ON THE I—CENTERS OF A TRIANGLE

Nathan Altshiller Court, 1917.

(Abstract)

The sides of a triangle are touched by four circles the centers of which are referred to as the I-centers of the triangle.

Each I-center of a triangle lies on three of the six circles which pass through the pairs of vertices of the triangle and have their centers at the midpoints of the arcs substended by the respective sides of the triangle on its circumcircle.

Each of the six circles contains two I-centers, and the two points are the extremities of a diameter.

If these propositions are considered in connection with the four triangles obtained from an inscribed quadrilateral by omitting in turn one of its four vertices, the following properties are found:

The sixteen I-centers of the four triangles obtained by taking the vertices of an inscribed quadrilateral three at a time, lie on twelve circles, each I-center lies on three circles. Each circle contains four I-centers. The centers of those twelve circles lie on the circucircle of the quadrilateral.

The four incenters of the four triangles determined by the vertices of an inscribed quadrilateral taken three at a time, form a rectangle.

If of the four triangles determined by the vertices of an inscribed quadrilateral taken three at a time, the three triangles are taken having a vertex in common, the three excenters relative to this vertex in the three triangles, are the vertices of a rectangle, the fourth vertex of which is the incenter of the fourth triangle.

The sixteen I-centers of the four triangles determined by the vertices of an inscribed quadrilateral taken three at a time, lie by groups of four on eight straight lines. These eight lines consist of two sets of four parallel lines, and the lines of one set are perpendicular to the lines of the other.

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