A LABORATORY EXPERIMENT ON THE MEASUREMENT OF MASS AND FORCE BY A KINETICAL METHOD

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One of the weaknesses of the experiments on mass and force in elementary mechanics is that they are faulty from a logical standpoint. The experiment described in this paper is, on the other hand, logically correct. This experiment does not involve any radically new idea, for it presents the concepts of mass and force essentially in the manner advocated by the great critic of the scientific method—Ernst Mach*. Furthermore, it does not provide the student with a more accurate method of measuring mass, for there is no more accurate method for ordinary purposes than the beam balance employed in the traditional experiments. But the beam balance measures gravitational mass directly, whereas the present method measures inertial mass; and it is the latter that is the fundamental concept in kinetics. It is only by the performance of an additional experiment that one is able to establish the principle of the equivalence of inertial and gravitational mass.

Newton defined mass as "quantity of matter." As Mach has pointed out, this is not definition of mass for it does not have the requisite clearness and it does not provide a method for measuring mass independently of the measurement of force. If we consider placing together a number of atoms of the same kind so as to obtain a chemically homogeneous body, the conception of "quantity of matter" then becomes clear, for we can simply count the number of atoms and then perceive that the acceleration of the body is inversely proportional to this number. However, in the case of heterogeneous bodies, in which the atoms are not alike, the acceleration is no longer proportional to the number of atoms and therefore the definition of mass as "quantity of matter" becomes meaningless. On the other hand, according to Mach, nothing prohibits us from using our mechanical experiences to define mass in the following manner: All those bodies are bodies of equal mass, which mutually acting on each other in any manner whatever, produce in each other accelerations equal in magnitude but opposite in direction. This means essentially that we can employ Newton's third law to define and measure mass kinetically.

Having defined mass independently of force, the latter is then defined as proportional to the time rate of change of momentum; that is, by Newton's second law. Thus we are led to the conclusion that the most fundamental and logically correct kinetical experiment on mass and force is that one in which masses are compared by the accelerations they produce and force is measured in terms of the time rate of change of momentum.

The apparatus for the first part of the experiment consists essentially of two cars and horizontal track. The cars are set into motion by means of a compressed spring placed between them. The cars are initially held together by a thread. When the thread is cut, the cars are set into motion and experiment shows that, if the friction is negligible, the cars move with constant velocities after the spring has ceased to act. By trial and error, the student is to find a starting point on the track such as that the two cars will reach the end of the track simultaneously. Since the two cars traverse distances d_1 and d_2 in equal times, we have

$$\frac{d_1}{d_2} = \frac{v_1}{v_2}, \qquad (1)$$

where v_1 and v_2 are the respective speeds of the cars. But, from Newton's third law of motion,

$$\frac{m_1}{m_2} = - \frac{v_2}{v_1}, \qquad (2)$$

where m_1 and m_2 are the masses of the cars. Hence

$$\frac{m_1}{m_2} = - \frac{d_2}{d_1},$$

which is the laboratory equation for determining the relative masses of the two cars and their contents.

The second part of the experiment utilizes an apparatus that can be any one of several variations of the Atwood machine principle. Specifically, we use an apparatus that consists essentially of a car and a horizontal track. In order to accelerate the car, a weight is attached to it by means of a cord running over a pulley. The student finds by experiment that the quantity mv/t is constant as long as the weight producing acceleration remains the same, and, moreover, that this quantity is proportional to the weight producing the acceleration. The definition of force as a quantity which is proportional to the time rate of change of momentum is therefore justified.

The present experiment is obviously an improvement over the traditional ones which attempt to justify Newton's second law by keeping the mass constant and showing that force is proportional to the acceleration, and then keeping the force constant and showing that the mass is inversely proportional to the acceleration, thus employing the same equation to define both mass and force. The experiment also has the advantage that it

*Ernst Mach, Science of Mechanics, 4th ed., Open Court, 1901, p. 243.

never involves the acceleration. It identifies force with time rate of change of momentum, rather than with the product of mass and acceleration; hence, the student has nothing to unlearn when he begins his study of the special theory of relativity. The experiment has been tested over a period of two years in a technical school and has proved satisfactory for freshmen engineers.