C. PHYSICAL SCIENCES

I. THE ECONOMIC THEORY OF RURAL LINE DESIGN.

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Introduction:

We are on the eve of a great rural electrification in Oklahoma as well as throughout the United States. State Committees in 23 states are studying the problems of the application of electricity to agriculture and much progress has been made. In Oklahoma a State Committee has been working for more than two years, an experimental line has been erected at Pauls Valley for observation, and about ten experimental research projects are being carried on. The writer is the project director for the committee and is in active charge of the investigational work.

One of these projects is a study of the economic and mathematical theory underlying rural line design. This project was undertaken in order that attention might be called to engineers charged with the design of rural lines of the economic theory underlying the design of such lines.

This paper deals with the economic theory of rural line design and points out the factors which must be applied in the various line calculations to take account of the fact that the rural load is not concentrated at the end of the line but is distributed along the line. Practically all tables and calculations developed and used at the present time are for lines having their entire load concentrated at one end with the source at the other end. This paper is the first presentation of which the writer is aware in which a uniformly distributed load along the line in place of a concentrated load at the end of the line is considered.

Kelvin's Law. That Kelvin's Law uttered in 1881 should also apply to rural lines is taken for granted for its application is universal. This law stated in words when applied to an electric line is as follows:

"The economical line conductor is that which makes the cost of the annual wasted energy equal to the annual cost of interest, depreciation and taxes."

In other words the fixed costs must be equal to the variable costs.

This principle is easily proved as follows:

IF C=cross section of conductor, then,

$$\text{Total Annual Charge} = K_1 C + \frac{K_2}{C}$$

Differentiating Annual Charge with respect to C, gives,

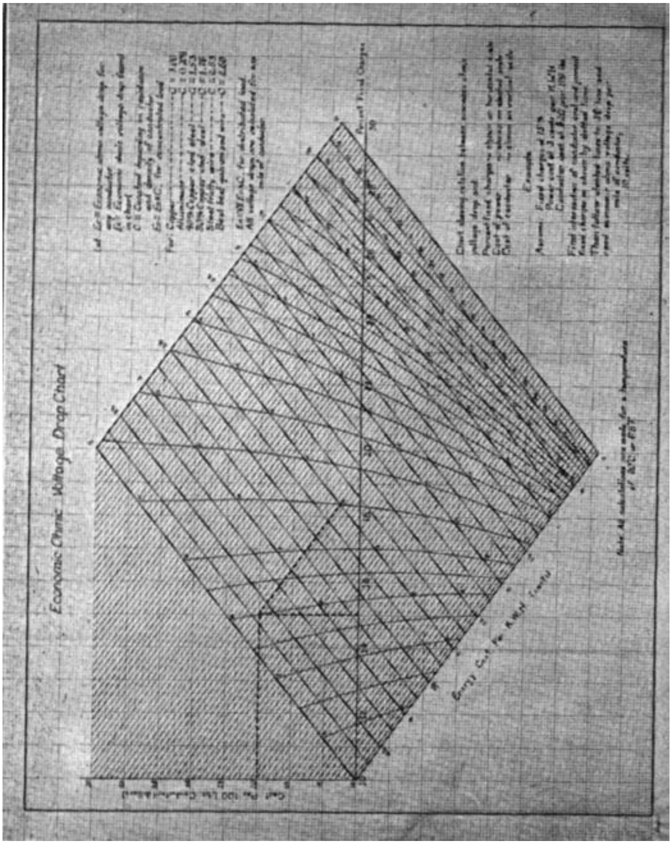
$$\frac{d\,AC}{dC} = K_1 - \frac{K_2}{C^2}$$

For minimum value the differential should equal zero, thus:

$$K_1 - \frac{K_2}{C^2} = 0 \quad \text{or}$$

$$K_1 = \frac{K_2}{C^2} \text{ or } K_1 C = \frac{K_2}{C}$$

or Fixed Cost=Variable cost.



Economic Ohmic Voltage Drop. The principle expressed by Kelvin can be stated in another form which makes possible computing the economic ohmic voltage drop per mile if the following three determining factors are known—these are:

p=dollars conductor cost per hundred pounds.
a=percentage covering interest, depreciation and taxes.
p_1=cost of energy per KW-year.
Then Annual Fixed Charge=%xInvestment

$$= \frac{a}{100} \times \left(p \frac{K}{R} \right)$$

Where R is the ohms per mile of conductor, and $\frac{K}{R}$ is weight conductor per mile.
For copper K=8.76
For Aluminum K=4.32

Annual copper loss per mile $= p_1 \times$ Kw loss per mile of conductor

$$= p_1 \times \frac{I^2R}{1000}$$

$$= p_1 \times \frac{I^2R \times R}{1000 \times R} = p_1 \times \frac{Er^2}{100 \times R}$$

From Kelvin's Law Maximum Economy requires that Fixed Costs=Variable costs, therefore,

$$\frac{a \times p \times K}{100 \times R} = \frac{p_1 \times Er^2}{1000 \times R}$$

from which

$$Er^2 = \frac{a \times p \times K \times 1000 \times R}{p_1 \times R \times 100} = \frac{a \times p \times K \times 10}{p_1}$$

$$= 10 \times K \times \frac{(a \times p)}{p_1}$$

$$Er = \sqrt{10\,K} \times \frac{\sqrt{a \times p}}{\sqrt{p_1}}$$

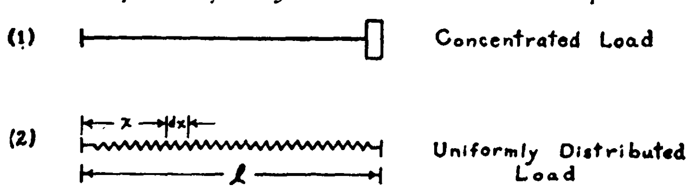
Substituting for Copper and Aluminum respectively, gives,

$$Er = \sqrt{87.6}\ \frac{\sqrt{a \times p}}{\sqrt{p_1}} = 9.35\ \frac{\sqrt{a \times p}}{\sqrt{p_1}}$$

$$Er = \sqrt{43.2}\ \frac{\sqrt{a \times p}}{\sqrt{p_1}} = 6.57\ \frac{\sqrt{a \times p}}{\sqrt{p_1}}$$

Economic Ohmic Voltage Drop Chart. In the accompanying chart are given the economic ohmic voltage drops for various combinations of conductor costs, energy cost and fixed charge percentage over their working ranges. It is not necessary, therefore, to evaluate the formula, but merely to refer to the chart for the conductor cost and fixed percentage assumed, and then from their intersection follow the guide line to the energy cost line where the economic voltage drop per mile can be read. It should be remembered that this chart is based on concentrated loads and therefore, the economic drops are not correct for rural lines with distributed loads. The chart, however, is correct for any kind of line namely: direct current, single phase, two phase, three phase, etc., when delivering energy to a load at the end of the line.

Line Loss for Uniformly Distributed Load as Compared to Con-



centrated Load. Before proceeding with the application of the chart shown above to rural line design, it is necessary to determine the ratio of the line losses with the load uniformly distributed along the line to the losses with the same load concentrated at the end of the line. This ratio is exactly 1-3 as the proof given below shows:

Line Loss (1)=$(Il)^2 \times Rl = I^2R \times l^3$
Where I=current per mile and R=Resistance per mile

$$\int_0^l (Il - Ix)^2 R\,dx = RI^2 \int_0^l (l-x)^2 dx$$

$$= RI^2 \int_0^l (l^2 - 2lx + x^2) dx$$

$$= RI^2 \left[l^2x - \frac{2lx^2}{2} + \frac{x^3}{3} \right]_0^l$$

$$= RI^2 \left[l^3 - l^3 + \frac{l^3}{3} \right]$$

$$= RI^2 \left[\frac{l^3}{3} \right]$$

Therefore

Line Loss (1) for Concentrated Load=$I^2R \times l^3$

Line Loss (2) for Distributed Load=$I^2R \times \frac{l^3}{3}$

and the loss on basis of uniformly distributed load is exactly 1-3 of the loss calculated on basis of a concentrated load at the end of the line.

Economic Ohmic Voltage Drop Factor for Uniformly Distributed Load. In examining the econcmic voltage drop formula it will be evident that allowance must be made for the reduced line loss for the distributed load. This can be taken care of by reducing the line loss by the factor 1-3 and again equating the two costs and solving. These operations are carried out below with the result that the economic voltage drop can be increased by $\sqrt{3}$ as compared to the drop for the concentrated load, thus:

$$\frac{a \times p \times K}{100 \times R} = \frac{l}{3} \left(\frac{pl \times I^2R}{1000} \right)$$

$$= \frac{pl \times Er^2}{3000 \times R}$$

from which

$$Er^2 = \frac{30\,K \times a \times P}{pl}$$

$$= \frac{3 \times 10\,K \times a \times P}{pl}$$

or

$$Er = \sqrt{3} \times \sqrt{10K}\ \frac{\sqrt{a \times p}}{\sqrt{pl}}$$

Use of Voltage Drop Factor. The first step in designing a rural line is to decide on the cost of wasted power, the cost of conductor material, and the percentage fixed charge to cover interest depreciation, and taxes. With these three quantities fixed, reference is made to the chart for the given energy cost, conductor cost and percentage fixed rharge and ther the econcmic ohmic voltage drop per mile is read. If the line has a uniformly distributed load this drop must be multiplied by $\sqrt{3}=1.73$ to give the allowable economic drop for such line as was shown in the previous paragraph.

Example: Assume:

Single phase line 10 miles long
Conductor material—cooper at 20c per lb.
Line voltage 2300 volts.
Total distributed load 30 KVA.
Percentage fixed charges—15%
Cost of wasted power—3c per Kilowatt-Hour.

For values of $20 conductor cost, 15% fixed charge and 3c per Kilowatt-Hour for wasted power the chart gives 10 volts as the economic voltage drop per mile. Multiply this value by $\sqrt{3}$ gives 17.3 volts.

$$I=\frac{30 \times 1000}{2300}=13.05 \text{ amperes}$$

$$R=\frac{17.3}{13.05}=1.325 \text{ ohms-mile}$$

Use No. 4 wire which has weight of 668 lbs.-mile or 13,360 lbs. for 20 miles of wire.

Total Fixed charge=15% (.20 x 13360)=$400.00

$$\text{Variable charge}=\frac{13.05^2 \times 1.325 \times 20 \times 8760 \times .03}{1000 \times 3}=\$396.00$$

The equality between the fixed and variable charges shows that Kelvin's Law has been satisfied.

The writer wish esto acknowledge the assistance of Mr. Geo. W. Recer, and Harold Shinn, senior students in Electrical Engineering, for assistance rendered in the preparation of the Economic Voltage Drop Chart.