Solar Constants to Sun-Spot Numbers Preceeding and Following Months for Periods of High and Low Solar Spottedness.

	Mo. Before							Current Mo.	Mo. After					
	-7	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6
Sun-spots Under 45, 1921-1924 n = .56	.30	.38	.47	.40	.39	.43	.44	.43	.41	.59	.27	.22	.25	.16
Sun-spots Over 50, 1926-1928 n = .36	.10	.09	-.05	.12	.12	.12	.12	.17	-.09	-.13	-.28	-.23	.05	-.09

The correlation coefficients are perhaps too low to warrant general conclusions. However, the negative correlation between monthly sun-spots and solar radiation of the four following months during the maxima supports the hypothesis of a secondary radiation minimum at the time of high solar spottedness, while at times of low solar activity sun-spots are an indication of high solar radiation on both preceding and subsequent months, the greatest influence being on the second month following the month of solar activity.

A PROBABLE TWENTY-TWO YEAR RADIATION CIRCLE

A comparison of the solar constants for the period 1918-1930 with the Mt. Wilson values for the preceding cycle suggests a twenty-two year cycle of solar radiation in which there is a general decline in the solar constant values associated with a given number of sun-spots. This conclusion is supported by the low solar constants of the latter part of the preceding double cycle. The average for the period 1909-1913 was only 1.922 and presents a striking contrast with 1.9518 for an equal period 1914-1918 early in the next double cycle. If such a cycle exist four formulae, one for each of the four insolation hemicycles of the double cycle, are required

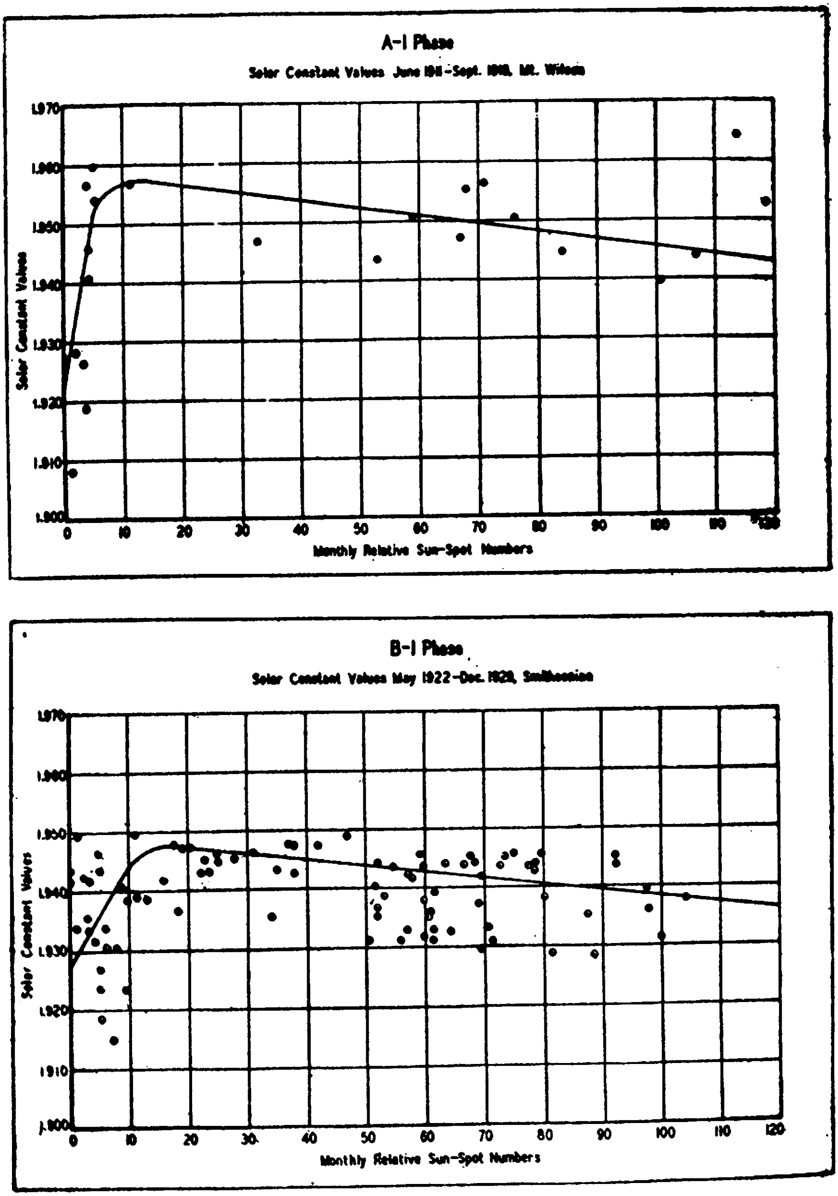
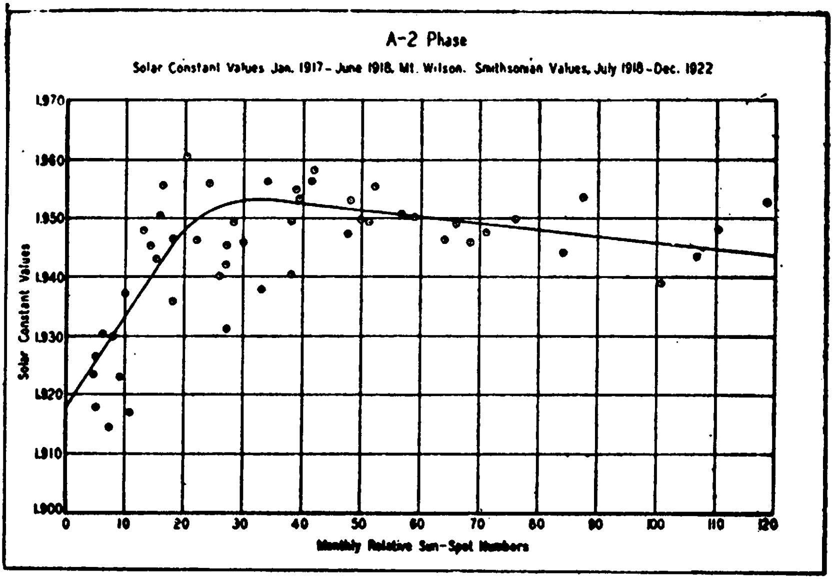


Fig. 3a. RELATION OF MONTHLY SUN-SPOT NUMBERS TO AVERAGE SOLAR CONSTANTS OF THE THREE FOLLOWING MONTHS DURING THE PRE-MAXIMA PHASES OF THE TWENTY-TWO YEAR DOUBLE CYCLE.



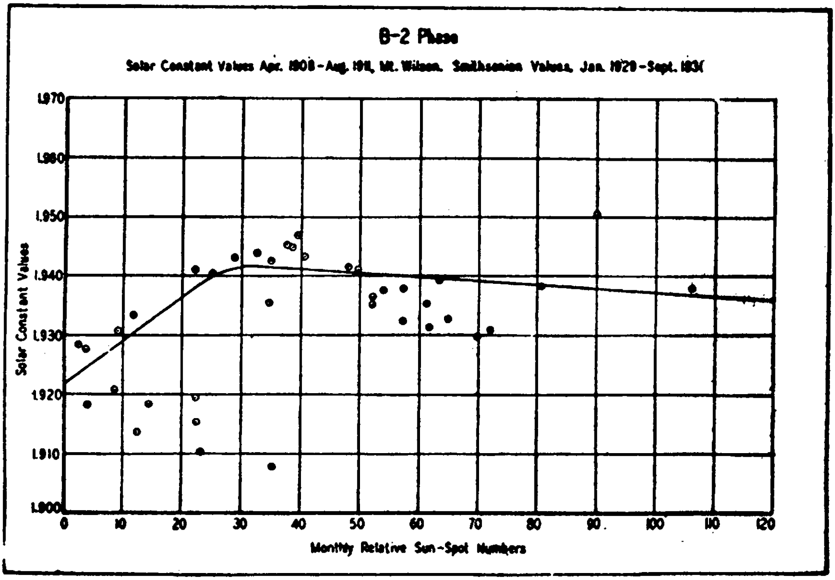


Fig. 3b. RELATION OF MONTHLY SUN-SPOT NUMBERS TO AVERAGE SOLAR CONSTANTS OF THE THREE FOLLOWING MONTHS DURING THE POST-MAXIMA PHASES OF THE TWENTY-TWO YEAR DOUBLE CYCLE.

to adequately express the relationship between sun-spots and solar radiation.

Separate graphs representing the relation of sun-spot numbers for each month to average solar constant values for the three succeeding months were drawn for the period 1908-1930 as follows: (1) Minimum to maximum of the eleven year cycle. (2) Maximum to minimum of the first cycle. (3) Minimum to maximum of the second cycle. (4) Maximum to minimum of the second cycle. These phases of the double cycle, designated respectively A-1, A-2, B-1, B-2, were taken to include the following years:

A-1............1911 to 1917
A-2............1917 to 1922
B-1............1922 to 1929
B-2............1908-1911 and 1929-1930

The relation of sun-spots to solar constants in each of these four phases of the double cycle are shown in Fig. 3. Using the four graphs expressing the relation of solar constants to sun-spots for the period 1908-1930 for the hypothetical four phases of the double cycle the solar constants indicated by the Wolf and Wolfer sun-spot numbers were determined for the four double cycles 1843-1934. The value for a given month being taken as one-fourth of the sum of the solar constant indicated by sun-spot numbers for that month and each of the preceding months. The mean of the monthly values, thus determined was taken to represent the indicated solar constant value for the year. These computed solar values had a mean value for the ninety-one year period 1843-1933 of 1940.69. The double cycles of radiation exhibit a notable similarity of pattern and may be of value in interpreting weather cycles prior to the period of solar constant determination.

The yearly solar constant computed from sun-spots using the double cycle four phase graphs had a positive .63 correlation with the measured values for the period 1905-1930 where both the Smithsonian and the Mt. Wilson values were included. The signs of the departures being incorrect only on three years, 1909, 1912, and 1925. The correlation with the annual Smithsonian values for the period 1919-1930 was .805 and proves a rather close association between the sun-spot cycle and solar radiation.

♣ ♣ ♣ ♣