XXII. NOTE ON THE ENERGY DISSIPATION IN BRANCHED ELECTRIC CIRCUITS WITH DIRECT AND ALTERNATING CURRENT

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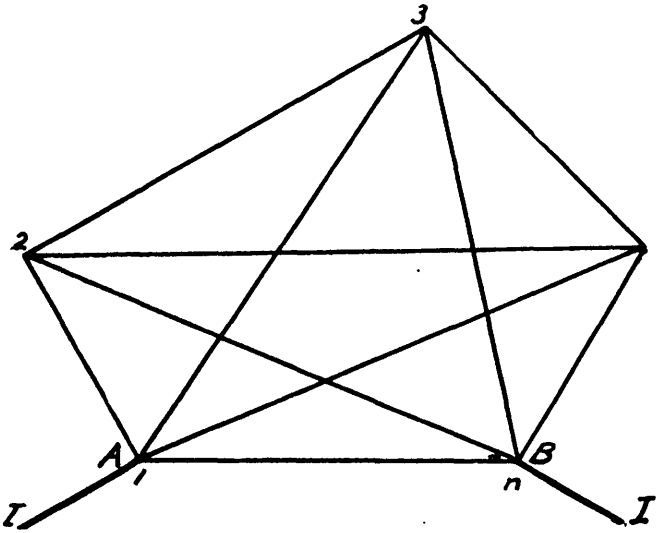
For a given effective value of the current in an induction coil AB (Fig. a), the energy dissipation will not depend upon



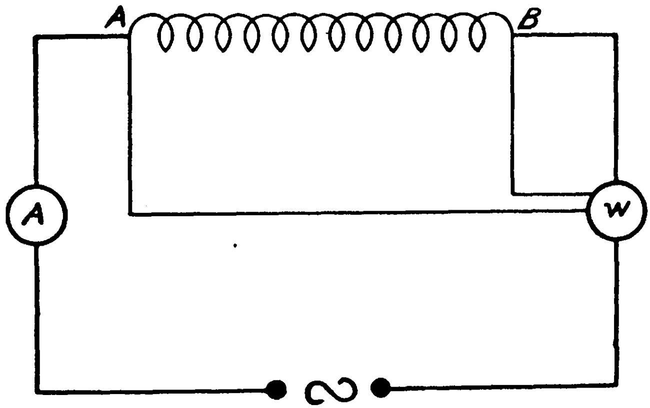
the frequency of the current. In particular, the energy dissipation will be the same whether the current is direct or alternating. If AB is not a simple coil consisting of resistance and self-inductance in series, the energy dissipation will, in general, not be the same for DC and AC, and it has been suggested by Dodge to make use of this fact to test whether an induction coil is a simple coil or is short-circuited or branched in any way.

If the DC and AC energy dissipations in a coil are found different, it may be concluded that the coil is not a simple coil. But the problem now arises whether this conclusion may be inversed. In other words: if the DC and AC dissipations are found to be equal, does that necessarily imply that the coil is not branched? The answer to this question is found in the following general theorem:

For a given effective value of the total current the energy

dissipation in a branched circuit, which does not contain capacities will be greater for AC than for DC, except in the special case when the ratio of self-inductance to resistance is the same for every branch. In this case the DC and AC current distributions and energy dissipations are equal.

To prove this theorem let us consider a network with n junction and number the junctions so that A is number 1 and B number n (Fig. b). If we assume that a simple sine current



enters the network at A and is taken out at B, we can represent the maximum value and phase angle of the current in any branch hk of the network by a two dimensional vector. For brevity, we shall simply call this vector the current in the branch hk. We denote the current entering the network at A by I, omitting the customary subscript m.

The total energy dissipation W in the network is given by equation (1). The expression under the double summation sign is the product of the resistance in the branch hk by the square of the maximum value of the current or, which is the same thing, the resistance multiplied by the scaler product of the current by itself.

For any possible current distribution Kirchhoff's First Law must be satisfied for every junction. This fact is expressed by the n-1 vector equations (2).

We will now find the current distribution for which W has its minimum value subject to the conditions (2). The variation of W is given by (3). The variations of the currents in this expression must satisfy the n-1 vector equations (4). We place the varation of W equal to zero and, in analogy with a method first employed by Lagrange, multiply the equations (4) scalarly by arbitrary vectors A, the values of which we shall later on dispose of, and add all the equations thus formed to (3).

In the expression thus obtained for the variation of W, n-1

$$(1) \qquad W = \frac{1}{2}\sum_{k=1}^{n-1}\sum_{h=k+1}^{n} R_{kh}\,(I_{kh}\,I_{kh})$$

$$(2) \qquad \sum_{h=2}^{n} I_{1h} = I\,; \qquad \sum_{h=1}^{n} I_{kh} = 0, \quad k = 2, 3, \cdots, n-1$$

$$(3) \qquad \delta W = \sum_{k=1}^{n-1}\sum_{h=k+1}^{n} R_{kh}\,(I_{kh}\;\delta I_{kh}) = 0$$

$$(4) \qquad \sum_{h=1}^{n} \delta I_{kh} = 0, \qquad k = 1, 2, \cdots, n-1$$

$$(5) \qquad R_{kh}\,I_{kh} + A_k - A_h = 0$$

$$\left.\begin{matrix} k \\ h \end{matrix}\right\} = 1, 2, 3, \cdots, n\,;\; h > k\,;\; A_n = 0$$

$$(6) \qquad R_{kh}I_{kh} + R_{hn}I_{hn} + R_{nk}I_{nk} = 0$$

$$\left.\begin{matrix} k \\ h \end{matrix}\right\} = 1, 2, 3, \cdots, n-1\,;\; h > k$$

of the variations of the currents may be considered as functions of the remaining variations, these being quite arbitrary. If we now assign such values to the A's that the coefficients to all the n-1 dependent variations vanish, then the remaining coefficients must also vanish by virtue of the arbitrariness of the corresponding variations. We thus obtain the equations (5). Eliminating the A's from (5), we finally get (6). This system of equations, together with (2), determines the current distribution for which the energy dissipation is a minimum.

The equatons (6) are perfectly analogous to the equations obtained by Kirchhoff's Second Law for direct current. On solving (2) and (6), we see that in case of minimum energy dissipation all the currents are in phase with I. The current distribution given by (2) and (6) is identical with the DC distribution and can be obtained with alternating current only in the special case when the radio of self-inductance to resistance is the same in all branches. This proves our theorem.

If every junction is not connected with every other junction, as assumed above, that will only mean that we have a certain number of terms less in the expression for W and in (2), (3) and (4), and the same number of equations less in (5) and (6). If there is more than one branch between, say, junctions h and k, that is equivalent to having one or more junctions connected to h or k through zero resistance. The circuit considered above is thus quite general.

The theorem does not hold when the circuit contains open branches with capacities, as may be easily seen by considering a circuit consisting of a capacity in parallel with a large resistance