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Physical Sciences

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A PSEUDO-EXAMPLE OF THE CONSERVATION OF ANGULAR MOMENTUM

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We propose to point out here some of the unusually neglected aspects of the conservation of angular momentum, centering our discussion about an alleged example which has found its way into at least two recent text books.

L. C. Little writes as follows in his "College Physics:" "A mass, tied to the end of a string and whirled about a stick, speeds up as the string winds itself around the stick. Although the angular velocity of the mass is increased, its angular momentum remains constant, for simultaneously with the increase in angular velocity there is a decrease in the moment of inertia as the distance from the mass to the axis becomes less and less." The same erroneous example is cited in Knowlton's "Physics for College Students," Page 437.

The passage in L. C. Little's text book contains two errors. First, it states that the mass "speeds up," which it cannot possibly do, since there is no source of energy; whence its linear speed cannot increase, but rather must remain constant. This is also seen from the fact that the instantaneous path of the mass is an element of a circle about the point where the string meets the circumference of the rod, and is, in the limit, exactly at right angles to the string. Therefore the force between rod and particle has no component in the direction of the particle's path, and does no work on the particle.

Second, the passage asserts that the angular momentum of the mass "remains constant," which also is not true. Its angular momentum with respect to any arbitrary point will steadily decrease, inasmuch as the centrifugal reaction of the particle exerts a torque about the rod, thereby steadily imparting its angular momentum to the rod and the system to which it is clamped.

Now the angular momentum of a particle with respect to a given point is $A.M. = I\omega = m\ r^2\omega = m\ v\ r$, in which r is the perpendicular distance of the point to the line of the instantaneous direction of the particle's path. For conservation of angular momentum, this quantity $m\ v\ r$, also called the moment of momentum, must be constant. For the case under discussion, v cannot change, while r steadily decreases as the string winds up on the rod. For r decreased by half, the angular momentum of the particle is decreased by half. This statement can also be checked by a simple integration of the angular impulse imparted to the rod about its center by the particle while winding up the string. The radius of the rod cancels during the calculation, showing that the decrease in angular momentum of the particle, for a given decrease in r, is independent of the size of the rod.

Nevertheless, conservation of angular momentum still holds, provided we take a system sufficiently large. The angular momentum of the particle is only imparted to the system in which the rod is clamped. The situation is analogous to that of a man on a piano stool. If he jerks his head in one direction, his feet must twist in the opposite direction.

It is evident that for any case of changing moment of inertia, angular momentum and kinetic energy of motion cannot simultaneously remain constant. Thus in the classical lecture room experiment of the rotating sliding masses, which when pulled toward their center of rotation by cords are greatly accelerated, angular momentum is really conserved, but work must be done upon the masses against their outward centrifugal reaction. In this case, as in the case of a planet approaching a minimum apse of its orbit about the sun, there is a component of the central force in the instantaneous direction of motion which does work upon the body by accelerating it. The work done may be found by evaluating a line integral. For the case of the radius vector decreased by half, the integration shows that the kinetic energy of the body does become four times as great, which of course must be the case if angular momentum, or m v r, is to be conserved; for if the radius is halved, the linear velocity must double and the kinetic energy become four times as great.

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