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A Linear Programming Model for Optimal Check Airmen Allocation to Minimize Travel Costs

João S. D. Garcia
Embry-Riddle Aeronautical University

Christian K. Jädicke
Embry-Riddle Aeronautical University

Dothang Truong
Embry-Riddle Aeronautical University

Across the globe, civil aviation authorities (CAA) require pilots to be examined upon completion of their flight training and at regular intervals to uphold their pilot's license. These flight examinations, or checkrides, are conducted by designated flight examiners and CAA pilots. While government employees are dispatched to different locations to conduct such exams, designated check airmen may only conduct checkrides that have limited coverage in the geographic area in which those exams are allowed. Thus, if the demand for checkrides at a given location is higher than the number of available designated flight examiners, those employed by the CAA may have to travel to satisfy the need for checkrides, incurring additional costs to these organizations. This paper aims at developing an optimization model using linear programming to find the optimal number of checkrides at different locations that minimizes the total travel cost of government check airmen (GCA) conducting checkrides, considering specific travel costs between locations. Based on a realistic set of initialization parameters, the optimal solution showed a minimal travel cost of $35,827.30 for six months. This model could be applied to other areas that may face a similar decision-making process.

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Proficiency examinations are constant in a pilot's career, starting with the check flight conducted as a requirement for receiving a private pilot's license. Experienced captains flying international routes for global airlines are also subject to periodic proficiency checks. The International Civil Aviation Organization (ICAO) is the institution responsible for establishing global safety standards for aviation activities (ICAO, 2019). In Annex 1 to the Convention on International Civil Aviation, ICAO requires that all applicants to pilot's licenses "shall have demonstrated a degree of skill appropriate to the license" (ICAO,2011,p. 2-3) or have "demonstrated the skill and knowledge required for the safe operation of the applicable type of aircraft" (ICAO, 2011, p. 2-4). As this is done through a formal examination with regulatory implications, the activity is usually conducted by an employee of the civil aviation authority from the country where the pilot intends to obtain the license, or by a professional accredited by that organization. The use of designated pilot examiners or designated check airmen is common practice in the global aviation industry, and authorities frequently require candidates to provide an excellent safety record as a pilot (e.g., accidents, incident, and violations), to present a reputation for integrity and professionalism within the aviation industry, and to have experience as a flight instructor (FAA, 2018).

CAAs face one issue because the allocation of these scarce professionals is not usually geographically aligned with the demand. One option for authorities to satisfy local demands is to designate pilot examiners to conduct exams in particular locations. If not enough of these professionals are available to satisfy the local demand, the CAA will need to dispatch a government employee to conduct these proficiency exams. Figure 1 provides a graphical representation of the relation between checkrides required and check airmen available. The varying demands of checkrides in the different cities may require the available governmental check airmen to travel, thus generating associated travel cost. Considering that states are continually searching for ways to increase the quality and effectiveness of operations, increased efficiency in travel resources can be relocated to other flight safety assurance initiatives.

To reduce these costs, governments have signed contracts with specific airlines and for specific destinations, such as airfare rates in the City Pair Program (GSA, 2019), which allow secure scheduling of personnel allocation. However, even doing so, travel expenses can be further reduced if the assignment to an exam of personnel from a far home base is reduced. Nonetheless, the requirement for a particular flight examination which demands a governmental check airmen may result in additional travel cost.

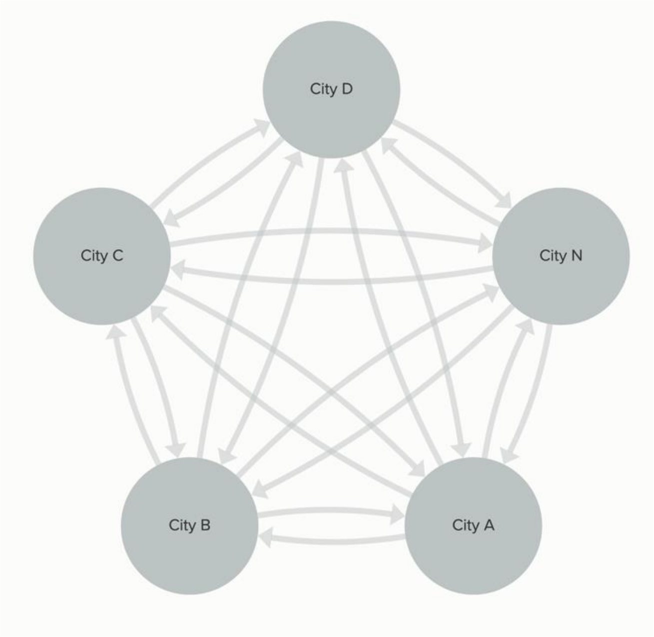

Figure 1. Simplified graph of the relationship between cities (circles) and the distribution of exam-related cost-incurring travels (arrows) for a reduced example set of five cities.

Within the aviation domain, conducting checkrides is a mandatory requirement for flight crew licensing. This task is performed by governmental agencies who either provide or contract the check airmen to administer flight examinations. As the authority responsible for aviation safety within a state, the CAA is responsible for enforcing legal requirements and minimizing associated costs in its operations while optimizing the use of highly-specialized limited-availability personnel. Hence, CAAs have three tasks: satisfying the demand for flight examinations, optimizing the assignment of the available check airmen, and reducing the associated travel costs to the extent possible.

Purpose

This study aims to develop an optimization model using linear programming that addresses the requirement of the CAAs to minimize travel costs while optimizing the allocation of government and designated check airmen. This model's optimal solution can create a comprehensive tool that could be applied in other aviation-related areas involving specialized personnel and the requirement to travel to execute specific demands for activities. This paper's finding provides a tool for decision-making to support the assignment of check airmen to conduct flight examinations at various locations while minimizing the associated travel costs. The provided assignment model offers decision-makers an option to optimize the assignment of key personnel and develop assignment models that consider the average travel costs between city pairs to minimize travel costs. Although developed for air transportation, the model could be applied to other areas that may face a similar decision-making process.

Research Question

The research question investigated in this paper is: Considering the required personnel allocation to fulfill the required proficiency exams at various locations, what is the optimal solution for allocating governmental check airmen to minimize the total travel costs?

Literature Review

The focus of the research presented in this paper lies in optimizing the allocation of check airmen to minimize travel costs. Two main themes can be identified: the allocation of resources and the associated travel or transportation cost. The review of relevant literature includes a short excursion into the subject of flight examination followed by exploring sources addressing the topic of resource allocation. The last part will address the subject of travel costs. The focus will be placed on solving the transportation problem in order to achieve cost minimization.

Flight Examination and Examiners

A flight examination, commonly referred to as “checkride” among aircrews, is part of the regulatory requirements to obtain, regain, or uphold an aviation rating (Flight Examiner, 2019). As such, the aircraft rating supports the capability to legally fly a specific type of aircraft, for instance, single-engine or multi-engine airplanes, seaplanes, or helicopters. Furthermore, additional endorsements may be required, such as high performance, complex, or high-altitude flying operations (Martin, 2016).

Aviation regulations require a flight examiner to conduct the examinations and grant some endorsements. This professional is also referred to as a *check airman*. In general, to qualify as a flight examiner, the applicant is required to provide proof of a minimum amount of total flying hours and an experience as a flight instructor, also measured in flying hours (Central Flight Training, 2019). Detailed requirements to become a flight examiner are provided in the regulatory documents governing flight crew licensing within the respective CAA (FAA, 2019; European Commission, 2011). Although flight examination is a governmental requirement to enforce a standardization level in exams, the flight examiner does not have to be a government employee. Any qualified personnel who is the holder of a flight examiner license can apply to obtain a license as a check airman to conduct checkrides as an associated representative for the respective CAA (European Commission, 2011; Luftfahrt-Bundesamt, 2019).

Resource Allocation

The allocation or assignment of resources can be defined as “the process of assigning and managing assets in a manner that supports an organization’s strategic goals” (Rouse, 2014). These resources – also referred to as assets – can be jobs, agents, or contracts assigned to complete a task or achieve the desired result (Anderson, Sweeney, Williams, Camm, & Martin, 2012). Within the business process management domain, human resource (HR) allocation is a crucial element affecting the overall process performance and the result (Arias, Saavedra, Marques, Munoz-Gama, & Sepúlveda, 2018).

At the center of such a process is the respective assignment problem model to optimize resource allocation. It does reflect the requirements to achieve the goal, which is either the minimization or maximization of that objective. A specific characteristic of such assignment problems is that one single asset is assigned to one task at a time (Anderson et al., 2012). This type of resource-task allocation is also referred to as a mono-objective assignment problem (Bouajaja & Dridi, 2017).

In general, the classical allocation or assignment problem can be regarded as a linear problem. If the count of resources matches that of the tasks, the resources and tasks are balanced or symmetric (Bouajaja & Dridi, 2017; Pentico, 2007). If, however, any side constraints are present (e.g., unavailability for travel), the balanced linear problem-solving approach shows susceptibility to these noise factors (Mazzola and Neebe, 1986). When put into relation to the model presented in this paper, the number of check airmen and required checkrides can be described as asymmetric or imbalanced, since the two interacting sets have different proportions (Bouajaja & Dridi, 2017).

Regarding the effect of side constraints, Pentico states that, based on the assumption of any number of given tasks not being accomplished simultaneously, sequencing requirements exist (Pentico, 2007). Concerning allocating check airmen to conduct checkrides, this problem is a factor to be considered as there are considerably fewer HRs than tasks available. In combination with side constraints, for example, “budgetary limitations [or] degree of technical training of personnel” (Pentico, 2007, p. 782), the allocation of the resource can be affected significantly, for example, by the limitation of availability for traveling due to the form of employment (governmental versus designated).

It is critical to minimize the total travel costs while making optimum use of the available check airmen. since there are more checkrides to perform than check airmen are available, the latter constitutes a limiting factor or a bottleneck. Within management science, the bottleneck problem is defined as a limitation imposed on a system’s performance through a single or a limited number of assets (Avudari, 2013). Concerning the presented assignment problem, the classification as a bottleneck assignment problem (BAP) would be feasible, since its goal is to “minimize the maximum of the costs” (Pentico, 2007, p. 777). However, to achieve a BAP solution, a constraint is required, since the “linear programming version of this problem does not guarantee a binary solution” (Pentico, 2007; p. 777).

Travel Cost

Transportation itself describes either the tool to move something from the point of origin to a destination or merely describes the process of moving something between locations. As the presented study addresses the transportation of governmental check airmen from location A to location B, it can be defined as a transportation problem, where “goods and services from several supply locations [are distributed] to several demand locations” (Anderson et al., 2012, p. 256).

According to Kostoglou (2012), a transportation problem requires three sets: capacity, demand, and unit shipping cost. In the present study, capacity is represented by the total number of check airmen available per location. The required checkrides denote the demand, and the unit shipping cost illustrates the travel cost from the point of origin to the destination, limited to the governmental check airmen who can be ordered to travel.

Since the capacity in the current study is comprised of government and non-government check airmen and the demand outweighs the capacity considerably, a simple linear programming model to satisfy supply and demand as described by Anderson et al. (2012) or Kashyap (2017) would be insufficient to formulate the transportation problem. Considering the imbalance and the fact that not all of the check airmen can travel, a potential conflict arises.

An approach to address such an imbalanced constellation with multiple parameters is the bi-criteria transportation problem. A study conducted by Singh and Singh (2018) investigated a multi-objective problem consisting of shipment cost and time required, of which the latter constituted a bottleneck. Applying multiple-choice programming, the researchers found that incorporating multiple-choice parameters resulted in a more flexible transportation model. Concerning the study presented in this paper, the governmental check airmen could function as multiple-choice parameters combined with the associated travel cost from the origin to the destination.

Finally, Bazargan (2010) presents a model for solving crew scheduling problems in airlines. The author argues that flight-crew expenses are controllable and thus subject to airlines' optimization, which could result in significant competitive advantages if adequately addressed. However, such problems' computational costs usually require that it be split into a crew pairing and a crew rostering phase. The crew pairing step yields sequences of flights that start and end at a crew base. While the check airmen problem discussed in the current paper may resemble the crew pairing problem presented by (Bazargan, 2010), decision variables, cost functions, and constraints differ, as check airmen are not restricted by crew duty time limitations and are subject to travel costs, among other differences.

Methodology

Optimization model

An optimization model was developed to support the minimization of travel costs associated with proficiency checks demanded from a CAA by deploying government employees from different geographic locations to complement the checkrides conducted by designated check airmen in each of a set of locations. The parameters used in the model were calculated from information obtained from an imaginary CAA. They included the demand for checkrides in a list of cities and the average prices of flights between cities for six months.

The used parameters were:

- checkride demand in the city;
- average airfare between cities;
- number of available designated check airmen in the city;
- number of available government check airmen in the city;
- maximum number of checkrides per government check airman in the simulated period; and

- the maximum number of checkrides per designated check airman in the simulated period.

Additional constraints for the model included the limitation of the number of active check airmen. Differences between designated check airmen and CAA employees engaged in the activity were also considered, as only the latter is allowed to travel to conduct such exams. In the model, it wasalso considered that, although the current total number of government employees conducting proficiency checks cannot change, the CAA may dispatch them to other locations according to need. One important constraint was the need for all of the demand for checkrides in each location to be satisfied, once upholding their flight currency is essential for pilots, particularly for those flying professionally.

Assumptions and limitations

The LP model's baseline calculations were based on absolute numbers, that is, influencing factors such as leave or illness of check airmen, fluctuating ticket prices, or multiple check rides performed in one day by a single check airman were not considered. Additionally, the historical data used did not specify if every travel was successful, meaning, if the flight examination was performed. Some assumptions were considered in developing the model. The price of air tickets between two cities were considered constant and equal to the average of tickets purchased by the CAA between the same two cities in a particular period. Although this may not result in an accurate representation of the price for a specific flight, these variations tend to have little influence on the average costs in the longer term. Another assumption was that only one exam was performed in each travel and that all government and designated check airmen had a limited number of checkrides they could perform.

Additionally, it was considered that both check airmen were allowed to perform all checkrides. Although this consideration may not hold in reality since some checkrides may need the check pilot to have valid, specific type ratings, the proposed model can be replicated to other sets of categories of type ratings by adjusting the model's parameters accordingly. This adjustment would be facilitated by the simplicity of the proposed model, which resulted in reduced calculation times. However, further enhancements of the proposed model could implement additional complexity in a single run of the calculation, and attention should be placed on the impact on computational costs.

Linear programming model

The proposed linear programming model for the problem is presented below.

Sets

i, j, I: cities i and j in a set of cities I;

The only set considered in the model is that of the cities with information on the local demand for checkrides and government and designated check airmen's availability. This set is referenced as I in the model. A particular city is referenced as i or j, depending on its role as origin or destination in a particular equation.

Decision Variables
$x_{i,j}$: number of checkrides by government check airmen from city i to city j;
y_i: number of checkrides by designated check airmen in city i;

Two main groups of decision variables were used in the model. While y_i represents the number of exams performed by designated check airmen in city i, variable $x_{i,j}$ represents the number of exams performed by government check airmen from city i to city j. While government employees may travel to perform exams in other cities, they can also conduct such exams in their home base without incurring travel costs.

Parameters
CD_i: Checkride demand in city i;
$ATC_{i,j}$: Average ticket cost per ride between cities i and j;
DCA_i: Number of available designated check airmen in city i;
GCA_i: Number of available government check airmen in city i;
$MCRG$: Maximum number of checkrides per government check airman in the simulated period;
$MCRD$: Maximum number of checkrides per designated check airman in the simulated period;

From a total of six types of parameters, three relate to the set of cities I for which the simulation were be run. They included the local demand for checkrides (CD_i) and the number of designated (DCA_i) and government check airmen (GCA_i) for each city i. Additional parameters included the limitations for the number of checkrides each government ($MCRG$) and designated examiners ($MCRD$) could perform and averaged travel costs between two cities i and j. To increase the availability of cost information for all city pairs, the associated costs wereassumed to be the same in both directions ($i \leftrightarrow j$).

Objective Function
Minimize $\sum_i \sum_j x_{i,j} \cdot ATC_{i,j}$
Subject to

$$y_i + \sum_j x_{i,j} = CD_i$$
$$\sum_j x_{j,i} \leq GCA_i \times MCRG$$
$$y_i \leq DCA_i \times MCRD$$
$$x_{i,j},\ y_i \geq 0, \forall i,j \in I$$

The objective function reflects the need to meet the local demand for checkrides in all cities while minimizing travel costs. The total cost is simplified as the sum of the number of flights between two cities multiplied by the average ticket cost. An additional constraint relates to the individual capacity of check airmen. To account for the limitation of personnel, the model considered that the number of travels from city i should be less than the number of government check airmen in that city multiplied by the limit of exams an individual government check airman could perform. A similar constraint was added to account for the number of exams locally designated check airmen could perform. Finally, a constraint was included to ensure that all decision variables were nonnegative numbers.

Data collection

Data generated from six months of demand for checkrides for a set of cities supported the calculation of model parameters, including average flight costs between these different cities and the current availability of government and designated check airmen and their geographic locations. The 26 cities considered in solving the proposed model along with the respective values for parameters CD_i, GCA_i, and DCA_i are presented in Table 1.

Table 1
Cities with Associated Data Regarding Required Checkrides, and Designated and Governmental Check Airmen

City	CD	DCA	GCA		City	CD	DCA	GCA
City A	1	0	1		City N	8	0	0
City B	8	0	1		City O	0	0	1
City C	35	5	10		City P	5	0	1
City D	28	0	7		City Q	0	0	1
City E	18	0	0		City R	13	0	4
City F	1	0	1		City S	3	0	0
City G	17	0	1		City T	73	7	8
City H	16	0	3		City U	5	6	1
City I	5	0	2		City V	8	0	3
City J	3	0	0		City W	173	10	3
City K	22	0	1		City X	2	0	0
City L	9	0	0		City Y	2	0	0
City M	3	0	2		City Z	5	0	0

Note. CD = checkride demand; DCA = designated check airmen; GCA = government check airmen

The demand for check rides was higher in some cities, notably City W, City T, and City C, as visualized in Figure 3. These cities represented the most important economic activity centers in the presented scenario and, thus, presenting more flight training activity. However, the availability of examiners did not necessarily reflect that demand and could bemore elevated where aviation authorities and significant governmental flight departments are located.

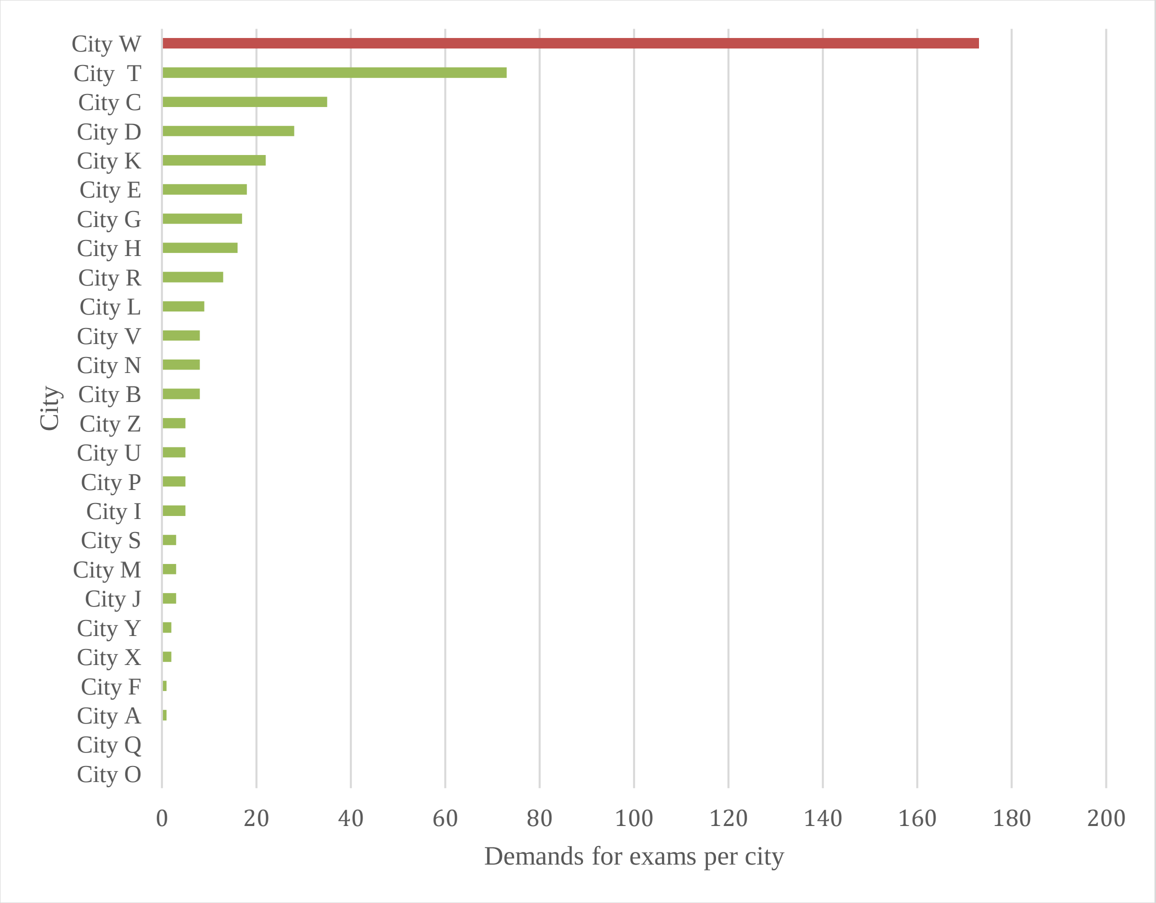

Figure 2. Demand for flight exams in the cities considered for the model.

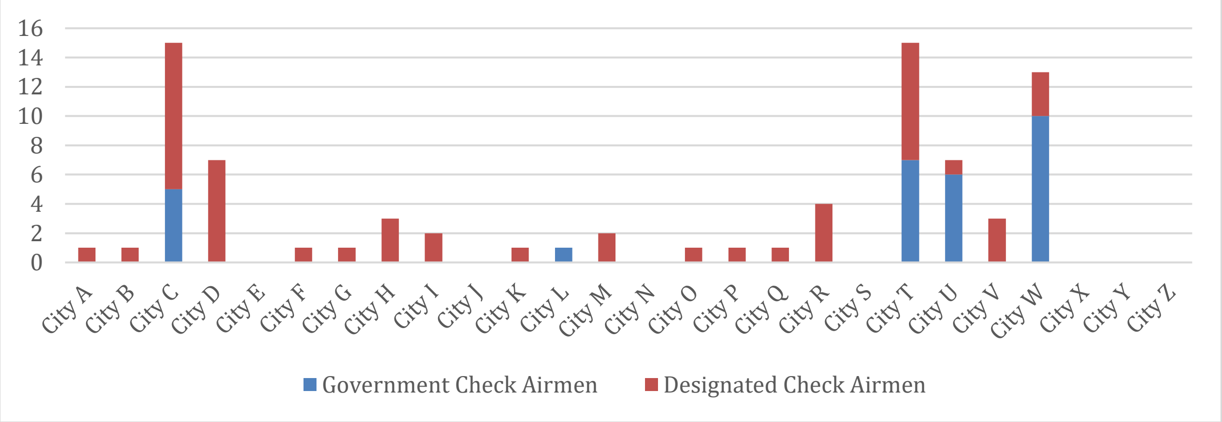

Figure 3. Availability of Government and Designated Check airmen per city in the model.

Solving the linear programming model

The literature supported the development of a Python code used to model the optimization problem. The code utilized an API library for the Gurobi Optimizer 8.1 optimization platform, a mathematical programming solver for linear programming, quadratic programming, and mixed-integer programming problems used extensively by industry (Gurobi Optimization, 2019). Several studies have compared performance among different solvers for mixed-integer linear and nonlinear mathematical optimization problems and included Gurobi in the analysis. Gurobi was consistently ranked highly among competitors due to its CPU-efficiency and has provided significant performance updates in each version update (Anand, Aggarwal, & Kumar, 2017; Jablonský, 2015; Meindl & Templ, 2013; Mittelmann, 2017). It was calculated using Jupyter in a MacBook Pro notebook with 2.9 GHz Intel Core i9 and 32 GB 2400 MHz DDR4.

Gurobi Optimizer's solver indicated that the initial model had 78 rows, 702 columns, and 1404 nonzeros. The solver applies an efficient presolver that reduces unused or redundant variables or unnecessary complexity, which, for the proposed case, removed 64 rows and 660 columns, resulting in a model with 14 rows, 42 columns, and 82 nonzeros. The presolver transforms the original model to a simplified equivalent version, facilitating its computational solution by employing constraint aggregation and reducing sparsity. The optimal solution was obtained in 2 iterations, and both the presolver and solver computations were concluded in under 0.01 seconds.

Results

The proposed model resulted in a practical solution for reducing the costs of the check airman allocation. The application of the proposed model to a set of constraints created from a realistic conjecture of model parameter values yielded a feasible optimal solution while satisfying all of the problem's constraints. The optimal solution resulted in a total travel cost of $35,827.30, meeting the demand for 463 exams in all 26 cities. Locally designated check airmen conducted a total of 298 exams while government employees performed 165 checkrides. Local exams conducted by government employees in City W accounted for 113 exams. Table 2 presents the non-zero values.

Table 2

Nonzero results of checkrides flown by government check airmen after applying the linear programming model

Variable	x	Variable	x
$y_i[City\ A]$	1	$y_i[City\ V]$	8
$y_i[City\ B]$	8	$y_i[City\ W]$	60
$y_i[City\ C]$	35		
$y_i[City\ D]$	28		
$y_i[City\ F]$	1	$x_{i,j}[City\ C, City\ K]$	2
$y_i[City\ G]$	17	$x_{i,j}[City\ C, City\ Y]$	2
$y_i[City\ H]$	16	$x_{i,j}[City\ L, City\ L]$	9
$y_i[City\ I]$	5	$x_{i,j}[City\ T, City\ N]$	8
$y_i[City\ K]$	20	$x_{i,j}[City\ T, City\ S]$	3
$y_i[City\ M]$	3	$x_{i,j}[City\ T, City\ Z]$	5
$y_i[City\ P]$	5	$x_{i,j}[City\ U, City\ E]$	18
$y_i[City\ R]$	13	$x_{i,j}[City\ W, City\ J]$	3
$y_i[City\ T]$	73	$x_{i,j}[City\ W, City\ W]$	113
$y_i[City\ U]$	5	$x_{i,j}[City\ W, City\ X]$	2

Note. x = number of checkrides; $x_{i,j}$ = number of checkrides by government check airmen from city i in city j; y_i: number of checkrides by designated check airmen in city i

The results mentioned above indicated lower ticket prices leaving from City C, City T, and City W favor dispatching government employees from these locations to different cities to perform exams. This finding supports results that show significant slack in several locations, both for the government and designated check pilots. This result is indicated in the shadow price and slack information obtained in the calculations for all three constraints reflecting demand (CD) and government (GCA) and designated check airmen (DCA) limitations. These sensitivity measures indicated the model calculation to be robust to changes in parameter values, but also reflected that the check airman allocation and resulting costs depend heavily on $ATC_{i,j}$, which should receive proper consideration in the model parameter setup. For instance, noticing the higher travel costs to City L, a new simulation was run considering the designation of an additional government check airman to the city. As a result, the total cost was reduced significantly compared to that in the standard scenario to $17,827.30. This finding indicates the versatility of the model supporting decision-makers by facilitating prospecting scenarios and opportunities for gains in efficiency.

Discussion

The linear programming model application using the Gurobi optimization algorithm to an instantiation of the proposed problem provided a single optimum solution, fulfilling the requirement to conduct checkrides in all locations. Considering the parameters presented in the previous sections, the results presented in Table 2 illustrate that the demand constraint pressures total cost for the model in seven cities, with City L, City N, and City Y representing higher costs, and should demand an increase in check airmen allocation in these locations. When assessing the sensitivity metrics associated with the constraint, it should be noticed that some locations such as City C (91 for GCA;165 for DCA), City D (0; 112), City T (120;87), and City U (120;15) present significant levels of slack concerning the limitation of personnel. As mentioned previously, City L, City N, and City Y are examples of locations not supplied with enough local examiners, which pressure total costs for the model.

The review of the defined constraints allows the argument that, concerning the qualification of the check airmen, no differentiation regarding the type of airframe was made. The result of such constraint would be a possible change in check airmen allocation to assign the adequately qualified person to conduct the corresponding checkride. This argument can be countered with the assumption that flight examiners can conduct checkrides for multiple aircraft types. Nonetheless, implementing such a constraint would add to the robustness of the proposed model since it would constitute additional decision variables in the model.

Regarding the data used to develop the minimization model, it could be argued that it is non-representative as it was limited to a particular CAA's method of dispatching examiners. Although this observation is correct, one must consider that the mix of dedicated and government check airmen is not a localized phenomenon but common practice around the globe. However, there may be differences between CAAs regarding the number of available government check airmen and the regulations governing their travel. Applying the proposed model would, therefore, produce adequate results.

Conclusions

This study proposed the application of an assignment and transportation linear programming model using the Gurobi optimization algorithm. It was developed to provide a solution to minimize travel costs of check airmen operating in a CAA's area of responsibility. With a defined set of decision variables and constraints based on available data, the model application for check airmen resulted in an optimal solution. This means that the application of the model to other sectors of general aviation could produce relevant results. Through manipulations of the constraints, the decision-maker could also define requirements regarding the number of necessary check airmen.

Although using a simplified approach by observing only the constraint of being able to travel or not, the proposed model has been successful in minimizing the travel cost through optimization of check airmen allocation. Therefore, the presented LP model offers decision-makers in aviation authorities an easy-to-use tool adaptable to accommodate similar situations such as the presented one. The combination of optimizing the use of human resources and minimizing the related-travel costs will allow the efficient and responsible use of two commodities that lack abundance. If additional resources are available for hiring additional government check airmen, rerunning the simulation indicates that City L would be the right place for that resource to be placed. Adding the new post there would reduce the total costs significantly.

Recommendations

Similar data from different CAAs should be used to create a basis for comparison to validate the applicability of the proposed model. Furthermore, to improve the robustness and adaptability to various scenarios, the implantation of additional constraints affecting the assignment process could be beneficial. At the same time, more detailed information would be required regarding the check airmen qualification and type of airframe on which the checkride must be conducted.

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