THE VANISHING OF THE MAGNETIC POLE

F. W. WARBURTON
University of Oklahoma

As early as 1825 Ampere suggested that the behavior of magnets is due entirely to "molecular" currents set up in the iron, these currents in the case of the electromagnet contributing to and supplying the major part of the magnetic field. Although this explanation seems almost obvious now, in view of the successes of the Bohr atomic model in which the electron orbit becomes visualized, it was considered highly speculative at first, and Maxwell's equations of the electromagnetic field in 1873 were based on the definition of magnetic field as the force acting on the magnetic unit pole, while the mutual effect of current and magnetic field was taken as an additional experimental fact. But Ampere's explanation has been gaining weight, slowly but steadily and surely, and Langevin[1, 2] in 1905 laid the foundation of the electron theory of magnetism.

The most useful theories are built on as few experimental phenomena as possible, and the theory is used to determine new facts, which when checked by experiment, substantiate the theory. Maxwell's electromagnetic theory was based on three experimental laws, Coulomb's inverse square law of attraction of electric charges, the similar inverse square law for magnetic poles, and the interaction of magnets and currents. In recent years the mathematical description of magnetism has been dealing more and more with Amperian currents and less and less with magnetic poles. Magnetic quantities can be defined in terms of currents, and the behavior of iron described in terms of Amperian currents. Mason and Weaver,[3] for example, do not in their definitions even mention the magnetic pole. This amounts to a considerable simplification since the number of experimental laws on which the electromagnetic theory is founded is reduced from three to two; the law of attraction or repulsion between two charges, $f=q'q/r^2$ (Eq. 1), and the law of attraction or repulsion between two elements of current, $df=i\ ds'\ i\ ds/c^2r^2$ (Eq. 11); while the behavior of the magnets becomes one of the deduced facts which when verified experimentally substantiates the theory.

Since an electric current is a succession of moving charges, the two basic laws express the forces between stationary charges and the forces between moving charges respectively. If the ordinary laws should hold when two electrons moving along parallel paths attain the speed of light then the magnetic attraction between the moving charges would just balance their electrostatic repulsion, while on a frame of reference moving with the electrons, the electrons would be stationary and there would be electrostatic repulsion alone. This suggests that electric forces and magnetic forces are different aspects of the same thing, the explanation of which involves relativity.

The purpose of this paper is to present the physical picture of electric polarization and of magnetization, to develop briefly the expressions for fields and potentials corresponding to that picture in terms of charges and currents only, then to outline the derivation of the Maxwell equations, and finally to discuss a few of the disadvantages of using the concept of the non-existing magnetic poles. For the sake of clearness and simplicity the expressions for the electric and magnetic fields in vacuum and in dielectric and magnetic media will be developed for uniform fields (the differential expressions hold also for non-uniform fields), and the parallel plate condenser and the long solenoid chosen as the sources of the fields. The Gaussian system of units will be used with the modification that whenever current appears it is always divided by c, and c is the velocity of light when all quantities are expressed in electrostatic units, and $c=1$ cm/sec when it is desired to express the magnitude of magnetic quantities in electromagnetic units, the dimensions in this case remaining those of electrostatic units, i.e. charge has dimensions $M^{1/2}L^{3/2}T^{-1}$. The dielectric constant ϵ and the magnetic permeability u are dimensionless. It is interesting to note in passing that if we consider ϵ and u to have dimensions and c to be dimensionless and then arbitrarily assign to electric charge the dimensions of mass and frequency MT^{-1}—charge is known to have mass and wave motion—then the dimensions of the dielectric constant become ML^{-3}, a mass density, and the reciprocal af the permeability (which as we shall see corresponds rather to more ϵ than does u itself) has dimensions $MLT^{-2}L^{-3}$, an energy density.

ELECTROSTATICS

The experimental law of attraction or repulsion between two unlike or like charges, Coulomb's law,

$$f = q'q/r^2 \qquad (1)$$

represents the fundamental physical fact underlying electrostatics. When $f=1$ dyne and $r=1$ cm, q' and q are unit charges in electrostatic units. The electric field E is defined as the force that acts on a unit test charge q'.

$$E = f/q' \qquad (2)$$

The force on the charge q' due to a number of charges q is by (1)

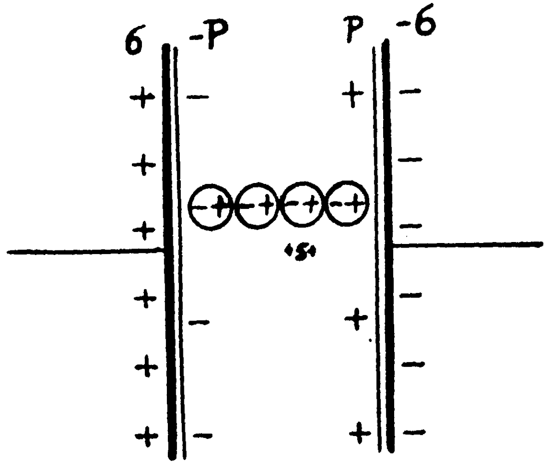
$$E = \Sigma q/r^2 \qquad (3)$$

Between the plates of a parallel plate condenser of area large compared to the distance between the plates, this summation is found by direct integration to be, in vacuum

$$D = E_0 = 4\pi\sigma \qquad (4)$$

where σ is the charge per square centimeter on the condenser plates, the subscript $_0$ refers to vacuum, and E_0 is replaced by D, the so-called electric "induction," which is nothing more than the electric field in the absence of dielectric material. (D would differ from E° dimensionally if the dielectric constant were not dimensionless).

If a dielectric material be placed between the plates of the condenser of Fig. 1, electrons are considered to be displaced on the average a short distance s from the positive nuclei toward the positive plate of the condenser. The electrons are moving about in orbits and by the position of an electron we mean the average position over a period of time. The more loosely bound electrons are displaced farther than those closely bound. The sum of the product of the electric charge of each electron and its mean



displacement is equal to the sum of the charges of all the electrons in an atom multiplied by their average displacement and is called the electric moment of the atom. The sum of the electric moments of all the atoms in a cubic centimeter is the electric moment per unit volume and is termed the polarization P. $P = \Sigma es/V$ where Σes represents the total electric moment in volume V. $\Sigma es = S_{av}\Sigma e$ where S_{av} is the mean displacement of all the electrons in volume V, and a charge—Σe multiplied by d, the distance

S_{av}

the electrons in volume V, and a charge $\frac{S_{av}}{d}\Sigma e$ multiplied by d, the distance between the plates of the condenser, gives the same total electric moment. $\frac{S_{av}}{d}\Sigma e$ is the charge q_p appearing at each surface of the dielectric medium adjacent to the plates of the condenser and this is equal to $PV/d = PA$ where A is the area of the plates. The polarization P, defined as electric moment per unit volume, is then also the charge per unit area appearing at the edge of the dielectric. This surface density of charge P reduces the net charge density at the condenser plates from σ to $\sigma - P$ and the resultant field inside is by integration

$$E = 4\pi\,(\sigma - P) = 4\pi\sigma/\epsilon \qquad (5)$$

where $\epsilon = \sigma/\sigma(\sigma - P)$ and is called the dielectric constant. The electrons are displaced until the forces opposing the displacement just balance the (reduced) field E. This determines P.

$$E = D\text{-}4\pi\ P = D/\epsilon \qquad (6)$$

In the case of the spherical condenser, if the inner sphere be positive, a negative charge $q_p = -\frac{S_{av}}{d}\Sigma e$ appears at the inner spherical surface of the dielectric medium, and an equal positive charge at the outer surface. The polarization, like the field D, varies inversely with the square of the distance from the center of the sphere. If Q be the charge put on the condenser,

$$D = Q/r^2 \text{ and } P = q_p/4\pi r^2$$
$$E = Q/r^2 - q_p/r^2 = D - 4\pi P$$

In any non-uniform field

$$E = \Sigma q/r^2 - \Sigma q_p/r^2 = D - \Sigma q_p/r^2 \qquad (7)$$

D can always represent the field due to static charges and aside from displacement charges while the polarization term may be a complicated function involving several dielectrics.

In a crystal P may not have the direction of D or E but Eq. (6) still holds if E be considered the vector sum of D and $-4\pi P$.

Electric potential is defined as the work done or the potential energy per unit charge

$$V = W/q = \int E ds \text{ and conversely, } E = dV/ds \qquad (8)$$

In general

$$V = \int \Sigma \frac{q}{r^2} dr - \int \Sigma \frac{q}{r^2} dr = \Sigma q/r + \Sigma q_p/r \qquad (9)$$

Since the potential V is a scalar quantity and involves a lower power of r, it is frequently easier to compute the potential and from it the field than it is to compute the field directly.

If we connect a condenser to a generator or storage battery and then change the dielectric between the plates is the potential V and not the charge density σ which remains constant. Were the reverse true it would be useful todefine a potential

$$V_o = \int D ds. \quad \text{Conversely } D = dV_o/ds \qquad (10)$$

V_o is the work that would be required to carry a unit charge between the plates after the dielectric were were removed.

ELECTROKINETICS OR MAGNETOSTATICS

The fundamental law for the attractive forces between two parallel currents is expressed in the elemental form

$$df = \frac{i'ds' \ ids}{c^2 r^2} \perp \qquad (11)$$

where df is the force acting on the short length ds′ of current i′ due to the short length ds of current i at a distance r from ds′. The symbol $\perp$ represents that r is perpendicular to ds′ and to ds. c is the ratio between the electromagnetic and the electrostatic units, found experimentally to be 2.99796×10^{10}cm/sec. If ds′=1 cm, ds=1 cm, i′=i=1 electrostatic unit of charge per second and r=10 cm, then the force between the currents is $1/100c^2$ dyne. If we let c=1 cm/sec, ds′=ds=1 cm, r=10 cm and df= 1/100 dyne, i′ and i each have the magnitude of the electromagnetic unit of current, the dimensions being still those of the electrostatic unit of current, $M^{1/2}L^{1/2}T^{-2}$. For an experimental check of the law, Eq. (11) must be integrated for the particular shape of the currents, i′ and i, since in general r is a function of ds′ and ds, and has a different numerical value for different parts of the circuit, and the angles between ds′ and r and ds and r must be introduced. In the example just chosen r varies a very little from 10 cm as we consider the force on any point of ds′ due to each point of ds.

In the description of magnetic phenomena it is found useful to define a field B, perpendicular to i and perpendicular to the plane containing i and r. B then gives the direction in which the end of a solenoid or the end of a magnet would move. B is defined as equal to the force per unit length on the test current i′, when ds′ is perpendicular to B. The direction of B is determined by the current i which produces it, and when i′ is not perpendicular to B the force on i′ds′ depends on the angle between ds′ and B.

$$B = \frac{c \ df}{i' \ ds'} \perp \qquad (12)$$

The force per unit length on i′ due to a number of current elements is equal to

$$B=\int\frac{i\ ds}{cr^2}\perp=\int\frac{i\ ds}{cr^2}\sin\theta \tag{13}$$

where θ is the angle between r and ds. In the absence of magnetic media

$$H=B_o=\int\frac{i\ ds}{cr^2}\sin\theta \tag{13a}$$

This is known as LaPlace's law. When iron is present and besides the current i in the wire there is the current i_n induced in the iron

$$B=\int\frac{i\ ds\sin\theta}{cr^2}+\int\frac{i_m ds\sin_m\theta}{cr^2} \tag{13b}$$

$$=H+\int\frac{i_m ds\sin_m\theta}{cr^2}$$

It is customary to say that the field H between the pole pieces of a motor is equal to B, the socalled "induction" of the iron. The above definitions, however, state that the force per unit length on the armature conductors between the pole pieces is B. "Field" is a better name for B than "induction." Frenkel[4] and Kennard[5] show that B corresponds to the electric field E rather more than does H, while Terry[6] defines B as the force per unit length on a current, as above.

In a uniform field B, the torque on a coil carrying a current i′ is found by integration of Eq. (12)

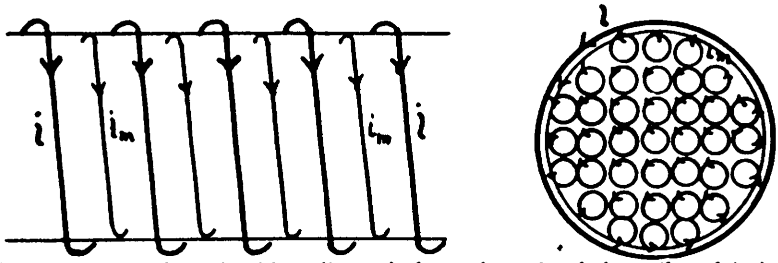
$$L=Bi'A\sin\theta$$

where A is the area of the coil and θ is the angle between B and the normal to the plane of the coil. The quantity i′ A is called the magnetic moment of the coil, for when $\theta=90°$ it represents the ratio between the torque L and the field B. (A simple substitute for the integration is to consider the coil made up of a large number of rectangular coils and the torques on the small rectangular coils found from the forces on two sides of the coils, the effects of the other two sides cancelling.)

On the axis within a long straight solenoid at a distance from the ends, we find by integration of Eq. (13a)

$$H=B_o=4\pi ni/lc \tag{14}$$

where n is the number of turns of wire carrying current i, and l is the



length of the solenoid. H is directed along the axis of the coil and is in the direction of the thumb of the right hand when the fingers indicate the direction of the current in the solenoid.

When we introduce iron into the solenoid of Fig. 2, the electron orbits of the atoms which were oriented at random or in neutral groups[7] in the unmagnetized state, behave much like a coil carrying current, and experience a torque tending to line them up in planes perpendicular to the axis of the coil. The magnetic moment of the atom is the vector sum of the product of each electronic current and the area of its orbit. The vector sum of the magnetic moments of all the atoms in a cubic centimeter is the magnetic moment per unit volume, and is termed the intensity of magnetization I. $I = n_a \Sigma iA/cV$ where $n_a \Sigma iA/c$ is the total magnetic moment in volume V, n_a is the number of planes of atoms normal to B, and the summation Σ refers to area. $\Sigma iA = i_a \Sigma A_a$ where A_a is the mean area of the atoms and i_a is the equivalent current flowing around the periphery of the atom. A current

$$i_m = \frac{n_a \Sigma A_a}{n\ A} i_a$$

per turn of wire in the solenoid is the equivalent current appearing at the cylindrical surface of the iron, giving a magnetic moment equal to the total magnetic moment of the atoms. The current i_m asumes greater physical significance when we consider A, the cross section of the solenoid and iron, to be the vector sum ΣA_a, and that within the iron the electronic current of one atom just cancels that of the neighboring atoms, the currents $i_a (= i_m n/n_a)$ remaining at the surface only. I, then, defined as the magnetic moment per unit volume is also equal to $n i_m A/cV = n i_n/cl$ where l is the length of the iron.

The foregoing discussion describes ferromagnetism and paramagnetism. Ferromagnetic substances are distinguished by the fact that when certain neutral groupings of atoms in equilibrium are broken up by the application of an external field, the atoms form new equilibrium groups which add strongly to the field. In paramagnetic substances the orbits are rotated only a small amount.

While the iron is being brought into the solenoid or while the current in the solenoid is being built up, there is induced in the iron a small current in the opposite direction, a slowing down of electrons in the orbits which are contributing to the para- or ferromagnetic effects or a speeding up of electrons which at the moment are traveling in the opposite direction. This is the diamagnetic effect and is present in all magnetic materials. When this diamagnetic effect is predominate the material is diamagnetic, and when the para- or ferromagnetic effect is greater than the diamagnetic effect, the substance is classed as paramagnetic or ferromagnetic. The symbols I and i_m now refer to the combined effects of both para- and diamagneism or ferro- and diamagnetism. In a diamagnetic substance I and i_m are small and negative.

The field B due to the combined effects of the current in the solenoid i and the current induced in the iron i_m is found by integration of Eq. (13b)

$$B=4\pi_n(i+i_m)/lc=u4\pi ni/2c \qquad (15)$$

where $1/u=i/(i+i_m)$, provided we assume i_m constant throughout the length of iron l. (For comparison with the electrostatic case see Eq. (5).)

$$B=H+4\pi\ I=uH \qquad (16)$$

since $I=ni_m/lc$. The extraordinarily large values of i_m as compared to i in iron and other ferromagnetic materials rests in the mutual magnetic effect of the adjacent electron orbits. When the external applied field H rotates the orbits slightly the effect of each orbit on its neighbors rotates them more until they line up, one might say, almost spontaneously. At the ends of the iron this mutual effect exists on one side of the atoms only, and the orbits of these atoms are less well lined up and the resultant current i_m weaker at the ends. If we use for i_m its value over the major part of the iron except for the ends we should use a length somewhat shorter than the geometrical length of the iron. This corresponds, in the pole theory, to considering the magnetic poles located near but not at the extreme ends. Again because even a very narrow slot cut through the iron breaks up the mutual action of the atoms near the slot, the average i_m and I are decidedly less with the slot than without. If two magnetic media be in contact in the solenoid, the field B depends on the magnetization currents i_m of each media and also on the shape and extent of each media. B is a function of I_1 and I_2.

In permanent magnets the orbits remain lined up and

$$B=4\pi ni_m/lc \qquad (17)$$

The field due to a long straight wire is found by integration of Eq. (13a) to be

$$H=2i/rc$$

and from Eq. (13b)

$$B=2(i+i_m)/rc$$
$$=H+4\pi I$$

since $I—i_mA/Vc$, and using as an element of volume a cylindrical shell of thickness r_2-r_1, and length l, $A=(r_2-r_1)l$, $V=\pi(r_2^2-r_1^2)l$, we see that

$$4\pi I=\frac{4\pi i_m l(r_2-r_2)}{c\pi l(r_2^2-r_1^2)}=\frac{4\pi i_m}{2\pi c\left(\frac{r_1+r_2}{2}\right)}=2i_m/rc$$

The magnetic scalar potential, like the electric potential, is defined as

$$\phi=\int Bds \quad \text{and in vacuum } \phi=\int Hds \qquad (18)$$

ϕ (or ϕ_o) is not the work done to move an element of current since B (or H) is perpendicular to the force acting on the current and work is not done if the current is moved along B. ϕ_o however is a useful quantity because it is related to the engineer's "Ampere turns" of his magnetic circuit, and it involves the current i which is externally controlled, rather than the sum of the currents $i+i_m$ involved in ϕ.

ϕ and ϕ_o are not so closely related to the currents which produce the fields B and H, as are the electric potentials V and V_o to the charges $q-q_p$ and q, and there is another potential A, derived from the currents $i+i_m$, which is a vector.

$$A=\int\frac{j+j_m}{rc}dV \qquad (19)$$

where j and j_m are the densities of current i and i_m in electrostatic (or electromagnetic) units of current per square centimeter, and dV is an element of volume. B is found from A by the vector operation

$$B=\text{curl } A$$

Similarly in the absence of magnetc media, the magnetic vector potential for the field H due to currents in wires alone is

$$A_o=\int\frac{j}{rc}dV \text{ and } H=\text{curl } A_o \qquad (20)$$

In the foregoing discussion the equations have emphasized the displacement charges q_p (P) and the curent i_m (j_m) corresponding to the physical picture of the processes, rather than the more or less blanket terms dielectic constant E and the permeability u.

THE MAXWELL EQUATIONS

It follows from the principle of the conservation of energy that the work to carry a charge from one point to another is independent of the path chosen. The work to carry a charge from one point to another and back to the starting point again is zero. Around a closed loop, then

$$\int Eds=0$$

The distinction between the nature of the vectors E and D is less emphasized of late years, and if we define D as the force on a test charge q′ with dielectric media absent, but with the same distribution of "real" charges q as were present with dieelectrics of different constants ϵ_1, ϵ_2, adjacent and present, rather than define D as simply $D=\epsilon E$, then also, around a closed loop,

$$\int Dds=0$$

The vector differential form of these line integrals are

$$\text{curl } E=0$$
$$\text{curl } D=0 \qquad (21)$$

Integrating B, however, around a closed loop enclosing a current i and the induced magnetization current i_m, gives

$$\int Bds=4\pi(i+i_m)/c.$$

Likewise

$$\int H ds = 4\pi i/c$$

These, written in vector differential form, are

$$\text{curl } B = 4\pi(j + j_m)/c$$
$$\text{curl } H = 4\pi j/c \qquad (22)$$

Taking the surface integral of the normal component of the electric field over a surface S, enclosing charge q and the charge q_p induced in the dielectric (Gauss' Theorem),

$$\int E_n dS = 4\pi(q - q_p),$$

Likewise

$$\int D_n dS = 4\pi q,$$

or in vector differential form

$$\text{div } E = 4\pi(p - p_p)$$
$$\text{div } D = 4\pi p \qquad (23)$$

where p and p_p are the volume densities of the charge q and q_p.

From the theorem of vector analysis that the divergence of the curl of any vector is zero, we can write down directly

$$\text{div } B = \text{div curl } A = 0$$
$$\text{div } H = \text{div curl } A_o = 0 \qquad (24)$$

By computing the work done in carrying a charge around a loop enclosing a changing magnetic field B, both in terms of the electric field E and the magnetic field B, we have, using the law, well known to engineers, that the induced electromotive force is equal to the rate of change of magnetic flux, where flux is defined as the field B multiplied by the area S of the coil.

$$\int E' ds = \frac{S}{c} \frac{dB}{dt}$$

or

$$\text{curl } E' = -\frac{1}{c} \frac{dB}{dt} = -\frac{1}{c} \frac{dH}{dt} - \frac{4\pi}{c} \frac{dI}{dt}$$

also $\text{div } E' = 0$ (25)

We should note here that we need to use the actual electric and magnetic forces E and B rather than D and H. The primes are added merely to distinguish this field E′ due to changing magnetic field from the field E due to stationary and induced charges.

By considering the current charging a condenser and rate of change of the field D, we see that the current i is equal to the rate of increase of the charge on the condenser, dq/dt, or

$$j' = d\sigma/dt$$

$$\frac{4\pi j'}{c} = \frac{4\pi}{c} \frac{d\sigma}{dt} = \frac{1}{c} \frac{dD}{dt} \quad \text{by Eq. (4)}$$

$$\text{curl } H' = \frac{1}{c} \frac{dD}{dt} \quad \text{by Eq. (22)}$$

also $\quad \text{div } H' = 0 \qquad (26)$

This process is independent of the media and involves D and H.

The total electric field E_T is the (vector) sum of the fields due to the three sources, stationary or "real" charges q, induced or displacement charges p_p, and changing magnetic field dB/dt.

$$E_T = D - 4\pi P + E'$$

$$\text{curl } E_T = 0 + 0 - \frac{1}{c} \frac{dH}{dt} - \frac{4\pi}{c} \frac{dI}{dt} \qquad (27)$$

$$\text{div } E_T = 4\pi(p - p_p) + 0$$

Also the total magnetic field B_T is due to the three sources, currents in wires i, induced currents around the surface of the iron i_m, and changing electric field dD/dt.

$$\text{div } B_T = 0 + 0 + 0$$

$$\text{curl } B_T = \frac{4\pi(j + j_m)}{c} + \frac{1}{c} \frac{dD}{dt} \qquad (28)$$

$$\text{div } B_T = 0 + 0 + 0$$

In general whenever D (p) $=0$, P $(p_p)=0$ also, except for permament electric polarization, and whenever H (j)$=0$, I $(j_m)=0$ also, except for permanent magnets, but in the absence of dielectric or magnetic material, P and I vanish independently of D and H.

In free space, $j=0$, $j_m=0$, $P=0$, $P_p=0$, and, dropping the primes, equations (27) and (28) reduce to

$$\text{curl } E = -\frac{1}{c} \frac{dB}{dt} \qquad \text{curl } H = \frac{1}{c} \frac{dD}{dt}$$

$$\text{div } E = 0 \qquad \text{div } H = 0 \qquad (29)$$

By the vector operation

$$\text{curl }^2E = \text{grad div } E - \text{Lap} = \text{Lap } E = -\frac{1}{c} \text{curl} \frac{dB}{dt} = \frac{i}{c} \frac{d}{dt}$$

(curl B) we find

$$\text{Lap. } E = \frac{ue}{c^2} \frac{d^2E}{dt^2} \qquad (30)$$

which is the equation of wave motion with the velocity $v = c\sqrt{\frac{1}{ue}}$

THE MAGNETIC POLE

In the development of the theory based on the magnetic pole, considerable emphasis is placed on the analogy between magnetic fields and electric fields, and on the supposed correspondance of H to E and B to D, as is implied in the names given these quantities.

With Coulomb's law for poles, $f=m'm/ur^2$, which expresses the attraction or repulsion between the ends of two long magnets, as basis, the magnetic field in vacuum (u=1) is defined correctly as the force acting on a unit north pole, $H=f/m'$. In a magnetic medium, H is still regarded as the force acting on the unit pole because of the definition, and the socalled "magnetic induction" B is defined as B=uH. In the narrow slot cut in the iron perpendicular to B, however, the force on the unit pole is considered to be B. B is greater than H and D is greater than E and we are glad that they are so. With E and H defined as the force on the charge and on the pole respectively, and D and B as "induction," the analogy of B=uH to $D=\epsilon E$ seemed complete. But on investigating them more closely, we see a fundamental difference. We are glad that E is much less than D in a good dielectric because that allows us to store more electrons on a condenser with less work. If we were interested in storing poles with less work we should want the force on the pole in the presence of iron to be less than in vacuum, $B<H$. But we are not interested in storing poles, but in getting more force on a pole or on a current with less current in our copper wire, and hence with less applied H. We want $B>H$. Because we are happy that $D>E$ and $B>H$ we seem to have overlooked the fact that E is the electric field which is dependent on dielectric matter, while H is the magnetic field which does not depend on the presence of magnetic matter, that introducing matter reduces D to E but increases H to B, and that the equation B=uH and $D=\epsilon E$ represent tendancies more opposite than similar.

There is a certain justification of course in defining H rather than B as the force on a unit pole, if we wish to consider the poles of a permanent magnet as fixed and invariable. When a permanent magnet is placed in a paramagnetic medium the field H due to the invariable poles is believed to be unchanged, and were the force on the test pole or current element equal to B, the "magnetic induction" of the paramagnetic medium, we would be getting something for nothing, since there is no battery source, as there is in the electromagnet, from which to draw the extra energy to move the pole (or current) along (or across) B, above that needed to move the pole (or current) along (or across) H. If we were to permit the poles of the magnet and hence H to become sufficiently weaker, when placed in the paramagnetic medium, to conform to the principle of conservation of energy—and this would be more in keeping with the picture of magnetism in terms of the magnetic moment, ΣiA, of the electrons revolving in orbits in the atoms—then B rather than H could be defined as the force on a unit pole, and Coulomb's law for poles (including u, the characteristic of the medium) would be $f=um'm/r^2$ rather than $f=m'm/ur^2$, but there is no advantage in using this expression with u in the numerator when we have the quite similar law, Eq. (12), as the fundamental expression for the forces between currents.

Another disadvantage of the now vanishing pole theory is that in a short piece of iron, because B is found to be much less than the applied field H multiplied by the permeability u, it was necessary, in order to retain the equation $B=uH'$, to postulate a demagnetizing field reducing H to H'. In the theory based on currents this demagnetizing field does not exist, the small B in a short piece of iron being due to the less mutual action between the atoms than is possible in the longer piece in which the end effects are negligible.

With the definitions of B and H as the force per unit length on a conductor carrying current in the presence of iron and in vacuum respectively, there is no necessity for postulating Kelvin's narrow "slot" in a magnetic medium for the definition of B, nor of the slender "tunnel" for the definition of H. H is the field in vacuum due to the current i. When we wish the partial field in iron due to the current i, we can compute it from Eq. (13a) or Eq. (14) tunnel or no tunnel.

REFERENCES

1. Langevin, Journal de Physique (4), Vol. 4, p. 678, 1905.
2. Stoner, Magnetism and Atomic Structure, pp. 64-74.
3. Mason and Weaver, The Electromagnetic Field, pp. 173-186.
4. Frenkel, Lehrbuch der Elektrodynamik, Part II, Chap. I, pp. 13-16. Frenkel actually retains H as the force on currents per unit length, but is careful to explain that his H means what was foremrly called B, and his B what is usually called H.
5. Kennard, Notes used in course in Advanced Electricity and Magnetism, at Cornell University.
6. Terry, Advanced Laboratory Practice in Electricity and Magnetism, p. 15.
7. Wall, Applied Magneism, Chap. VII.